REVISION

The density functions of two independent R.V.s x and Y are given by $f(x) = \frac{2x}{a^2}$ in $0 \le x < a$ and $f(y) = \frac{2y}{b^2}$ in $0 \le y \le b$. Find the coefficient of correlation between $U = X + Y$ and $V = X - Y$

$f(x) = \frac{2x}{a^2}$ & $f(y) = \frac{2y}{b^2}$

$$\int \frac{2x}{a^2}\, dy = \left[\frac{2xy}{a^2}\right]_0^a = \frac{2ax}{a^2} = \frac{2a}{3}$$

$$\int \frac{2y}{b^2}\, dx = \left[\frac{2xy}{b^2}\right]_0^b = \frac{2b}{3}$$

~~coefficient of correlation =~~

Second momentum $= \int_0^a x^2 f(x)\, dx$ & $\int_0^b y^2 f(y)\, dy$

$$= \int_0^a x^2 \cdot \frac{2x}{a^2}\, dx = \left[\frac{2x^4}{4a^2}\right]_0^a = \frac{a^4}{2a^2} = \frac{a^2}{2}$$

$$= \int_0^b y^2 \frac{2y}{b^2}\, dy = \left[\frac{2y^4}{4b^2}\right]_0^b = \frac{2b^4}{4b^2} = \frac{b^2}{2}$$

Mean $= E(x) = \int_0^a x f(x)\, dx = \int_0^a \frac{2x^2}{a^2}\, dx = \left[\frac{2x^3}{3a^2}\right]_0^a = \frac{2a}{3}$

& $E(y) = \frac{2b}{3}$

$$\sigma_x^2 = E(x^2) - (E(x))^2$$

$$= \frac{a^2}{2} - \left(\frac{2a}{3}\right)^2 = \frac{a^2}{18}$$

$$\sigma_y^2 = \frac{b^2}{18}$$

$$\text{cov}(UV) = \frac{a^2 - b^2}{18}$$

$$; \; r(U,V) = \frac{\frac{a^2 - b^2}{18}}{\sqrt{\frac{a^2}{3\sqrt2} + \frac{b^2}{3\sqrt2}}} = \frac{a^2 - b^2}{a^2 + b^2}$$

$=$

(2) If the joint P.d.f of (X, Y) is given by $f(x,y) = \frac{1}{3}(x+y)$ in $0 \le x \le 1$, $0 \le y \le 1$. Find the coefficient of corr b/w x & Y.

$$\cancel{\int_0^2 f(x)\,dy = \int_0^2 \frac{1}{3}(x+y)\,dy = \frac{x}{3}y + \frac{y^2}{2}}$$

Marginal density $f^n \rightarrow$

$$\Rightarrow \int_0^1 \frac{1}{3}(x+y)\,dy = \left[\frac{xy}{3} + \frac{y^2}{2}\right]_0^1$$

$$= \frac{x}{3} + \frac{1}{2}$$

$$\Rightarrow \int_0^2 \frac{1}{3}(x+y)\,dx = \left[\frac{x^2}{6} + xy\right]_0^2 = \frac{\cancel{24}}{36} + 2y = \frac{2}{3} + 2y$$

$$\Rightarrow f(y/x)(y) = \frac{f(x,y)}{f_X(x)}$$

$$= \frac{\frac{2}{3}(x+y)}{\frac{2}{3}(x+1)} = \frac{x+y}{2x+2}$$

$$f(x/y)(x) = \frac{f(x,y)}{f_Y(y)} = \frac{2(x+y)}{1+2y}$$

Conditional distribution

$$E(Y/x) = \int_0^2 y \cdot \frac{x+y}{2(x+1)}\,dy$$

$$= \frac{1}{2(x+1)} \int_0^2 xy + y^2\,dy = \frac{1}{2(x+1)} \left[\frac{xy^2}{2} + \frac{y^3}{3}\right]_0^2$$

$$\Rightarrow \frac{4x}{2} + \frac{8}{3} = y = \frac{3x+4}{3(x+1)}$$

Date / /
Page No.

