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1. Taylor's polynomial for the function

   i)$f(x) = e^x sin(x)$

   would be $P_n(x) = f(a) + \sum_{j=1}^{n} \frac{f^{(j)}(a)(x-a)^j}{j!}$

   We have a = 0, n = 5; Hence the polynomial reduces to
   $P_5(x) = f(0) + \sum_{j=1}^{5} \frac{f^{(j)}(0)x^j}{j!}$
   $= f(0) + xf^{(1)}(0) + \frac{x^2}{2!}f^{(2)}(0) + \frac{x^3}{3!}f^{(3)}(0) + \frac{x^4}{4!}f^{(4)}(0) + \frac{x^5}{5!}f^{(5)}(0)$

   $f(0) = 0$

   Using the Matlab program *hw1_part1.m* we derive manually the values of the derivatives(upto 5th) at the point a = 0.

   We have,
   $f^{(1)}(0) = 1$
   $f^{(2)}(0) = 2$
   $f^{(3)}(0) = 2$
   $f^{(4)}(0) = 0$
   $f^{(5)}(0) =$ -4

   On substituting the values, the polynomial becomes
   $P_5(x) = x + x^2 + \frac{x^3}{3}$ - $\frac{x^5}{30}$

   Similarly, for ii)$f(x) = \log(1 + x) - \log(1 - x)$, we follow the same procedure.
   We have,
   $f(0) = 0$
   Using the Matlab program *hw1_part1.m* we obtain the values upto 5th derivative at the point a = 0

   $f^{(1)}(0) = 2$
   $f^{(2)}(0) = 0$
   $f^{(3)}(0) = 4$
   $f^{(4)}(0) = 0$
   $f^{(5)}(0) = 48$

On substituting the values, the polynomial for the given function becomes
$P_5(x) = 2x + \frac{2}{3}x^3 + \frac{2}{5}x^5$

Observed pattern:
We increased the value of n and tried to find the pattern of the general Taylor's formula for the given functions. For the function $f(x) = \log(1+x) - \log(1-x)$ is $P_n(x) = \sum_{j=1}^{n} (j \bmod 2)\frac{2x^j}{j}$
Computation of log(3) :
We can calculate the value of log(3) using the general Taylor's polynomial for $f(x) = \log(1+x) - \log(1-x)$
We need to substitute x with 0.5 in the general polynomial for the function.
$\log(3) = 1.098612288668110$

We used the same Matlab program for evaluating the polynomial for different values of n, and the error. We found out that, for n= 13, the observed error=5.226242687728089e-06 which is within an accuracy of 10^{-5}.

2. Homework Part 2
From what the formulas that we derived manually, the functions were evaluated for different values of x. For $f(x) = e^x sin(x)$, the values of x that has been considered are 1 to 8. Using our program, the function value observed is given below

2.300000000000000
7.600000000000000
12.900000000000000
7.200000000000000
-32.500000000000000
-1.452000000000000e+02
-3.899000000000000e+02
-8.496000000000000e+02

This is exactly same as the value calculated by the program given in the assignment when n=5.

For $f(x) = \log(1+x) - \log(1-x)$, the values of x that has been considered are 0.5, 0.6, 0.7, 0.8 and 0.9. Using our program, the function value observed is given below

1.095833333333333
1.375104000000000

1.695894666666667
2.072405333333333
2.522196000000000

This is exactly same as the value calculated by the program given in the assignment when n=5.

Modifying the error formula:
We use the absolute value of $P_m(x)$ - $P_n(x)$ as the error for the formula $f(x) = \log(1+x) - \log(1-x)$. The results have been listed in the file Matlab_results.docx. The program used is *taylor_script2.m*.
We noticed that it might not be a good idea always to use this as the error formula. In our polynomial, the coefficients of all the even powered terms are 0, hence if we choose m=16 and n=15, we will not get two different polynomials and hence the error would be 0, which is wrong.