CONTENTS

PREFACE

When I hear, I forget
When I see, I remember
When I do, I understand
ANCIENT CHINESE PROVERB

This text is intended for the kinematics and dynamics of machinery topics. The usual prerequisites are first courses in statics, dynamics, and calculus. Usually, the first semester, or portion, is devoted to kinematics, and the second to dynamics of machinery. These courses are ideal vehicles for introducing the mechanical engineering student to the process of design, since mechanisms tend to be intuitive for the typical mechanical engineering student to visualize and create.

While this text attempts to be thorough and complete on the topics of analysis, it also emphasizes the synthesis and design aspects of the subject to a greater degree than most texts in print on these subjects. While the mathematical level of this text is aimed at second- or third-year university students, it is presented *de novo* and should be understandable to the technical school student as well.

Part I of this text is suitable for a one-semester or one-term course in kinematics. Part II is suitable for a one-semester or one-term course in dynamics of machinery. Alternatively, both topic areas can be covered in one semester with less emphasis on some of the topics covered in the text.

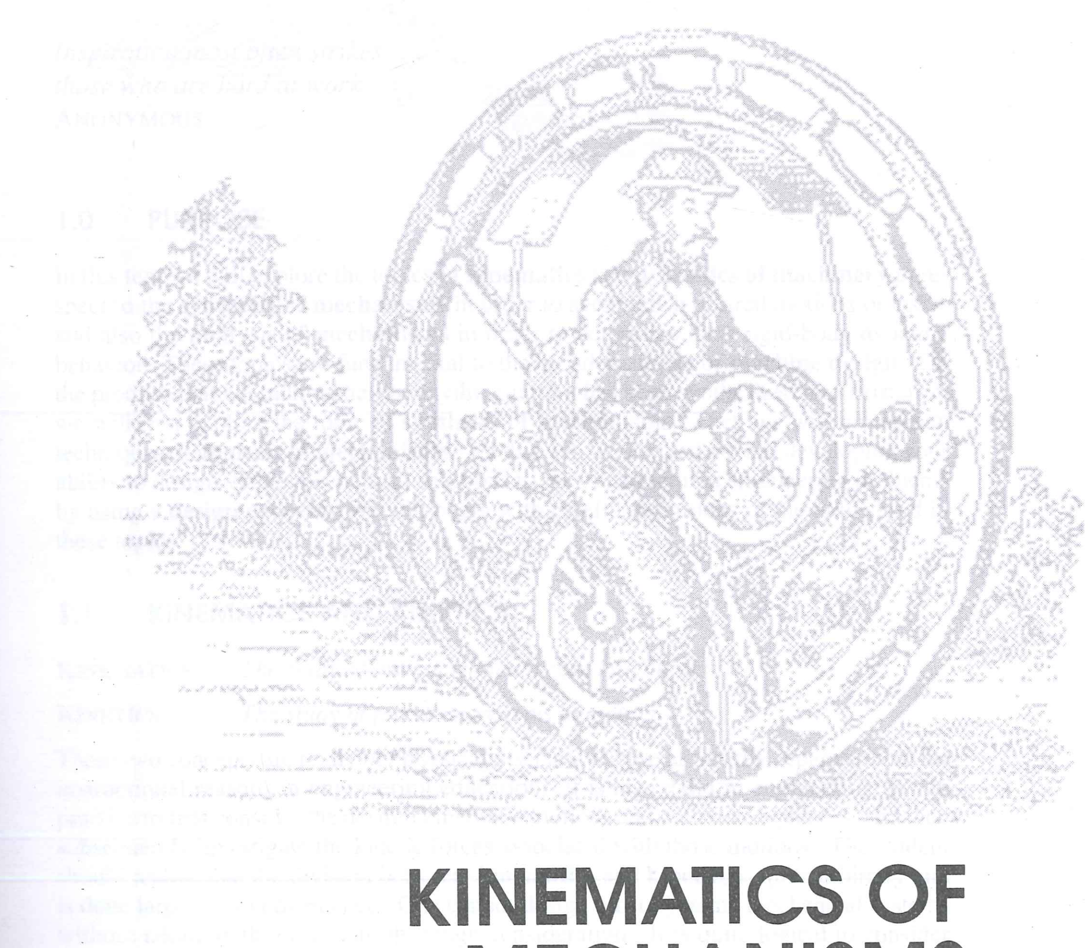
The writing and style of presentation in the text is designed to be clear, informal, and easy to read. Many example problems and solution techniques are presented and spelled out in detail, both verbally and graphically. Many suggested readings are provided in the bibliography. Short problems, and where appropriate, many longer, unstructured design project assignments are provided at the ends of chapters. These projects provide an opportunity for the students *to do and understand.*

The author's approach to these courses and this text is based on over 45 years' experience in mechanical engineering design, both in industry and as a consultant. He has taught these subjects since 1967, both in evening school to practicing engineers and in day school to younger students. His approach to the course has evolved a great deal in that time, from a traditional approach, emphasizing graphical analysis of

Take to Kinematics. It will repay you. It is more fecund than geometry; it adds a fourth dimension to space.

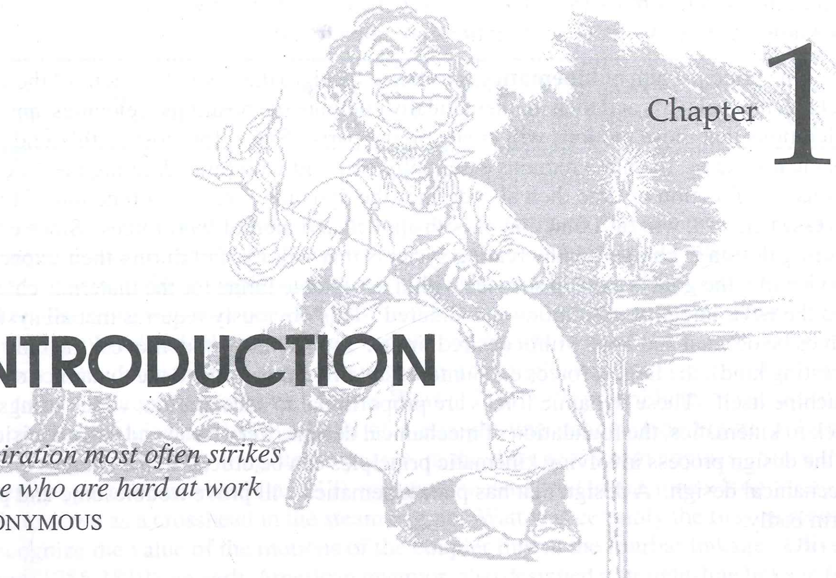
CHEBYSCHEV TO SYLVESTER, 1873

PART I



KINEMATICS OF MECHANISMS

Chapter 1



INTRODUCTION

Inspiration most often strikes
those who are hard at work
ANONYMOUS

1.0 PURPOSE

In this text we will explore the topics of **kinematics** and **dynamics of machinery** in respect to the **synthesis of mechanisms** in order to accomplish desired motions or tasks, and also the **analysis of mechanisms** in order to determine their rigid-body dynamic behavior. These topics are fundamental to the broader subject of **machine design**. On the premise that we cannot analyze anything until it has been synthesized into existence, we will first explore the topic of **synthesis of mechanisms**. Then we will investigate techniques of **analysis of mechanisms**. All this will be directed toward developing your ability to design viable mechanism solutions to real, unstructured engineering problems by using a **design process**. We will begin with careful definitions of the terms used in these topics.

1.1 KINEMATICS AND KINETICS

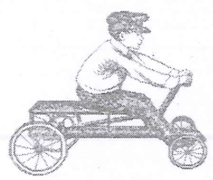
KINEMATICS *The study of motion without regard to forces.*

KINETICS *The study of forces on systems in motion.*

These two concepts are really *not* physically separable. We arbitrarily separate them for instructional reasons in engineering education. It is also valid in engineering design practice to first consider the desired kinematic motions and their consequences, and then subsequently investigate the kinetic forces associated with those motions. The student should realize that the division between **kinematics** and **kinetics** is quite arbitrary and is done largely for convenience. One cannot design most dynamic mechanical systems without taking both topics into thorough consideration. It is quite logical to consider them in the order listed since, from Newton's second law, $\mathbf{F} = m\mathbf{a}$, one typically needs to know the **accelerations** (**a**) in order to compute the dynamic **forces** (**F**) due to the motion of the system's **mass** (m). There are also many situations in which the applied forces are known and the resultant accelerations are to be found.

One principal aim of **kinematics** is to create (design) the desired motions of the subject mechanical parts and then mathematically compute the positions, velocities, and accelerations that those motions will create on the parts. Since, for most earthbound mechanical systems, the mass remains essentially constant with time, defining the accelerations as a function of time then also defines the dynamic forces as a function of time. **Stresses**, in turn, will be a function of both applied and inertial (m**a**) forces. Since engineering design is charged with creating systems that will not fail during their expected service life, the goal is to keep stresses within acceptable limits for the materials chosen and the environmental conditions encountered. This obviously requires that all system forces be defined and kept within desired limits. In machinery that moves (the only interesting kind), the largest forces encountered are often those due to the dynamics of the machine itself. These dynamic forces are proportional to acceleration, which brings us back to kinematics, the foundation of mechanical design. Very basic and early decisions in the design process involving kinematic principles can be crucial to the success of any mechanical design. A design that has poor kinematics will prove troublesome and perform badly.

1.2 MECHANISMS AND MACHINES

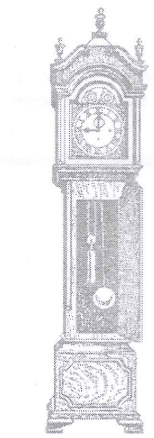

A mechanism


A machine

A **mechanism** is a device that transforms motion to some desirable pattern and typically develops very low forces and transmits little power. Hunt[13] defines a mechanism as "a means of *transmitting, controlling, or constraining relative movement*." A **machine** typically contains mechanisms that are designed to provide significant forces and transmit significant power.[1] Some examples of common mechanisms are a pencil sharpener, a camera shutter, an analog clock, a folding chair, an adjustable desk lamp, and an umbrella. Some examples of machines that possess motions similar to the mechanisms listed above are a food blender, a bank vault door, an automobile transmission, a bulldozer, a robot, and an amusement park ride. There is no clear-cut dividing line between mechanisms and machines. They differ in degree rather than in kind. If the forces or energy levels within the device are significant, it is considered a machine; if not, it is considered a mechanism. A useful working **definition of a mechanism** is *A system of elements arranged to transmit* **motion** *in a predetermined fashion*. This can be converted to a definition of a **machine** by adding the words **and energy** after **motion**.

Mechanisms, if lightly loaded and run at slow speeds, can sometimes be treated strictly as kinematic devices; that is, they can be analyzed kinematically without regard to forces. Machines (and mechanisms running at higher speeds), on the other hand, must first be treated as mechanisms, a kinematic analysis of their velocities and accelerations must be done, and then they must be subsequently analyzed as dynamic systems in which their static and dynamic forces due to those accelerations are analyzed using the principles of kinetics. **Part I** of this text deals with **Kinematics of Mechanisms**, and **Part II** with **Dynamics of Machinery**. The techniques of mechanism synthesis presented in Part I are applicable to the design of both mechanisms and machines, since in each case some collection of movable members must be created to provide and control the desired motions and geometry.

1.3 A BRIEF HISTORY OF KINEMATICS



Machines and mechanisms have been devised by people since the dawn of history. The ancient Egyptians devised primitive machines to accomplish the building of the pyramids and other monuments. Though the wheel and pulley (on an axle) were not known to the Old Kingdom Egyptians, they made use of the lever, the inclined plane (or wedge), and probably the log roller. The origin of the wheel and axle is not definitively known. Its first appearance seems to have been in Mesopotamia about 3000 to 4000 B.C.

A great deal of design effort was spent from early times on the problem of timekeeping as more sophisticated clockworks were devised. Much early machine design was directed toward military applications (catapults, wall scaling apparatus, etc.). The term **civil engineering** was later coined to differentiate civilian from military applications of technology. **Mechanical engineering** had its beginnings in machine design as the inventions of the industrial revolution required more complicated and sophisticated solutions to motion control problems. **James Watt** (1736-1819) probably deserves the title of first kinematician for his synthesis of a straight-line linkage (see Figure 3-29 on p. 137) to guide the very long stroke pistons in the then new steam engines. Since the planer was yet to be invented (in 1817), no means then existed to machine a long, straight guide to serve as a crosshead in the steam engine. Watt was certainly the first on record to recognize the value of the motions of the coupler link in the fourbar linkage. **Oliver Evans** (1755-1819), an early American inventor, also designed a straight-line linkage for a steam engine. **Euler** (1707-1783) was a contemporary of Watt, though they apparently never met. Euler presented an analytical treatment of mechanisms in his *Mechanica Sive Motus Scienta Analytice Exposita* (1736-1742), which included the concept that planar motion is composed of two independent components, namely, translation of a point and rotation of the body about that point. Euler also suggested the separation of the problem of dynamic analysis into the "geometrical" and the "mechanical" in order to simplify the determination of the system's dynamics. Two of his contemporaries, **d'Alembert** and **Kant**, also proposed similar ideas. This is the origin of our division of the topic into kinematics and kinetics as described on p. 3.

In the early 1800s, L'Ecole Polytechnic in Paris, France, was the repository of engineering expertise. **Lagrange** and **Fourier** were among its faculty. One of its founders was **Gaspard Monge** (1746-1818), inventor of descriptive geometry (which incidentally was kept as a military secret by the French government for 30 years because of its value in planning fortifications). Monge created a course in elements of machines and set about the task of classifying all mechanisms and machines known to mankind! His colleague, **Hachette**, completed the work in 1806 and published it as what was probably the first mechanism text in 1811. **Andre Marie Ampere** (1775-1836), also a professor at L'Ecole Polytechnic, set about the formidable task of classifying "all human knowledge." In his *Essai sur la Philosophie des Sciences*, he was the first to use the term **cinematique**, from the Greek word for motion,* to describe the study of motion without regard to forces, and suggested that "this science ought to include all that can be said with respect to motion in its different kinds, independently of the forces by which it is produced." His term was later anglicized to *kinematics* and germanized to *kinematik*.

Robert Willis (1800-1875) wrote the text *Principles of Mechanism* in 1841 while a professor of natural philosophy at the University of Cambridge, England. He attempted to systematize the task of mechanism synthesis. He counted five ways of obtaining

* Ampere is quoted as writing "(The science of mechanisms) must therefore not define a machine, as has usually been done, as an instrument by the help of which the direction and intensity of a given *force* can be altered, but as an instrument by the help of which the direction and *velocity* of a given motion can be altered. To this science . . . I have given the name Kinematics from Κινμα—motion." in Maunder, L. (1979). "Theory and Practice." *Proc. 5th World Cong. on Theory of Mechanisms and Machines*, Montreal, p. 1.

relative motion between input and output links: rolling contact, sliding contact, linkages, wrapping connectors (belts, chains), and tackle (rope or chain hoists). **Franz Reuleaux** (1829-1905), published *Theoretische Kinematik* in 1875. Many of his ideas are still current and useful. **Alexander Kennedy** (1847-1928) translated Reuleaux into English in 1876. This text became the foundation of modern kinematics and is still in print! (See bibliography at end of chapter.) He provided us with the concept of a kinematic pair (joint), whose shape and interaction define the type of motion transmitted between elements in the mechanism. Reuleaux defined six basic mechanical components: the link, the wheel, the cam, the screw, the ratchet, and the belt. He also defined "higher" and "lower" pairs, higher having line or point contact (as in a roller or ball bearing) and lower having surface contact (as in pin joints). Reuleaux is generally considered the father of modern kinematics and is responsible for the symbolic notation of skeletal, generic linkages used in all modern kinematics texts.

In the 20th century, prior to World War II, most theoretical work in kinematics was done in Europe, especially in Germany. Few research results were available in English. In the United States, kinematics was largely ignored until the 1940s when **A. E. R. deJonge** wrote "What Is Wrong with 'Kinematics' and 'Mechanisms'?,"[2] which called upon the U.S. mechanical engineering education establishment to pay attention to the European accomplishments in this field. Since then, much new work has been done, especially in kinematic synthesis, by American and European engineers and researchers such as **J. Denavit, A. Erdman, F. Freudenstein, A. S. Hall, R. Hartenberg, R. Kaufman, B. Roth, G. Sandor**, and **A. Soni** (all of the U.S.) and **K. Hain** (of Germany). Since the fall of the "iron curtain" much original work done by Soviet Russian kinematicians has become available in the United States, such as that by **Artobolevsky**.[3] Many U.S. researchers have applied the computer to solve previously intractable problems, both of analysis and synthesis, making practical use of many of the theories of their predecessors.[4] This text will make much use of the availability of computers to allow more efficient analysis and synthesis of solutions to machine design problems.

1.4 APPLICATIONS OF KINEMATICS

One of the first tasks in solving any machine design problem is to determine the kinematic configuration(s) needed to provide the desired motions. Force and stress analyses typically cannot be done until the kinematic issues have been resolved. This text addresses the design of kinematic devices such as linkages, cams, and gears. Each of these terms will be fully defined in succeeding chapters, but it may be useful to show some examples of kinematic applications in this introductory chapter. You probably have used many of these systems without giving any thought to their kinematics.

Virtually any machine or device that moves contains one or more kinematic elements such as links, cams, gears, belts, chains. Your bicycle is a simple example of a kinematic system that contains a chain drive to provide torque multiplication and simple cable-operated linkages for braking. An automobile contains many more examples of kinematic devices. Its steering system, wheel suspensions, and piston-engine all contain linkages; the engine's valves are opened by cams; and the transmission is full of gears. Even the windshield wipers are linkage-driven. Figure 1-1a shows a spatial linkage used to control the rear wheel movement over bumps of a modern automobile.

(*a*) Spatial linkage rear suspension *Courtesy of Daimler Benz Co.* (*b*) Utility tractor with backhoe *Courtesy of John Deere Co.* (*c*) Linkage-driven exercise mechanism *Courtesy of ICON Health & Fitness, Inc.*

FIGURE 1-1
Examples of kinematic devices in general use

Construction equipment such as tractors, cranes, and backhoes all use linkages extensively in their design. Figure 1-1b shows a small backhoe that is a linkage driven by hydraulic cylinders. Another application using linkages is that of exercise equipment as shown in Figure 1-1c. The examples in Figure 1-1 are all of consumer goods that you may encounter in your daily travels. Many other kinematic examples occur in the realm of producer goods—machines used to make the many consumer products that we use. You are less likely to encounter these outside of a factory environment. Once you become familiar with the terms and principles of kinematics, you will no longer be able to look at any machine or product without seeing its kinematic aspects.

1.5 THE DESIGN PROCESS

Design, Invention, Creativity

These are all familiar terms but may mean different things to different people. These terms can encompass a wide range of activities from styling the newest look in clothing, to creating impressive architecture, to engineering a machine for the manufacture of facial tissues. **Engineering design**, which we are concerned with here, embodies all three of these activities as well as many others. The word **design** is derived from the Latin **designare**, which means *"to designate, or mark out."* Webster's gives several definitions, the most applicable being *"to outline, plot, or plan, as action or work. . . . to conceive, invent – contrive."* **Engineering design** has been defined as "*. . . the process of applying the various techniques and scientific principles for the purpose of defining a device, a process or a system in sufficient detail to permit its realization . . . Design may be simple or enormously complex, easy or difficult, mathematical or nonmathematical; it may involve a trivial problem or one of great importance.*" **Design** is a universal constituent of engineering practice. But the complexity of engineering subjects usually

TABLE 1-1
A Design Process

1. Identification of Need
2. Background Research
3. Goal Statement
4. Performance Specifications
5. Ideation and Invention
6. Analysis
7. Selection
8. Detailed Design
9. Prototyping and Testing
10. Production

requires that the student be served with a collection of **structured**, **set-piece problems** designed to elucidate a particular concept or concepts related to the particular topic. These textbook problems typically take the form of "*given A, B, C, and D, find E.*" Unfortunately, real-life engineering problems are almost never so structured. Real design problems more often take the form of "*What we need is a framus to stuff this widget into that hole within the time allocated to the transfer of this other gizmo.*" The new engineering graduate will search in vain among his or her textbooks for much guidance to solve such a problem. This **unstructured problem** statement usually leads to what is commonly called "**blank paper syndrome**." Engineers often find themselves staring at a blank sheet of paper pondering how to begin solving such an ill-defined problem.

Much of engineering education deals with topics of **analysis**, which means *to decompose, to take apart, to resolve into its constituent parts.* This is quite necessary. The engineer must know how to analyze systems of various types, mechanical, electrical, thermal, or fluid. Analysis requires a thorough understanding of both the appropriate mathematical techniques and the fundamental physics of the system's function. But, before any system can be analyzed, it must exist, and a blank sheet of paper provides little substance for analysis. Thus the first step in any engineering design exercise is that of **synthesis**, which means *putting together.*

The design engineer, in practice, regardless of discipline, continuously faces the challenge of *structuring the unstructured problem.* Inevitably, the problem as posed to the engineer is ill-defined and incomplete. Before any attempt can be made to *analyze the situation* he or she must first carefully define the problem, using an engineering approach, to ensure that any proposed solution will solve the right problem. Many examples exist of excellent engineering solutions that were ultimately rejected because they solved the wrong problem, i.e., a different one than the client really had.

Much research has been devoted to the definition of various "design processes" intended to provide means to structure the unstructured problem and lead to a viable solution. Some of these processes present dozens of steps, others only a few. The one presented in Table 1-1 contains 10 steps and has, in the author's experience, proven successful in over 40 years of practice in engineering design.

Blank paper syndrome

ITERATION Before we discuss each of these steps in detail, it is necessary to point out that this is not a process in which one proceeds from step one through ten in a linear fashion. Rather it is, by its nature, an iterative process in which progress is made haltingly, two steps forward and one step back. It is inherently *circular*. To **iterate** means *to repeat, to return to a previous state.* If, for example, your apparently great idea, upon analysis, turns out to violate the second law of thermodynamics, you can return to the ideation step and get a better idea! Or, if necessary, you can return to an earlier step in the process, perhaps the background research, and learn more about the problem. With the understanding that the actual execution of the process involves iteration, for simplicity, we will now discuss each step in the order listed in Table 1-1.

Identification of Need

This first step is often done for you by someone, boss or client, saying, "What we need is . . .." Typically this statement will be brief and lacking in detail. It will fall far short of providing you with a structured problem statement. For example, the problem statement might be "We need a better lawn mower."

Background Research

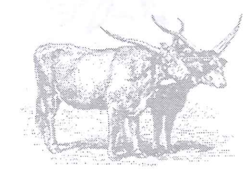
This is the most important phase in the process, and is unfortunately often the most neglected. The term **research**, used in this context, should *not* conjure up visions of white-coated scientists mixing concoctions in test tubes. Rather this is research of a more mundane sort, gathering background information on the relevant physics, chemistry, or other aspects of the problem. Also it is desirable to find out if this, or a similar problem, has been solved before. There is no point in reinventing the wheel. If you are lucky enough to find a ready-made solution on the market, it will no doubt be more economical to purchase it than to build your own. Most likely this will not be the case, but you may learn a great deal about the problem to be solved by investigating the existing "art" associated with similar technologies and products. Many companies purchase, disassemble, and analyze their competitors' products, a process sometimes referred to as "**benchmarking**."

Identifying the need

The **patent** literature and technical publications in the subject area are obvious sources of information and are accessible via the World Wide Web. The U.S. Patent and Trademark Office operates a web site at www.uspto.gov where you can search patents by keyword, inventor, title, patent number, or other data. You can print a copy of the patent from the site. A commercial site at www.delphion.com also provides copies of extant patents including those issued in European countries. The "disclosure" or "specification" section of a patent is required to describe the invention in such detail that anyone "skilled in the art" could make the invention. In return for this full disclosure the government grants the inventor a 20-year monopoly on the claimed invention. After that term expires, anyone can use it. Clearly, if you find that the solution exists and is covered by a patent still in force, you have only a few ethical choices: buy the patentee's existing solution, design something that does not conflict with the patent, or drop the project.

Reinventing the wheel

Technical publications in engineering are numerous and varied and are provided by a large number of professional organizations. For the subject matter of this text, the *American Society of Mechanical Engineers* (ASME), which offers inexpensive student memberships, and the *International Federation for the Theory of Machines and Mechanisms* (IFToMM) both publish relevant journals, the ASME *Journal of Mechanical Design* and *Mechanism and Machine Theory*, respectively. Your school library may subscribe to these, and you can purchase copies of articles from their web sites at www.asme.org/pubs/journals/ and www.elsevier.com/inca/publications, respectively.



Grass shorteners

The World Wide Web provides an incredibly useful resource for the engineer or student looking for information on any subject. The many search engines available will deliver a wealth of information in response to selected keywords. The web makes it easy to find sources for purchased hardware such as gears, bearings, and motors, for your machine designs. In addition, much machine design information is available from the web. A number of useful web sites are catalogued in the bibliography of this chapter.

It is very important that sufficient energy and time be expended on this research and preparation phase of the process in order to avoid the embarrassment of concocting a great solution to the wrong problem. Most inexperienced (and some experienced) engineers give too little attention to this phase and jump too quickly into the ideation and invention stage of the process. *This must be avoided!* You must discipline yourself to *not* try to solve the problem before thoroughly preparing yourself to do so.

TABLE 1-2
Performance Specifications

1 Device to have self-contained power supply.
2 Device to be corrosion resistant.
3 Device to cost less than $100.00.
4 Device to emit < 80 dB sound intensity at 10 m.
5 Device to shorten 1/4 acre of grass per hour.
6 etc. . . . etc.

Goal Statement

Once the background of the problem area as originally stated is fully understood, you will be ready to recast that problem into a more coherent goal statement. This new problem statement should have three characteristics. It should be concise, be general, and be uncolored by any terms that predict a solution. It should be couched in terms of **functional visualization,** *meaning to visualize its function,* rather than any particular embodiment. For example, if the original statement of need was *"Design a Better Lawn Mower,"* after research into the myriad of ways to cut grass that have been devised over the ages, the wise designer might restate the goal as **"Design a Means to Shorten Grass."** The original problem statement has a built-in trap in the form of the *colored* words "lawn mower." For most people, this phrase will conjure up a vision of something with whirring blades and a noisy engine. For the **ideation** phase to be most successful, it is necessary to avoid such images and to state the problem generally, clearly, and concisely. As an exercise, list 10 ways to shorten grass. Most of them would not occur to you had you been asked for 10 better lawn mower designs. You should use **functional visualization** to avoid unnecessarily limiting your creativity!

