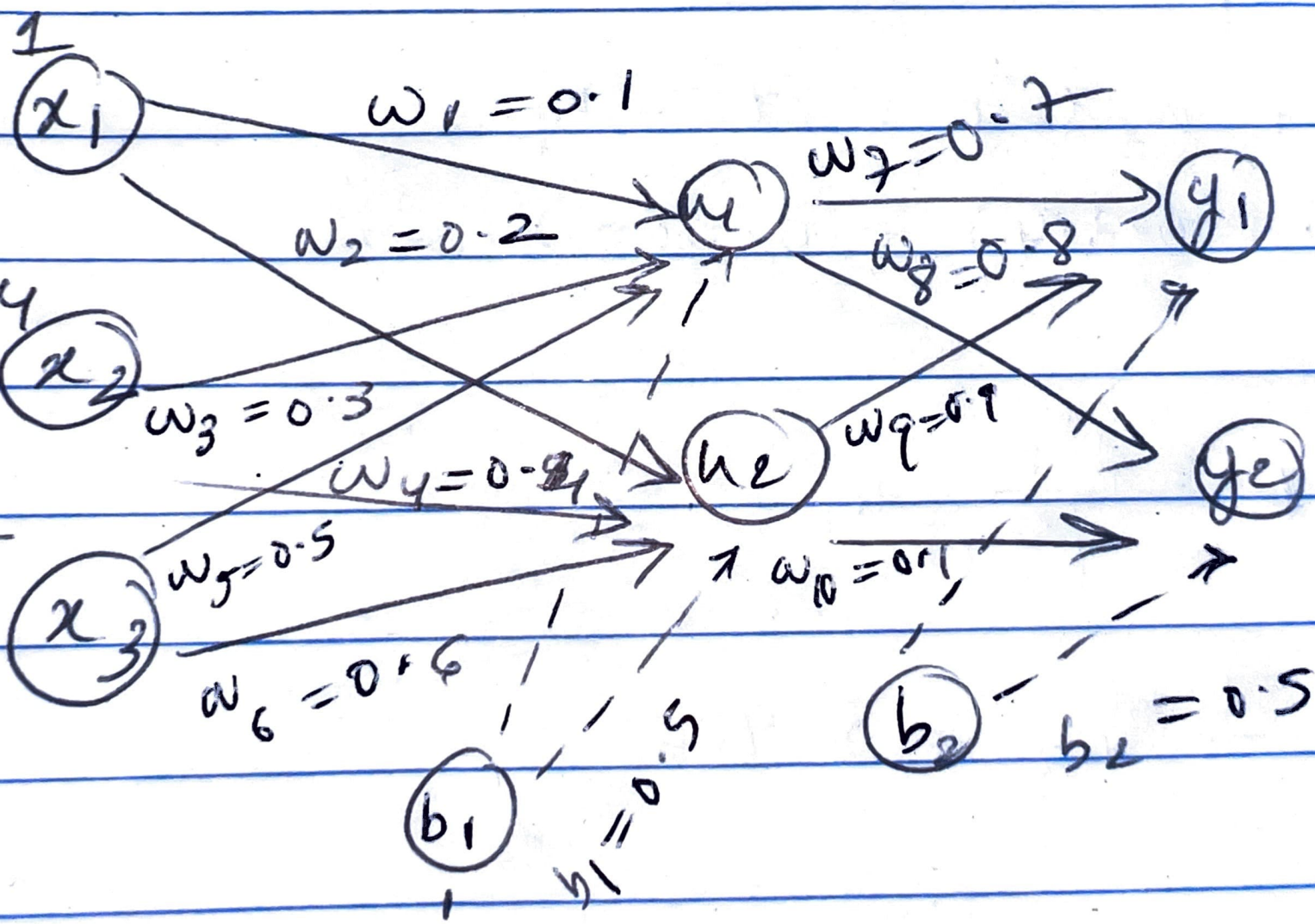
# Backpropagation.



input of $H_1$

$H_1 = w_1 x_1 + w_3 x_2 + w_5 x_3 + b_1$

$= 4.3$

output of $H_1 = 1/1+e^{-H_1} = 0.986$

input of $H_2$

$H_2 = w_2 x_1 + w_4 x_2 + w_6 x_3 + b_1$

$= 5.3$

output of $H_2 = 0.9950$

input of $y_1$

$y_1 = w_7 h_1 + w_9 h_2 + b_2$

$= 0.7(0.986) + 0.9(0.9886) + 0.5$

$= 2.0862$

output of $y_1 = 0.8896.$

input of $y_2$

$y_2 = w_8 H_1 + w_{10} H_2 + b_2 = 1.388$

output of $y_2 = 0.8004.$

$t_1 = 0.01 \quad t_2 = 0.99$

$E_{total} = \sum 1/2 (t - output)^2$

$= 1/2 (t_1 - out_{y_1})^2 + 1/2 (t_2 - out_{y_2})^2$

$= 0.3868 + 0.0179 = 0.4047$

Learning rate $= 0.5$

since $E_{total} = 1/2(T_1 - out_{y1})^2 + 1/2(T_2 - out_{y2})^2$

$\frac{\partial E_{total}}{\partial out_{y1}} = 2 \times 1/2 (T_1 - out_{y1}) * (-1)$

$= 0.8796$

$\frac{\partial E_{total}}{\partial out_{y2}} = 2 \times 1/2 (T_2 - out_{y2}) * (-1)$

$= -(0.99 - 0.8004)$

$= -0.1896$

$\frac{\partial E}{\partial w_7} = \frac{\partial E}{\partial out_{y1}} \frac{\partial out_{y1}}{\partial y_1} \frac{\partial y_1}{\partial w_7}$

