Pentagon

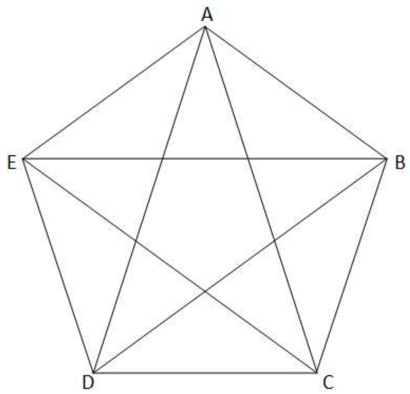
Let $AD \cap CE = M$

Here, we can easily see that $\triangle AED \cong \triangle AMC$

Hence, $AM = MC = a$ and $EM = DM = b - a$

Again, we can check that $\triangle ACM \sim \triangle DEM$

So, $\frac{b}{a} = \frac{a}{b-a} \Rightarrow a^2 = b^2 - ab \Rightarrow a^2 - b^2 + ab = 0$



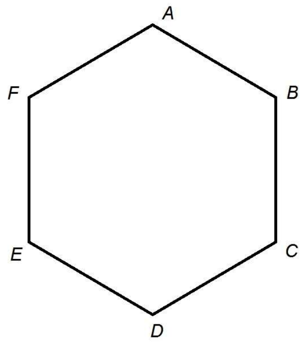
Hexagon

We got that $b^2 = 3a^2$

Now, we see $\triangle AFD$ is right-angled.

So, $c^2 = b^2 + a^2 = 4a^2 \Rightarrow c = 2a$

We can conclude it by considering that a hexagon is the sum of six equilateral triangle whose sidelengths are *a*. So, *c*=2*a* is obvious.



Heptagon

Here $\triangle ABC \sim \triangle AHD$

[because the angles are equal]

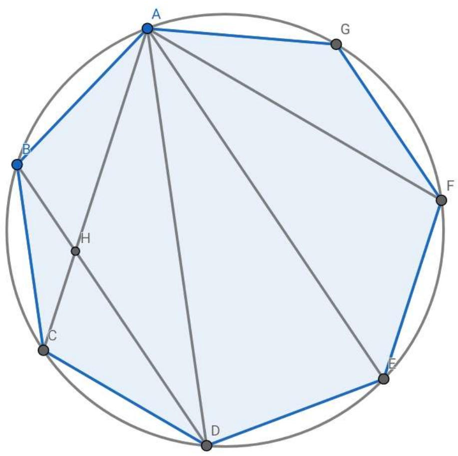
Let $AH = m$

So, $\frac{b}{c} = \frac{a}{m} \Rightarrow m = \frac{ac}{b}$

Here $\triangle AHC \sim \triangle AHD$

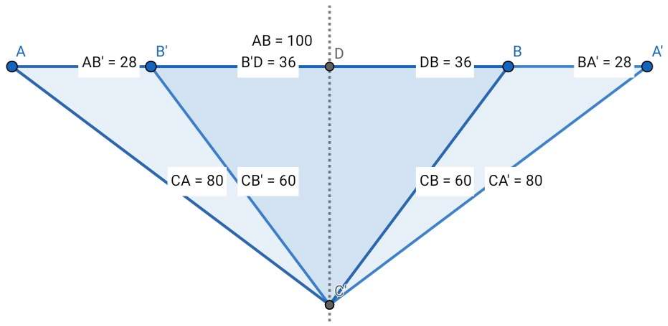
So, $\frac{c}{a} = \frac{m}{b-m} = \frac{\frac{ac}{b}}{b-\frac{ac}{b}} = \frac{ac}{b^2-ac} \Rightarrow cb^2 - ac^2 = a^2c$

So, $b^2 = ac + a^2 = a(a+c)$



Now, the integer length problem.

We get a condition after reflecting a right-angled triangle (with side lengths $25k$, $20k$ and $15k$, where $k \in N$, I took k=4) along the perpendicular line on the hypotenus through the opposite vertex.



Now we can choose any four points and they all have integer distances from each other.

Now another condition.

We flip that triangle along its hypotenus once, and then along the hypotenus' perpendicular bisector. And we get several of our required condition.

(G is the intersection of CF and BD, got hidden somehow)

ABDC, *BCDF*, *CEAD*, *CBAG*, *CAGD*, *BECA*, *BADF*

and the cases I previously showed separately.