HW-3

1. Matlab has built in functions for computing and evaluating interpolation C=polyfit(xp,yp,n-1); Interp=polyval(C,x); The following program interpolates the function f(x) =1/(1 +10 * x^2).

Use these programs to graphically show the interpolation of the function

  i. $f(x) = \cos(x), \ \ [-\pi, \pi]$
  ii. $f(x) = \frac{1}{1+10x^2}, \ [-4, 4]$

For different number of points, $n = 2, 4, 8, \ldots$ and $x_i = a + h * (i - 1), i = 1,2, \ldots, n, h = \frac{b-a}{n}$. Also plot pictures of the error $f(x) - P_n(x)$. Explain your results and correlate your conclusions with the discussion in class.

Addendum to HW-3(see http://en.wikipedia.org/wiki/Chebyshev_nodes)

Assume that Chebyshev interpolation nodes are used ($x_i = \frac{b-a}{2}\cos[(2i+1)\,\pi/(2n+2)] + \frac{a+b}{2}, i = 0,1,2, \ldots, n$)
Repeat the same experiment for the above functions (i) and (ii). And provide conclusions of your results.

```
function  runge(n)
%Function that shows the Runge phenomena
%
% The Polynomial interpolation of f(x) =1/(1 +10 * x^2)
% on equal distributed n nodes on [-1 1]
% causes  extreme variation near the ends
% as n increases
%INPUT n the # of equally distributed nodes in [-1 1]
%
%
close all
x=-1:0.02:1; % mesh points on [-1 1] used for plotting
y=1./(1 + 10 * x.^2);
plot(x,y) % a plot of f
xp=linspace(-1,1,n); % the interpolation nodes
yp=1./(1 + 10 * xp.^2); %y_i=f(x_i)
hold on
plot(xp,yp,'o') %plotting the data points on the graph of f
C=polyfit(xp,yp,n-1); %computes the coefficients of P_{n-1}
Interp=polyval(C,x); %evaluates the Interpolant  P_{n-1} at the mesh points
plot(x,Interp,'r')
end
```

2. Use both Lagrange interpolation and Newton's interpolation formulae to find the polynomials for the following data.
   i. $\{(0,1), (1,2), (2,3)\}$
   ii. $\{(0,1), (1,1), (2,1)\}$

3. As generalized interpolation problem, find the quadratic polynomial $q(x)$ for which

$$q(0) = -1, q(1) = -1, q'(1) = 4.$$

HW-3

4. For an interval $[a,b]$ define $h = (b-a)/n$ and the evenly spaced points $x_j = a + jh, \quad j = 0,1,\dots,n.$ Consider the polynomial

$$\Omega_n(x) = (x - x_0)(x - x_1)\dots(x - x_n).$$

Show that

$$|\Omega_n(x)| \leq n!\,h^{n+1}, \qquad a \leq x \leq b$$

Hint: Consider the $n = 2$ *or* 3. Also consider separately $x_0 < x < x_1, or\ x_1 < x < x_2 \dots or\ x_{n-1} < x < x_n$ and with each case bound the factors of $x - x_j$ with multiples of h.

The error for interpolation polynomials is given by

$$\max_{a\leq x\leq b} |f(x) - p_n(x)| \leq \max_{a\leq x\leq b} \frac{|\Omega_n(x)|}{(n+1)!} \max_{a\leq x\leq b} |f^{(n+1)}(x)|$$

What will be the error bound for equidistant points?

Can you use these error bounds to explain the behavior of the Runge function $f(x) = \frac{1}{1+10x^2}$, $[-4,4]$?

5. The Langrage interpolation Polynomials is defined by

$$P_n(x) = \sum_{j=0}^{n} L_j(x) f_j, \qquad L_j(x) = \prod_{i=0,i\neq j}^{n} (x - x_i) / \prod_{i=0,i\neq j}^{n} (x_j - x_i)$$

a. Show that

$$\sum_{j=0}^{n} L_j(x) = 1$$

b. And in general

$$\sum_{j=0}^{n} x_j^m L_j(x) = x^m, \qquad m \leq n$$

6. A. We want to interpolate the following data:

x	0	1	2	3	4
$f(x)$	1	1	7	25	61

i. Determine Newton's divided difference Table.

ii. Give the interpolation polynomials $P_1(x), P_2(x), P_3(x), P_4(x)$ using the Table.

iii. Do these data come from a polynomial? Explain.

B. Given two points $(x_0, y_0), (x_1, y_1)$. The Lagrange interpolation polynomial is given by $P_1(x) = L_0(x)y_0 + L_1(x)y_1$.

HW-3

i. What is $L_0(x) =?, L_1(x) =?$

ii. What is $L_0(x) + L_1(x) =?$