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1 (a)(i)Check reflexive:

$$(x, x) \in R \Rightarrow f(x) = k f(x) \Rightarrow k=1 \quad , \therefore k \in N$$

$(x, x) \in R$ and R is reflexive

(ii) Check anti-symmetric:

let $(x,y) \in R \Rightarrow f(y) = k f(x) \Rightarrow$ ①

if $(y,x) \in R$, -then,

$$y(x) = k f(y) \longrightarrow ②$$

substitute $k = \dfrac{f(y)}{f(x)}$ from ①

$$[f(x)]^2 = [f(y)]^2$$

$$\therefore f(x) = f(y)$$

taking inverse, $f^{-1} f(x) = f^{-1} f(y)$

$$= x = y$$

$\therefore (y, x) \in R$ iff $y = x$

Hence, it is anti-symmetric

(iii) Check for transitive:

let $(x, y) \in R$ or k_1 be a natural number.

$$f(y) = k_1 f(x)$$

let $(y, z) \in R$ and $k_2 \in N$

$$f(z) = k_2 f(y)$$

$$\therefore f(z) = k_2 k_1 f(x)$$

and $k_1 k_2 = k \quad \therefore k \in N.$

$$\therefore f(z) = k f(x) \Rightarrow (x, z) \in R$$

Hence the relation is transitive.

from ⓘ, ⓘⓘ & ⓘⓘⓘ, it is a partial order relation.

2) $A = \{4, 5, 6, 7, 8, 9\}$

$R = \{(x, y) : f(y) = k f(x), k \in N\}$

$f(x) = 10 - x$

at $x = 4$, $f(y) = 6 \Rightarrow y = 4$

$f(5) = 5$, $y = 5$

$f(6) = 2 \Rightarrow y = 4, 6, 8$

$f(8) = 4 \Rightarrow y = 6$

$f(7) = 3 \Rightarrow y = 4, 7$

$f(1) = y = 9, 8, 7, 6, 5, 4$

$\therefore R = \{(4,4), (5,5), (6,6), (7,7), (7,4), (8,4), (8,6), (8,8), (9,9), (9,8)$

$(9,7) (9,6), (9,5)\}$