$$E(X/Y) = \int_0^1 \frac{x(2x+y)}{1+2y}\,dx$$

$$= \frac{1}{1+2y}\int_0^1 2x^2 + xy\,dx$$

$$= \frac{1}{1+2y}\left[\frac{2x^3}{3} + \frac{x^2 y}{2}\right]_0^1 = \left[\frac{2}{3} + \frac{y}{2}\right] \times \frac{1}{1+2y}$$

$$x = \frac{2+3y}{3(1+2y)}$$

coeff of reg $b_{XY} = \dfrac{2}{3(1+2y)}$

=

Reg

$\Sigma = E(X/Y) = \int_0$

$= \int_0^1 x^2 f(x)\,dx$ & $\int_0^2 y^2 f(y)\,dy$

$= \int_0^1 x^2 \frac{1}{3}(x+y)\,dx$ $= \int_0^1 y^2 \frac{1}{3}(x+y)\,dy$

$= \int_0^1 \frac{x^3}{3} + \frac{x^2 y}{3}\,dx$ $= \int_0^1 \frac{xy^2}{3} + \frac{y^3}{3}\,dy$

$= \left[\frac{x^4}{12} + \frac{x^3 y}{9}\right]_0^1$ $= \left[\frac{xy^3}{9} + \frac{y^4}{12}\right]_0^1$

$= \frac{1}{12} + \frac{y}{9}$ $= \frac{x}{9} + \frac{1}{12}$

$\text{Mean} = E(X) = \int_0^1 \frac{x^2}{3} + \frac{xy}{3}\,dx = \left[\frac{x^3}{9} + \frac{x^2 y}{9}\right]_0^1 = \frac{1}{9} + \frac{y}{9}$

$E(Y) = \frac{1}{9} + \frac{x}{9}$

~~$\sigma x^2 = \frac{1}{9} \frac{5y}{18} = \frac{1}{9} + \frac{x}{9} - \frac{1}{9} + \frac{y}{9} = 1.6$~~

$\sigma_x^2 = \cancel{\sigma_y^2} = \frac{1}{12} + \cancel{\frac{y}{9}} - \frac{1}{9} - \cancel{\frac{y}{9}} = \frac{9-12}{108} = \frac{-3}{108}$

$\sigma_y^2 = \cancel{\frac{x}{9}} + \frac{1}{12} - \frac{1}{9} - \cancel{\frac{x}{9}} = \frac{3}{108}$

$= \frac{6}{108} = 2.96$

$=$

$\frac{3}{108} + \frac{3}{108}$

$= \int_0^1 x^2 f(x)\, dx$ & $\int_0^2 y^2 f(y)\, dy$

$= \int_0^1 x^2 \frac{1}{3}(x+y)\, dx$ $= \int_0^1 y^2 \frac{1}{3}(x+y)\, dy$

$= \int_0^1 \frac{x^3}{3} + \frac{x^2 y}{3}\, dx$ $= \int_0^1 \frac{x y^2}{3} + \frac{y^3}{3}\, dy$

$= \left[\frac{x^4}{12} + \frac{x^3 y}{9}\right]_0^1$ $= \left[\frac{x y^3}{9} + \frac{y^4}{12}\right]_0^1$

$= \frac{1}{12} + \frac{y}{9}$ $= \frac{x}{9} + \frac{1}{12}$

Mean $= E(x) = \int_0^1 \frac{x^2}{3} + \frac{xy}{3}\, dx = \left[\frac{x^3}{9} + \frac{x^2 y}{9}\right]_0^1 = \frac{1}{9} + \frac{y}{9}$

$E(y) = \frac{1}{9} + \frac{x}{9}$

~~$\sigma x^2 = \frac{1}{9} \cdot \frac{5}{18} = \frac{1}{9} + \frac{x}{9} - \frac{1}{9} + \frac{y}{9} = 1.6$~~

$\sigma x^2 = \cancel{\sigma y^2} = \frac{1}{12} + \cancel{\frac{y}{9}} - \frac{1}{9} - \cancel{\frac{y}{9}} = \frac{9-12}{108} = \frac{-3}{108}$

$\sigma y^2 = \cancel{\frac{x}{9}} + \frac{1}{12} - \frac{1}{9} - \cancel{\frac{x}{9}} = \frac{3}{108}$

