Maths Assignment

1 Q1 : 5W + 5B

Total Probability : $P(W) = P(W/A)P(A) + P(W/B)P(B) + P(W/C)P(C) + P(W/D)P(D) + P(W/E)P(E) + P(W/F)P(F)$

$$= 1 \times \frac{1}{^{10}C_5} + \frac{4}{5} \times \frac{^5C_4 \times {^5C_1}}{^{10}C_5} + \frac{3}{5} \times \frac{^5C_3 \times {^5C_2}}{^{10}C_5}$$

$$+ \frac{2}{5} \times \frac{^5C_2 \times {^5C_3}}{^{10}C_5} + \frac{1}{5} \times \frac{^5C_1 \times {^5C_4}}{^{10}C_5} + 0 \times \frac{1}{^{10}C_5}$$

$$= \frac{1}{252} + \frac{20}{252} + \frac{60}{252} + \frac{40}{252} + \frac{5}{252} = \frac{126}{252} = \frac{1}{2}$$

Now, $P(A/W)P(W) = P(W/A)P(A)$

$$= P(A/W) = \frac{1}{252} \times 1 \times 2 = \frac{1}{126}$$

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(1, 6)

2 $A \rightarrow 2W + 4R + 6B$ 2, 3, 4, 5

$B \rightarrow 3R + 5W$

$P(Red) = P(A) \cdot P(red/A) + P(B) \cdot P(red/B)$

$= \frac{2}{6} \cdot \frac{4C_1}{12C_1} + \frac{4}{6} \cdot \frac{3C_1}{8C_1}$

$= \frac{\cancel{2}}{\cancel{6}_3} \times \frac{\cancel{4}}{\cancel{12}_3} + \frac{\cancel{4}}{\cancel{6}_2} \times \frac{\cancel{3}}{\cancel{8}_2} = \frac{1}{9} \times \frac{1}{4} = \frac{1}{36}$

$\therefore P(six/red) = \frac{P(six \cap red)}{P(red)} = \frac{4/24}{1/36} = \frac{2\cancel{4}}{\cancel{24}_2} \times \cancel{36}^3 = \frac{1}{6}$

3 $P(\text{True coin}) = 3/4$

$P(\text{False coin}) = 1/4$

$P(\text{all heads} / \text{true coin}) = \frac{1}{16}$

$P(\text{all heads} / \text{False coin}) = 1$

$\therefore P(\text{False coin} / \text{all heads}) = \frac{1/4 \times 1}{(1/4 \times 1) + (3/4 \times 1/16)} = \frac{1}{\frac{19}{16}} = \frac{16}{19}$

4 $P(N=2) + P(N=3) + P(N=4) + \cancel{P(N=5)}$

4 $P(1) = P(2) = P(3) = P(4) = P(5) = \frac{1}{6} = P(0)$

$\therefore \left(\frac{1}{6}\right) \Big/ \left[\frac{1}{6}\left({}^2C_2 + {}^3C_2 + {}^4C_2 + {}^5C_2\right)\right]$

$= \frac{1}{6} \Big/ \frac{1}{6} \times \left(1 + 3 + \frac{\cancel{4}^{2} \times 3}{1 \times \cancel{2}} + \frac{5 \times 4 \times 3 \times \cancel{2}}{1 \times \cancel{2}}\right)$

$= \frac{1}{6} \Big/ \frac{1}{6} \times \left(4 + 6 + 60\right)$

$= \frac{1}{20}$

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$f(x) = kx(2-x)$, $0 < x < 2$

$$\int f(x)\,dx = 1$$

$$\therefore \int_0^2 kx(2-x) = 1$$

$$= \int_0^2 2kx - kx^2 = 1 = 2k\left[\frac{x^2}{2}\right]_0^2 - k\left[\frac{x^3}{3}\right]_0^2 = 1$$

$$= k[4-0) - \frac{k}{3}[8-0] = 1$$

$$= 4k - \frac{8k}{3} = 1 \Rightarrow \frac{(12-8)k}{3} = 1$$

$$\Rightarrow 4k = 3 \;; \quad k = \frac{3}{4}$$

$$\text{Mean} = \int_0^2 x f(x)\,dx = \int_0^2 x \cdot kx(2-x) = E(X)$$

$$= \int_0^2 2kx^2 - kx^3 = \frac{2k}{3}\left[x^3\right]_0^2 - \frac{k}{4}\left[x^4\right]_0^2$$

$$= \frac{2k}{3} \times 8 - \frac{k}{\cancel{4}} \times \cancel{16}^{\,4} = \frac{16k}{3} - 4k = \frac{16k - 12k}{3} = \frac{4k}{3} = \frac{4}{3} \times \frac{3}{4} = 1 \;//$$

$$E(X) = x.\int_0^2 \frac{3}{4} x \, x(2-x)\, dx$$

$$= \frac{3}{4}\int_0^2 2x^2 - x^3\, dx$$

$$= \frac{3}{4}\left(\frac{2}{3}\left[x^3\right]_0^2 - \frac{1}{4}\left[x^4\right]_0^2\right)$$

$$= \frac{3}{4}\left(\frac{2}{3}\times 8 - \frac{1}{\cancel{4}}\times \overset{4}{\cancel{16}}\right)$$

$$= \frac{3}{4}\times\left(\frac{16}{3} - 4\right) = \frac{3}{4}\times\frac{4}{3} = 1$$

$$E(X^2) = \frac{3}{4}\, x^2 \int_0^2 x(2-x)\, dx$$

$$= \frac{3}{4}\left[\cancel{x^2}\int_0^2 2x^3 - x^4\, dx\right]$$

$$= \frac{3}{4}\left[\frac{2}{4}\left[x^4\right]_0^2 - \frac{1}{5}\left[x^5\right]_0^2\right] = \frac{3}{4}\left[\frac{1}{\cancel{2}}\times \overset{8}{\cancel{16}} - \frac{1}{5}\times 32\right]$$

$$= \frac{3}{\cancel{4}}\times\overset{2}{\cancel{8}} - \frac{3}{\cancel{4}}\times\frac{\overset{8}{\cancel{32}}}{5} = 6 - \frac{24}{5} = \frac{30-24}{5} = \frac{6}{5}$$

Variance $=$ ~~E(X^2) - E(X^2) = 1~~

$$E(X^2) - (E(X))^2 = \frac{6}{5} - 1 = \frac{1}{5}$$

S.D $= 1/\sqrt{5}$

6 A = transmit 1

$\bar{A}$ = transmit 0

B = receive 1

$\bar{B}$ = receive 0

(i) $P(A/B) = \dfrac{P(A)\,P(B/A)}{P(B)} = \dfrac{0.6 \times 0.9}{0.56} = \dfrac{27}{28}$

(ii) $P(B) = P(A)\,P(B/A) + P(\bar{A})\,P(B/\bar{A}) = (1-0.4)\,0.9 + 0.4(1-0.95)$

$= 0.6 \times 0.9 + 0.4 \times 0.05$

$= 0.56$

—— x —— x —— x ——