Performance Specifications *

When the background is understood, and the goal clearly stated, you are ready to formulate a set of *performance specifications* (also called *task specifications*). These should **not** be design specifications. The difference is that **performance specifications** define **what** *the system must do,* while **design specifications** define **how** *it must do it.* At this stage of the design process it is unwise to attempt to specify *how* the goal is to be accomplished. That is left for the **ideation** phase. The purpose of the performance specifications is to carefully define and constrain the problem so that it both *can be solved* and *can be shown to have been solved* after the fact. A sample set of performance specifications for our "grass shortener" is shown in Table 1-2.

Note that these specifications constrain the design without overly restricting the engineer's design freedom. It would be inappropriate to require a gasoline engine for specification 1, because other possibilities exist that will provide the desired mobility. Likewise, to demand stainless steel for all components in specification 2 would be unwise, since corrosion resistance can be obtained by other, less-expensive means. In short, the performance specifications serve to define the problem in as complete and as general a manner as possible, and they serve as a contractual definition of what is to be accomplished. The finished design can be tested for compliance with the specifications.

Ideation and Invention

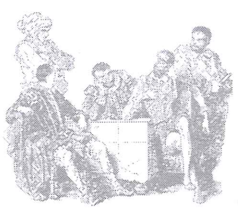
This step is full of both fun and frustration. This phase is potentially the most satisfying to most designers, but it is also the most difficult. A great deal of research has been done to explore the phenomenon of "**creativity**." It is, most agree, a common human trait. It is certainly exhibited to a very high degree by all young children. The rate and degree of development that occurs in the human from birth through the first few years of life certainly requires some innate creativity. Some have claimed that our methods of Western education tend to stifle children's natural creativity by encouraging conformity and restricting individuality. From "coloring within the lines" in kindergarten to imitating the

* Orson Welles, famous author and filmmaker, once said, *The enemy of art is the absence of limitations.* We can paraphrase that as *The enemy of design is the absence of specifications.*

textbook's writing patterns in later grades, individuality is suppressed in favor of a socializing conformity. This is perhaps necessary to avoid anarchy but probably does have the effect of reducing the individual's ability to think creatively. Some claim that creativity can be taught, some that it is only inherited. No hard evidence exists for either theory. It is probably true that one's lost or suppressed creativity can be rekindled. Other studies suggest that most everyone underutilizes his or her potential creative abilities. You can enhance your creativity through various techniques.

TABLE 1-3
The Creative Process

5a	Idea Generation
5b	Frustration
5c	Incubation
5d	Eureka!


Brainstorming


Frustration


Eureka!

CREATIVE PROCESS Many techniques have been developed to enhance or inspire creative problem solving. In fact, just as design processes have been defined, so has the *creative process* shown in Table 1-3. This creative process can be thought of as a subset of the design process and to exist within it. The ideation and invention step can thus be broken down into these four substeps.

IDEA GENERATION is the most difficult of these steps. Even very creative people have difficulty inventing "on demand." Many techniques have been suggested to improve the yield of ideas. The most important technique is that of *deferred judgment,* which means that your criticality should be temporarily suspended. Do not try to judge the quality of your ideas at this stage. That will be taken care of later, in the **analysis** phase. The goal here is to obtain as large a *quantity* of potential designs as possible. Even superficially ridiculous suggestions should be welcomed, as they may trigger new insights and suggest other more realistic and practical solutions.

BRAINSTORMING is a technique for which some claim great success in generating creative solutions. This technique requires a group, preferably 6 to 15 people, and attempts to circumvent the largest barrier to creativity, which is *fear of ridicule.* Most people, when in a group, will not suggest their real thoughts on a subject, for fear of being laughed at. Brainstorming's rules require that no one be allowed to make fun of or criticize anyone's suggestions, no matter how ridiculous. One participant acts as "scribe" and is duty bound to record all suggestions, no matter how apparently silly. When done properly, this technique can be fun and can sometimes result in a "feeding frenzy" of ideas that build upon each other. Large quantities of ideas can be generated in a short time. Judgment on their quality is deferred to a later time.

When you are working alone, other techniques are necessary. **Analogies** and **inversion** are often useful. Attempt to draw analogies between the problem at hand and other physical contexts. If it is a mechanical problem, convert it by analogy to a fluid or electrical one. Inversion turns the problem inside out. For example, consider what you want moved to be stationary and vice versa. Insights often follow. Another useful aid to creativity is the use of **synonyms.** Define the action verb in the problem statement, and then list as many synonyms for that verb as possible. For example:

Problem statement: Move this object from point A to point B.

The action verb is "move." Some synonyms are push, pull, slip, slide, shove, throw, eject, jump, spill.

By whatever means, the aim in this **ideation** step is to generate a large number of ideas without particular regard to quality. But, at some point, your "mental well" will go dry. You will have then reached the step in the creative process called **frustration.** It is time to leave the problem and do something else for a time. While your conscious mind is occupied with other concerns, your subconscious mind will still be hard at work on the problem. This is the step called **incubation.** Suddenly, at a quite unexpected time and place, an idea will pop into your consciousness, and it will seem to be the obvious and "right" solution to the problem . . . **Eureka!** Most likely, later analysis will discover some flaw in this solution. If so, back up and **iterate!** More ideation, perhaps more research, and possibly even a redefinition of the problem may be necessary.

In "Unlocking Human Creativity"[5] Wallen describes three requirements for creative insight:

- *Fascination with a problem.*
- *Saturation with the facts, technical ideas, data, and the background of the problem.*
- *A period of reorganization.*

The first of these provides the motivation to solve the problem. The second is the background research step described on p. 9. The period of reorganization refers to the frustration phase when your subconscious works on the problem. Wallen[5] reports that testimony from creative people tells us that in this period of reorganization they have no conscious concern with the particular problem and that the moment of insight frequently appears in the midst of relaxation or sleep. So to enhance your creativity, saturate yourself in the problem and related background material. Then relax and let your subconscious do the hard work!

Analysis

Once you are at this stage, you have structured the problem, at least temporarily, and can now apply more sophisticated analysis techniques to examine the performance of the design in the **analysis phase** of the design process. (Some of these analysis methods will be discussed in detail in the following chapters.) Further iteration will be required as problems are discovered from the analysis. Repetition of as many earlier steps in the design process as necessary must be done to ensure the success of the design.

	Cost	*Safety*	*Performance*	*Reliability*	RANK
Weighting Factor	.35	.30	.15	.20	1.0
Design 1	3 / 1.05	6 / 1.80	4 / .60	9 / 1.80	5.3
Design 2	4 / 1.40	2 / .60	7 / 1.05	2 / .40	3.5
Design 3	1 / .35	9 / 2.70	4 / .60	5 / 1.00	4.7
Design 4	9 / 3.15	1 / .30	6 / .90	7 / 1.40	5.8
Design 5	7 / 2.45	4 / 1.20	2 / .30	6 / 1.20	5.2

FIGURE 1-2
A decision matrix

Selection

When the technical analysis indicates that you have some potentially viable designs, the best one available must be **selected** for **detailed design, prototyping,** and **testing.** The selection process usually involves a comparative analysis of the available design solutions. A **decision matrix** sometimes helps to identify the best solution by forcing you to consider a variety of factors in a systematic way. A decision matrix for our better grass shortener is shown in Figure 1-2. Each design occupies a row in the matrix. The columns are assigned categories in which the designs are to be judged, such as cost, ease of use, efficiency, performance, reliability, and any others you deem appropriate to the particular problem. Each category is then assigned a **weighting factor**, which measures its relative importance. For example, reliability may be a more important criterion to the user than cost, or vice versa. You as the design engineer have to exercise your judgment as to the selection and weighting of these categories. The body of the matrix is then filled with numbers that rank each design on a convenient scale, such as 1 to 10, in each of the categories. Note that this is ultimately a *subjective ranking* on your part. You must examine the designs and decide on a score for each. The scores are then multiplied by the weighting factors (which are usually chosen so as to sum to a convenient number such as 1) and the products summed for each design. The weighted scores then give a ranking of designs. Be cautious in applying these results. Remember the source and subjectivity of your scores and the weighting factors! There is a temptation to put more faith in these results than is justified. After all, they look impressive! They can even be taken out to several decimal places! (But they shouldn't be.) The real value of a decision matrix is that it breaks the problem into more tractable pieces and forces you to think about the relative value of each design in many categories. You can then make a more informed decision as to the "best" design.

Detailed Design

This step usually includes the creation of a complete set of assembly and detail drawings or **computer-aided design** (CAD) part files, for *each and every part* used in the design. Each detail drawing must specify all the dimensions and the material specifications necessary to make that part. From these drawings (or CAD files) a prototype test model (or models) must be constructed for physical testing. Most likely the tests will discover more flaws, requiring further **iteration.**

Prototyping and Testing

MODELS Ultimately, one cannot be sure of the correctness or viability of any design until it is built and tested. This usually involves the construction of a prototype physical model. A mathematical model, while very useful, can never be as complete and accurate a representation of the actual physical system as a physical model, due to the need to make simplifying assumptions. Prototypes are often very expensive but may be the most economical way to prove a design, short of building the actual, full-scale device. Prototypes can take many forms, from working scale models to full-size, but simplified, representations of the concept. Scale models introduce their own complications in regard to proper scaling of the physical parameters. For example, volume of material varies as the cube of linear dimensions, but surface area varies as the square. Heat transfer to the environment may be proportional to surface area, while heat generation may be proportional to volume. So linear scaling of a system, either up or down, may lead to behavior different from that of the full-scale system. One must exercise caution in scaling physical models. You will find as you begin to design linkage mechanisms that a **simple cardboard model** of your chosen link lengths, coupled together with thumbtacks for pivots, will tell you a great deal about the quality and character of the mechanism's motions. You should get into the habit of making such simple articulated models for all your linkage designs.

TESTING of the model or prototype may range from simply actuating it and observing its function to attaching extensive instrumentation to accurately measure displacements, velocities, accelerations, forces, temperatures, and other parameters. Tests may need to be done under controlled environmental conditions such as high or low temperature or humidity. The microcomputer has made it possible to measure many phenomena more accurately and inexpensively than could be done before.

Production

Finally, with enough time, money, and perseverance, the design will be ready for production. This might consist of the manufacture of a single final version of the design, but more likely will mean making thousands or even millions of your widget. The danger, expense, and embarrassment of finding flaws in your design after making large quantities of defective devices should inspire you to use the greatest care in the earlier steps of the design process to ensure that it is properly engineered.

The **design process** is widely used in engineering. Engineering is usually defined in terms of what an engineer does, but engineering can also be defined in terms of *how* the engineer does what he or she does. **Engineering** is *as much a method, an approach, a process, a state of mind for problem solving, as it is an activity*. The engineering approach is that of thoroughness, attention to detail, and consideration of all the possibilities. While it may seem a contradiction in terms to emphasize "attention to detail" while extolling the virtues of open-minded, freewheeling, creative thinking, it is not. The two activities are not only compatible, they are symbiotic. It ultimately does no good to have creative, original ideas if you do not, or cannot, carry out the execution of those ideas and "reduce them to practice." To do this you must discipline yourself to suffer the nitty-gritty, nettlesome, tiresome details that are so necessary to the completion of any one phase of the creative design process. For example, to do a creditable job in the design of anything, you must *completely* define the problem. If you leave out some detail of the problem definition, you will end up solving the wrong problem. Likewise, you must *thoroughly* research the background information relevant to the problem. You must *exhaustively* pursue conceptual potential solutions to your problem. You must then *extensively* analyze these concepts for validity. And, finally, you must *detail* your chosen design down to the last nut and bolt to be confident it will work. If you wish to be a good designer and engineer, you must discipline yourself to do things thoroughly and in a logical, orderly manner, even while thinking great creative thoughts and iterating to a solution. Both attributes, creativity and attention to detail, are necessary for success in engineering design.

1.6 OTHER APPROACHES TO DESIGN

In recent years, an increased effort has been directed toward a better understanding of design methodology and the design process. Design methodology is the study of the process of designing. One goal of this research is to define the design process in sufficient detail to allow it to be encoded in a form amenable to execution in a computer, using "artificial intelligence" (AI).

Dixon[6] defines a design as a *state of information* which may be in any of several forms:

> . . . words, graphics, electronic data, and/or others. It may be partial or complete. It ranges from a small amount of highly abstract information early in the design process to a very large amount of detailed information later in the process sufficient to perform manufacturing. It may include, but is not limited to, information about size and shape, function, materials, marketing, simulated performance, manufacturing processes, tolerances, and more. Indeed, any and all information relevant to the physical or economic life of a designed object is part of its design.

He goes on to describe several generalized states of information such as the *requirements* state that is analogous to our **performance specifications**. Information about the physical concept is referred to as the *conceptual* state of information and is analogous to our **ideation** phase. His *feature configuration* and *parametric* states of information are similar in concept to our **detailed design** phase. Dixon then defines a design process as:

> The series of activities by which the information about the designed object is changed from one information state to another.

Axiomatic Design

N. P. Suh[7] suggests an *axiomatic approach* to design in which there are four domains: **customer** domain, **functional** domain, **physical** domain, and the **process** domain. These represent a range from "what" to "how," i.e., from a state of defining what the customer wants through determining the functions required and the needed physical embodiment, to how a process will achieve the desired end. He defines two axioms that need to be satisfied to accomplish this:

1 Maintain the independence of the functional requirements.

2 Minimize the information content.

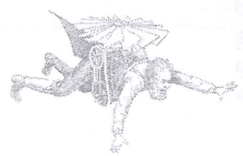
The first of these refers to the need to create a complete and nondependent set of performance specifications. The second indicates that the best design solution will have the lowest information content (i.e., the least complexity). Others have earlier referred to this second idea as *KISS,* which stands, somewhat crudely, for "*keep it simple, stupid.*"

The implementation of both Dixon's and Suh's approaches to the design process is somewhat complicated. The interested reader is referred to the literature cited in the bibliography to this chapter for more complete information.

1.7 MULTIPLE SOLUTIONS

Note that by the nature of the design process, there is **not** any **one** correct answer or solution to any design problem. Unlike the structured "engineering textbook" problems, which most students are used to, there is no right answer "in the back of the book" for any real design problem.* There are as many potential solutions as there are designers willing to attempt them. Some solutions will be better than others, but many will work. Some will not! There is no "one right answer" in design engineering, which is what makes it interesting. The only way to determine the relative merits of various potential design solutions is by thorough analysis, which usually will include physical testing of constructed prototypes. Because this is a very expensive process, it is desirable to do as much analysis on paper, or in the computer, as possible before actually building the device. Where feasible, mathematical models of the design, or parts of the design, should be created. These may take many forms, depending on the type of physical system involved. In the design of mechanisms and machines it is usually possible to write the equations for the rigid-body dynamics of the system, and solve them in "closed form" with (or without) a computer. Accounting for the elastic deformations of the members of the mechanism or machine usually requires more complicated approaches using **finite difference** techniques or the **finite element method** (FEM).

1.8 HUMAN FACTORS ENGINEERING



Make the machine fit the man

With few exceptions, all machines are designed to be used by humans. Even robots must be programmed by a human. **Human factors engineering** is the study of the human-machine interaction and is defined as *an applied science that coordinates the design of devices, systems, and physical working conditions with the capacities and requirements of the worker*. The machine designer must be aware of this subject and design devices to "fit the man" rather than expect the man to adapt to fit the machine. The term **ergonomics** is synonymous with *human factors engineering*. We often see reference to the good or bad ergonomics of an automobile interior or a household appliance. A machine designed with poor ergonomics will be uncomfortable and tiring to use and may even be dangerous. (Have you programmed your VCR lately, or set its clock?)

There is a wealth of human factors data available in the literature. Some references are noted in the bibliography. The type of information that might be needed for a machine design problem ranges from dimensions of the human body and their distribution among the population by age and gender, to the ability of the human body to withstand accelerations in various directions, to typical strengths and force generating ability in various positions. Obviously, if you are designing a device that will be controlled by a human (a grass shortener, perhaps), you need to know how much force the user can exert with hands held in various positions, what the user's reach is, and how much noise the ears can stand without damage. If your device will carry the user on it, you need data on the limits of acceleration that the body can tolerate. Data on all these topics exist. Much of it was developed by the government which regularly tests the ability of military personnel to withstand extreme environmental conditions. Part of the background research of any machine design problem should include some investigation of human factors.

* A student once commented that "*Life is an odd-numbered problem.*" This (slow) author had to ask for an explanation, which was: "*The answer is not in the back of the book.*"

1.9 THE ENGINEERING REPORT

Communication of your ideas and results is a very important aspect of engineering. Many engineering students picture themselves in professional practice spending most of their time doing calculations of a nature similar to those they have done as students. Fortunately, this is seldom the case, as it would be very boring. Actually, engineers spend the largest percentage of their time communicating with others, either orally or in writing. Engineers write proposals and technical reports, give presentations, and interact with support personnel and managers. When your design is done, it is usually necessary to present the results to your client, peers, or employer. The usual form of presentation is a formal engineering report. Thus, it is very important for the engineering student to develop his or her communication skills. *You may be the cleverest person in the world, but no one will know that if you cannot communicate your ideas clearly and concisely.* In fact, if you cannot explain what you have done, you probably don't understand it yourself. To give you some experience in this important skill, the design project assignments in later chapters are intended to be written up in formal engineering reports. Information on the writing of engineering reports can be found in the suggested readings in the bibliography at the end of this chapter.

1.10 UNITS

There are several systems of units used in engineering. The most common is the **Systeme International (SI)**. All systems are created from the choice of three of the quantities in the general expression of Newton's second law

$$F = \frac{ml}{t^2} \tag{1.1a}$$

where F is force, m is mass, l is length, and t is time. The units for any three of these variables can be chosen and the other is then derived in terms of the chosen units. The three chosen units are called *base units*, and the remaining one is then a *derived unit*. The gravitational constant (g) in the SI system is approximately 9.81 m/s^2.

The SI system chooses *mass*, *length*, and *time* as the base units and force is the derived unit. SI is then referred to as an *absolute system* since the mass is a base unit whose value is not dependent on local gravity. The **SI** system requires that lengths be measured in meters (m), mass in kilograms (kg), and time in seconds (s). Force is derived from Newton's law, equation 1.1b, and the units are:

kilogram-meters per second^2 (kg-m/s^2) = newtons

The only system of units used in this textbook will be the **SI** system. Table 1-4 shows some of the variables used in this text and their units. Note that dynamic calculations must be done in "pure" units. **Do not use mm for length in dynamic calculations.**

The student is cautioned to always check the units in any equation written for a problem solution, whether in school or in professional practice after graduation. If properly written, an equation should cancel all units across the equal sign. If it does not, then you can be ***absolutely sure it is incorrect.*** Unfortunately, a unit balance in an equation does not guarantee that it is correct, as many other errors are possible. Always double-check your results. You might save a life.

Table 1-4 Variables and Units
Base Units in Boldface – Abbreviations in ( )

Variable	Symbol	SI unit
Force	F	newtons (N)
Length	l	**meters (m)**
Time	t	**seconds (s)**
Mass	m	**kilograms (kg)**
Weight	W	newtons (N)
Velocity	v	m / s
Acceleration	a	m / s^2
Jerk	j	m / s^3
Angle	θ	degrees (deg)
Angle	θ	radians (rad)
Angular velocity	ω	rad / s
Angular acceleration	α	rad / s^2
Angular jerk	φ	rad / s^3
Torque	T	N-m
Mass moment of inertia	I	N-m-s^2
Energy	E	joules (J)
Power	P	watts (W)
Volume	V	m^3
Weight density	γ	N / m^3
Mass density	ρ	kg / m^3

1.11 A DESIGN CASE STUDY

Of all the myriad activities that the practicing engineer engages in, the one that is at once the most challenging and potentially the most satisfying is that of design. Doing calculations to analyze a clearly defined and structured problem, no matter how complex, may be difficult, but the exercise of creating something from scratch, to solve a problem that is often poorly defined, is *very* difficult. The sheer pleasure and joy at conceiving a viable solution to such a design problem is one of life's great satisfactions for anyone, engineer or not.

Some years ago, a very creative engineer of the author's acquaintance, George A. Wood Jr., heard a presentation by another creative engineer of the author's acquaintance, Keivan Towfigh, about one of his designs. Years later, Mr. Wood himself wrote a short paper about creative engineering design in which he reconstructed Mr. Towfigh's presumed creative process when designing the original invention. Both Mr. Wood and Mr. Towfigh have kindly consented to the reproduction of that paper here. It serves, in this author's opinion, as an excellent example and model for the student of engineering design to consider when pursuing his or her own design career.

Educating for Creativity in Engineering[9]

by GEORGE A. WOOD JR.

One facet of engineering, as it is practiced in industry, is the creative process. Let us define creativity as Rollo May does in his book, The Courage to Create.[10] It is "the process of bringing something new into being." Much of engineering has little to do with creativity in its fullest sense. Many engineers choose not to enter into creative enterprise, but prefer the realms of analysis, testing and product or process refinement. Many others find their satisfaction in management or business roles and are thus removed from engineering creativity as we shall discuss it here.

From the outset, I wish to note that the less creative endeavors are no less important or satisfying to many engineers than is the creative experience to those of us with the will to create. It would be a false goal for all engineering schools to assume that their purpose was to make all would-be engineers creative and that their success should be measured by the "creative quotient" of their graduates.

On the other hand, for the student who has a creative nature, a life of high adventure awaits if he can find himself in an academic environment which recognizes his needs, enhances his abilities and prepares him for a place in industry where his potential can be realized.

In this talk I will review the creative process as I have known it personally and witnessed it in others. Then I shall attempt to indicate those aspects of my training that seemed to prepare me best for a creative role and how this knowledge and these attitudes toward a career in engineering might be reinforced in today's schools and colleges.

During a career of almost thirty years as a machine designer, I have seen and been a part of a number of creative moments. These stand as the high points of my working life. When I have been the creator I have felt great elation and immense satisfaction. When I have been with others at their creative moments I have felt and been buoyed up by their delight. To me, the creative moment is the greatest reward that the profession of engineering gives.

Let me recount an experience of eight years ago when I heard a paper given by a creative man about an immensely creative moment. At the First Applied Mechanisms Conference in Tulsa, Oklahoma, was a paper entitled The Four-Bar Linkage as an Adjustment Mechanism.[11] It was nestled between two "how to do it" academic papers with graphs and equations of interest to engineers in the analysis of their mechanism problems. This paper contained only one very elementary equation and five simple illustrative figures; yet, I remember it now more clearly than any other paper I have ever heard at mechanism conferences. The author was Keivan Towfigh and he described the application of the geometric characteristics of the instant center of the coupler of a four bar mechanism.

His problem had been to provide a simple rotational adjustment for the oscillating mirror of an optical galvanometer. To accomplish this, he was required to rotate the entire galvanometer assembly about an axis through the center of the mirror and perpendicular to the pivot axis of the mirror. High rigidity of the system after adjustment was essential with very limited space available and low cost required, since up to sixteen of these galvanometer units were used in the complete instrument.

1

His solution was to mount the galvanometer elements on the coupler link of a one-piece, flexure hinged, plastic four bar mechanism so designed that the mirror center was at the instant center of the linkage at the midpoint of its adjustment. (See Fig 4.) It is about this particular geometric point (see Fig 1.) that pure rotation occurs and with proper selection of linkage dimensions this condition of rotation without translation could be made to hold sufficiently accurately for the adjustment angles required.*

Unfortunately, this paper was not given the top prize by the judges of the conference. Yet, it was, indirectly, a description of an outstandingly creative moment in the life of a creative man.

Let us look at this paper together and build the steps through which the author probably progressed in the achievement of his goal. I have never seen Mr. Towfigh since, and I shall therefore describe a generalized creative process which may be incorrect in some details but which, I am sure, is surprisingly close to the actual story he would tell.

The galvanometer problem was presented to Mr. Towfigh by his management. It was, no doubt, phrased something like this: "In our new model, we must improve the stability of the adjustment of the equipment but keep the cost down. Space is critical and low weight is too. The overall design must be cleaned up, since customers like modern, slim-styled equipment and we'll lose sales to others if we don't keep ahead of them on all points. Our industrial designer has this sketch that all of us in sales like and within which you should be able to make the mechanism fit."

Then followed a list of specifications the mechanism must meet, a time when the new model should be in production and, of course, the request for some new feature that would result in a strong competitive edge in the marketplace.

I wish to point out that the galvanometer adjustment was probably only one hoped-for improvement among many others. The budget and time allowed were little more than enough needed for conventional redesign, since this cost must be covered by the expected sales of the resulting instrument. For every thousand dollars spent in engineering, an equivalent increase in sales or reduction in manufacturing cost must be realized at a greater level than the money will bring if invested somewhere else.

(research) *In approaching this project, Mr. Towfigh had to have a complete knowledge of the equipment he was designing. He had to have run the earlier models himself. He must have adjusted the mirrors of existing machines many times. He had to be able to visualize the function of each element in the equipment in its most basic form.*

Secondly, he had to ask himself (as if he were the customer) what operational and maintenance requirements would frustrate him most. He had to determine which of these might be improved within the design time available. In this case he focused on the mirror adjustment. He considered the requirement of rotation without translation. He determined the maximum angles that would be necessary and the allowable translation that would not affect the practical accuracy of the equipment. He recognized the desirability of a one screw adjustment. He spent a few hours thinking of all the ways he had seen of rotating an assembly about an arbitrary point. He kept rejecting each solution as it came to him as he felt, in each case, that there was a better way. His ideas had too many parts, involved slides, pivots, too many screws, were too vibration sensitive or too large.

(ideation)

(frustration)

(incubation) *He thought about the problem that evening and at other times while he proceeded with the design of other aspects of the machine. He came back to the problem several times during the next few days. His design time was running out. He was a mechanism specialist and visualized a host of cranks and bars moving the mirrors. Then one day, probably after a period when he had turned his attention elsewhere, on rethinking of the adjustment device, an image of the system based on one of the elementary characteristics of a four bar mechanism came to him.*

(Eureka!) *I feel certain that this was a visual image, as clear as a drawing on paper. It was probably not complete but involved two inspirations. First was the characteristics of the instant center.* (See Figs 1, 2, 3.) Second was the use of flexure hinge joints which led to a one-piece plastic molding. (See Fig 4.) I am sure that at this moment he had a feeling that this solution was right. He knew it with certainty. The whole of his engineering background told him. He was elated. He was filled with joy. His pleasure was not because of the knowledge that his superiors would be impressed or that his security in the company would be enhanced. It was the joy of personal victory, the awareness that he had conquered.*

The creative process has been documented before by many others far more qualified to analyze the working of the human mind than I. Yet I would like to address, for the remaining minutes, how education can enhance this process and help more engineers, designers and draftsmen extend their creative potential.

