Assignment 4
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Q.1
i) Refer to kjiform1.m.

$$L = \begin{bmatrix} 1.0000 & 0 & 0 & 0 \\ 0.2000 & 1.0000 & 0 & 0 \\ 0.2000 & 0.1667 & 1.0000 & 0 \\ 0.2000 & 0.1667 & 0.1429 & 1.0000 \end{bmatrix}$$

$$U = \begin{bmatrix} 5.0000 & 1.0000 & 1.0000 & 1.0000 \\ 0 & 4.8000 & 0.8000 & 0.8000 \\ 0 & 0 & 4.6667 & 0.6667 \\ 0 & 0 & -0.0000 & 4.5714 \end{bmatrix}$$

This is the same as the L, U matrix that we get from using matlab function [L U P]= lu(A).
ii) the kji form(column wise) and kij form(row wise) are kjiform1.m and ijkform1.m respectively.
Time taken: Kij form (seconds) 0.4799(n = 500) 5.2293(n = 1000) 52.4398(n = 2000) Time taken: Kji form (seconds) 0.4236((n = 500) 3.8846(n = 1000) 32.5909(n = 2000)

this is almost 10 times when we double the value of n, hence we can say cubic growth. Column-wise method (kjiform) is growing at a speed slightly less than 10 times. Row wise method is growing at a speed slightly more than 10 times as we double the value of n. iii) Matlab stores matrices data in column-major order, i.e., the consecutive elements of a column reside next to each other. That is why columnwise operations are much faster compared to rowwise operations where accessing every next element in the row could result in a miss. **Q.2**
i) Refer to the program kjipivot1.m, Backsub1.m and Forsub1.m
ii)When run for n=5 the result from both the programs yielded ones(5,1) as answer. However as the value of n= 10, 20 40, the results started to deviate from the real answer(ones(n,1)). For n=10, the values are slightly deviated from correct answer, but for n=20 and 40, the answers are completely skewed. The number of operations to calculate these values is of complexity O(). As the value of n increases, the number of calculations increase and so does the magnitude of error with each calculation (there is a good amount of approximation involved in matrix computation). This leads to bad scaling of Matrices or we can say that the matrices get badly conditioned, which gets reflected in the value of rcond nearing 0.
Q.3 Refer to pdf Q3.pdf
Q.4
i)
The initial matrix is $a = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & -2 \\ -2 & 1 & 1 \end{bmatrix}$ The given program stores the multipliers in the same matrix.

We show the transformation of the matrix at each step below:
The operations performed here are:$(R2 - R1 * m_{21})$ and m_{21} is stored at a_{21}
$$\begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -2 & 1 & 1 \end{bmatrix}$$

The operations performed here are:$(R3 - R1 * m_{31})$ and m_{31} is stored at a_{31}
$$\begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -2 & 3 & -1 \end{bmatrix}$$

The operations performed here are:$(R3(2:3) - R1(2:3) * m_{32})$ and m_{32} is stored at a_{32} Here $(R3(2:3)$ means the 2nd and 3rd entry of the vector $R3$ and similarly for $R1$.
$$\begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -2 & 3 & 2 \end{bmatrix}$$
At this step the lower triangular matrix is extracted from the resulting matrix with the operation tril(a, -1)
$$L = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ -2 & 3 & 0 \end{bmatrix}$$
$$U = a - L = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix}$$
$$L = L + \text{eye}(3) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -2 & 3 & 1 \end{bmatrix}$$

ii) Refer to the matlab program kji.m

iii) Refer to the matlab program lux_solve.m

iv) Tested the lux_solve.m program with the system given in question (i). We tried different values the vector b as [0, -1, 3], [1, 1, 0], [4, 5, -3]. We obtained the x values as [1, 2, 3], [1, 1, 1] and [3, 2, 1] respectively which are correct solutions.

Q.5 In this program the matrix A is reduced to an upper triangular matrix U at the end of execution and the multipliers are stored in a matrix L which was initially an identity matrix of order 4.
The initial given matrix is:
A = [2 1 -1 -2;4 4 1 3;-6 -1 10 10;-2 1 8 4];
We show the transformations of this matrix below:
R3 is exchanged with R1

$$A = \begin{bmatrix} -6 & -1 & 10 & 10 \\ 4 & 4 & 1 & 3 \\ 2 & 1 & -1 & -2 \\ -2 & 1 & 8 & 4 \end{bmatrix}$$

Now we start the usual Gaussian elimination on the rows 2, 3 and 4 for column 1.

At the end of this step $A = \begin{bmatrix} -6 & -1 & 10 & 10 \\ 0 & 10/3 & 23/3 & 29/3 \\ 0 & 2/3 & 7/3 & 4/3 \\ 0 & 4/3 & 14/3 & 2/3 \end{bmatrix}$

The lower triangular matrix after this step:

$$l = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2/3 & 1 & 0 & 0 \\ -1/3 & 0 & 1 & 0 \\ 1/3 & 0 & 0 & 1 \end{bmatrix}$$

In next step, there is no exchange between rows as a_{22} contains the element with max absolute value in that column.

We use this as the pivot column and make the entries a_{32} and $a_{42} = 0$ by performing the elimination operation. At the end of this step $A = \begin{bmatrix} -6 & -1 & 10 & 10 \\ 0 & 10/3 & 23/3 & 29/3 \\ 0 & 0 & 4/5 & -3/5 \\ 0 & 0 & 8/5 & -16/5 \end{bmatrix}$

The lower triangular matrix after this step:

$$l = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2/3 & 1 & 0 & 0 \\ -1/3 & 1/5 & 1 & 0 \\ 1/3 & 2/5 & 0 & 1 \end{bmatrix}$$

The next step performs an exchange between the rows R3 and R4 of a as the max element the 3rd column of a is present in R4. After exchange the matrix becomes:

$$A = \begin{bmatrix} -6 & -1 & 10 & 10 \\ 0 & 10/3 & 23/3 & 29/3 \\ 0 & 0 & 8/5 & -16/5 \\ 0 & 0 & 4/5 & -3/5 \end{bmatrix}$$

After the next step of elimination the matrix is reduced to:

$$A = \begin{bmatrix} -6 & -1 & 10 & 10 \\ 0 & 10/3 & 23/3 & 29/3 \\ 0 & 0 & 8/5 & -16/5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The lower triangular matrix after this step:

$$l = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2/3 & 1 & 0 & 0 \\ -1/3 & 1/5 & 1 & 0 \\ 1/3 & 2/5 & 1/2 & 1 \end{bmatrix}$$

After this step, we have the final upper triangular matrix U = $\begin{bmatrix} -6 & -1 & 10 & 10 \\ 0 & 10/3 & 23/3 & 29/3 \\ 0 & 0 & 8/5 & -16/5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

lower triangular matrix L = = $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -2/3 & 1 & 0 & 0 \\ -1/3 & 1/5 & 1 & 0 \\ 1/3 & 2/5 & 1/2 & 1 \end{bmatrix}$

and the permutation matrix P = $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

The program given in this assignment doesn't give the correct L. The actual L matrix should be:

L = $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -2/3 & 1 & 0 & 0 \\ 1/3 & 2/5 & 1 & 0 \\ -1/3 & 1/5 & 1/2 & 1 \end{bmatrix}$

Using the P, L, U of our program we get P*L*U = $\begin{bmatrix} 2 & 1 & -0.2 & -4.6 \\ 4 & 4 & 1 & 3 \\ -6 & -1 & 10 & 10 \\ -2 & 1 & 7.2 & 6.6 \end{bmatrix}$

where 4 entries are different from the original matrix.