MATHS ASSIGNMENT

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Name: Ananya Prasad

Reg No: 20BCE10093

$\boxed{(3)}$ $F(x,y) = \begin{cases} 8xy & 0<x<y<1 \\ 0 & \text{else} \end{cases}$

(a) $E(Y/X=X)$

$$f(Y/X=X) = \frac{8xy}{f_X(X)}$$

$$f_X(X) = 4x(1-x^2)$$

$$\therefore \quad = \frac{2y}{1-x^2}$$

$$= \int_0^1 \int_0^y xy \frac{2y}{1-x^2} \, dx \, dy$$

$$1-x^2 = t$$

$$-2x\,dx = dt$$

$$\int_0^1 \int_0^y \frac{y^2}{t} - dt \, dx \, dy$$

$$\int_0^1 y^2 [\ln t]_0^y \, dy = \int_0^1 [y^2 \ln(1-x^2)]_0^y$$

$$= \int_0^1 y^2 \ln(1-y^2) \, dy$$

$$= 1-y^2 = p \Rightarrow -2y\,dy = dp \Rightarrow \int_0^1 (p-1) \ln p \, dp$$

$$= \left[(p-1)\ln p - \int 1.\ln p \, dp\right]_0^1 = \left[(p-1)\ln p - \ln p\right]_0^1$$

$$= -1$$

(b) $E(XY/X = X)$

$$f(xy/x) = \frac{F_{t/x}(t)}{F_X(x)}$$

$$\because (p = 8xy) \times (1-x^2) \times$$

$$t = 4 \times (1-x^2) \times y \times 2$$

$$(1-x^2)\, p = 8xy(1-x^2)$$

$$2y\, t = 8xy(1-x^2)$$

$$\frac{p}{t} = \frac{2y}{1-x^2}$$

$$= \int_0^1 \int_0^y xy \frac{2y}{1-x^2}\, dx\, dy$$

$$= \int_0^1 \frac{y}{2} \ln(1-y^2) \frac{}{2}\, dy$$

$$= \left[\ln(1-y^2) \cdot 1 \Big|_0^1 - \int_0^1 \ln(1-y^2) \cdot \frac{y^2}{2}\, dy \right]$$

$$=$$

(c) $Var(Y/X = x)$

$$= \int_0^1 \int_0^y \frac{8x\cancel{y}}{4\cancel{y}(1-y^2)}\, dx\, dy$$

$$= \int_0^1 x^2 \ln(1-x^2)\, dx$$

$$= -1$$

$$\Rightarrow$$

4) $f(x,y) = \begin{cases} k & (4-x-y) \quad 0 \le x, y \le 2 \\ 0 & \text{else} \end{cases}$

k ; $\iint f(xy)\,dx\,dy = 1$

$= \int_0^2 \int_0^2 k(4-x-y)\,dx\,dy = 1$

$= \int_0^2 \int_0^2 4k - xk - yk \,dx\,dy = 1$

$= \int_0^2 \left[4kx - \frac{kx^2}{2} - kxy\right]_0^2 = 1$

$= \int_0^2 8k - 2k - 2ky \,dy = 1$

$\left[6ky - \frac{2ky^2}{2}\right]_0^2 = 1$

$12k - 4k = 1$; $k = 1/8$

Marginal density function of $X \rightarrow$

$f_X(x) = \int_0^2 \frac{1}{8}(4-x-y)\,dy$

$= \int_0^2 \frac{1}{2} - \frac{x}{8} - \frac{y}{8}\,dy$

$= \left[\frac{4}{2} - \frac{xy}{8} - \frac{y^2}{16}\right]_0^2 = 1 - \frac{x}{4} - \frac{1}{4}$

Marginal density function of $Y \rightarrow$

$f_Y(y) = \int_0^2 \frac{1}{8}(4-x-y)\,dx = \int_0^2 \left[\frac{1}{2} - \frac{x}{8} - \frac{y}{8}\,dx\right]$

$= \left[\frac{x}{2} - \frac{x^2}{16} - \frac{xy}{8}\right]_0^2 = 1 - \frac{1}{4} - \frac{y}{4}$