The key elements I see in creativity that have greatest bearing on the quality that results from the creative effort are visualization and basic knowledge that gives strength to the feeling that the right solution has been achieved. There is no doubt in my mind that the fundamental mechanical principles that apply in the area in which the creative effort is being made must be vivid in the mind of the creator. The words that he was given in school must describe real elements that have physical, visual significance. $F = ma$ *must bring a picture to his mind vivid enough to touch.*

If a person decides to be a designer, his training should instill in him a continuing curiosity to know how each machine he sees works. He must note its elements and mentally see them function together even when they are not moving. I feel that this kind of solid, basic knowledge couples with physical experience to build ever more critical levels at which one accepts a tentative solution as "right."

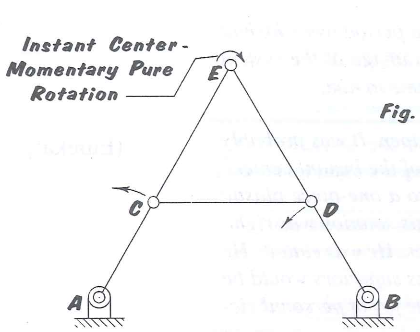
It should be noted that there have been times for all of us when the inspired "right" solution has proven wrong in the long run. That this happens does not detract from the process but indicates that creativity is based on learning and that failures build toward a firmer judgment base as the engineer matures. These failure periods are only negative, in the growth of a young engineer, when they result in the fear to accept a new challenge and beget excessive caution which then stifles the repetition of the creative process.

What would seem the most significant aspects of an engineering curriculum to help the potentially creative student develop into a truly creative engineer?

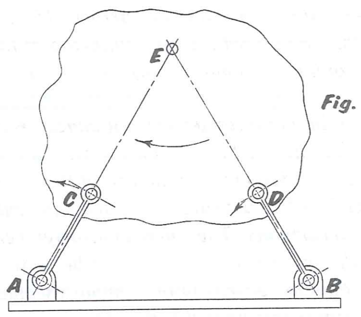
(analysis) *First is a solid, basic knowledge in physics, mathematics, chemistry and those subjects relating to his area of interest. These fundamentals should have physical meaning to the student and a vividness that permits him to explain his thoughts to the untrained layman. All too often technical words are used to cover cloudy concepts. They serve the ego of the user instead of the education of the listener.*

Second is the growth of the student's ability to visualize. The creative designer must be able to develop a mental image of that which he is inventing. The editor of the book Seeing with the Mind's Eye,[12] by Samuels and Samuels, says in the preface:

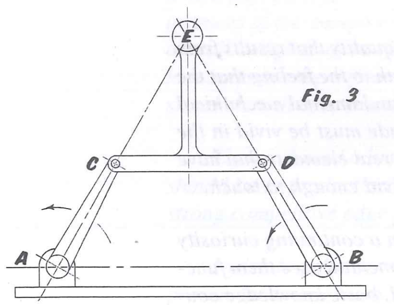
* The theory of instant centers will be thoroughly explained in Chapter 6.



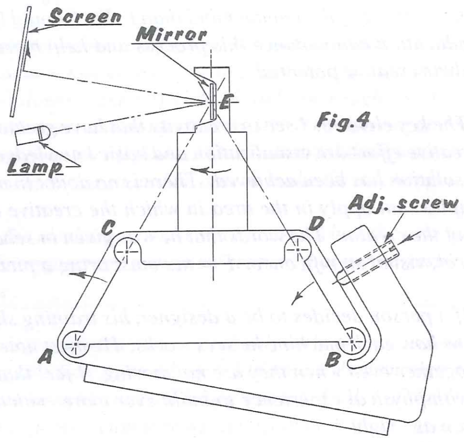
The Theory



The Development



The Mechanism



The Final Product of Keivan Towfigh

". . . visualization is the way we think. Before words, images were. Visualization is the heart of the bio-computer. The human brain programs and self-programs through its images. Riding a bicycle, driving a car, learning to read, baking a cake, playing golf - all skills are acquired through the image making process. Visualization is the ultimate consciousness tool."

Obviously, the creator of new machines or products must excel in this area.

To me, a course in Descriptive Geometry is one part of an engineer's training that enhances one's ability to visualize theoretical concepts and graphically reproduce the result. This ability is essential when one sets out to design a piece of new equipment. First, he visualizes a series of complete machines with gaps where the problem or unknown areas are. During this time, a number of directions the development could take begin to form. The best of these images are recorded on paper and then are reviewed with those around him until, finally, a basic concept emerges.

The third element is the building of the student's knowledge of what can be or has been done by others with different specialized knowledge than he has. This is the area to which experience will add throughout his career as long as he maintains an enthusiastic curiosity. Creative engineering is a building process. No one can develop a new concept involving principles about which

he has no knowledge. The creative engineer looks at problems in the light of what he has seen, learned and experienced and sees new ways for combining these to fill a new need.

Fourth is the development of the ability of the student to communicate his knowledge to others. This communication must involve not only skills with the techniques used by technical people but must also include the ability to share engineering concepts with untrained shop workers, business people and the general public. The engineer will seldom gain the opportunity to develop a concept if he cannot pass on to those around him his enthusiasm and confidence in the idea. Frequently, truly ingenious ideas are lost because the creator cannot transfer his vivid image to those who might finance or market it.

Fifth is the development of a student's knowledge of the physical result of engineering. The more he can see real machines doing real work, the more creative he can be as a designer. The engineering student should be required to run tools, make products, adjust machinery and visit factories. It is through this type of experience that judgement grows as to what makes a good machine, when approximation will suffice and where optimization should halt.

It is often said that there has been so much theoretical development in engineering during the past few decades that the colleges and universities do not have time for the basics I have outlined above. It is suggested that industry should fill in the practice areas that colleges have no time for, so that the student can be exposed to the latest technology. To some degree I understand and sympathize with this approach, but I feel that there is a negative side that needs to be recognized. If a potentially creative engineer leaves college without the means to achieve some creative success as he enters his first job, his enthusiasm for creative effort is frustrated and his interest sapped long before the most enlightened company can fill in the basics. Therefore, a result of the "basics later" approach often is to remove from the gifted engineering student the means to express himself visually and physically. Machine design tasks therefore become the domain of the graduates of technical and trade schools and the creative contribution by many a brilliant university student to products that could make all our lives richer is lost.

As I said at the start, not all engineering students have the desire, drive and enthusiasm that are essential to creative effort. Yet I feel deeply the need for the enhancement of the potential of those who do. That expanding technology makes course decisions difficult for both student and professor is certainly true. The forefront of academic thought has a compelling attraction for both the teacher and the learner. Yet I feel that the development of strong basic knowledge, the abilities to visualize, to communicate, to respect what has been done, to see and feel real machinery, need not exclude or be excluded by the excitement of the new. I believe that there is a curriculum balance that can be achieved which will enhance the latent creativity in all engineering and science students. It can give a firm basis for those who look towards a career of mechanical invention and still include the excitement of new technology.

I hope that this discussion may help in generating thought and providing some constructive suggestions that may lead more engineering students to find the immense satisfaction of the creative moment in the industrial environment. In writing this paper I have spent considerable time reflecting on my years in engineering and I would close with the following thought. For those of us who have known such times during our careers, the successful culminations of creative efforts stand among our most joyous hours.

Mr. Wood's description of his creative experiences in engineering design and the educational factors which influenced them closely parallels this author's experience as well. The student is well advised to follow his prescription for a thorough grounding in the fun-

* Defined in Chapter 6.

damentals of engineering and communication skills. A most satisfying career in the design of machinery can result.

1.12 WHAT'S TO COME

In this text we will explore the **design of machinery** in respect to the **synthesis of mechanisms** in order to accomplish desired motions or tasks, and also the **analysis of mechanisms** in order to determine their rigid-body dynamic behavior. On the premise that we cannot analyze anything until it has been synthesized into existence, we will first explore the synthesis of mechanisms. Then we will investigate the analysis of those and other mechanisms for their kinematic behavior. Finally, in Part II we will deal with the **dynamic analysis** of the forces and torques generated by these moving machines. These topics cover the essence of the early stages of a design project. Once the kinematics and kinetics of a design have been determined, most of the conceptual design will have been accomplished. What then remains is **detailed design**—sizing the parts against failure. The topic of *detailed design* is discussed in other texts such as reference [8].

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http://www.motionsystemdesign.com

Motion System Design magazine's site with articles and reference information for design and data on motors, bearings, etc. (searchable).

http://www.thomasregister.com

Thomas Register is essentially a national listing of companies by product or service offered (searchable).

http://www.howstuffworks.com

Much useful information on a variety of engineering devices (searchable).

http://www.manufacturing.net/dn/index.asp

Design News magazine's site with articles and information for design (searchable).

http://iel.ucdavis.edu/design/

University of California Davis Integration Engineering Laboratory site with applets that animate various mechanisms.

http://kmoddl.library.cornell.edu/

A collection of mechanical models and related resources for teaching the principles of kinematics including the Reuleaux Collection of Mechanisms and Machines, an important collection of 19th-century machine elements held by Cornell's Sibley School of Mechanical and Aerospace Engineering.

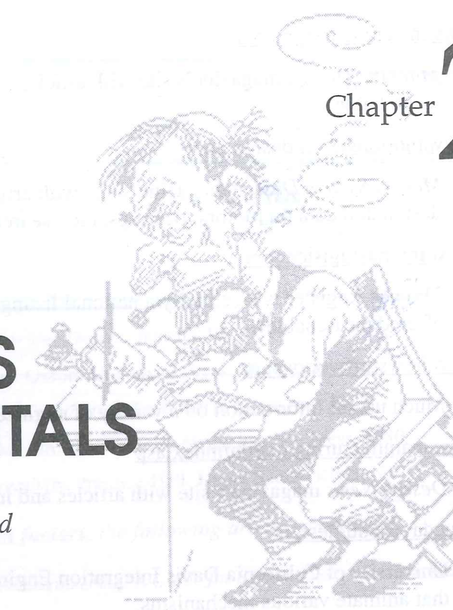
http://www.mech.uwa.edu.au/DANotes/design/home.html

A good description of the design process from Australia.

Suggested ***keywords*** *for searching the web for more information:*

machine design, mechanism, linkages, linkage design, kinematics, cam design

Chapter 2



KINEMATICS FUNDAMENTALS

Chance favors the prepared mind
PASTEUR

2.0 INTRODUCTION

This chapter will present definitions of a number of terms and concepts fundamental to the synthesis and analysis of mechanisms. It will also present some very simple but powerful analysis tools that are useful in the synthesis of mechanisms.

2.1 DEGREES OF FREEDOM (DOF) OR MOBILITY

A mechanical system's **mobility** (*M*) can be classified according to the number of **degrees of freedom** (*DOF*) that it possesses. The system's *DOF* is equal to *the number of independent parameters (measurements) that are needed to uniquely define its position in space at any instant of time*. Note that *DOF* is defined with respect to a selected frame of reference. Figure 2-1 shows a pencil lying on a flat piece of paper with an *x, y* coordinate system added. If we constrain this pencil to always remain in the plane of the paper, three parameters (*DOF*) are required to completely define the position of the pencil on the paper, two linear coordinates (*x, y*) to define the position of any one point on the pencil and one angular coordinate (θ) to define the angle of the pencil with respect to the axes. The minimum number of measurements needed to define its position is shown in the figure as *x, y*, and θ. This system of the pencil in a plane then has **three** *DOF*. Note that the particular parameters chosen to define its position are not unique. Any alternate set of three parameters could be used. There is an infinity of sets of parameters possible, but in this case there must be three parameters per set, **such as two lengths and an angle**, to define the system's position because *a rigid body in plane motion has three DOF*.

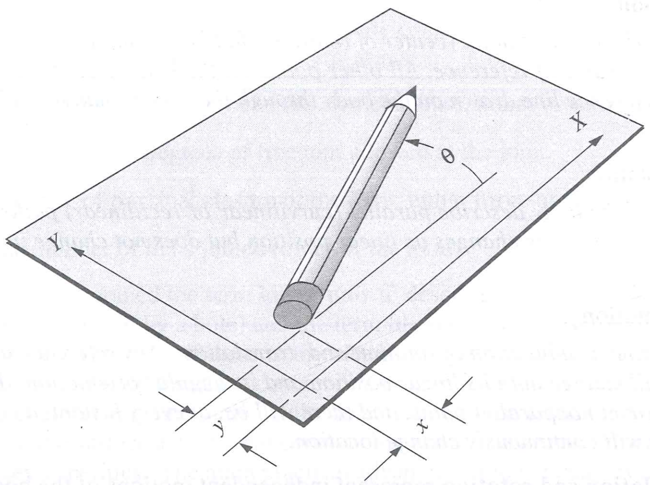


FIGURE 2-1
A rigid body in a plane has three *DOF*

Now allow the pencil to exist in a three-dimensional world. Hold it above your desktop and move it about. You now will need six parameters to define its **six** *DOF*. One possible set of parameters that could be used is **three lengths**, (*x, y, z*), plus **three angles** (θ, ϕ, ρ). *Any rigid body in three-space has six degrees of freedom.* Try to identify these six *DOF* by moving your pencil or pen with respect to your desktop.

The pencil in these examples represents a **rigid body**, or **link**, which for purposes of kinematic analysis we will assume to be incapable of deformation. This is merely a convenient fiction to allow us to more easily define the gross motions of the body. We can later superpose any deformations due to external or inertial loads onto our kinematic motions to obtain a more complete and accurate picture of the body's behavior. But remember, we are typically facing a *blank sheet of paper* at the beginning stage of the design process. We cannot determine deformations of a body until we define its size, shape, material properties, and loadings. Thus, at this stage we will assume, for purposes of initial kinematic synthesis and analysis, that *our kinematic bodies are rigid and massless*.

2.2 TYPES OF MOTION

A rigid body free to move within a reference frame will, in the general case, have **complex motion**, which is a simultaneous combination of **rotation** and **translation**. In three-dimensional space, there may be rotation about any axis (any skew axis or one of the three principal axes) and also simultaneous translation that can be resolved into components along three axes. In a plane, or two-dimensional space, complex motion becomes a combination of simultaneous rotation about one axis (perpendicular to the plane) and also translation resolved into components along two axes in the plane. For simplicity, we will limit our present discussions to the case of **planar (2-D) kinematic systems**. We will define these terms as follows for our purposes, in planar motion:

2

Pure rotation

The body possesses one point (center of rotation) that has no motion with respect to the "stationary" frame of reference. All other points on the body describe arcs about that center. A reference line drawn on the body through the center changes only its angular orientation.

Pure translation

All points on the body describe parallel (curvilinear or rectilinear) paths. A reference line drawn on the body changes its linear position but does not change its angular orientation.

Complex motion

A simultaneous combination of rotation and translation. Any reference line drawn on the body will change both its linear position and its angular orientation. Points on the body will travel nonparallel paths, and there will be, at every instant, a center of rotation, which will continuously change location.

Translation and **rotation** represent independent motions of the body. Each can exist without the other. If we define a 2-D coordinate system as shown in Figure 2-1 (p. 29), the x and y terms represent the translation components of motion, and the θ term represents the rotation component.

2.3 LINKS, JOINTS, AND KINEMATIC CHAINS

We will begin our exploration of the kinematics of mechanisms with an investigation of the subject of **linkage design**. Linkages are the basic building blocks of all mechanisms. We will show in later chapters that all common forms of mechanisms (cams, gears, belts, chains) are in fact variations on a common theme of linkages. Linkages are made up of links and joints.

A **link**, as shown in Figure 2-2, is an (assumed) rigid body that possesses at least two **nodes** that are *points for attachment to other links.*

Binary link - *one with two nodes.*

Ternary link - *one with three nodes.*

Quaternary link - *one with four nodes.*

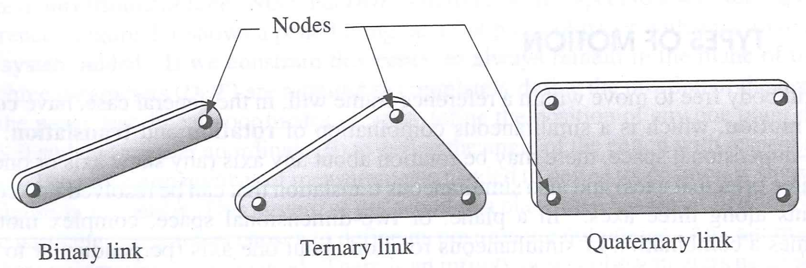


FIGURE 2-2
Links of different order

2

A **joint** is *a connection between two or more links (at their nodes), which allows some motion, or potential motion, between the connected links.* **Joints** (also called **kinematic pairs**) can be classified in several ways:

1 By the type of contact between the elements, line, point, or surface.

2 By the number of degrees of freedom allowed at the joint.

3 By the type of physical closure of the joint: either **force** or **form** closed.

4 By the number of links joined (order of the joint).

Reuleaux [1] coined the term **lower pair** to describe joints with surface contact (as with a pin surrounded by a hole) and the term **higher pair** to describe joints with point or line contact. However, if there is any clearance between pin and hole (as there must be for motion), so-called surface contact in the pin joint actually becomes line contact, as the pin contacts only one "side" of the hole. Likewise, at a microscopic level, a block sliding on a flat surface actually has contact only at discrete points, which are the tops of the surfaces' asperities. The main practical advantage of lower pairs over higher pairs is their better ability to trap lubricant between their enveloping surfaces. This is especially true for the rotating pin joint. The lubricant is more easily squeezed out of a higher pair, nonenveloping joint. As a result, the pin joint is preferred for low wear and long life, even over its lower pair cousin, the prismatic or slider joint.

lower pairs
full joint (DOF=1).
higher pairs?

Figure 2-3a (p. 32) shows the six possible lower pairs, their degrees of freedom, and their one-letter symbols. The revolute (R) and the prismatic (P) pairs are the only lower pairs usable in a planar mechanism. The screw (H), cylindric (C), spherical (S), and flat (F) lower pairs are all combinations of the revolute and/or prismatic pairs and are used in spatial (3-D) mechanisms. The R and P pairs are the basic building blocks of all other pairs that are combinations of those two as shown in Table 2-1.

the screw & the nut
DOF: ?.

A more useful means to classify joints (pairs) is by the number of degrees of freedom that they allow between the two elements joined. Figure 2-3 (p. 32) also shows examples of both one- and two-freedom joints commonly found in planar mechanisms. Figure 2-3b shows two forms of a planar, **one-freedom** joint (or pair), namely, a rotating (revolute) pin joint (R) and a translating (prismatic) slider joint (P). These are also referred to as **full joints** (i.e., full = 1 *DOF*) and are **lower pairs**. The pin joint allows one rotational *DOF*, and the slider joint allows one translational *DOF* between the joined links. These are both contained within (and each is a limiting case of) another common, one-freedom joint, the screw and nut (Figure 2-3a). Motion of either the nut or the screw with respect to the other results in helical motion. If the helix angle is made zero, the nut rotates without advancing and it becomes the pin joint. If the helix angle is made 90 degrees, the nut will translate along the axis of the screw, and it becomes the slider joint.

Figure 2-3c shows examples of two-freedom joints (higher pairs) that simultaneously allow two independent, relative motions, namely translation and rotation, between the joined links. Paradoxically, this **two-freedom joint** is sometimes referred to as a "**half joint**," with its two freedoms placed in the denominator. The **half joint** is also called a **roll-slide joint** because it allows both rolling and sliding. A spherical, or ball-and-socket joint, (Figure 2-3a) is an example of a three-freedom joint, which allows three independent angular motions between the two links joined. This *joystick* or *ball joint* is typically used in a three-dimensional mechanism, one example being the ball joints in an automotive suspension system.

TABLE 2-1
The Six Lower Pairs

Name (Symbol)	*DOF*	Contains
Revolute (R)	1	R
Prismatic (P)	1	P
Helical (H)	1	RP
Cylindric (C)	2	RP
Spherical (S)	3	RRR
Planar (F)	3	RPP

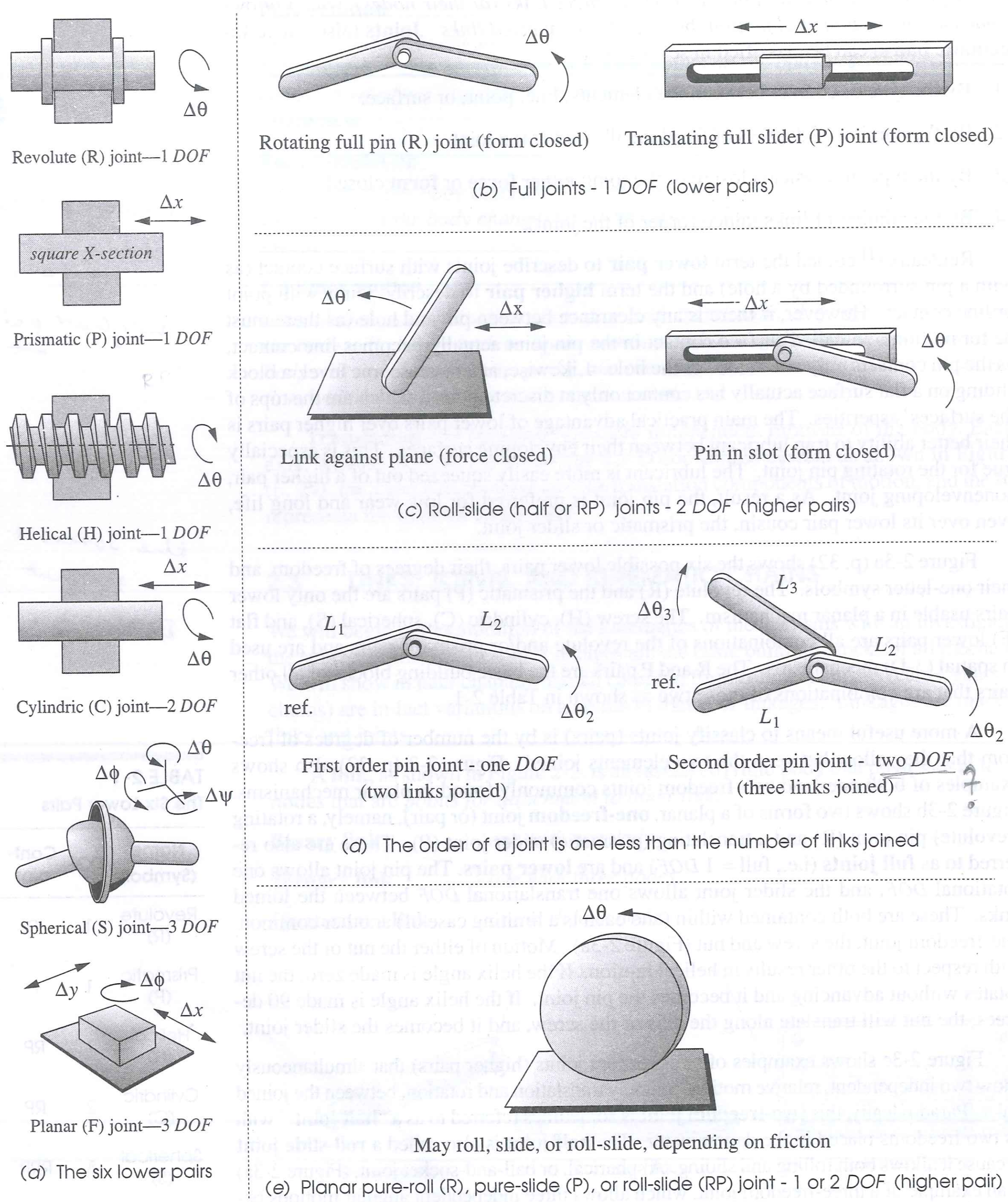


FIGURE 2-3
Joints (pairs) of various types

A joint with more than one freedom may also be a **higher pair** as shown in Figure 2-3c. Full joints (lower pairs) and half joints (higher pairs) are both used in planar (2-D), and in spatial (3-D) mechanisms. Note that if you do not allow the two links in Figure 2-3c connected by a roll-slide joint to slide, perhaps by providing a high friction coefficient between them, you can "lock out" the translating (Δx) freedom and make it behave as a full joint. This is then called a **pure rolling joint** and has rotational freedom ($\Delta\theta$) only. A common example of this type of joint is your automobile tire rolling against the road, as shown in Figure 2-3e. In normal use there is pure rolling and no sliding at this joint, unless, of course, you encounter an icy road or become too enthusiastic about accelerating or cornering. If you lock your brakes on ice, this joint converts to a pure sliding one like the slider block in Figure 2-3b. Friction determines the actual number of freedoms at this kind of joint. It can be **pure roll**, **pure slide**, or **roll-slide**.

To visualize the degree of freedom of a joint in a mechanism, it is helpful to "mentally disconnect" the two links that create the joint from the rest of the mechanism. You can then more easily see how many freedoms the two joined links have with respect to one another.

Figure 2-3c also shows examples of both **form-closed** and **force-closed** joints. A **form-closed** joint is kept together or *closed by its geometry*. A pin in a hole or a slider in a two-sided slot are form closed. In contrast, a **force-closed** joint, such as a pin in a half-bearing or a slider on a surface, *requires some external force to keep it together or closed*. This force could be supplied by gravity, a spring, or any external means. There can be substantial differences in the behavior of a mechanism due to the choice of force or form closure, as we shall see. The choice should be carefully considered. In linkages, form closure is usually preferred, and it is easy to accomplish. But for cam-follower systems, force closure is often preferred. This topic will be explored further in later chapters.

Figure 2-3d shows examples of joints of various orders, where **joint order** is defined as *the number of links joined minus one*. It takes two links to make a single joint; thus the simplest joint combination of two links has joint order one. As additional links are placed on the same joint, the joint order is increased on a one-for-one basis. Joint order has significance in the proper determination of overall degree of freedom for the assembly. We gave definitions for a **mechanism** and a **machine** in Chapter 1. With the kinematic elements of links and joints now defined, we can define those devices more carefully based on Reuleaux's classifications of the kinematic chain, mechanism, and machine. [1]

A kinematic chain is defined as:

An assemblage of links and joints, interconnected in a way to provide a controlled output motion in response to a supplied input motion.

A mechanism is defined as:

A kinematic chain in which at least one link has been "grounded," or attached, to the frame of reference (which itself may be in motion).

A machine is defined as:

A combination of resistant bodies arranged to compel the mechanical forces of nature to do work accompanied by determinate motions.