$= \frac{6}{108} = 2.96$

$\frac{3}{108} + \frac{3}{108}$

(3) Write the statement of Poisson distribution and find the mean and variance using poisson distribution

Poisson Distribution: $P(X) = \frac{e^{-m} m^r}{r!}$

$E(x) =$ Mean: $\mu_1' = E(x) = \sum_{r=0}^{\infty} r \cdot \frac{e^{-m} m^r}{r!}$

$= 0 + e^{-m} m + \frac{\cancel{2} e^{-m} m^2}{\cancel{2}} + \frac{\cancel{3} e^{-m} m^3}{2 \cancel{6}} + \frac{\cancel{4} e^{-m} m^4}{\cancel{24}\ 6} + \frac{5 e^{-m} m^5}{120} + \ldots$

$= e^{-m} m \left[1 + m + \frac{m^2}{2!} + \frac{m^3}{3!} + \frac{m^4}{4!} \ldots \right]$

$= m =$ mean

$(E(x))^2 = \quad r^2 = r(r-1) + r$

Replacing

already calculated

$= \sum_{r=0}^{\infty} \frac{r(r-1) e^{-m} m^r}{r!} + m$

$= 0 + 0 + \frac{2 e^{-m} m^2}{2!} + \frac{3 e^{-m} m^3}{3!} + \frac{4 e^{-m} m^4}{4!} + \ldots$

$= \cancel{e^{-m}} m^2 \cancel{e^{m}} = m^2 + m$

$= m^2 + m$

Variance $=$ ~~$\{E(x)\}^2 - E(x) = m^2 + m - m = m^2$~~

$E(x^2) - (E(x))^2 = \cancel{m^2} + m - \cancel{m^2} = m$

Date / /
Page No.

(Q) If X & Y are standardized R.V.s such that the coefficient of correlation b/w (2X + Y) & (X + 2Y) is equal to 1/2. Find the coefficient of correlation b/w X and Y.

coeff of correlation $= \frac{1}{2}$

$(2X+Y)\ (X+2Y)$ $\qquad U = 2X+Y \ , \ V = X+2Y$

$$\cancel{\frac{1}{2} = \frac{E(XY) - E(X)E(Y)}{\sigma_X \cdot \sigma_Y}} \qquad \frac{1}{2} = \frac{E(UV) - E(U)E(V)}{\sigma_U \cdot \sigma_V}$$

$$\Rightarrow r_{X,Y} = \frac{\operatorname{cov}(X,Y)}{\sigma_X \sigma_Y}$$

$$\sigma_U = E(U^2) - (E(U))^2$$

$$\Rightarrow \sigma_U + \sigma_V \mp 2\operatorname{cov}(U,V)$$

$\Rightarrow$

(5) Write the statement of binomial distribution, find the mean and variance of the above distribution

$$p(x) = {}^nC_r\, p^r q^{n-r} \qquad (p+q=1)$$

$$\mu_1' = E(x) = \sum_{r=0}^{n} r\, {}^nC_r\, p^r q^{n-r}$$

$$= 0q^n + n\alpha\, pq^{n-1} + \cdots np^{n-r}$$

$$= npq^{n-1} + \cdots n^{n-r}$$

$$= np\left(q^{n-1} + {}^{m-1}C_1\, pq^{n-1} + \cdots p^{n-1}\right)$$

$$= np(p+q)^{n-1}$$

$$= np \qquad (p+q=1)$$

$$\mu_2' = E(x^2) = \sum_{r=0}^{n} r^2\, {}^nC_r\, p^r q^{n-r}$$

$$= \sum_{r=0}^{n} [r(r-1)+r]\, {}^nC_r\, p^r q^{n-r}$$

$$= np + n(n-1)p^2 \left[\sum_{r=0}^{n} {}^{n-1}C_{n-2}\, p^n q^{n-r}\right]$$

$$\downarrow 0$$

Variance $= E(x^2) - (E(x))^2 = \cancel{np + n(n-1)p^2} - n^2p^2$

$= \mu_1 = 0$ (always)

$$\mu_2 = \mu_2' - (\mu_1)^2$$

$$= npq$$

$$\sigma^2 = \mu_1 = npq$$