✓ ↓

$\frac{\partial E}{\partial out_{y1}} = 0.8796$

$out_{y1} = \frac{1}{1 - e^{-y_1}}$

$\frac{\partial out_{y1}}{\partial y_1} = y_1(1 - y_1)$

$= (0.8896)(1 - 0.8896)$

$y_1 = w + h_4 + w_4 h_2$

$\frac{\partial y_1}{\partial w_7} = h_4 = 0.8896$

$\frac{\partial E_{total}}{\partial w_7} = 0.0765$

$$\frac{\partial E}{\partial w_8} = \frac{\partial E}{\partial out_{y_2}} \frac{\partial out_{y_2}}{\partial y_2} \frac{\partial y_2}{\partial w_8}$$

$$\frac{\partial E}{\partial w_8} = (out_{y_2} - t_4)(out_{y_2}(1 - out_{y_2})) \cdot w_4$$

$$= 0.1183$$

$$\frac{\partial E}{\partial w_9} = \frac{\partial E}{\partial out_{y_1}} \frac{\partial out_{y_1}}{\partial y_1} \frac{\partial y_1}{\partial w_9}$$

$$= 0.0772$$

$$\frac{\partial E}{\partial w_{10}} = 0.1193$$

$$\frac{\partial E}{\partial b_2} = \frac{\partial E}{\partial out_{y_1}} \frac{\partial out_{y_1}}{\partial y_1} \frac{\partial y_1}{\partial b_2} + \frac{\partial E}{\partial out_{y_2}} \frac{\partial out_{y_2}}{\partial y_2} \frac{\partial y_2}{\partial b_2}$$

$$= 0.1972$$

$$\frac{\partial E}{\partial w_1} = \frac{\partial E}{\partial out_{h_4}} \frac{\partial out_{h_4}}{\partial h_4} \frac{\partial h_4}{\partial w_1}$$

- since $w_4$ effects errors through $o_1$ & $o_2$

$$\frac{\partial E}{\partial w_{int}} = \frac{\partial E}{\partial out_{y_1}} \frac{\partial out_{y_1}}{\partial y_1} \frac{\partial y_1}{\partial h_4}$$

$$\frac{\partial E}{\partial out_{w_4}} = 0.1502.$$

$$\frac{\partial E}{\partial w_1} = (0.1502)(0.0132)(1)$$

$$= 0.0020$$

$$\frac{\partial E}{\partial w_3} = \frac{\partial E}{\partial out_{H_4}} \frac{\partial out_{H_4}}{\partial H_4} \frac{\partial H_4}{\partial w_3}$$

$$= (0.1502)(0.0134)(4)$$

$$= 0.0079$$

$$\frac{\partial E}{\partial w_5} = (0.1502)(0.0132)(5)$$

$$= 0.007$$

~~$\frac{\partial E}{\partial w_2} \quad \frac{\partial E}{\partial out_{H_2}} \quad \frac{\partial E}{\partial out_{H_2}} = \frac{\partial E}{\partial out_{y_1}} \frac{\partial out_{y_1}}{\partial y_1} \frac{\partial y_1}{\partial out_{H_2}}$~~

~~$= (0.1896)(0.0983)(0.9)$~~

~~$(0.7504)(0.1)(0.1) =$~~

$$\frac{\partial E}{\partial H_{2_{out}}} = \frac{\partial E}{\partial out_{H_2}} = \frac{\partial E}{\partial out_{y_1}} \frac{\partial out_{y_1}}{\partial y_1} \frac{\partial y_1}{\partial out_{H_2}} +$$

$$\frac{\partial E}{\partial out_{y_2}} \frac{\partial out_{y_2}}{\partial y_2} \frac{\partial y_2}{\partial out_{H_2}}$$

$$= 0.1896 \times 0.0983 \times 0.9 +$$

$$0.7504 \times 0.1 \times 0.1$$

$$= 0.9950$$

$\frac{d out_{h_2}}{d h_2} = h_2(1-h_2)$

$= 0.0049$

$\frac{d\epsilon}{dw_2} = \frac{d\epsilon}{d out_{h_2}} \frac{d out_{h_2}}{d h_2} \frac{d h_2}{d w_2}$

$= 0.0818 \times 0.0049 \times 1.$

$= 0.00040082$

$\frac{d\epsilon}{dw_4} = \frac{d\epsilon}{d out_{h_2}} \cdot \frac{d out_{h_2}}{d h_2} \frac{d h_2}{d w_4} = 0.0016$

learning rate $= \eta$

$\eta = 0.01$

$\frac{d\epsilon}{dw_6} = 0.0020$

$\frac{d\epsilon}{dg} = 0.0008$

$w_1 = w_1 - \eta \frac{d\epsilon}{dw_1} = 0.1 - (0.01)(0.0020)$

$= 0.1000$

$w_2 = w_2 - \eta \frac{d\epsilon}{dw_2} = 0.2 - (0.01)(0.0004)$

$= 0.200$

$w_3 = 0.3 - (0.01)(0.0079)$

$= 0.2999$

$w_4 = 0.4 - (0.01)(0.0016) = 0.4000$

$w_5 = 0.5 - (0.01)(0.0099) = 0.4999$

$w_6 = 0.6 - (0.01)(0.002d) = 0.6000$

$w_7 = 0.7 - (0.01)(0.0865) = 0.6992$

$w_8 = 0.7988 \quad w_9 = 0.8992 \quad w_{10} = 0.9988$

$b_1 = 0.5 \quad b_2 = 0.4980$