By Reuleaux's* definition [1] a machine is *a collection of mechanisms arranged to transmit forces and do work.* He viewed all energy or force transmitting devices as machines that utilize mechanisms as their building blocks to provide the necessary motion constraints.

We will now define a **crank** as *a link that makes a complete revolution and is pivoted to ground*, a **rocker** as *a link that has oscillatory (back and forth) rotation and is pivoted to ground*, and a **coupler** (or connecting rod) *as a link that has complex motion and is not pivoted to ground.* **Ground** is defined as *any link or links that are fixed* (nonmoving) with respect to the reference frame. Note that the reference frame may in fact itself be in motion.

2.4 DRAWING KINEMATIC DIAGRAMS

Analyzing the kinematics of mechanisms requires that we draw clear, simple, schematic kinematic diagrams of the links and joints of which they are made. Sometimes it can be difficult to identify the kinematic links and joints in a complicated mechanism. Beginning students of this topic often have this difficulty. This section defines one approach to the creation of simplified kinematic diagrams.

Real links can be of any shape, but a "kinematic" link, or link edge, is defined as a line between joints that allow relative motion between adjacent links. Joints can allow rotation, translation, or both between the links joined. The possible joint motions must be clear and obvious from the kinematic diagram. Figure 2-4 shows recommended schematic notations for binary, ternary, and higher-order links, and for movable and grounded joints of rotational and translational freedoms plus an example of their combination. Many other notations are possible, but whatever notation is used, it is critical that your diagram indicate which links or joints are grounded and which can move. Otherwise nobody will be able to interpret your design's kinematics. Shading or crosshatching should be used to indicate that a link is solid.

Figure 2-5a shows a photograph of a simple mechanism used for weight training called a leg press machine. It has six pin-jointed links labeled L_1 through L_6 and seven pin joints. The moving pivots are labeled A through D; O_2, O_4 and O_6 denote the grounded pivots of their respective link numbers. Even though its links are in parallel

* Reuleaux created a set of 220 models of mechanisms in the 19th century to demonstrate machine motions. Cornell University acquired the collection in 1892 and has now put images and descriptions of them on the web at: http://kmoddl.library.cornell.edu. The same site also has depictions of three other collections of machines and gear trains.

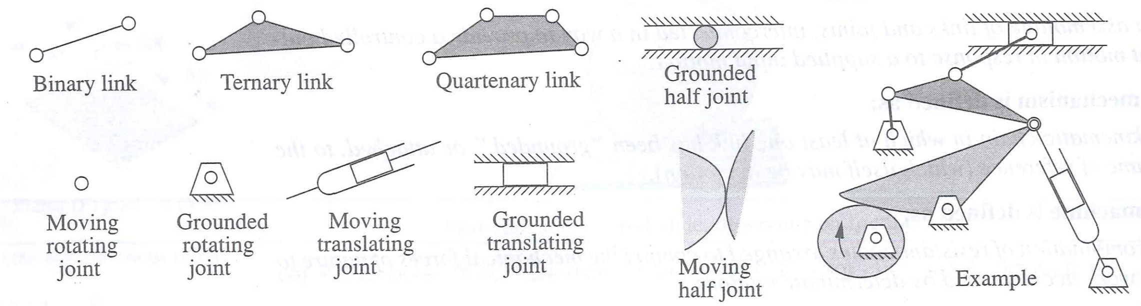
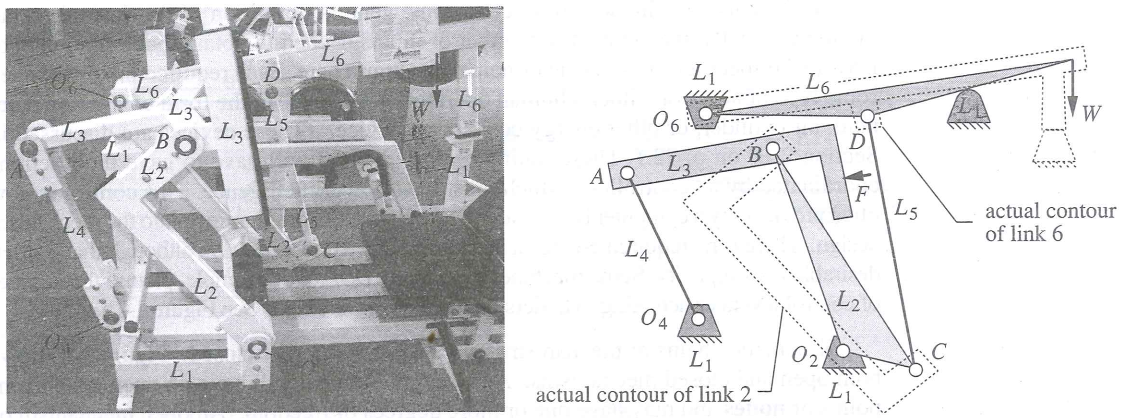


FIGURE 2-4
Schematic notation for kinematic diagrams



(*a*) Weight-training mechanism (*b*) Kinematic diagram

FIGURE 2-5
A mechanism and its kinematic diagram

planes separated by some distance in the z-direction, it can still be analyzed kinematically as if all links were in a common plane.

To use the leg press machine, the user loads some weights on link 6 at top right, sits in the seat at lower right, places both feet against the flat surface of link 3 (a coupler) and pushes with the legs to lift the weights through the linkage. The linkage geometry is designed to give a variable mechanical advantage that matches the human ability to provide force over the range of leg motion. Figure 2-5b shows a kinematic diagram of its basic mechanism. Note that here all the links have been brought to a common plane. Link 1 is the ground. Links 2, 4, and 6 are rockers. Links 3 and 5 are couplers. The input force F is applied to link 3. The "output" resistance weight W acts on link 6. Note the difference between the actual and kinematic contours of links 2 and 6.

The next section discusses techniques for determining the mobility of a mechanism. That exercise depends on an accurate count of the number of links and joints in the mechanism. Without a proper, clear, and complete kinematic diagram of the mechanism, it will be impossible to get the count, and thus the mobility, correct.

2.5 DETERMINING DEGREE OF FREEDOM OR MOBILITY

The concept of **degree of freedom** (*DOF*) is fundamental to both the synthesis and analysis of mechanisms. We need to be able to quickly determine the *DOF* of any collection of links and joints that may be suggested as a solution to a problem. Degree of freedom (also called the **mobility** *M*) of a system can be defined as:

Degree of Freedom

the number of inputs that need to be provided in order to create a predictable output;

also:

the number of independent coordinates required to define its position.

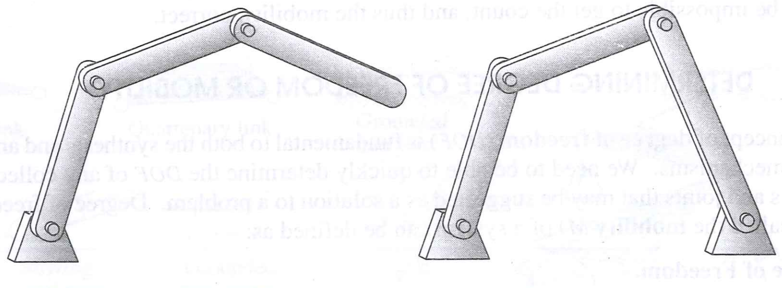
At the outset of the design process, some general definition of the desired output motion is usually available. The number of inputs needed to obtain that output may or may not be specified. Cost is the principal constraint here. Each required input will need some type of actuator, either a human operator or a "slave" in the form of a motor, solenoid, air cylinder, or other energy conversion device. (These devices are discussed in Section 2.19 on p. 72.) These multiple input devices will have to have their actions coordinated by a "controller," which must have some intelligence. This control is now often provided by a computer but can also be mechanically programmed into the mechanism design. There is no requirement that a mechanism have only one *DOF*, although that is often desirable for simplicity. Some machines have many *DOF*. For example, picture the number of control levers or actuating cylinders on a bulldozer or crane. See Figure 1-1b (p. 7).

Kinematic chains or mechanisms may be either **open** or **closed**. Figure 2-6 shows both open and closed mechanisms. A closed mechanism will have no open attachment points or **nodes** and may have one or more degrees of freedom. An open mechanism of more than one link will always have more than one degree of freedom, thus requiring as many actuators (motors) as it has *DOF*. A common example of an open mechanism is an industrial robot. *An open kinematic chain of two binary links and one joint* is called a **dyad**. The sets of links shown in Figure 2-3b and c (p. 32) are **dyads**.

Reuleaux limited his definitions to closed kinematic chains and to mechanisms having only one *DOF*, which he called *constrained*. [1] The somewhat broader definitions above are perhaps better suited to current-day applications. A multi-*DOF* mechanism, such as a robot, will be constrained in its motions as long as the necessary number of inputs is supplied to control all its *DOF*.

Degree of Freedom (Mobility) in Planar Mechanisms

To determine the overall *DOF* of any mechanism, we must account for the number of links and joints, and for the interactions among them. The *DOF* of any assembly of links can be predicted from an investigation of the **Gruebler condition**. [2] Any link in a plane has 3 *DOF*. Therefore, a system of L unconnected links in the same plane will have $3L$ *DOF*, as shown in Figure 2-7a where the two unconnected links have a total of six *DOF*.



(*a*) Open mechanism chain (*b*) Closed mechanism chain

FIGURE 2-6

Mechanism chains

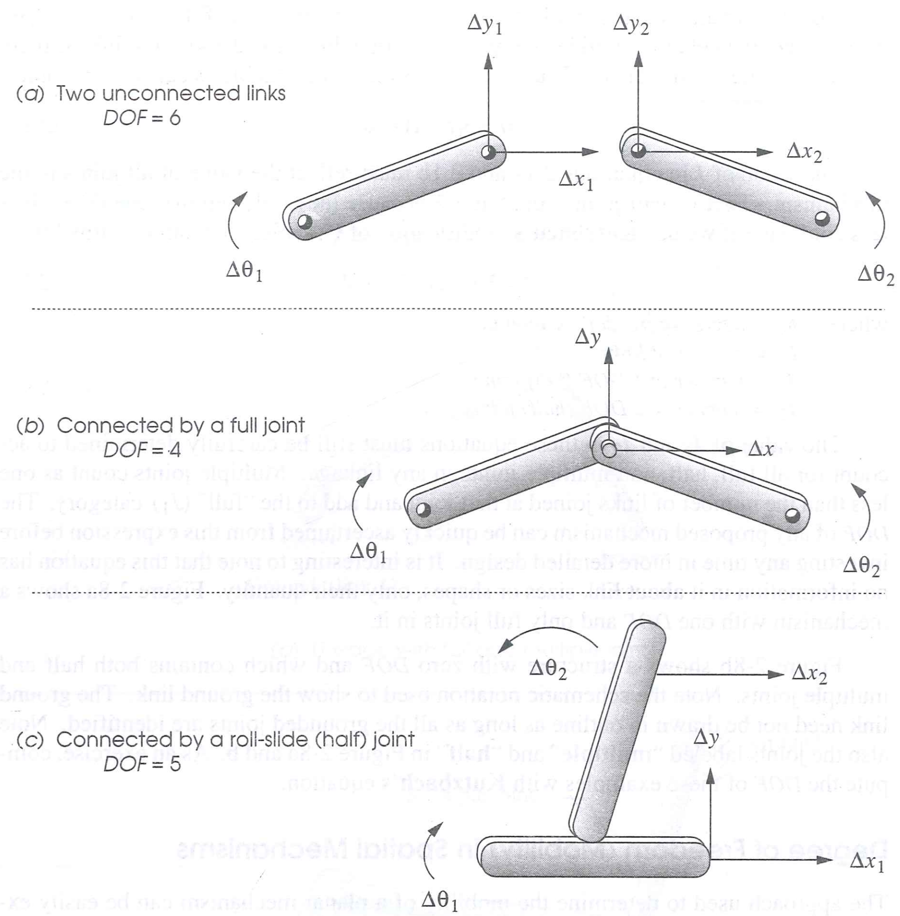


FIGURE 2-7

Joints remove degrees of freedom

When these links are connected by a **full joint** in Figure 2-7b, Δy_1 and Δy_2 are combined as Δy, and Δx_1 and Δx_2 are combined as Δx. This removes two *DOF*, leaving four *DOF*. In Figure 2-7c the half joint removes only one *DOF* from the system (because a half joint has two *DOF*), leaving the system of two links connected by a half joint with a total of five *DOF*. In addition, when any link is grounded or attached to the reference frame, all three of its *DOF* will be removed. This reasoning leads to **Gruebler's equation**:

$$M = 3L - 2J - 3G \tag{2.1a}$$

where: M = *degree of freedom or mobility*
L = *number of links*
J = *number of joints*
G = *number of grounded links*

Note that in any real mechanism, even if more than one link of the kinematic chain is grounded, the net effect will be to create one larger, higher-order ground link, as there can be only one ground plane. Thus G is always one, and Gruebler's equation becomes:

$$M = 3(L-1) - 2J \tag{2.1b}$$

The value of J in equations 2.1a and 2.1b must reflect the value of all joints in the mechanism. That is, half joints count as 1/2 because they only remove one *DOF*. It is less confusing if we use **Kutzbach**'s modification of Gruebler's equation in this form:

$$M = 3(L-1) - 2J_1 - J_2 \tag{2.1c}$$

where: M = *degree of freedom or mobility*
L = *number of links*
J_1 = *number of 1 DOF (full) joints*
J_2 = *number of 2 DOF (half) joints*

The value of J_1 and J_2 in these equations must still be carefully determined to account for all full, half, and multiple joints in any linkage. Multiple joints count as one less than the number of links joined at that joint and add to the "full" (J_1) category. The *DOF* of any proposed mechanism can be quickly ascertained from this expression before investing any time in more detailed design. It is interesting to note that this equation has no information in it about link sizes or shapes, only their quantity. Figure 2-8a shows a mechanism with one *DOF* and only full joints in it.

Figure 2-8b shows a structure with zero *DOF* and which contains both half and multiple joints. Note the schematic notation used to show the ground link. The ground link need not be drawn in outline as long as all the grounded joints are identified. Note also the joints labeled "**multiple**" and "**half**" in Figure 2-8a and b. As an exercise, compute the *DOF* of these examples with **Kutzbach**'s equation.

Degree of Freedom (Mobility) in Spatial Mechanisms

The approach used to determine the mobility of a planar mechanism can be easily extended to three dimensions. Each unconnected link in three-space has 6 *DOF*, and any one of the six lower pairs can be used to connect them, as can higher pairs with more freedom. A one-freedom joint removes 5 *DOF*, a two-freedom joint removes 4 *DOF*, etc. Grounding a link removes 6 *DOF*. This leads to the Kutzbach mobility equation for spatial linkages:

$$M = 6(L-1) - 5J_1 - 4J_2 - 3J_3 - 2J_4 - J_5 \tag{2.2}$$

where the subscript refers to the number of freedoms of the joint. We will limit our study to 2-D mechanisms in this text.

2.6 MECHANISMS AND STRUCTURES

The degree of freedom of an assembly of links completely predicts its character. There are only three possibilities. *If the DOF is positive, it will be a **mechanism**,* and the links will have relative motion. *If the DOF is exactly zero, then it will be a **structure**,* and no motion is possible. *If the DOF is negative, then it is a **preloaded structure**,* which means

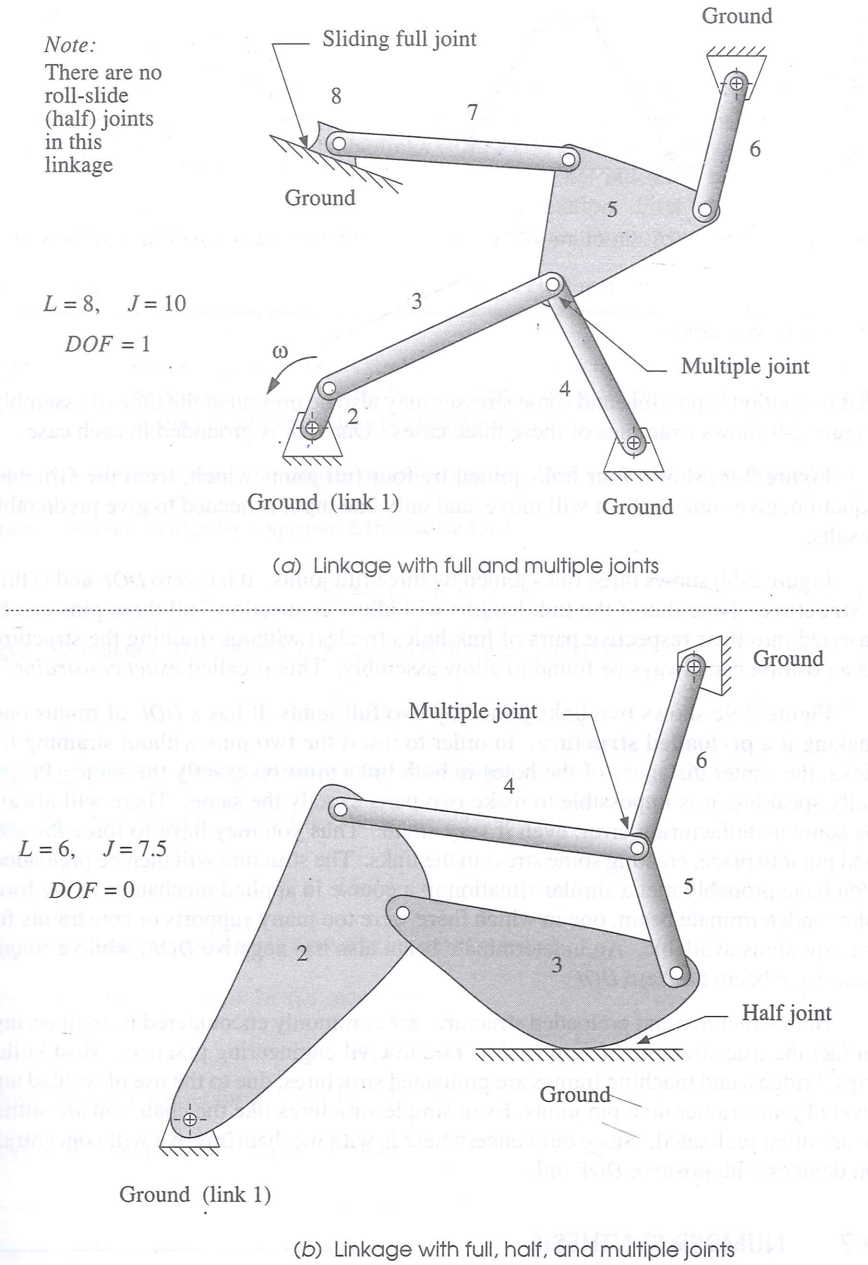
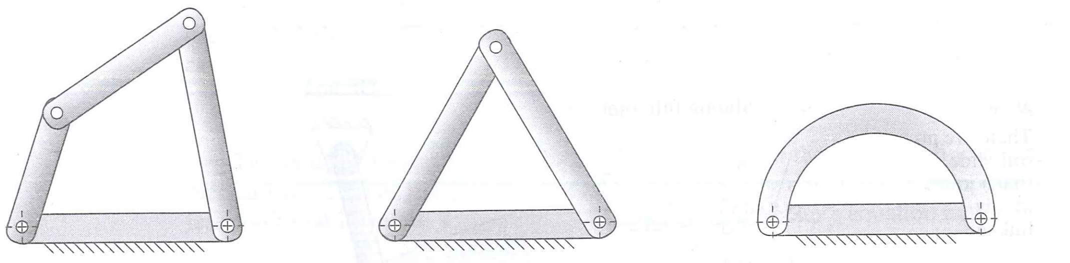


FIGURE 2-8
Linkages containing joints of various types



(a) Mechanism—*DOF* = +1 (b) Structure—*DOF* = 0 (c) Preloaded structure—*DOF* = −1

FIGURE 2-9
Mechanisms, structures, and preloaded structures

that no motion is possible and some stresses may also be present at the time of assembly. Figure 2-9 shows examples of these three cases. One link is grounded in each case.

Figure 2-9a shows four links joined by four full joints which, from the Gruebler equation, gives one *DOF*. It will move, and only one input is needed to give predictable results.

Figure 2-9b shows three links joined by three full joints. It has zero *DOF* and is thus a **structure**. Note that if the link lengths will allow connection,* all three pins can be inserted into their respective pairs of link holes (nodes) without straining the structure, as a position can always be found to allow assembly. This is called *exact constraint*.†

Figure 2-9c shows two links joined by two full joints. It has a *DOF* of minus one, making it a **preloaded structure**. In order to insert the two pins without straining the links, the center distances of the holes in both links must be exactly the same. Practically speaking, it is impossible to make two parts exactly the same. There will always be some manufacturing error, even if very small. Thus you may have to force the second pin into place, creating some stress in the links. The structure will then be preloaded. You have probably met a similar situation in a course in applied mechanics in the form of an indeterminate beam, one in which there were too many supports or constraints for the equations available. An indeterminate beam also has negative *DOF*, while a *simply supported* beam has zero *DOF*.

Both structures and preloaded structures are commonly encountered in engineering. In fact the true structure of zero *DOF* is rare in civil engineering practice. Most buildings, bridges, and machine frames are preloaded structures, due to the use of welded and riveted joints rather than pin joints. Even simple structures like the chair you are sitting in are often preloaded. Since our concern here is with mechanisms, we will concentrate on devices with positive *DOF* only.

2.7 NUMBER SYNTHESIS

The term **number synthesis** has been coined to mean *the determination of the number and order of links and joints necessary to produce motion of a particular DOF*. **Link**

* If the sum of the lengths of any two links is less than the length of the third link, then their interconnection is impossible.

† The concept of *exact constraint* also applies to mechanisms with positive *DOF*. It is possible to provide redundant constraints to a mechanism (e.g., making its theoretical *DOF* = 0 when 1 is desired) yet still have it move because of particular geometry (see Section 2.8 *Paradoxes*). Non-exact constraint should be avoided in general as it can lead to unexpected mechanical behavior. For an excellent and thorough discussion of this issue see Blanding, D. L., *Exact Constraint: Machine Design Using Kinematic Principles*, ASME Press, 1999.

order in this context refers to the number of nodes per link,* i.e., **binary**, **ternary**, **quaternary**, etc. The value of number synthesis is to allow the exhaustive determination of all possible combinations of links that will yield any chosen *DOF*. This then equips the designer with a definitive catalog of potential linkages to solve a variety of motion control problems.

As an example we will now derive all the possible link combinations for one *DOF*, including sets of up to eight links, and link orders up to and including hexagonal links. For simplicity we will assume that the links will be connected with only single, full rotating joints (i.e., a pin connecting two links). We can later introduce half joints, multiple joints, and sliding joints through linkage transformation. First let's look at some interesting attributes of linkages as defined by the above assumption regarding full joints.

Hypothesis: If all joints are full joints, an odd number of *DOF* requires an even number of links and vice versa.

Proof: **Given:** All even integers can be denoted by $2m$ or by $2n$, and all odd integers can be denoted by $2m-1$ or by $2n-1$, where n and m are any positive integers. The number of joints must be a positive integer.

Let : L = number of links, J = number of joints, and $M = DOF = 2m$ (i.e., all even numbers)

Then: rewriting Gruebler's equation 2.1b to solve for J,

$$J = \frac{3}{2}(L-1) - \frac{M}{2} \tag{2.3a}$$

Try: Substituting $M = 2m$, and $L = 2n$ (i.e., both any even numbers):

$$J = 3n - m - \frac{3}{2} \tag{2.3b}$$

This cannot result in J being a positive integer as required.

Try: $M = 2m - 1$ and $L = 2n - 1$ (i.e., both any odd numbers):

$$J = 3n - m - \frac{5}{2} \tag{2.3c}$$

This also cannot result in J being a positive integer as required.

Try: $M = 2m - 1$, and $L = 2n$ (i.e., odd-even):

$$J = 3n - m - 2 \tag{2.3d}$$

This is a positive integer for $m \geq 1$ and $n \geq 2$.

Try: $M = 2m$ and $L = 2n - 1$ (i.e., even-odd):

$$J = 3n - m - 3 \tag{2.3e}$$

This is a positive integer for $m \geq 1$ and $n \geq 2$.

So, for our example of one-*DOF* mechanisms, we can only consider combinations of 2, 4, 6, 8 . . . links. Letting the order of the links be represented by:

* Not to be confused with "joint order" as defined earlier, which refers to the number of *DOF* that a joint possesses.

B = *number of binary links*
T = *number of ternary links*
Q = *number of quaternaries*
P = *number of pentagonals*
H = *number of hexagonals*

the total number of links in any mechanism will be:

$$L = B + T + Q + P + H + \cdots \tag{2.4a}$$

Since *two link nodes* are needed to make *one joint*:

$$J = \frac{nodes}{2} \tag{2.4b}$$

and

$$nodes = order\ of\ link \times no.\ of\ links\ of\ that\ order \tag{2.4c}$$

then

$$J = \frac{(2B + 3T + 4Q + 5P + 6H + \cdots)}{2} \tag{2.4d}$$

Substitute equations 2.4a and 2.4d into Gruebler's equation (2.1b, on p. 38)

$$M = 3(B + T + Q + P + H - 1) - 2\left(\frac{2B + 3T + 4Q + 5P + 6H}{2}\right)$$
$$M = B - Q - 2P - 3H - 3 \tag{2.4e}$$

Note what is missing from this equation! The ternary links have dropped out. The *DOF* is independent of the number of ternary links in the mechanism. But because each ternary link has three nodes, it can only create or remove 3/2 joints. So we must add or subtract ternary links in pairs to maintain an integer number of joints. *The addition or subtraction of ternary links in pairs will not affect the DOF of the mechanism.*

In order to determine all possible combinations of links for a particular *DOF*, we must combine equations 2.3a (p. 41) and 2.4d:*

$$\frac{3}{2}(L-1) - \frac{M}{2} = \frac{(2B + 3T + 4Q + 5P + 6H)}{2}$$
$$3L - 3 - M = 2B + 3T + 4Q + 5P + 6H \tag{2.5}$$

Now combine equation 2.5 with equation 2.4a to eliminate B:

$$L - 3 - M = T + 2Q + 3P + 4H \tag{2.6}$$

We will now solve equations 2.4a and 2.6 simultaneously (by progressive substitution) to determine all compatible combinations of links for *DOF* = 1, up to eight links. The strategy will be to start with the smallest number of links, and the highest-order link possible with that number, eliminating impossible combinations.

(Note: *L must be even for odd DOF*.)

* Karunamoorthy [17] defines some useful rules for determining the number of possible combinations for any number of links with a given degree of freedom.

CASE 1. $L = 2$

$$L - 4 = T + 2Q + 3P + 4H = -2 \tag{2.7a}$$

This requires a negative number of links, so $L = 2$ is impossible.