Conditional probability →

$$f_{Y/X}(y) = \frac{2-x-y}{1-\frac{x}{4}-\frac{1}{4}} = \frac{4(2-x-y)}{(3-x)}$$

$$f_{(X/Y)}(x) = \frac{4(2-x-y)}{3-y}$$

Var(x)

$$E(x) = \int_0^2 x\left(\frac{3-x}{4}\right)dx$$

$$= \int_0^2 \frac{3}{4}x - \frac{x^2}{4}\,dx$$

$$= \left[\frac{3}{4}\times\frac{x^2}{2} - \frac{x^3}{12}\right]_0^2 = \frac{3}{\cancel{4}}\times\frac{\cancel{4}}{2} - \frac{\cancel{8}^{2}}{\cancel{12}\,3} = \frac{3}{2} - \frac{2}{3} = \frac{9-4}{6} = \frac{5}{6}$$

$$E(x^2) = \int_0^2 \frac{3x^2}{4} - \frac{x^3}{4}\,dx = \left[\frac{x^3}{4} - \frac{x^4}{16}\right]_0^2 = \frac{8}{4} - 1 = \underline{\underline{1}}$$

$$Var(x) = 1 - \frac{25}{36} = \underline{\underline{\frac{11}{36}}}$$

$$\therefore Var(Y) = \underline{\underline{\frac{11}{36}}}$$

$$Cov(x,y) = \int_0^2\int_0^2 \frac{xy}{2} - \frac{x^2y}{8} - \frac{xy^2}{8}\,dx\,dy$$

$$= \int_0^2 \left[\frac{x^2}{4}y - \frac{x^3 y}{24} - \frac{x^2y^2}{16}\right]_0^2 dy$$

$$= \int_0^2 y - \frac{y}{3} - \frac{y^2}{4}\,dy$$

$$= \left[\frac{y^2}{2} - \frac{y^2}{6} - \frac{y^3}{12}\right]_0^2 = 2 - \frac{2}{3} - \frac{2}{3}$$

$$= \underline{\underline{\frac{2}{3}}}$$

$$\text{Cov}(x,y) = \frac{2}{3} - \frac{5}{6} \times \frac{5}{6} = \frac{2}{3} - \frac{25}{36} = \frac{24-25}{36} = \frac{-1}{36}$$

$\boxed{6}$ $f(x,y) = \frac{1}{3}x^2 e^{-y(1+x)}$ $\quad x \geq 0 \quad y \geq 0$

$$y = \int y \, f_{y/x}(y) \, dy$$

$$f_{y/x}(y) = \frac{1/3 \, x^2 e^{-y(1+x)}}{f_x(x)}$$

$$f_x(x) = \int \frac{1}{3} x^2 e^{-y(1+x)} \, dy$$

$$= \int \frac{1}{3}\left[x^2 \frac{e^{-y(1+x)}}{-(1+x)}\right]_0^{\infty}$$

$$y = \frac{1}{3} \frac{x^2}{1+x} \quad (y \text{ on } x)$$

$$y = \int_0^{\infty} y(1+x) e^{-y(1+x)} \, dy \qquad y(1+x) = t$$

$$dy = \frac{t}{(1+x)}$$

$$= \int_0^{\infty} \frac{t e^{-t} \, dt}{(1+x)}$$

$$= \frac{1}{1+x}\left[\frac{t e^{-t}}{-1} + \int \frac{e^{-t}}{\cdot}\right]$$

$$= \frac{1}{1+x}\left[-te^{-t} - e^{-t}\right]_0^{\infty}$$

$$= \frac{1}{1+x}$$

$\boxed{5}$ $f(x,y) = kxy \quad 0<x<y<1$

to get k, $\int_0^1 \int_0^y kxy \, dx \, dy = 1$

$$= \int_0^1 \left[\frac{kx^2y}{2}\right]_0^y dy = 1$$

$$= \int_0^1 \frac{ky^3}{2} dy = 1$$

$$= \left[\frac{ky^4}{8}\right]_0^1 = 1 \Rightarrow \underline{k=8}$$

$f_{(Y|X)}(Y) = \frac{kxy}{f_X(X)} \quad f_X(X) = \int_x^1 8xy = \frac{\cancel{8}^{4} x(1-x^2)}{\cancel{2}}$

$$= \frac{\cancel{8}^{2} xy}{\frac{\cancel{x}(1-x^2)}{2}}$$

$$\therefore y = \int_x^1 \frac{2y^2}{1-x^2} dy \Rightarrow y = \frac{2}{3}\left(\frac{1-x^3}{1-x^2}\right)$$

