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(1)

1) a $M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$, $M_S = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$

For $M_{R\circ S}$, $M_{R\circ S} = M_R \cdot M_S$

We know that $1+1=1$

This is the 'Binary OR'

$$\therefore M_{R\circ S} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Now, by simplifying each element cross

$$\therefore M_{R\circ S} = \begin{bmatrix} (1+1.0+1.1+1) & \ldots & (0+1.0+0.1+1) & 0+0+0 \\ 1+1+0 & \ldots & 0+0+0 \ldots & 0+1+0 \\ 1+1+1 & & 0+1+0 \quad 1+0+1 & 0+1+0 \end{bmatrix} = \begin{bmatrix} 1+0+1 & 0+0+1 & 0+0+0 & 1+0+1 & 0+0+0 \\ 1+1+0 & 0+0+0 & 0+1+0 & 1+0+0 & 0+1+0 \\ 1+1+1 & 0+0+1 & 0+1+0 & 1+0+1 & 0+1+0 \end{bmatrix}$$

$$M_{R\circ S} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

2

I(b) Total number of vertices $n = 7$

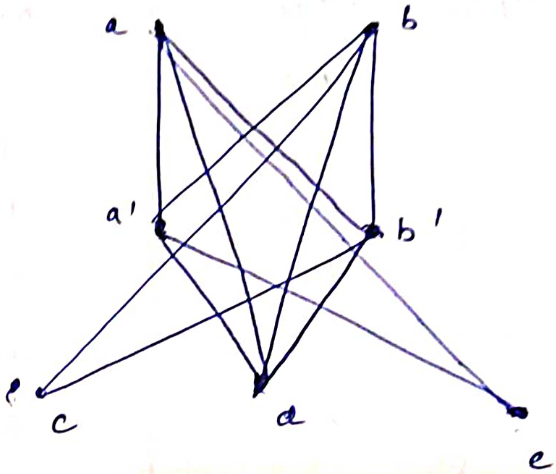
$\deg(a) + \deg(b) + \deg(a') + \deg(b') + \deg(c) + \deg(d) + \deg(e)$

$\deg(K_{2,2,3}) = 32$

Now by handshaking theorem, we know that

$\cancel{\deg(K_{2,2,3}) = 32}$

$$\sum_{i=1}^{n} \deg(v_i) = 2 \cdot e.$$



$\therefore$ $e = 16$.

each region is bounded by 3 edges.

$\therefore$ r-regions are bounded by 3r-edges.

$\therefore$ $2 \cdot e = 3 \cdot r$

$r = \frac{2e}{3}$

$\therefore$ By Euler's formula, $r = 2 - n + e$

$\frac{2e}{3} = 2 - n + e$

$2e = 6 - 3n + 3e$

$e = 3n - 6$

$\boxed{e = 16 \text{ and } n = 7}$

Now, substituting values,

$16 = 3 \times 7 - 6$

$16 = 21 - 6$

$16 = 15$

Now this is a contradiction.

S. $K(2,2,3)$ is non-planar.

3(b) Tree T with ~~8~~.

vertices = 3 , degree = 2

vertices 4 , degree = 3

vertices = 3, degree = 4

By Handshaking Theorem, sum of all degrees = 2 * sum of edges.

$(3V \times 2D) + (4V \times 3D) + (3V \times 4D) + (kV \times 1D) = 2 * \text{edges}$

k = pendant vertices

$3 \times 2 + 4 \times 3 + 3 \times 4 + k \times 1 = 2 \times \text{edges} \rightarrow ①$

number of edges = number of vertices − 1

$\therefore$ edges = $(3 + 4 + 3 + k) - 1$

$\therefore$ edges = $10 + k - 1$

edges = $9 + k \rightarrow ②$

$\therefore$ subsituting values of ② in ①,

$6 + 12 + 12 + k = 2(9 + k)$

$\cancel{30 = 2(9+k)}$ (15) $\qquad 30 + k = 2(9 + k)$

$\cancel{15 - 9 = k = 6}$ $\qquad 15 + \frac{k}{2} = 9 + k \Rightarrow 15 - 9 = k - \frac{k}{2} = 6 = \frac{k}{2} = 12$

$\therefore$ Total number of pendant vertices are = ~~6~~ 12

4. $f(x, y) = xy' + x'y + x'y' = \text{DNF}$.

to get CNF,

Here, $f' = xy$ as in the total function $f(x,y)$ has 4 terms.

Now, $f''(x,y) = (xy)'$

By demorgans law $\Rightarrow$ $(x.y)' = x' + y'$

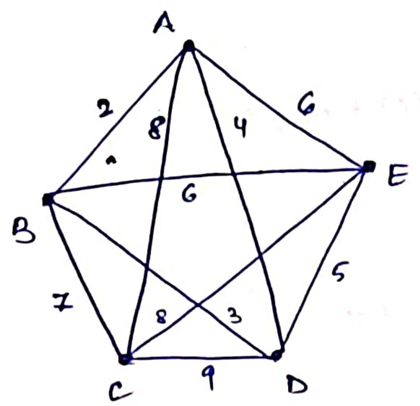
$(x+y)' = x'y'$

$\therefore f''(x,y) = (x' + y')$

$\therefore$ CNF $= (x' + y')$

PTO

5.



By Kruskal algorithm,

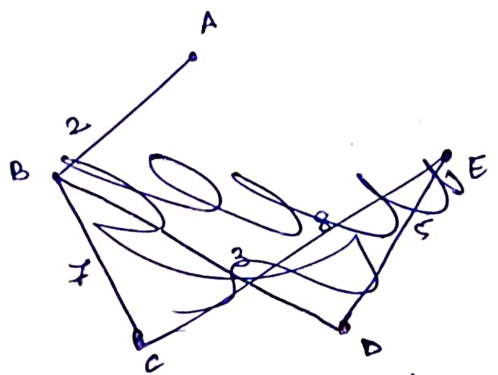
step 1

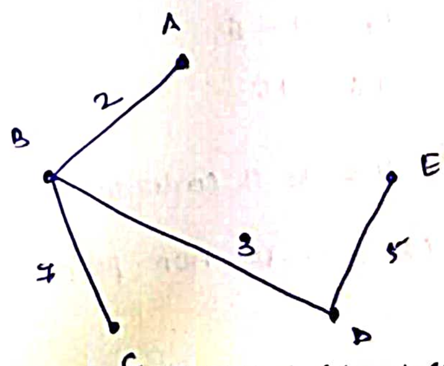
Edge	weight	
A B	~~2~~	→ 1 ✓
B C	7	→ 6 ✓
C D	9	→ 8 × forms a loop
D E	~~5~~	→ 4 ✓
E A	6×	→ 5 × forms a loop
A C	8	→ 7 × forms a loop
A D	4×	→ 3 × cant take this as it forms a loop.
B E	6×	→ 5 × forms a loop
B D	~~3~~	→ 2 ✓
C E	8	→ 7 × forms a cycle

step 2 Select the edge with minimum weight :

A B = First branch

step 3 Select the next edge having minimal weight and add that edge.





MINIMAL SPANNING TREE //

∴ Total weight = 7 + 2 + 3 + 5 = 17

——— X ——— X ——— X ———