CASE 2. $L = 4$

$$L - 4 = T + 2Q + 3P + 4H = 0; \qquad \text{so}: \ T = Q = P = H = 0$$
$$L = B + 0 = 4; \qquad B = 4 \tag{2.7b}$$

The simplest one-*DOF* linkage is four binary links—the **fourbar linkage**.

CASE 3. $L = 6$

$$L - 4 = T + 2Q + 3P + 4H = 2; \qquad \text{so}: \quad P = H = 0 \tag{2.7c}$$

T may only be 0, 1, or 2; Q may only be 0 or 1

If $Q = 0$ then T must be 2 and:

$$L = B + 2T + 0Q = 6; \qquad B = 4, \qquad T = 2 \tag{2.7d}$$

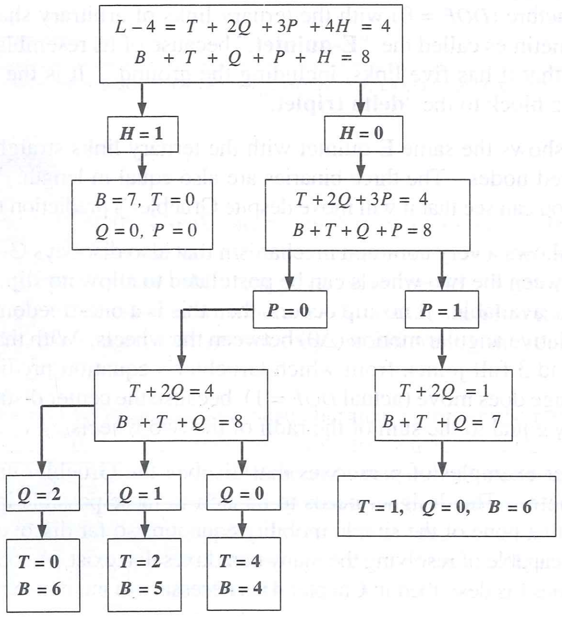
If $Q = 1$, then T must be 0 and:

$$L = B + 0T + 1Q = 6; \qquad B = 5, \qquad Q = 1 \tag{2.7e}$$

There are then two possibilities for $L = 6$. Note that one of them is in fact the simpler fourbar with two ternaries added as was predicted above.

CASE 4. $L = 8$

A tabular approach is needed with this many links:



(2.7f)

TABLE 2-2 1-*DOF* Planar Mechanisms with Revolute Joints and Up to 8 Links

Total Links	Link Sets				
	Binary	Ternary	Quaternary	Pentagonal	Hexagonal
4	4	0	0	0	0
6	4	2	0	0	0
6	5	0	1	0	0
8	7	0	0	0	1
8	4	4	0	0	0
8	5	2	1	0	0
8	6	0	2	0	0
8	6	1	0	1	0

From this analysis we can see that, for one *DOF*, there is only one possible four-link configuration, two six-link configurations, and five possibilities for eight links using binary through hexagonal links. Table 2-2 shows the so-called "link sets" for all the possible linkages for one *DOF* up to 8 links and hexagonal order.

2.8 PARADOXES

Because the Gruebler criterion pays no attention to link sizes or shapes, it *can give misleading results* in the face of unique geometric configurations. For example, Figure 2-10a shows a structure ($DOF = 0$) with the ternary links of arbitrary shape. This link arrangement is sometimes called the **"E-quintet,"** because of its resemblance to a capital **E** and the fact that it has five links, including the ground.* It is the next simplest **structural** building block to the **"delta triplet."**

Figure 2-10b shows the same E-quintet with the ternary links straight and parallel and with equispaced nodes. The three binaries are also equal in length. With this very unique geometry, you can see that it will move despite Gruebler's prediction to the contrary.

Figure 2-10c shows a very common mechanism that also disobeys Gruebler's criterion. The joint between the two wheels can be postulated to allow no slip, provided that sufficient friction is available. If no slip occurs, then this is a one-freedom, or full, joint that allows only relative angular motion ($\Delta\theta$) between the wheels. With that assumption, there are 3 links and 3 full joints, from which Gruebler's equation predicts zero *DOF*. However, this linkage does move (actual $DOF = 1$), because the center distance, or length of link 1, is exactly equal to the sum of the radii of the two wheels.

There are other examples of paradoxes that disobey the Gruebler criterion due to their unique geometry. The designer needs to be alert to these possible inconsistencies. Gogu† has shown that none of the simple mobility equations so far discovered (Gruebler, Kutzbach, etc.) are capable of resolving the many paradoxes that exist. A complete analysis of the linkage motions (as described in Chapter 4) is necessary to guarantee mobility.

* It is also called an Assur chain.

† Gogu, G., (2005) Mobility of Mechanisms: A Critical Review." *Mechanism and Machine Theory* (40) pp. 1068-1097

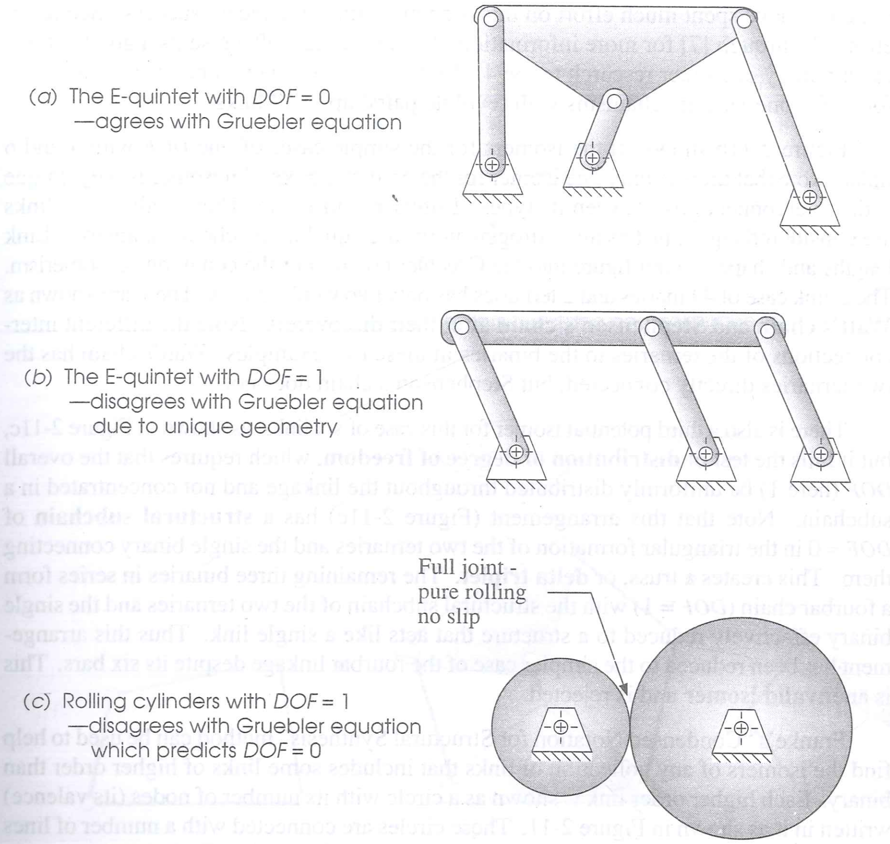


FIGURE 2-10

Gruebler paradoxes—linkages that do not behave as predicted by the Gruebler equation

2.9 ISOMERS

The word **isomer** is from the Greek and means *having equal parts*. Isomers in chemistry are compounds that have the same number and type of atoms but which are interconnected differently and thus have different physical properties. Figure 2-11a shows two hydrocarbon isomers, n-butane and isobutane. Note that each has the same number of carbon and hydrogen atoms (C_4H_{10}), but they are differently interconnected and have different properties.

Linkage isomers are analogous to these chemical compounds in that the **links** (like atoms) have various **nodes** (electrons) available to connect to other links' nodes. The assembled linkage is analogous to the chemical compound. Depending on the particular connections of available links, the assembly will have different motion properties. The number of isomers possible from a given collection of links (as in any row of Table 2-2 on p. 44) is far from obvious. In fact the problem of mathematically predicting the number of isomers of all link combinations has been a long-unsolved problem. Many re-

TABLE 2-3
Number of Valid Isomers

Links	Valid Isomers
4	1
6	2
8	16
10	230
12	6856

searchers have spent much effort on this problem with some recent success. See references [3] through [7] for more information. Dhararipragada [6] presents a good historical summary of isomer research to 1994. Table 2-3 shows the number of valid isomers found for one-*DOF* mechanisms with revolute pairs, up to 12 links.

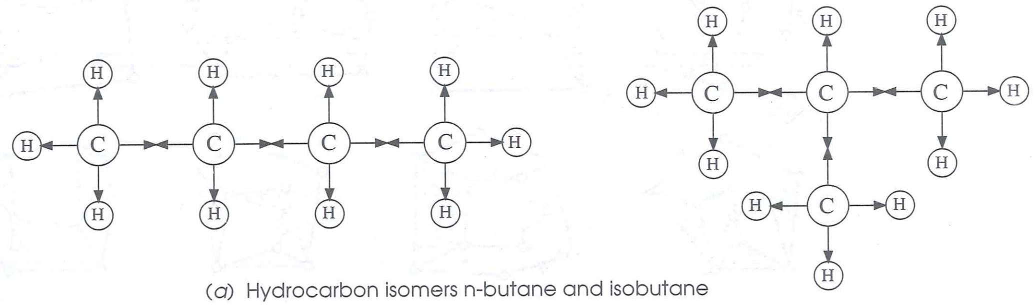
Figure 2-11b shows all the isomers for the simple cases of one *DOF* with 4 and 6 links. Note that there is only one isomer for the case of 4 links. An isomer is only unique if the interconnections between its types of links are different. That is, all binary links are considered equal, just as all hydrogen atoms are equal in the chemical analog. Link lengths and shapes do not figure into the Gruebler criterion or the condition of isomerism. The 6-link case of 4 binaries and 2 ternaries has only two valid isomers. These are known as **Watt's chain** and **Stephenson's chain** after their discoverers. Note the different interconnections of the ternaries to the binaries in these two examples. Watt's chain has the two ternaries directly connected, but Stephenson's chain does not.

There is also a third potential isomer for this case of six links, as shown in Figure 2-11c, but it fails the test of **distribution of degree of freedom**, which requires that the overall *DOF* (here 1) be uniformly distributed throughout the linkage and not concentrated in a subchain. Note that this arrangement (Figure 2-11c) has a **structural subchain** of $DOF = 0$ in the triangular formation of the two ternaries and the single binary connecting them. This creates a truss, or **delta triplet**. The remaining three binaries in series form a fourbar chain ($DOF = 1$) with the structural subchain of the two ternaries and the single binary effectively reduced to a structure that acts like a single link. Thus this arrangement has been reduced to the simpler case of the fourbar linkage despite its six bars. This is an **invalid isomer** and is rejected.

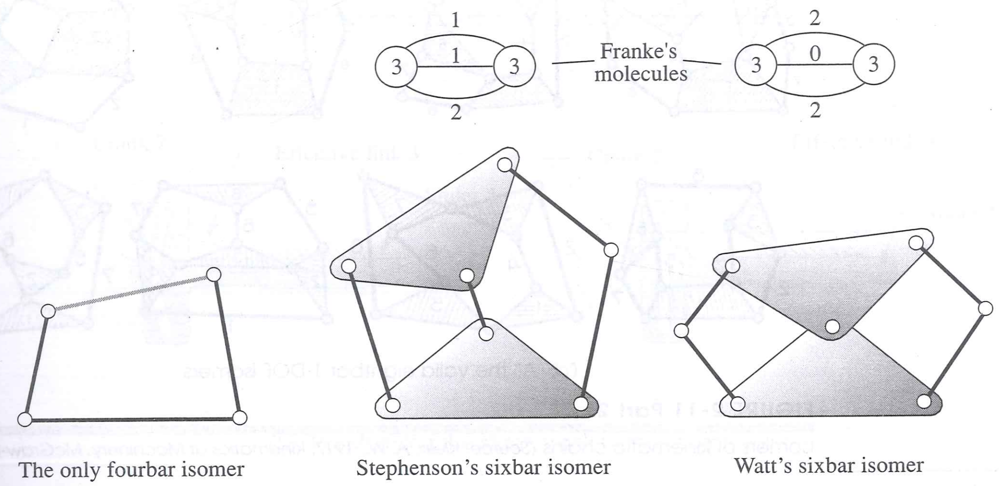
Franke's "Condensed Notation for Structural Synthesis" method can be used to help find the isomers of any collection of links that includes some links of higher order than binary. Each higher order link is shown as a circle with its number of nodes (its valence) written in it as shown in Figure 2-11. These circles are connected with a number of lines emanating from each circle equal to its valence. A number is placed on each line to represent the quantity of binary links in that connection. This gives a "molecular" representation of the linkage and allows exhaustive determination of all the possible binary link interconnections among the higher links. Note the correspondence in Figure 2-11b between the linkages and their respective Franke molecules. The only combinations of 3 integers (including zero) that add to 4 are: (1, 1, 2), (2, 0, 2), (0, 1, 3), and (0, 0, 4). The first two are, respectively, Stephenson's and Watt's linkages; the third is the invalid isomer of Figure 2-11c. The fourth combination is also invalid as it results in a 2-DOF chain of 5 binaries in series with the 5th "binary" comprised of the two ternaries locked together at two nodes in a preloaded structure with a subchain *DOF* of –1. Figure 2-11d shows all 16 valid isomers of the eightbar 1-DOF linkage.

2.10 LINKAGE TRANSFORMATION

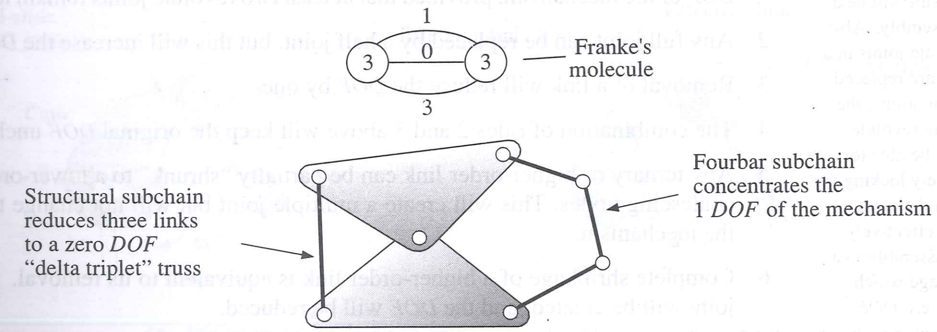
The number synthesis techniques described above give the designer a toolkit of basic linkages of particular *DOF*. If we now relax the arbitrary constraint that restricted us to only revolute joints, we can transform these basic linkages to a wider variety of mechanisms with even greater usefulness. There are several transformation techniques or rules that we can apply to planar kinematic chains.



(*a*) Hydrocarbon isomers n-butane and isobutane

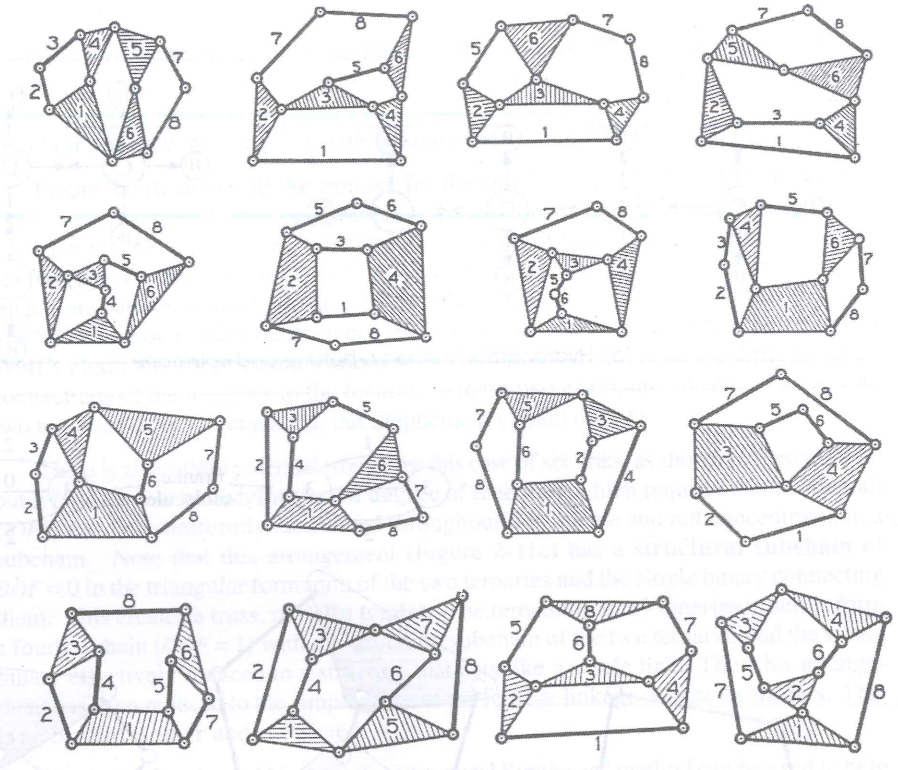


(*b*) All valid isomers of the fourbar and sixbar linkages



(*c*) An invalid sixbar isomer which reduces to the simpler fourbar

FIGURE 2-11 Part 1
Isomers of kinematic chains



(d) All the valid eightbar 1-DOF isomers

FIGURE 2-11 Part 2

Isomers of kinematic chains *(Source: Klein, A. W., 1917. Kinematics of Machinery, McGraw-Hill, NY)*

1 Revolute joints in any loop can be replaced by prismatic joints with no change in *DOF* of the mechanism, provided that at least two revolute joints remain in the loop.*

2 Any full joint can be replaced by a half joint, but this will increase the *DOF* by one.

3 Removal of a link will reduce the *DOF* by one.

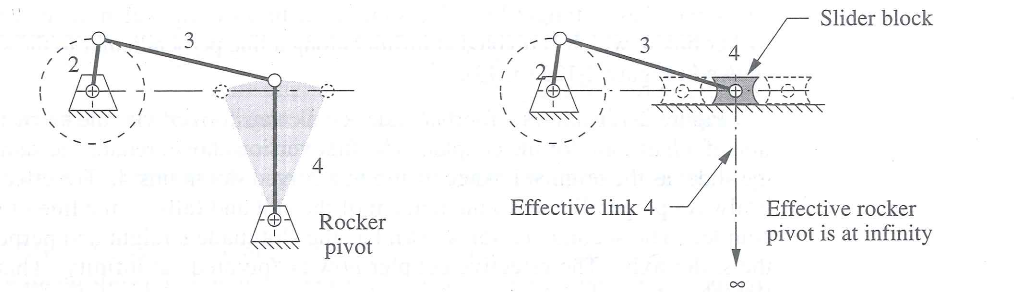
4 The combination of rules 2 and 3 above will keep the original *DOF* unchanged.

5 Any ternary or higher-order link can be partially "shrunk" to a lower-order link by coalescing nodes. This will create a multiple joint but will not change the *DOF* of the mechanism.

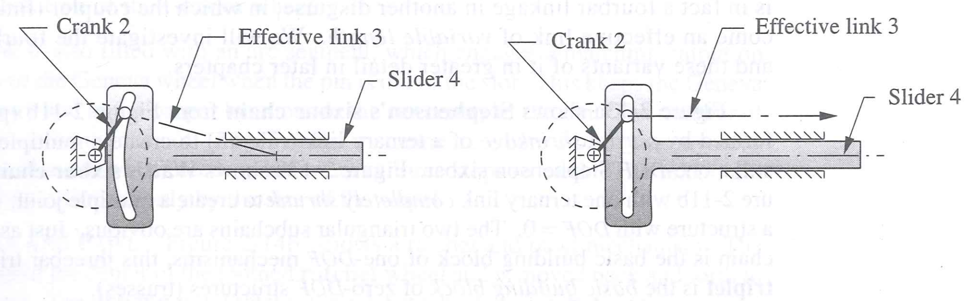
6 Complete shrinkage of a higher-order link is equivalent to its removal. A multiple joint will be created, and the *DOF* will be reduced.

Figure 2-12a shows a fourbar crank-rocker linkage transformed into the fourbar slider-crank by the application of rule #1. It is still a fourbar linkage. Link 4 has become a sliding block. Gruebler's equation is unchanged at one *DOF* because the slider

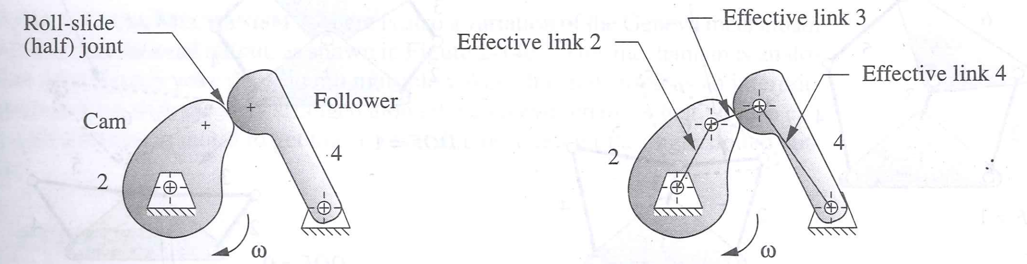
* If all revolute joints in a fourbar linkage are replaced by prismatic joints, the result will be a two-*DOF* assembly. Also, if three revolute joints in a fourbar loop are replaced with prismatic joints, the one remaining revolute joint will not be able to turn, effectively locking the two pinned links together as one. This effectively reduces the assembly to a threebar linkage which should have zero *DOF*. But, a delta triplet with three prismatic joints has one *DOF*—another Gruebler paradox.



(a) Transforming a fourbar crank-rocker to a slider-crank



(b) Transforming the slider-crank to the Scotch yoke



(c) The cam-follower mechanism has an effective fourbar equivalent

FIGURE 2-12

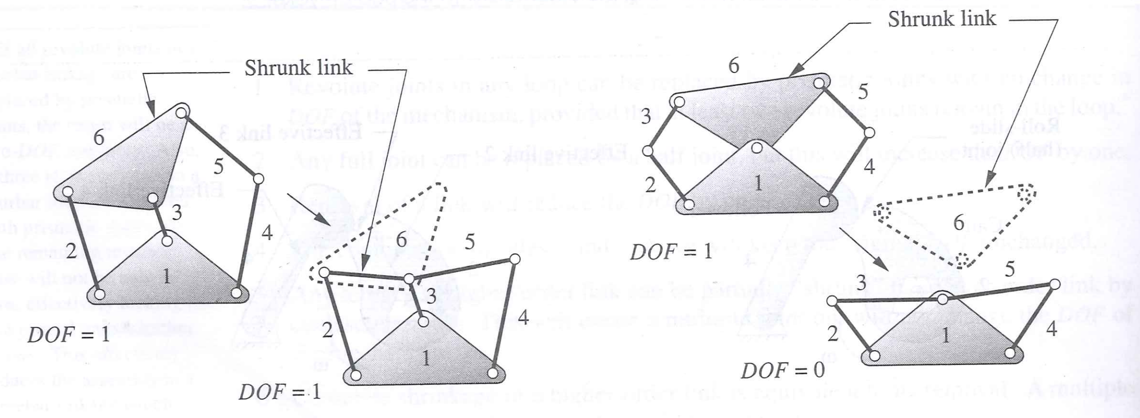
Linkage transformation

block provides a full joint against link 1, as did the pin joint it replaces. Note that this transformation from a rocking output link to a slider output link is equivalent to increasing the length (radius) of rocker link 4 until its arc motion at the joint between links 3 and 4 becomes a straight line. Thus the slider block is equivalent to an infinitely long rocker link 4, which is pivoted at infinity along a line perpendicular to the slider axis as shown in Figure 2-12a (p. 49).*

Figure 2-12b shows a fourbar slider-crank transformed via rule #4 by the substitution of a half joint for the coupler. The first version shown retains the same motion of the slider as the original linkage by use of a curved slot in link 4. The effective coupler is always perpendicular to the tangent of the slot and falls on the line of the original coupler. The second version shown has the slot made straight and perpendicular to the slider axis. The effective coupler now is "pivoted" at infinity. This is called a **Scotch yoke** and gives exact *simple harmonic motion* of the slider in response to a constant speed input to the crank.

Figure 2-12c shows a fourbar linkage transformed into a **cam-follower** linkage by the application of rule #4. Link 3 has been removed and a half joint substituted for a full joint between links 2 and 4. This still has one *DOF*, and the cam-follower is in fact a fourbar linkage in another disguise, in which the coupler (link 3) has become an effective link of *variable length*. We will investigate the fourbar linkage and these variants of it in greater detail in later chapters.

Figure 2-13a shows **Stephenson's sixbar chain** from Figure 2-11b (p. 47) transformed by *partial shrinkage* of a ternary link (rule #5) to create a multiple joint. It is still a one-*DOF* Stephenson sixbar. Figure 2-13b shows **Watt's sixbar chain** from Figure 2-11b with one ternary link *completely shrunk* to create a multiple joint. This is now a structure with *DOF* = 0. The two triangular subchains are obvious. Just as the fourbar chain is the basic building block of one-*DOF* mechanisms, this threebar triangle **delta triplet** is the *basic building block* of zero-*DOF* structures (trusses).



(*a*) Partial shrinkage of higher link retains original *DOF*

(*b*) Complete shrinkage of higher link reduces *DOF* by one

FIGURE 2-13
Link shrinkage

2.11 INTERMITTENT MOTION

Intermittent motion is *a sequence of motions and dwells*. A **dwell** is *a period in which the output link remains stationary while the input link continues to move*. There are many applications in machinery that require intermittent motion. The **cam-follower** variation on the fourbar linkage as shown in Figure 2-12c (p. 49) is often used in these situations. The design of that device for both intermittent and continuous output will be addressed in detail in Chapter 8. Other pure linkage **dwell mechanisms** are discussed in the next chapter.

GENEVA MECHANISM A common form of intermittent motion device is the **Geneva mechanism** shown in Figure 2-14a (p. 52). This is also a transformed fourbar linkage in which the coupler has been replaced by a half joint. The input crank (link 2) is typically motor driven at a constant speed. The **Geneva wheel** is fitted with at least three equispaced, radial slots. The crank has a pin that enters a radial slot and causes the Geneva wheel to turn through a portion of a revolution. When the pin leaves that slot, the Geneva wheel remains stationary until the pin enters the next slot. The result is intermittent rotation of the Geneva wheel.