$\boxed{2}$ $f(x,y) = \begin{cases} 2-x-y & ; \ 0 \le x \le 1, \ 0 \le y \le 1 \\ 0 & ; \ \text{otherwise} \end{cases}$

Marginal density f^n of x.

$$f_X(x) = \int_0^1 2-x-y \, dy$$

$$= \left[2y - xy - \frac{y^2}{2}\right]_0^1 = \frac{3}{2} - x$$

$$\therefore f_Y(Y) = \int_0^1 (2-x-y) \, dx$$

$$= \left[2x - \frac{x^2}{2} - xy\right]_0^1 = \frac{3}{2} - y$$

$\therefore f_{(Y/X)}(X) = \dfrac{2-x-y}{\frac{3}{2}-x} = \dfrac{2(2-x-y)}{(3-2x)}$

$f_{(X/Y)}(Y) = \dfrac{2(2-x-y)}{3-2y}$

(iii) $Var(X) = E(X^2) - E(X)^2$

$E(X) = \int_0^1 \frac{3}{2}x - x^2 \, dx$

$= \left[\frac{3}{2}\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{3}{4} - \frac{1}{3} = \frac{9-4}{12} = \frac{5}{12}$

$E(X^2) = \int_0^1 x^2\left(\frac{3}{2}-x\right)dx$

$= \left[\frac{3}{3} \times \frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

$var(X) = \frac{1}{4} - \frac{25}{144} = \frac{11}{144}$

$\because \int x f_x(x)\,dx = \int y f_y(y)\,dy \therefore Var(Y) = \frac{11}{144}$

$\boxed{(1)}$ X_1 and X_2 ; $V = max(X_1, X_2)$

so have joint distribution, X_1 & $y \ni X_1 \leq Y$

X_1 \ Y	1	2	3	4	5	6	
1	1/36	1/36	1/36	1/36	1/36	1/36	1/6
2	0	1/18	1/36	1/36	1/36	1/36	1/6
3	0	0	1/12	1/36	1/36	1/36	1/6
4	0	0	0	1/9	1/36	1/36	1/6
5	0	0	0	0	5/36	1/36	1/6
6	0	0	0	0	0	1/6	1/6

Mean & variance of Y

Mar. f^n of Y is

$\Rightarrow$ $Y=1$; $1/36$

$Y=2$; $1/12$

$Y=3$; $5/36$

$Y=4$; $7/36$

$Y=5$; $9/36$

$Y=6$ = $11/36$

$$= 1 \times \frac{1}{36} + \frac{1}{12} \times 2 + \frac{5}{36} \times 5 + \frac{7}{36} \times 7 + \frac{9}{36} \times 9 + \frac{11}{36} \times 11$$

$$= \frac{1}{36} + \frac{\cancel{6}}{\cancel{36}} + \frac{25}{36} + \frac{49}{36} + \frac{81}{36} + \frac{121}{36} = \frac{283}{36} = 7.86$$

Variance $\Rightarrow$ $E(x^2) = 1 \times \frac{1}{36} + 4 \times \frac{1 \times 3}{12 \times 3} + \frac{25 \times 5}{36} + \frac{49 \times 7}{36} + \frac{81 \times 9}{36} + \frac{121}{36} \times 11$

$$= \frac{1}{36} + \frac{12}{36} + \frac{125}{36} + \frac{343}{36} + \frac{729}{36} + \frac{1331}{36}$$

$= 70.58$

$\therefore$ Var $= 70.58 - (7.86)^2$

$= 70.58 - 61.77$

$= 8.81$

— x ——— x ——— x ——— x ——— x ——