MAT- 3003, Probability, Statistics and Reliability Theory

Assignment: 1 **Date: 22/03/2022**

(Conditional probability, Bayes' Theorem, Random variables)

1) The odds against A speaking the truth are 3 : 2 and the odds against B speaking the truth are 5 : 3. In what percentage of cases are they likely to contradict each other on an identical issue?
2) An urn contains 6 red balls and 3 blue balls. One ball is selected at random and is replaced by a ball of the other color. A second ball is then chosen. What is the conditional probability that the first ball selected is red, given that the second ball was red?
3) A family has five children. Assuming that the probability of a girl on each birth was 0.5 and that the five births were independent, what is the probability the family has at least one girl, given that they have at least one boy?
4) A box contains 2 green and 3 white balls. A ball is selected at random from the box. If the ball is green, a card is drawn from a deck of 52 cards. If the ball is white, a card is drawn from the deck consisting of just the 16 pictures. (a) What is the probability of drawing a king? (b) What is the probability of a white ball was selected given that a king was drawn?
5) Find the moment generating function of two parameter exponential distribution whose density function is given by $f(x) = \lambda e^{-\lambda(x-a)}$, $x \geq a$ hence find its mean and variance.
6) Let X be a random variable with PDF given by

$$f(x) = f(x) = \begin{cases} cx^2, & |x| \leq 1 \\ 0, & otherwise \end{cases}$$

   a) Find the constant c.
   b) Find E(x) and Var(X)
   c) Find $P(X \geq 1/2)$

7. You toss a fair coin three times:
   a) What is the probability of three heads, *HHH*?
   b) What is the probability that you observe exactly one heads?
   c) Given that you have observed at least one heads, what is the probability that you observe at least two heads?
8. For three events A, B, and C, we know that
   a) A and C are independent,
   b) B and C are independent,
   c) A and B are disjoint
   d) $P(A \cup C) = \frac{2}{3}$, $P(B \cup C) = \frac{3}{4}$, $P(A \cup B \cup C) = \frac{11}{12}$

   Find $P(A), P(B)$ & $P(C)$.
9. A box contains three coins: two regular coins and one fake two-headed coin (P(H)= 1),
   a) You pick a coin at random and toss it. What is the probability that it lands heads up?
   b) You pick a coin at random and toss it, and get heads. What is the probability that it is the two-headed coin?
10. In a factory there are 100 units of a certain product, 5 of which are defective. We pick three units from the 100units at random. What is the probability that none of them are defective?
11. I toss a coin twice. Let X be the number of observed heads. Find the CDF of X.
12. I roll a fair die and let X be the resulting number. Find E(X), Var (X), and σ_X.