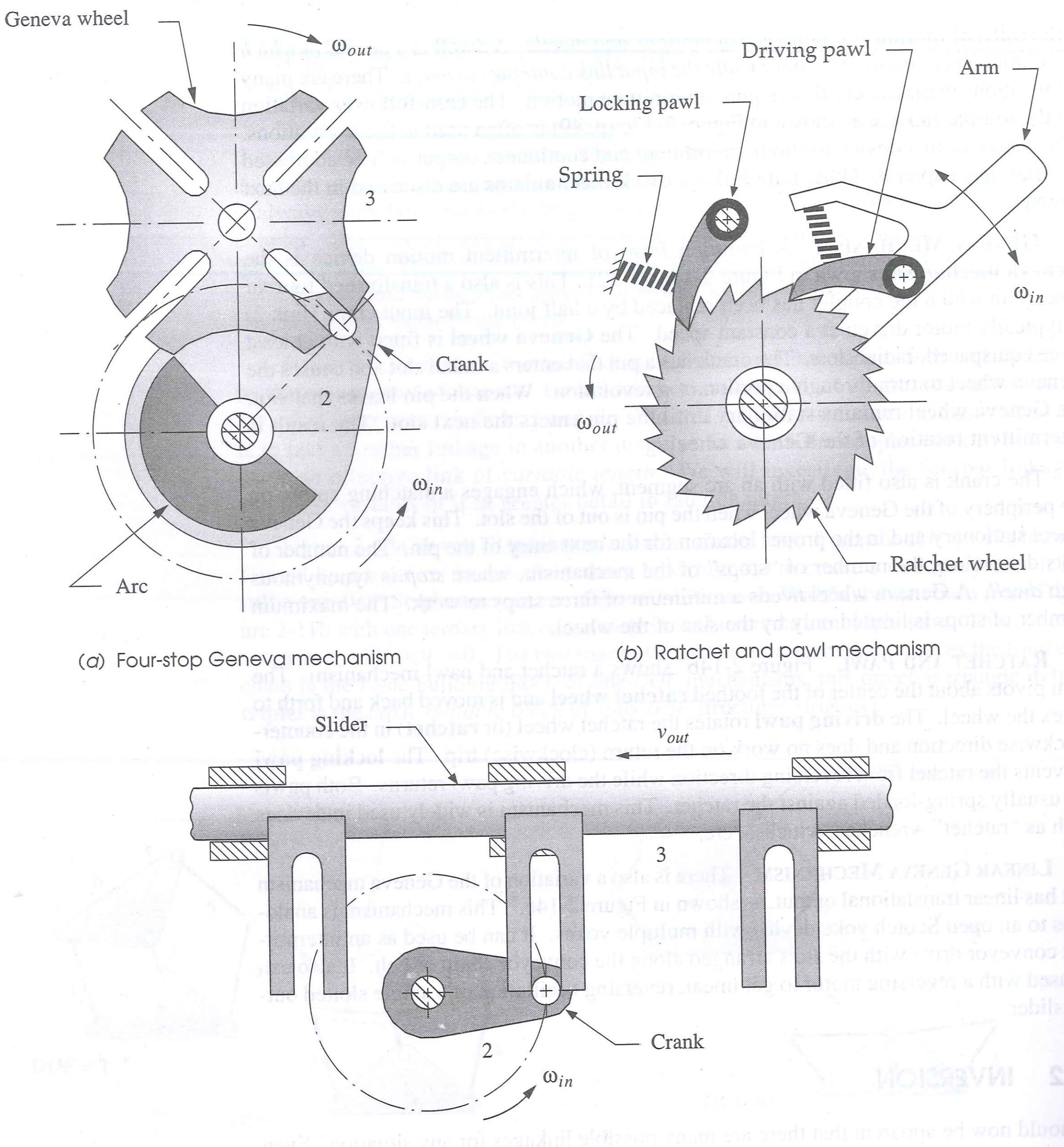
The crank is also fitted with an arc segment, which engages a matching cutout on the periphery of the Geneva wheel when the pin is out of the slot. This keeps the Geneva wheel stationary and in the proper location for the next entry of the pin. The number of slots determines the number of "stops" of the mechanism, where *stop* is synonymous with *dwell*. A Geneva wheel needs a minimum of three stops to work. The maximum number of stops is limited only by the size of the wheel.

RATCHET AND PAWL Figure 2-14b* shows a ratchet and pawl mechanism. The **arm** pivots about the center of the toothed **ratchet wheel** and is moved back and forth to index the wheel. The **driving pawl** rotates the ratchet wheel (or **ratchet**) in the counterclockwise direction and does no work on the return (clockwise) trip. The **locking pawl** prevents the ratchet from reversing direction while the driving pawl returns. Both pawls are usually spring-loaded against the ratchet. This mechanism is widely used in devices such as "ratchet" wrenches, winches, etc.

LINEAR GENEVA MECHANISM There is also a variation of the Geneva mechanism that has linear translational output, as shown in Figure 2-14c.* This mechanism is analogous to an open Scotch yoke device with multiple yokes. It can be used as an intermittent conveyor drive with the slots arranged along the conveyor chain or belt. It also can be used with a reversing motor to get linear, reversing oscillation of a single slotted output slider.

2.12 INVERSION

It should now be apparent that there are many possible linkages for any situation. Even with the limitations imposed in the number synthesis example (one *DOF*, eight links, up to hexagonal order), there are eight linkage combinations shown in Table 2-2 (p. 44), and these together yield 19 valid isomers as shown in Table 2-3 (p. 46). In addition, we can introduce another factor, namely mechanism inversion. An **inversion** is *created by grounding a different link in the kinematic chain*. Thus there are as many inversions of a given linkage as it has links.



(*a*) Four-stop Geneva mechanism

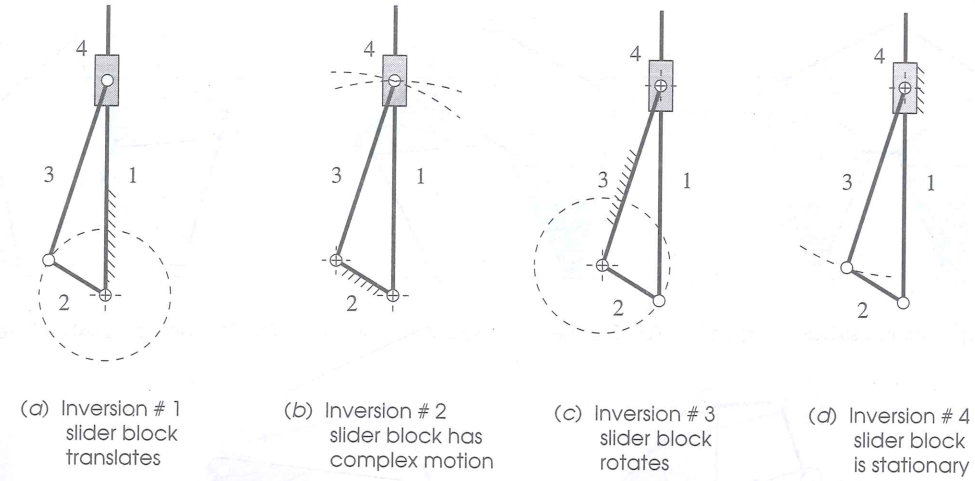
(*b*) Ratchet and pawl mechanism

(*c*) Linear intermittent motion "Geneva" mechanism

See also Figures P3-7 (p. 161) and P4-6 (p. 215) for other examples of linear intermittent motion mechanisms

FIGURE 2-14

Rotary and linear intermittent motion mechanisms



(*a*) Inversion # 1 slider block translates

(*b*) Inversion # 2 slider block has complex motion

(*c*) Inversion # 3 slider block rotates

(*d*) Inversion # 4 slider block is stationary

FIGURE 2-15

Four distinct inversions of the fourbar slider-crank mechanism (each black link is stationary—all red links move)

The motions resulting from each inversion can be quite different, but some inversions of a linkage may yield motions similar to other inversions of the same linkage. In these cases only some of the inversions may have distinctly different motions. We will denote the *inversions that have distinctly different motions* as **distinct inversions.**

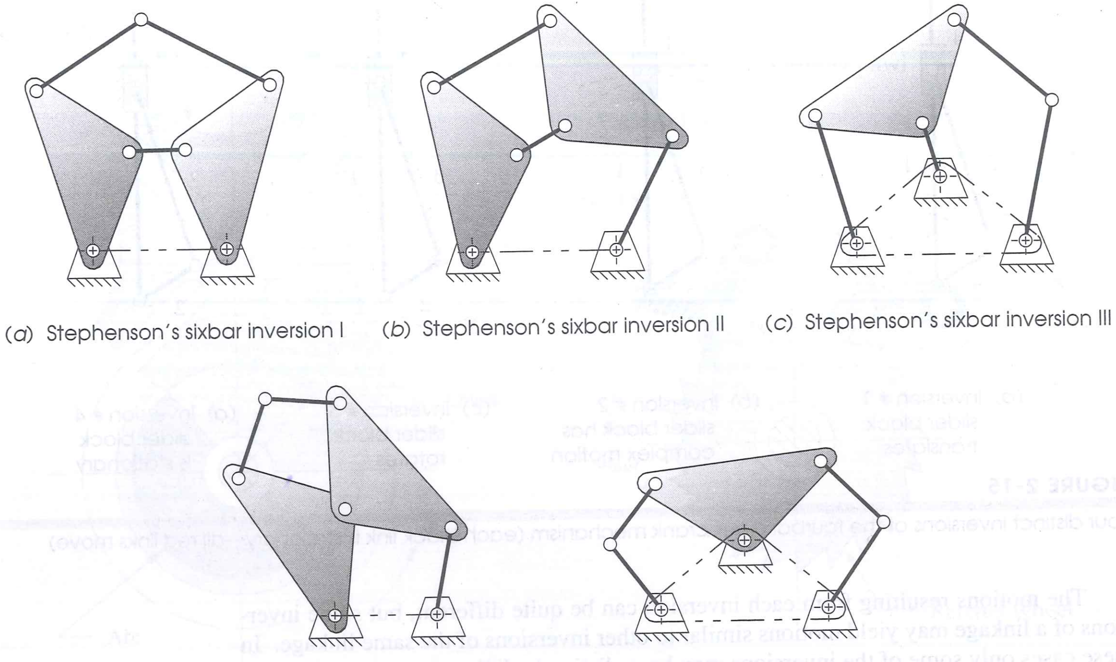
Figure 2-15* shows the four inversions of the fourbar slider-crank linkage, all of which have distinct motions. Inversion #1, with link 1 as ground and its slider block in pure translation, is the most commonly seen and is used for **piston engines** and **piston pumps**. Inversion #2 is obtained by grounding link 2 and gives the **Whitworth** or **crank-shaper** quick-return mechanism, in which the slider block has complex motion. (Quick-return mechanisms will be investigated further in the next chapter.) Inversion #3 is obtained by grounding link 3 and gives the slider block pure rotation. Inversion #4 is obtained by grounding the slider link 4 and is used in hand operated, **well pump** mechanisms, in which the handle is link 2 (extended) and link 1 passes down the well pipe to mount a piston on its bottom. (It is upside down in the figure.)

Watt's sixbar chain has two distinct inversions, and **Stephenson's sixbar** has three distinct inversions, as shown in Figure 2-16. The pin-jointed fourbar has four distinct inversions: the crank-rocker, double-crank, double-rocker, and triple-rocker which are shown in Figures 2-17* (p. 55) and 2-18 (p. 56).

2.13 THE GRASHOF CONDITION

The **fourbar linkage** has been shown above to be the *simplest possible pin-jointed mechanism* for single-degree-of-freedom controlled motion. It also appears in various disguises such as the **slider-crank** and the **cam-follower**. It is in fact the most common and ubiquitous device used in machinery. It is also extremely versatile in terms of the types of motion that it can generate.

* According to Hunt[18] (p. 84), Waldron proved that in a Grashof linkage, no two of the links other than the crank can rotate more than 180° with respect to one another, but in a non-Grashof linkage (which has no crank) links can have more than 180° of relative rotation.



(a) Stephenson's sixbar inversion I (b) Stephenson's sixbar inversion II (c) Stephenson's sixbar inversion III

(d) Watt's sixbar inversion I (e) Watt's sixbar inversion II

FIGURE 2-16
All distinct inversions of the sixbar linkage

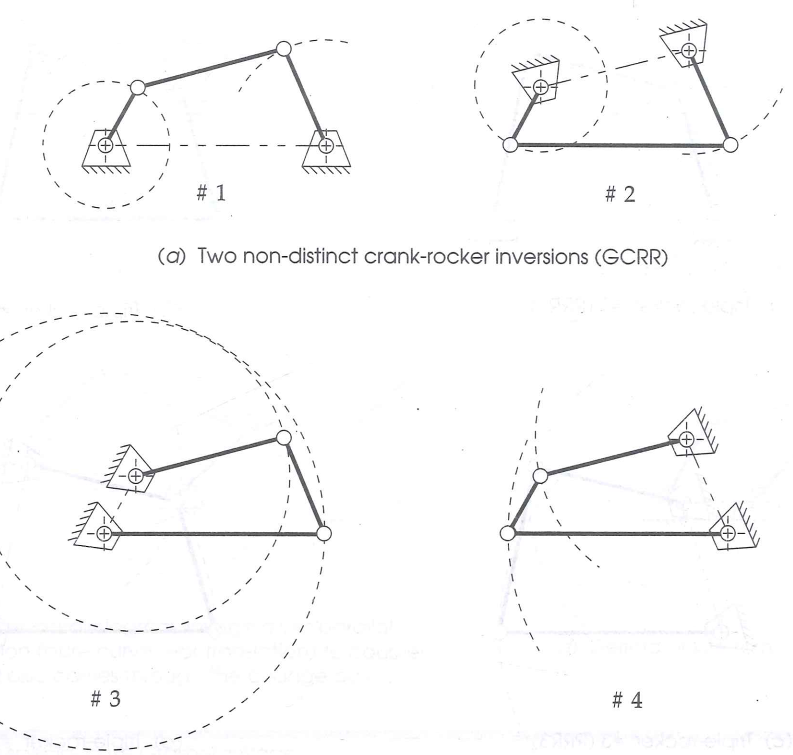
Simplicity is one mark of good design. The fewest parts that can do the job will usually give the least expensive and most reliable solution. Thus the **fourbar linkage** should be among the first solutions to motion control problems to be investigated. The **Grashof condition** [8] is a very simple relationship that predicts the *rotation behavior* or **rotatability** of a fourbar linkage's inversions based only on the link lengths.

Let: S = length of shortest link
L = length of longest link
P = length of one remaining link
Q = length of other remaining link

Then if:
$$S + L \le P + Q \tag{2.8}$$

the linkage is **Grashof** and at least one link will be capable of making a full revolution with respect to the ground plane. This is called a **Class I** kinematic chain. If the inequality is not true, then the linkage is **non-Grashof** and *no* link will be capable of a complete revolution relative to any other link.* This is a **Class II** kinematic chain.

Note that the above statements apply regardless of the order of assembly of the links. That is, the determination of the Grashof condition can be made on a set of unassembled links. Whether they are later assembled into a kinematic chain in S, L, P, Q, or S, P, L, Q or any other order, will *not* change the Grashof condition.



(a) Two non-distinct crank-rocker inversions (GCRR)

(b) Double-crank inversion (GCCC) (drag link mechanism)

(c) Double-rocker inversion (GRCR) (coupler rotates)

FIGURE 2-17
All inversions of the Grashof fourbar linkage

The motions possible from a fourbar linkage will depend on both the Grashof condition and the **inversion** chosen. The inversions will be defined with respect to the shortest link. The motions are:

For the Class I case, $S + L < P + Q$:

Ground either link adjacent to the shortest and you get a **crank-rocker**, in which the shortest link will fully rotate and the other link pivoted to ground will oscillate.

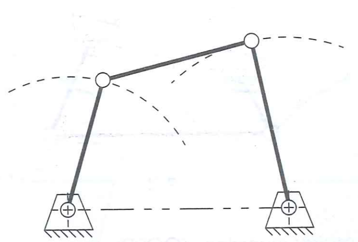
Ground the shortest link and you will get a **double-crank**, in which both links pivoted to ground make complete revolutions as does the coupler.

Ground the link opposite the shortest and you will get a **Grashof double-rocker**, in which both links pivoted to ground oscillate and only the coupler makes a full revolution.

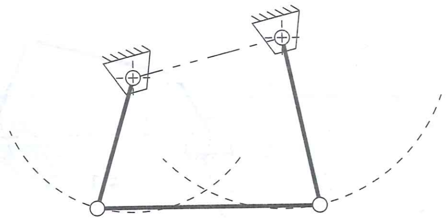
For the Class II case, $S + L > P + Q$:

All inversions will be **triple-rockers** [9] in which no link can fully rotate.

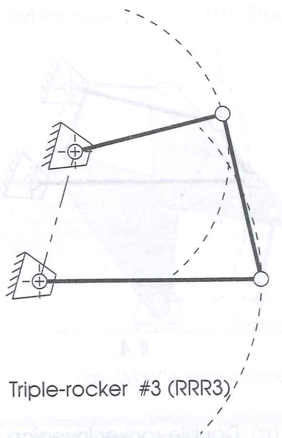
For the Class III case, $S + L = P + Q$:



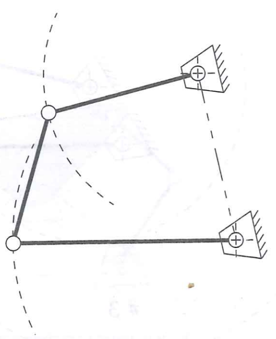
(a) Triple-rocker #1 (RRR1)



(b) Triple-rocker #2 (RRR2)



(c) Triple-rocker #3 (RRR3)



(d) Triple-rocker #4 (RRR4)

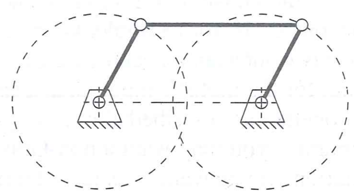
FIGURE 2-18

All inversions of the non-Grashof fourbar linkage are triple rockers

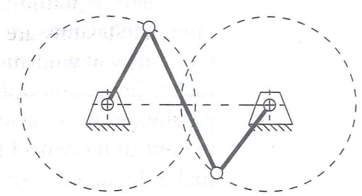
Referred to as **special-case Grashof** and also as a **Class III** kinematic chain, all inversions will be either **double-cranks** or **crank-rockers** but will have **"change points"** twice per revolution of the input crank when the links all become colinear. At these change points the output behavior will become indeterminate. Hunt[18] calls these **"uncertainty configurations."** At these colinear positions, the linkage behavior is unpredictable as it may assume either of two configurations. Its motion must be limited to avoid reaching the change points or an additional, out-of-phase link provided to guarantee a "carry through" of the change points. (See Figure 2-19c.)

Figure 2-17* (p. 55) shows the four possible inversions of the **Grashof case**: two crank-rockers, a double-crank (also called a drag link), and a double-rocker with rotating coupler. The two crank-rockers give similar motions and so are not distinct from one another. Figure 2-18 shows four non-distinct inversions, all triple-rockers, of a **non-Grashof linkage**.

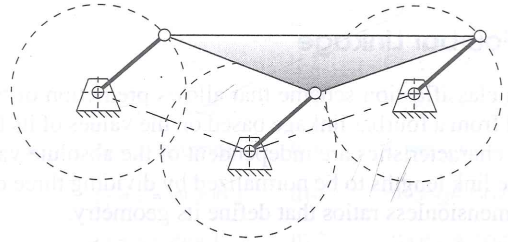
Figure 2-19a and b shows the **parallelogram** and **antiparallelogram** configurations of the **special-case Grashof** linkage. The parallelogram linkage is quite useful as it exactly duplicates the rotary motion of the driver crank at the driven crank. One common use is to couple the two windshield wiper output rockers across the width of the windshield on an automobile. The coupler of the parallelogram linkage is in curvilinear translation, remaining at the same angle while all points on it describe identical circular paths. It is often used for this parallel motion, as in truck tailgate lifts and industrial robots.



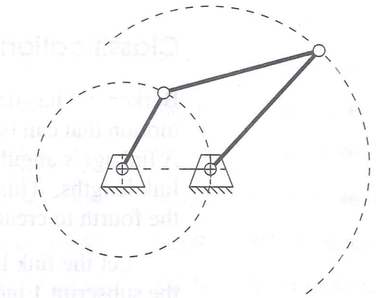
(a) Parallelogram form



(b) Antiparallelogram form



(c) Double-parallelogram linkage gives parallel motion (pure curvilinear translation) to coupler and also carries through the change points



(d) Deltoid or kite form

FIGURE 2-19

Some forms of the special-case Grashof linkage

The antiparallelogram linkage (also called "butterfly" or "bow-tie") is also a double-crank, but the output crank has an angular velocity different from the input crank. Note that the change points allow the linkage to switch unpredictably between the parallelogram and antiparallelogram forms every 180 degrees unless some additional links are provided to carry it through those positions. This can be achieved by adding an out-of-phase companion linkage coupled to the same crank, as shown in Figure 2-19c. A common application of this double parallelogram linkage was on steam locomotives, used to connect the drive wheels together. The change points were handled by providing the duplicate linkage, 90 degrees out of phase, on the other side of the locomotive's axle shaft. When one side was at a change point, the other side would drive it through.

The **double-parallelogram** arrangement shown in Figure 2-19c is quite useful as it gives a translating coupler that remains horizontal in all positions. The two parallelogram stages of the linkage are out of phase so each carries the other through its change points. Figure 2-19d shows the **deltoid** or **kite** configuration that is a double-crank in which the shorter crank makes two revolutions for each one made by the long crank. This is also called an **isoceles** linkage or a **Galloway** mechanism after its discoverer.

There is nothing either bad or good about the Grashof condition. Linkages of all three persuasions are equally useful in their place. If, for example, your need is for a motor driven windshield wiper linkage, you may want a non-special-case Grashof crank-rocker linkage in order to have a rotating link for the motor's input, plus a special-case parallelogram stage to couple the two sides together as described above. If your need is to control the wheel motions of a car over bumps, you may want a non-Grashof triple-rocker linkage for short stroke oscillatory motion. If you want to exactly duplicate some input motion at a remote location, you may want a special-case Grashof parallelogram linkage, as used in a drafting machine. In any case, this simply determined condition tells volumes about the behavior to be expected from a proposed fourbar linkage design prior to any construction of models or prototypes.*

Classification of the Fourbar Linkage

Barker [10] has developed a classification scheme that allows prediction of the type of motion that can be expected from a fourbar linkage based on the values of its link ratios. A linkage's angular motion characteristics are independent of the absolute values of its link lengths. This allows the link lengths to be normalized by dividing three of them by the fourth to create three dimensionless ratios that define its geometry.

Let the link lengths be designated r_1, r_2, r_3, and r_4 (all positive and nonzero), with the subscript 1 indicating the ground link, 2 the driving link, 3 the coupler, and 4 the remaining (output) link. The link ratios are then formed by dividing each link length by r_2 giving: $\lambda_1 = r_1 / r_2$, $\lambda_3 = r_3 / r_2$, $\lambda_4 = r_4 / r_2$.

Each link will also be given a letter designation based on its type of motion when connected to the other links. If a link can make a full revolution with respect to the other links, it is called a crank (C), and if not, a rocker (R). The motion of the assembled linkage based on its Grashof condition and inversion can then be given a letter code such as GCRR for a Grashof crank-rocker or GCCC for a Grashof double-crank (drag link) mechanism. The motion designators C and R are always listed in the order of input link, coupler, output link. The prefix G indicates a Grashof linkage, S a special-case Grashof (change point), and no prefix a non-Grashof linkage.

Table 2-4 shows Barker's 14 types of fourbar linkage based on this naming scheme. The first four rows are the Grashof inversions, the next four are the non-Grashof triple rockers, and the last six are the special-case Grashof linkages. He gives unique names to each type based on a combination of their Grashof condition and inversion. The traditional names for the same inversions are also shown for comparison and are less specific than Barker's nomenclature. Note his differentiation between the Grashof crank-rocker (subclass -2) and rocker-crank (subclass -4). To drive a GRRC linkage from the rocker requires adding a flywheel to the crank as is done with the internal combustion engine's slider-crank mechanism (which is a GPRC linkage). See Figure 2-12a (p. 49).

Barker also defines a "solution space" whose axes are the link ratios λ_1, λ_3, λ_4 as shown in Figure 2-20. These ratios' values theoretically extend to infinity, but for any practical linkages the ratios can be limited to a reasonable value.

In order for the four links to be assembled, the longest link must be shorter than the sum of the other three links,

$$L < (S + P + Q) \tag{2.9}$$

TABLE 2-4 Barker's Complete Classification of Planar Fourbar Mechanisms

Adapted from ref. (10). s = shortest link, l = longest link, Gxxx = Grashof, RRRx = non-Grashof, Sxx = Special case

Type	$s + l$ vs. $p + q$	Inversion	Class	Barker's Designation	Code	Also Known As
1	<	$L_1 = s$ = ground	I-1	Grashof crank-crank-crank	GCCC	double-crank
2	<	$L_2 = s$ = input	I-2	Grashof crank-rocker-rocker	GCRR	crank-rocker
3	<	$L_3 = s$ = coupler	I-3	Grashof rocker-crank-rocker	GRCR	double-rocker
4	<	$L_4 = s$ = output	I-4	Grashof rocker-rocker-crank	GRRC	rocker-crank
5	>	$L_1 = l$ = ground	II-1	Class 1 rocker-rocker-rocker	RRR1	triple-rocker
6	>	$L_2 = l$ = input	II-2	Class 2 rocker-rocker-rocker	RRR2	triple-rocker
7	>	$L_3 = l$ = coupler	II-3	Class 3 rocker-rocker-rocker	RRR3	triple-rocker
8	>	$L_4 = l$ = output	II-4	Class 4 rocker-rocker-rocker	RRR4	triple-rocker
9	=	$L_1 = s$ = ground	III-1	change point crank-crank-crank	SCCC	SC* double -crank
10	=	$L_2 = s$ = input	III-2	change point crank-rocker-rocker	SCRR	SC crank-rocker
11	=	$L_3 = s$ = coupler	III-3	change point rocker-crank-rocker	SRCR	SC double-rocker
12	=	$L_4 = s$ = output	III-4	change point rocker-rocker-crank	SRRC	SC rocker-crank
13	=	two equal pairs	III-5	double change point	S2X	parallelogram or deltoid
14	=	$L_1 = L_2 = L_3 = L_4$	III-6	triple change point	S3X	square

* SC = special case.

