HW

1. The power method for deriving the largest eigenvalue and eigenvector of a matrix $Ax = \lambda x$ is defined as follows $w^{(m+1)} = Az^{(m)}$ $\qquad z^{(m+1)} = \frac{w^{(m+1)}}{\left\|w^{(m+1)}\right\|_\infty}$ $\; m \geq 0.$

   a. Assume that $z^{(0)} = \sum_{j=1}^{n} a_j x_j$ where x_j are the eigenvectors corresponding to λ_i eigenvalues. Show that

   $A^m z^{(0)} = \sum_{j=1}^{n} a_j A^m x_j = \lambda_1^m [a_1 x_1 + \sum_{j=2}^{n} a_j\, x_j (\frac{\lambda_j}{\lambda_1})\hat{}m]$ and as $m \to \infty$

   $A^m z^{(0)} \to \lambda_1^m\, x_1$.

   b. Define $\lambda_1^{(m)} = w_k^{(m)}/z_k^{(m)}$ and show that it converges to λ_1 as m increase.
   c. Write a matlab program that computes that largest eigenvalue using the power method.
   d. Use the power method program to compute the eigenvalues of the following matrix:

   $$\begin{matrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{matrix}$$

   e. Find all eigenvalues and normalized eigenvectors by solving $\det(A - \lambda I) = 0$, $\; Ax = \lambda x$.

2. In class we wrote a matlab program for solving a two dimensional nonlinear system using Newton's method.

```
function [z0] =newtons2(n, x0,y0 )
syms x y
f(x,y)=x^2+4*y^2-9;
g(x,y)=18*y-14*x^2+45;


z0=[x0;y0];
fx(x,y)=diff(f,x);fy(x,y)= diff(f,y);gx(x,y)=diff(g,x);
gy(x,y)=diff(g,y);
A=double([fx(x0,y0) fy(x0,y0);gx(x0,y0) gy(x0,y0)]);

for i=1:n
   delta=double(-inv(A)*[f(x0,y0);g(x0,y0)]);
   z1=z0+delta;
   z0=z1;
   x0=z0(1);y0=z0(2);
   A=double([fx(x0,y0) fy(x0,y0);gx(x0,y0) gy(x0,y0)]);


end
```

Modify the above program so that for loop is replaced by while loop with the condition $norm(z1 - z0,2)$ becoming small at each step and stopping after it becomes less that eps=.000001.
Remember to avoid infinite loop you need to add a maximum number of iterations stopping criterion.

Solve the following equations:

a. $x^2 + y^2 = 4, x^2 - y^2 = 1$
b. $x^2 + y^2 - 2x - 2y + 1 = 0, x + y - 2xy = 0$
c. Generalize the program for 3 dimensional equations and solve $x^2 + y^2 + z^2 = 16, x^2 + y^2 - z^2 = 8, x - y + z = 2$