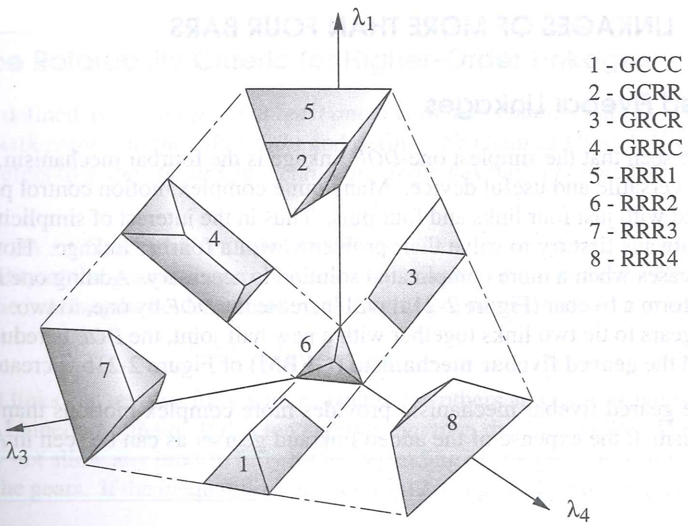


FIGURE 2-20

Barker's solution space for the fourbar linkage *Adapted from reference (10).*

If $L = (S + P + Q)$, then the links can be assembled but will not move, so this condition provides a criterion to separate regions of no mobility from regions that allow mobility within the solution space. Applying this criterion in terms of the three link ratios defines four planes of zero mobility that provide limits to the solution space.

$$\begin{aligned} 1 &= \lambda_1 + \lambda_3 + \lambda_4 \\ \lambda_3 &= \lambda_1 + 1 + \lambda_4 \\ \lambda_4 &= \lambda_1 + 1 + \lambda_3 \\ \lambda_1 &= 1 + \lambda_3 + \lambda_4 \end{aligned} \tag{2.10}$$

Applying the $S + L = P + Q$ Grashof condition (in terms of the link ratios) defines three additional planes on which the change-point mechanisms all lie.

$$\begin{aligned} 1 + \lambda_1 &= \lambda_3 + \lambda_4 \\ 1 + \lambda_3 &= \lambda_1 + \lambda_4 \\ 1 + \lambda_4 &= \lambda_1 + \lambda_3 \end{aligned} \tag{2.11}$$

The positive octant of this space, bounded by the λ_1–λ_3, λ_1–λ_4, λ_3–λ_4 planes and the four zero-mobility planes (equation 2.10) contains eight volumes that are separated by the change-point planes (equation 2.11). Each volume contains mechanisms unique to one of the first eight classifications in Table 2-4. These eight volumes are in contact with one another in the solution space, but to show their shapes, they have been "exploded" apart in Figure 2-20 (p. 59). The remaining six change-point mechanisms of Table 2-4 (p. 59) exist only in the change-point planes that are the interfaces between the eight volumes. For more detail on this solution space and Barker's classification system than space permits here, see reference [10].

2.14 LINKAGES OF MORE THAN FOUR BARS

Geared Fivebar Linkages

We have seen that the simplest one-*DOF* linkage is the fourbar mechanism. It is an extremely versatile and useful device. Many quite complex motion control problems can be solved with just four links and four pins. Thus in the interest of simplicity, designers should always first try to solve their problems with a fourbar linkage. However, there will be cases when a more complicated solution is necessary. Adding one link and one joint to form a fivebar (Figure 2-21a) will increase the *DOF* by one, to two. By adding a pair of gears to tie two links together with a new half joint, the *DOF* is reduced again to one, and the **geared fivebar mechanism (GFBM)** of Figure 2-21b is created.

The geared fivebar mechanism provides more complex motions than the fourbar mechanism at the expense of the added link and gearset as can be seen in Appendix D.

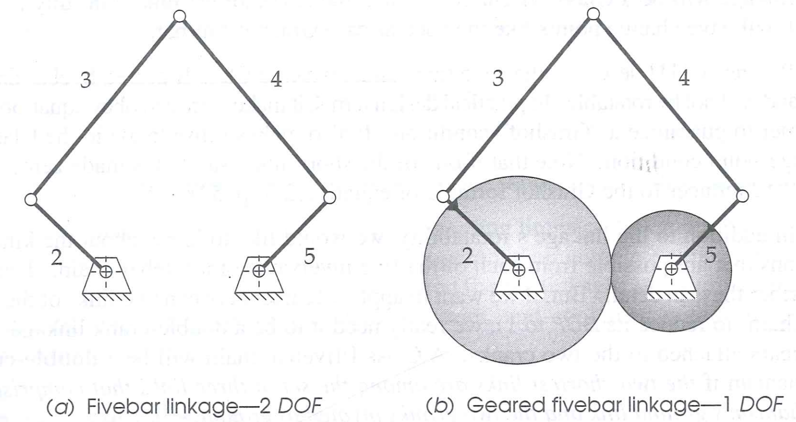


FIGURE 2-21
Two forms of the fivebar linkage

Sixbar Linkages

We already met Watt's and Stephenson's sixbar mechanisms. See Figure 2-16 (p. 54). **Watt's sixbar** can be thought of as *two fourbar linkages connected in series* and sharing two links in common. **Stephenson's sixbar** can be thought of as *two fourbar linkages connected in parallel* and sharing two links in common. Many linkages can be designed by the technique of combining multiple fourbar chains as *basic building blocks* into more complex assemblages. Many real design problems will require solutions consisting of more than four bars.

Grashof-Type Rotatability Criteria for Higher-Order Linkages

Rotatability is defined as *the ability of at least one link in a kinematic chain to make a full revolution with respect to the other links* and defines the chain as Class I, II or III. **Revolvability** refers to *a specific link in a chain and indicates that it is one of the links that can rotate.*

Rotatability of Geared Fivebar Linkages Ting [11] has derived an expression for rotatability of the geared fivebar linkage that is similar to the fourbar's Grashof criterion. Let the link lengths be designated L_1 through L_5 in order of increasing length,

then if : $$L_1 + L_2 + L_5 < L_3 + L_4 \tag{2.12}$$

the two shortest links can revolve fully with respect to the others and the linkage is designated a **Class I** kinematic chain. If this inequality is *not* true, then it is a **Class II** chain and may or may not allow any links to fully rotate depending on the gear ratio and phase angle between the gears. If the inequality of equation 2.12 is replaced with an equal sign, the linkage will be a **Class III** chain in which the two shortest links can fully revolve but it will have change points like the special-case Grashof fourbar.

Reference [11] describes the conditions under which a Class II geared fivebar linkage will and will not be rotatable. In practical design terms, it makes sense to obey equation 2.12 in order to guarantee a "Grashof" condition. It also makes sense to avoid the Class III change-point condition. Note that if one of the short links (say L_2) is made zero, equation 2.12 reduces to the Grashof formula of equation 2.8 (p. 54).

In addition to the linkage's rotatability, we would like to know about the kinds of motions that are possible from each of the five inversions of a fivebar chain. Ting [11] describes these in detail. But, if we want to apply a gearset between two links of the fivebar chain (to reduce its *DOF* to 1), we really need it to be a double-crank linkage, with the gears attached to the two cranks. A Class I fivebar chain will be a **double-crank** mechanism if *the two shortest links are among the set of three links that comprise the mechanism's ground link and the two cranks pivoted to ground.* [11]

Rotatability of *N*-bar Linkages Ting et al. [12], [13] have extended rotatability criteria to all single-loop linkages of *N*-bars connected with revolute joints and have developed general theorems for **linkage rotatability** and the **revolvability** of individual links based on link lengths. Let the links of an *N*-bar linkage be denoted by L_i ($i = 1, 2, \ldots N$), with $L_1 \le L_2 \le \cdots \le L_N$. The links need not be connected in any particular order as rotatability criteria are independent of that factor.

A single-loop, revolute-jointed linkage of *N* links will have $(N-3)$ *DOF*. The necessary and sufficient condition for the **assemblability** of an *N*-bar linkage is:

$$L_N \le \sum_{k=1}^{N-1} L_k \tag{2.13}$$

A link *K* will be a so-called *short* link if

$$\{K\}_{k=1}^{N-3} \tag{2.14a}$$

and a so-called *long* link if

$$\{K\}_{k=N-2}^{N} \tag{2.14b}$$

There will be three long links and $(N-3)$ short links in any linkage of this type.

A single-loop *N*-bar kinematic chain containing only first-order revolute joints will be a Class I, Class II, or Class III linkage depending on whether the sum of the lengths of its longest link and its $(N-3)$ shortest links is, respectively, less than, greater than, or equal to the sum of the lengths of the remaining two long links:

$$\begin{aligned}
&\text{Class I:} && L_N + (L_1 + L_2 + \cdots + L_{N-3}) < L_{N-2} + L_{N-1} \\
&\text{Class II:} && L_N + (L_1 + L_2 + \cdots + L_{N-3}) > L_{N-2} + L_{N-1} \\
&\text{Class III:} && L_N + (L_1 + L_2 + \cdots + L_{N-3}) = L_{N-2} + L_{N-1}
\end{aligned} \tag{2.15}$$

and, for a Class I linkage, there must be one and only one long link between two non-input angles. These conditions are necessary and sufficient to define the rotatability.

The **revolvability** of any link L_i is defined as its ability to rotate fully with respect to the other links in the chain and can be determined from:

$$L_i + L_N \le \sum_{k=1,\, k \ne i}^{N-1} L_k \tag{2.16}$$

Also, if L_i is a revolvable link, any link that is not longer than L_i will also be revolvable.

Additional theorems and corollaries regarding limits on link motions can be found in references [12] and [13]. Space does not permit their complete exposition here. Note that the rules regarding the behavior of geared fivebar linkages and fourbar linkages (the Grashof law) stated above are consistent with, and contained within, these general rotatability theorems.

2.15 SPRINGS AS LINKS

We have so far been dealing only with rigid links. In many mechanisms and machines, it is necessary to counterbalance the static loads applied to the device. A common example is the hood hinge mechanism on your automobile. Unless you have the (cheap) model with the strut that you place in a hole to hold up the hood, it will probably have either a fourbar or sixbar linkage connecting the hood to the body on each side. The hood may be the coupler of a non-Grashof linkage whose two rockers are pivoted to the body. A spring is fitted between two of the links to provide a force to hold the hood in the open position. The spring in this case is an additional link of variable length. As long as it can provide the right amount of force, it acts to reduce the *DOF* of the mechanism to zero, and holds the system in static equilibrium. However, you can force it to again be a one-*DOF* system by overcoming the spring force when you pull the hood shut.

Another example, which may now be right next to you, is the ubiquitous adjustable arm desk lamp, shown in Figure 2-22. This device has two springs that counterbalance the weight of the links and lamp head. If well designed and made, it will remain stable over a fairly wide range of positions despite variation in the overturning moment due to the lamp head's changing moment arm. This is accomplished by careful design of the geometry of the spring-link relationships so that, as the spring force changes with increasing length, its moment arm also changes in a way that continually balances the changing moment of the lamp head.

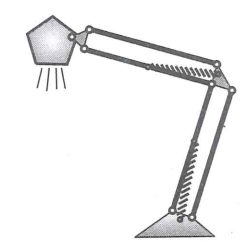


FIGURE 2-22
A spring-balanced linkage mechanism

A linear spring can be characterized by its spring constant, $k = F / x$, where F is force and x is spring displacement. Doubling its deflection will double the force. Most coil springs of the type used in these examples are linear.

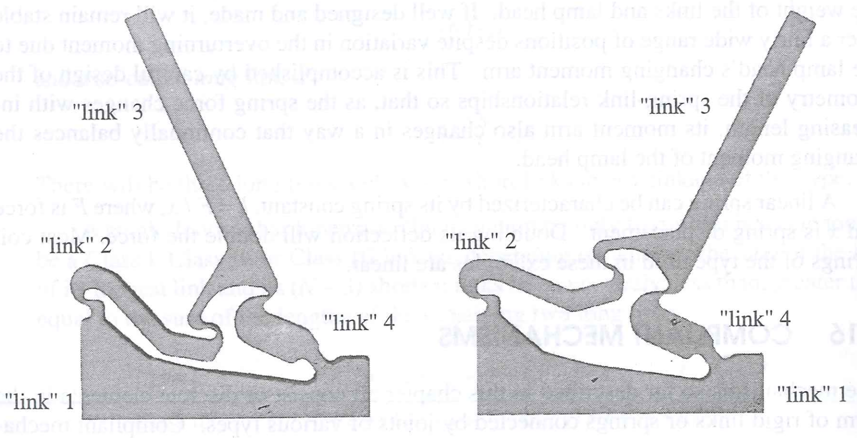
2.16 COMPLIANT MECHANISMS

The mechanisms so far described in this chapter all consist of discrete elements in the form of rigid links or springs connected by joints of various types. Compliant mechanisms can provide similar motions with fewer parts and fewer (even zero) physical joints. Compliance is the opposite of stiffness. A member or "link" that is compliant is capable of significant deflection in response to load. An ancient example of a compliant mechanism is the bow and arrow, in which the bow's deflection in response to the archer pulling back the bowstring stores elastic strain energy in the flexible (compliant) bow, and that energy launches the arrow.

The bow and bowstring comprise two parts, but in its purest form a compliant mechanism consists of a single link whose shape is carefully designed to provide areas of flexibility that serve as pseudo joints. Probably the most commonly available example of a simple compliant mechanism is the ubiquitous plastic tackle box or toolbox made with a "living hinge" as shown in Figure 2-23. This is a dyad or two-link mechanism (box and cover) with a thin section of material connecting the two. Certain thermoplastics, such as polypropylene, allow thin sections to be flexed repeatedly without failure. When the part is removed from the mold, and is still warm, the hinge must be flexed once to align the material's molecules. Once cooled, it can withstand millions of open-close cycles without failure. Figure 2-24 shows a prototype of a fourbar linkage toggle switch made in one piece of plastic as a compliant mechanism. It moves between the on and off positions by flexure of the thin hinge sections that serve as pseudo joints between the "links." The case study discussed in Chapter 1 describes the design of a compliant mechanism that is also shown in Figure 6-13 (p. 291).

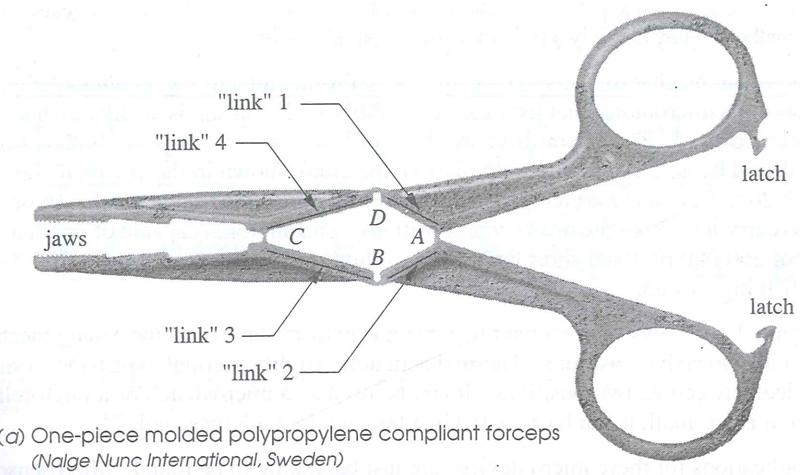
FIGURE 2-23
A tackle box with "living hinge" *Courtesy of Penn Plastics Inc, Bridgeport, CT*

Figure 2-25a shows a forceps designed as a one-piece compliant mechanism. Instead of the conventional two pieces connected by a pin joint, this forceps has small cross sections designed to serve as pseudo joints. It is injection molded of polypropylene thermoplastic with "living hinges." Note that there is a fourbar linkage 1, 2, 3, 4 at the center whose "joints" are the compliant sections of small dimension at A, B, C, and D. The compliance of the material in these small sections provides a built-in spring effect to hold it open in the rest condition. The other portions of the device such as the handles and jaws are designed with stiffer geometry to minimize their deflections. When the user closes the jaws, the hooks on the handles latch it closed, clamping the gripped item. Figure 2-25b shows a two-piece snap hook that uses the compliance of the spring closure that results from either ear of the wire spring being pivoted at different locations A_1 and A_2.

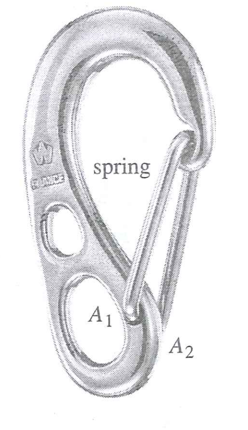


(*a*) Switch on (*b*) Switch off

FIGURE 2-24
One-piece compliant switch *Courtesy of Professor Larry L. Howell, Brigham Young University*



(*a*) One-piece molded polypropylene compliant forceps *(Nalge Nunc International, Sweden)*



(*b*) Compliant snap hook ™ *(Wichard USA, Portsmouth, RI)*

FIGURE 2-25
Compliant mechanisms

These examples show some of the advantages of compliant mechanisms over conventional ones. No assembly operations are needed, as there is only one part. The needed spring effect is built in by control of geometry in local areas. The finished part is ready to use as it comes out of the mold. These features all reduce cost.

Compliant mechanisms have been in use for a long time (e.g., the bow and arrow, fingernail clipper, paper clips), but found new applications in the late 20th century due in part to the availability of new materials and modern manufacturing processes. Some of their advantages over conventional mechanisms are the reduction of number of parts, elimination of joint clearances, inherent spring loading, and potential reductions in cost, weight, wear, and maintenance compared to conventional mechanisms. They are, however, more difficult to design and analyze because of their relatively large deflections that preclude the use of conventional small-deflection theory. This text will consider only the design and analysis of noncompliant (i.e., assumed rigid) links and mechanisms with physical joints. For information on the design and analysis of compliant mechanisms see reference 16.

2.17 MICRO ELECTRO-MECHANICAL SYSTEMS (MEMS)*

Recent advances in the manufacture of microcircuitry such as computer chips have led to a new form of mechanism known as micro electro-mechanical systems or MEMS. These devices have features measured in micrometers, and micromachines range in size from a few micrometers to a few millimeters. They are made from the same silicon wafer material that is used for integrated circuits or microchips. The shape or pattern of the desired device (mechanism, gear, etc.) is computer generated at large scale and then photographically reduced and projected onto the wafer. An etching process then removes the silicon material where the image either did or did not alter the photosensitive coating on the silicon (the process can be set to do either). What remains is a tiny reproduction

* More information on MEMS can be found at: http://www.sandia.gov/ and http://www.memsnet.org/mems/

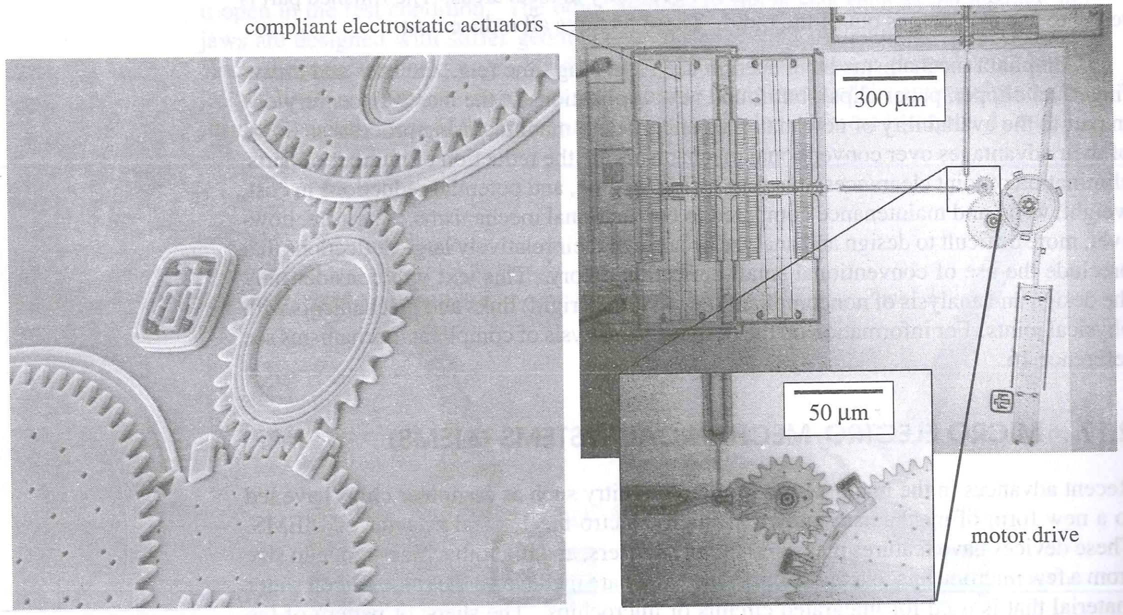
of the original geometric pattern in silicon. Figure 2-26a shows silicon microgears made by this method. They are only a few micrometers in diameter.

Compliant mechanisms are very adaptable to this manufacturing technique. Figure 2-26b shows a micromotor that uses the gears of Figure 2-26a and is smaller than a few millimeters overall. The motor drive mechanism is a series of compliant linkages that are oscillated by an electrostatic field to drive the crank shown in the enlarged view of Figure 2-26b. Two of these electrostatic actuators operate on the same crank, 90° out of phase to carry it through the dead center positions. This motor is capable of continuous speeds of 360 000 rpm and short bursts to a million rpm before overheating from friction at that high speed.

Figure 2-27 shows "a compliant bistable mechanism (known as the Young mechanism) in its two stable positions. Thermal actuators amplify thermal expansion to snap the device between its two positions. It can be used as a microswitch or a microrelay. Because it is so small, it can be actuated in a few hundred microseconds."†

Applications for these micro devices are just beginning to be found. Microsensors made with this technology are currently used in automobile airbag assemblies to detect sudden deceleration and fire the airbag inflator. MEMS blood pressure monitors that can be placed in a blood vessel have been made. MEMS pressure sensors are being fitted to automobile tires to continuously monitor tire pressure. Many other applications are being and will be developed to utilize this technology in the future.

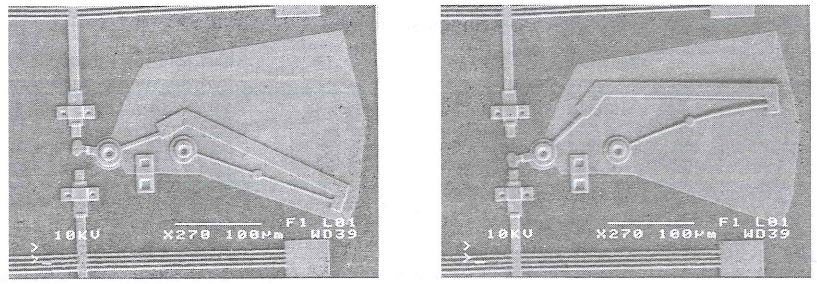
† Professor Larry L. Howell, (2002) personal communication.



(*a*) Microgears (*b*) Micromotor and gear train

FIGURE 2-26

MEMS of etched silicon (a) microgears *Courtesy of Sandia National Laboratories* (b) micromotor by Sandia Labs *SEM photos courtesy of Professor Cosme Furlong, Worcester Polytechnic Institute*



(*a*) (*b*)

FIGURE 2-27

Compliant bistable silicon micromechanism in two positions *Courtesy of Professor Larry L. Howell, Brigham Young University*

2.18 PRACTICAL CONSIDERATIONS

There are many factors that need to be considered to create good-quality designs. Not all of them are contained within the applicable theories. A great deal of art based on experience is involved in design as well. This section attempts to describe a few such practical considerations in machine design.

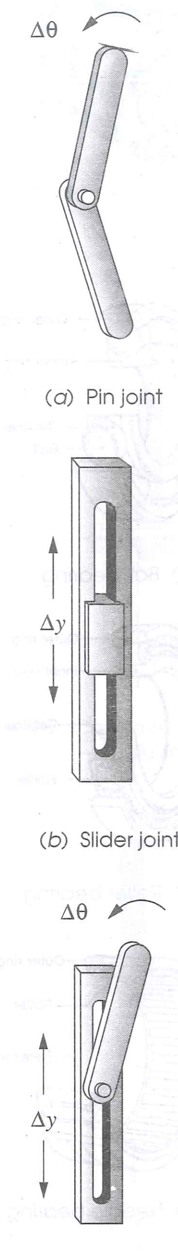
Pin Joints versus Sliders and Half Joints

Proper material selection and good lubrication are the key to long life in any situation, such as a joint, where two materials rub together. Such an interface is called a **bearing**. Assuming the proper materials have been chosen, the choice of joint type can have a significant effect on the ability to provide good, clean lubrication over the lifetime of the machine.

REVOLUTE (PIN) JOINTS The simple revolute or pin joint (Figure 2-28a) is the clear winner here for several reasons. It is relatively easy and inexpensive to design and build a good quality pin joint. In its pure form—a so-called **sleeve** or **journal** bearing—the geometry of pin-in-hole traps a lubricant film within its annular interface by capillary action and promotes a condition called *hydrodynamic lubrication* in which the parts are separated by a thin film of lubricant as shown in Figure 2-29 (p. 68). Seals can easily be provided at the ends of the hole, wrapped around the pin, to prevent loss of the lubricant. Replacement lubricant can be introduced through radial holes into the bearing interface, either continuously or periodically, without disassembly.

A convenient form of bearing for linkage pivots is the commercially available **spherical rod end** shown in Figure 2-30 (p. 68). This has a spherical, sleeve-type bearing that *self-aligns* to a shaft that may be out of parallel. Its body threads onto the link, allowing links to be conveniently made from round stock with threaded ends that allow adjustment of link length.

Relatively inexpensive **ball** and **roller bearings** are commercially available in a large variety of sizes for revolute joints as shown in Figure 2-31 (p. 68). Some of these



(*a*) Pin joint

(*b*) Slider joint

(*c*) Half joint

FIGURE 2-28

Joints of various types

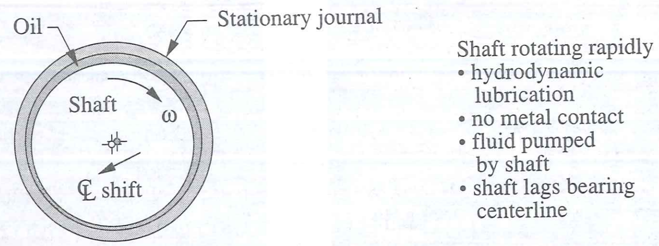
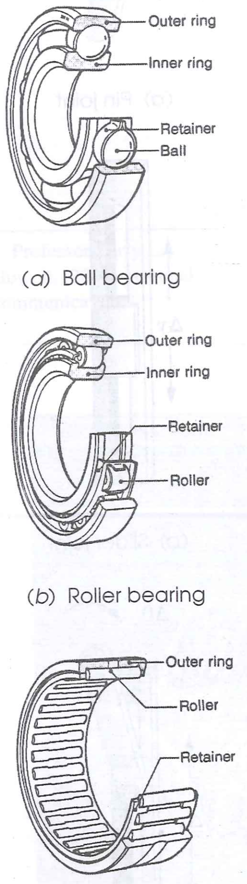


FIGURE 2-29
Hydrodynamic lubrication in a sleeve bearing—clearance and motions exaggerated

bearings (principally ball type) can be obtained prelubricated and with end seals. Their rolling elements provide low-friction operation and good dimensional control. Note that *rolling-element bearings* actually contain higher-joint interfaces (half joints) at each ball or roller, which is potentially a problem as noted below. However, the ability to trap lubricant within the roll cage (by end seals) combined with the relatively high rolling speed of the balls or rollers promotes elastohydrodynamic lubrication and long life. For more detailed information on bearings and lubrication, see reference [15].

For revolute joints pivoted to ground, several commercially available bearing types make the packaging easier. **Pillow blocks** and **flange-mount bearings** (Figure 2-32) are available fitted with either rolling-element (ball, roller) bearings or sleeve-type journal bearings. The pillow block allows convenient mounting to a surface parallel to the pin axis, and flange mounts fasten to surfaces perpendicular to the pin axis.

PRISMATIC (SLIDER) JOINTS require a carefully machined and straight slot or rod (Figure 2-28b, p .69). The bearings often must be custom made, though linear ball bearings (Figure 2-33) are commercially available but must be run over hardened and ground shafts. Lubrication is difficult to maintain in any sliding joint. The lubricant is not geometrically captured, and it must be resupplied either by running the joint in an oil bath or by periodic manual regreasing. An open slot or shaft tends to accumulate airborne dirt particles that can act as a grinding compound when trapped in the lubricant. This will accelerate wear.



(*a*) Ball bearing

(*b*) Roller bearing

(*c*) Needle bearing

FIGURE 2-31
Ball, roller, and needle bearings for revolute joints *Courtesy of NTN Corporation, Japan*

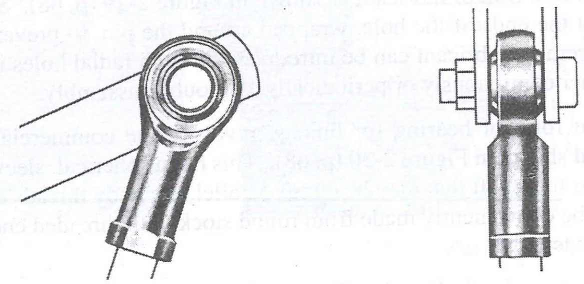
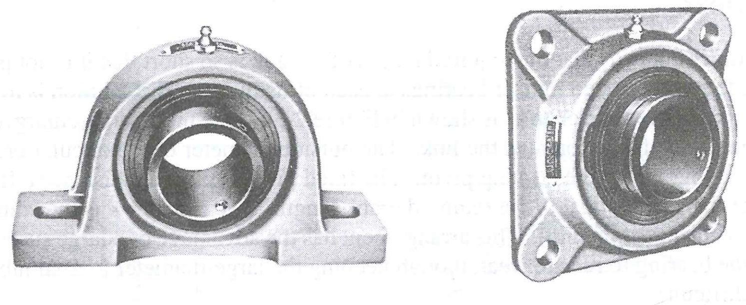


FIGURE 2-30
Spherical rod end *Courtesy of Emerson Power Transmission, Ithaca, NY*



(*a*) Pillow-block bearing (*b*) Flange-mount bearing

FIGURE 2-32
Pillow block and flange-mount bearing units. *Courtesy of Emerson Power Transmission, Ithaca, NY*

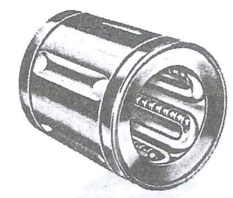


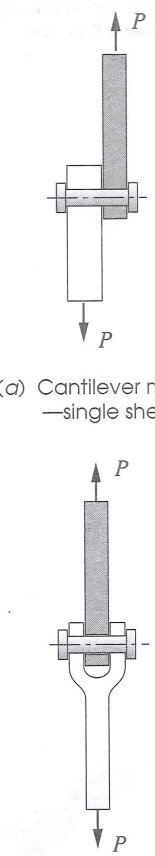
FIGURE 2-33
Linear ball bushing
Courtesy of Thomson Industries, Port Washington, NY

HIGHER (HALF) JOINTS such as a round pin in a slot (Figure 2-28c, p. 67) or a cam-follower joint (Figure 2-12c, p. 49) suffer even more acutely from the slider's lubrication problems, because they typically have two oppositely curved surfaces in line contact, which tend to squeeze any lubricant out of the joint. This type of joint needs to be run in an oil bath for long life. This requires that the assembly be housed in an expensive, oil-tight box with seals on all protruding shafts.

These joint types are all used extensively in machinery with great success. As long as the proper *attention to engineering detail* is paid, the design can be successful. Some common examples of all three joint types can be found in an automobile. The windshield wiper mechanism is a pure pin-jointed linkage. The pistons in the engine cylinders are true sliders and are bathed in engine oil. The valves in the engine are opened and closed by cam-follower (half) joints that are drowned in engine oil. You probably change your engine oil fairly frequently. When was the last time you lubricated your windshield wiper linkage? Has this linkage (not the motor) ever failed?

Cantilever or Straddle Mount?

Any joint must be supported against the joint loads. Two basic approaches are possible as shown in Figure 2-34. A cantilevered joint has the pin (journal) supported only, as a cantilever beam. This is sometimes necessary as with a crank that must pass over the coupler and cannot have anything on the other side of the coupler. However, a cantilever beam is inherently weaker (for the same cross section and load) than a straddle-mounted (simply supported) beam. The straddle mounting can avoid applying a bending moment to the links by keeping the forces in the same plane. The pin will feel a bending moment in both cases, but the straddle-mounted pin is in double shear—two cross sections are sharing the load. A cantilevered pin is in single shear. It is good practice to use straddle-mounted joints (whether revolute, prismatic, or higher) wherever possible. If a cantilevered pin must be used, then a commercial shoulder screw that has a hardened and ground shank as shown in Figure 2-35 (p. 70) can sometimes serve as a pivot pin.



(*a*) Cantilever mount —single shear

(*b*) Straddle mount —double shear

FIGURE 2-34
Cantilever, and straddle-mounted pin joints

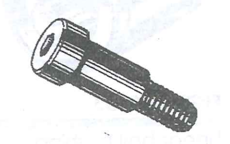


FIGURE 2-35
Shoulder screw *Courtesy of Cordova Bolt Inc., Buena Park, CA*

Short Links

It sometimes happens that the required length of a crank is so short that it is not possible to provide suitably sized pins or bearings at each of its pivots. The solution is to design the link as an **eccentric crank**, as shown in Figure 2-36. One pivot pin is enlarged to the point that it, in effect, contains the link. The outside diameter of the circular crank becomes the journal for the moving pivot. The fixed pivot is placed a distance *e* from the center of this circle equal to the required crank length. The distance *e* is the crank's eccentricity (the crank length). This arrangement has the advantage of a large surface area within the bearing to reduce wear, though keeping the large-diameter journal lubricated can be difficult.

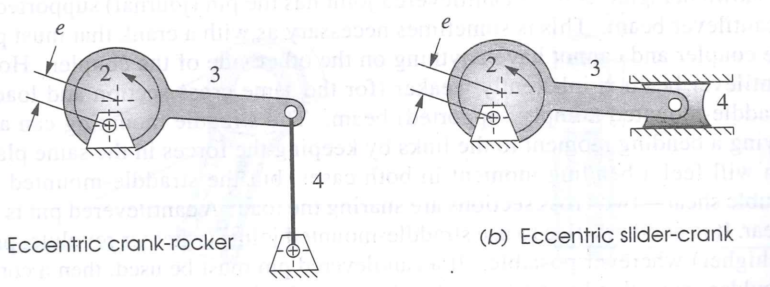
Bearing Ratio

The need for straight-line motion in machinery requires extensive use of linear translating slider joints. There is a very basic geometrical relationship called *bearing ratio*, which if ignored or violated will invariably lead to problems.

The **bearing ratio** (*BR*) is defined as *the effective length of the slider over the effective diameter of the bearing*: $BR = L / D$. For smooth operation **this ratio should be greater than 1.5, and never less than 1**. The larger it is, the better. **Effective length** is defined as *the distance over which the moving slider contacts the stationary guide*. There need not be continuous contact over that distance. That is, two short collars, spaced far apart, are effectively as long as their overall separation plus their own lengths and are kinematically equivalent to a long tube. **Effective diameter** is *the largest distance across the stationary guides*, in any plane perpendicular to the sliding motion.

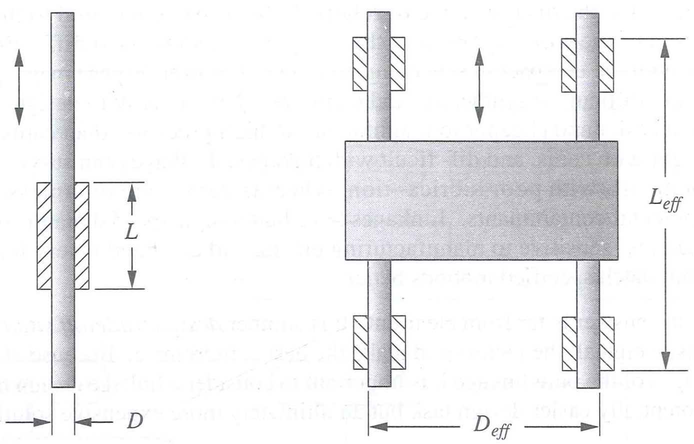
If the slider joint is simply a rod in a bushing, as shown in Figure 2-37a, the effective diameter and length are identical to the actual dimensions of the rod diameter and bushing length. If the slider is a platform riding on two rods and multiple bushings, as shown in Figure 2-37b, then the effective diameter and length are the overall width and length, respectively, of the platform assembly. It is this case that often leads to poor bearing ratios.

A common example of a device with a poor bearing ratio is a drawer in an inexpensive piece of furniture. If the only guides for the drawer's sliding motion are its sides



(a) Eccentric crank-rocker (b) Eccentric slider-crank

FIGURE 2-36
Eccentric cranks



(a) Single rod in bushing (b) Platform on two rods

FIGURE 2-37
Bearing ratio

running against the frame, it will have a bearing ratio less than 1, since it is wider than it is deep. You have probably experienced the sticking and jamming that occurs with such a drawer. A better-quality chest of drawers will have a center guide with a large L / D ratio under the bottom of the drawer and will slide smoothly.

Commercial Slides

Many companies provide standard linear slides that can be used for slider crank linkages and cam-follower systems with translating followers. These are available with linear ball bearings that ride on hardened steel ways giving very low friction. Some are preloaded to eliminate clearance and backlash error. Others are available with plain bearings. Figure 2-38 shows an example of a ball-bearing linear slide with two cars riding on a single rail. Mounting holes are provided for attaching the rail to the ground plane and in the cars for attaching the elements to be guided.

Linkages versus Cams

The pin-jointed linkage has all the advantages of revolute joints listed above. The cam-follower mechanism (Figure 2-12c, p. 49) has all the problems associated with the half joint listed above. But, both are widely used in machine design, often in the same machine and in combination (cams driving linkages). So why choose one over the other?

The "pure" pin-jointed linkage with good bearings at the joints is a potentially superior design, all else equal, and it should be the first possibility to be explored in any machine design problem. However, there will be many problems in which the need for a straight, sliding motion or the exact dwells of a cam-follower are required. Then the practical limitations of cam and slider joints will have to be dealt with accordingly.

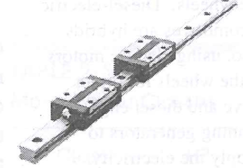


FIGURE 2-38
Ball bearing linear slide *Courtesy of THK America Inc., Schaumberg, IL*

Linkages have the disadvantage of relatively large size compared to the output displacement of the working portion; thus they can be somewhat difficult to package. Cams tend to be compact in size compared to the follower displacement. Linkages are rela- tively difficult to synthesize, and cams are relatively easy to design. But linkages are much easier and cheaper to manufacture to high precision than cams. Dwells are easy to get with cams, and dif- ficult with linkages. Linkages can survive very hostile environments, with poor lubrica- tion, whereas cams cannot, unless sealed from environmental contaminants. Linkages have better high-speed dynamic behavior than cams, are less sensitive to manufacturing errors, and can handle very high loads, but cams can match specified motions better.

So the answer is far from clear-cut. It is another *design trade-off situation* in which you must weigh all the factors and make the best compromise. Because of the potential advantages of the pure linkage it is important to consider a linkage design before choosing a potentially easier design task but an ultimately more expensive solution.

2.19 MOTORS AND DRIVERS

Unless manually operated, a mechanism will require some type of driver device to provide the input motion and energy. There are many possibilities. If the design requires a continuous rotary input motion, such as for a Grashof linkage, a slider-crank, or a cam-follower, then a motor or engine* is the logical choice. Motors come in a wide variety of types. The most common energy source for a motor is electricity, but compressed air and pressurized hydraulic fluid are also used to power air and hydraulic motors. Gasoline or diesel engines are another possibility. If the input motion is translation, as is common in earth-moving equipment, then a hydraulic or pneumatic cylinder is usually needed.

* The terms *motor* and *engine* are often used interchangeably, but they do not mean the same thing. Their difference is largely semantic, but the "purist" reserves the term *motor* for electrical, hydraulic and pneumatic motors and the term *engine* for thermodynamic devices such as external combustion (steam, stirling) engines and internal combustion (gasoline, diesel) engines. Thus, a conventional automobile is powered by a gasoline or diesel engine, but its windshield wipers and window lifts are run by electric motors. The newest hybrid automobiles have one or more electric motors to drive the wheels plus an engine to charge the battery and supply auxiliary power directly to the wheels. Diesel-electric locomotives are hybrids also, using electric motors at the wheels for direct drive and diesel engines running generators to supply the electricity. Modern commercial ships use a similar arrangement with diesel engines driving generators and electric motors turning the propellers.

Electric Motors

Electric motors are classified both by their function or application and by their electrical configuration. Some functional classifications (described below) are **gearmotors, servomotors**, and **stepping motors**. Many different electrical configurations as shown in Figure 2-39 are also available, independent of their functional classifications. The main electrical configuration division is between **AC** and **DC** motors, though one type, the **universal motor**, is designed to run on either AC or DC.

AC and **DC** refer to *alternating current* and *direct current* respectively. AC is typically supplied by the power companies and, in the United States, will be alternating sinusoidally at 60 hertz (Hz), at about ±120, ±240, or ±480 volts (V) peak. Many other countries supply AC at 50 Hz. Single-phase AC provides a single sinusoid varying with time, and 3-phase AC provides three sinusoids at 120° phase angles. DC is constant with time, supplied from generators or battery sources and is most often used in vehicles, such as ships, automobiles, aircraft, etc. Batteries are made in multiples of 1.5 V, with 6, 12, and 24 V being the most common. Electric motors are also classed by their rated power as shown in Table 2-5. Both AC and DC motors are designed to provide continuous rotary output. While they can be stalled momentarily against a load, they cannot tolerate a full-current, zero-velocity stall for more than a few minutes without overheating.

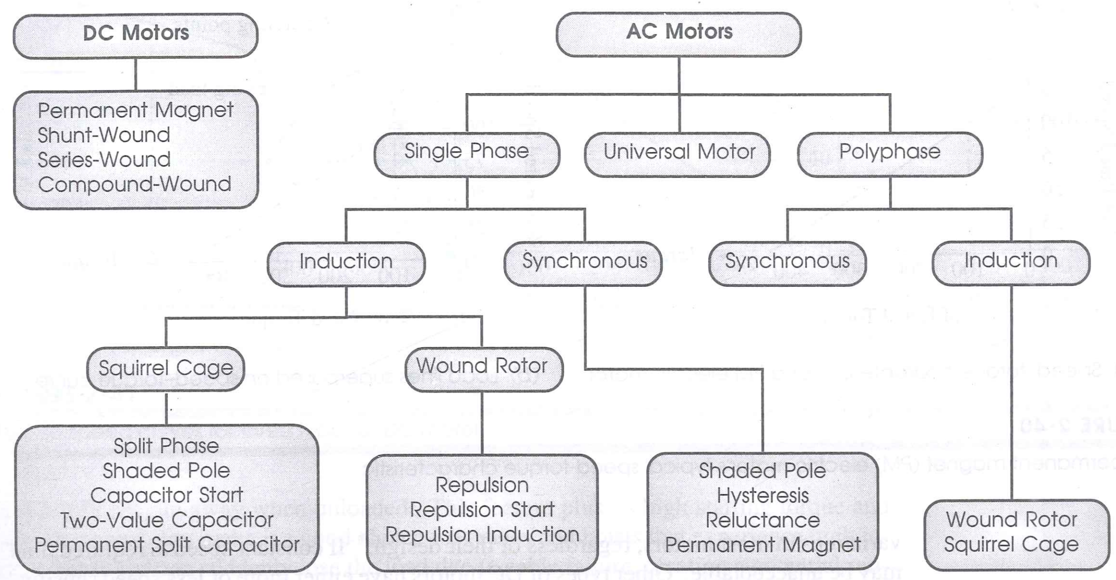


FIGURE 2-39
Types of electric motors *Source: Reference (14)*

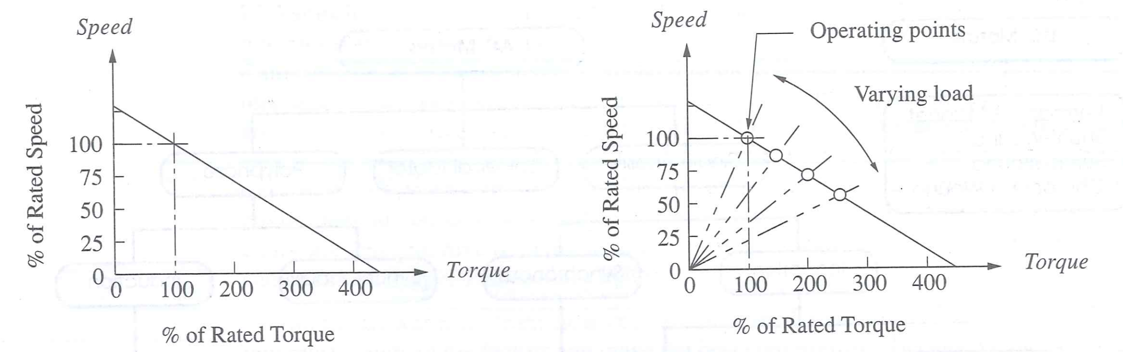
DC MOTORS are made in different electrical configurations, such as *permanent magnet (PM), shunt-wound, series-wound, and compound-wound.* The names refer to the manner in which the rotating armature coils are electrically connected to the stationary field coils—in parallel (shunt), in series, or in combined series-parallel (compound). Permanent magnets replace the field coils in a PM motor. Each configuration provides different *torque-speed* characteristics. The *torque-speed* curve of a motor describes how it will respond to an applied load and is of great interest to the mechanical designer as it predicts how the mechanical-electrical system will behave when the load varies dynamically with time.

PERMANENT MAGNET DC MOTORS Figure 2-40a (p. 74) shows a torque-speed curve for a permanent magnet (PM) motor. Note that its torque varies greatly with speed, ranging from a maximum (stall) torque at zero speed to zero torque at maximum (no-load) speed. This relationship comes from the fact that *power = torque* x *angular velocity*. Since the power available from the motor is limited to some finite value, an increase in torque requires a decrease in angular velocity and vice versa. Its torque is maximum at stall (zero velocity), which is typical of all electric motors. This is an advantage when starting heavy loads: e.g., an electric-motor-powered vehicle needs no clutch, unlike one powered by an internal combustion engine that cannot start from stall under load. An engine's torque increases rather than decreases with increasing angular velocity.

TABLE 2-5
Motor Power Classes

Class	HP
Subfractional	< 1/20
Fractional	1/20 – 1
Integral	> 1

Figure 2-40b shows a family of **load lines** superposed on the *torque-speed* curve of a PM motor. These load lines represent a time-varying load applied to the driven mechanism. The problem comes from the fact that *as the required load torque increases, the motor must reduce speed to supply it*. Thus, the input speed will vary in response to load



(*a*) Speed–torque characteristic of a PM electric motor (*b*) Load lines superposed on speed–torque curve

FIGURE 2-40

DC permanent magnet (PM) electric motor's typical speed-torque characteristic

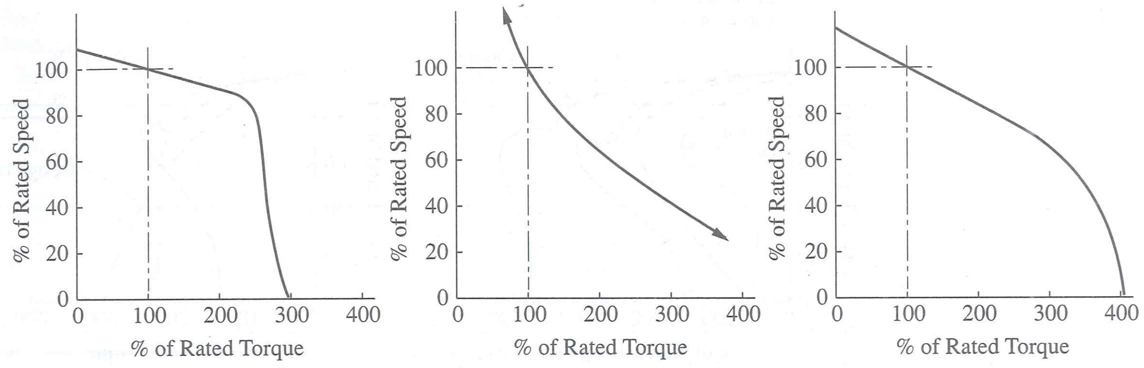
variations in most motors, regardless of their design.* If constant speed is required, this may be unacceptable. Other types of DC motors have either more or less speed sensitivity to load than the PM motor. *A motor is typically selected based on its torque-speed curve.*

SHUNT-WOUND DC MOTORS have a torque speed curve like that shown in Figure 2-41a. Note the flatter slope around the rated torque point (at 100%) compared to Figure 2-40. The shunt-wound motor is less speed-sensitive to load variation in its operating range, but stalls very quickly when the load exceeds its maximum overload capacity of about 250% of rated torque. Shunt-wound motors are typically used on fans and blowers.

SERIES-WOUND DC MOTORS have a torque-speed characteristic like that shown in Figure 2-41b. This type is more speed-sensitive than the shunt or PM configurations. However, its starting torque can be as high as 800% of full-load rated torque. It also does not have any theoretical maximum no-load speed, which makes it tend to run away if the load is removed. Actually, friction and windage losses will limit its maximum speed, which can be as high as 20,000 to 30,000 revolutions per minute (rpm). Overspeed detectors are sometimes fitted to limit its unloaded speed. Series-wound motors are used in sewing machines and portable grinders where their speed variability can be an advantage as it can be controlled, to a degree, with voltage variation. They are also used in heavy-duty applications such as vehicle traction drives where their high starting torque is an advantage. Also their speed sensitivity (large slope) is advantageous in high-load applications as it gives a "soft-start" when moving high-inertia loads. The motor's tendency to slow down when the load is applied cushions the shock that would be felt if a large step in torque were suddenly applied to the mechanical elements.

COMPOUND-WOUND DC MOTORS have their field and armature coils connected in a combination of series and parallel. As a result their torque-speed characteristic has aspects of both the shunt-wound and series-wound motors as shown in Figure 2-41c. Their speed sensitivity is greater than a shunt-wound but less than a series-wound motor

* The synchronous AC motor and the speed-controlled DC motor are exceptions.



(*a*) Shunt wound (*b*) Series wound (*c*) Compound wound

FIGURE 2-41

Torque-speed curves for three types of DC motor

and it will not run away when unloaded. This feature plus its high starting torque and soft-start capability make it a good choice for cranes and hoists that experience high inertial loads and can suddenly lose the load due to cable failure, creating a potential runaway problem if the motor does not have a self-limited no-load speed.

SPEED-CONTROLLED DC MOTORS If precise speed control is needed, as is often the case in production machinery, another solution is to use a speed-controlled DC motor that operates from a controller that increases and decreases the current to the motor in the face of changing load to try to maintain constant speed. These speed-controlled (typically PM) DC motors will run from an AC source since the controller also converts AC to DC. The cost of this solution is high, however. Another possible solution is to provide a **flywheel** on the input shaft, which will store kinetic energy and help smooth out the speed variations introduced by load variations. Flywheels will be investigated in Chapter 11.

AC MOTORS are the least expensive way to get continuous rotary motion, and they can be had with a variety of *torque-speed* curves to suit various load applications. They are limited to a few standard speeds that are a function of the AC line frequency (60 Hz in North America, 50 Hz elsewhere). The synchronous motor speed n_s is a function of line frequency f and the number of magnetic poles p present in the rotor.

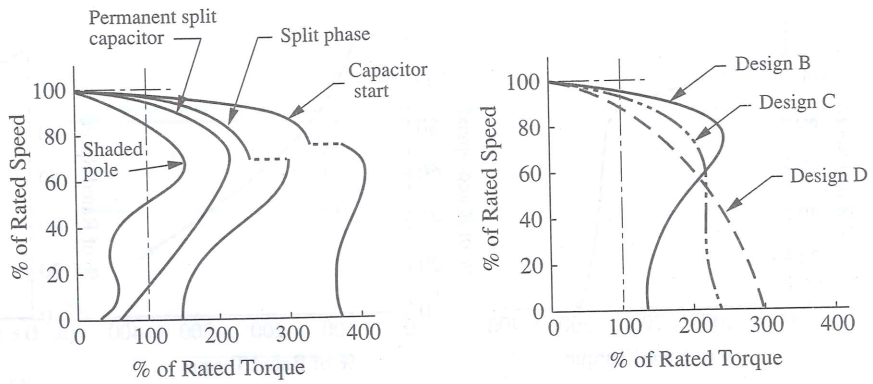
$$n_s = \frac{120f}{p} \tag{2.17}$$

Synchronous motors "lock on" to the AC line frequency and run exactly at synchronous speed. These motors are used for clocks and timers. Nonsynchronous AC motors have a small amount of slip that makes them lag the line frequency by about 3 to 10%.

Table 2-6 shows the synchronous and nonsynchronous speeds for various AC motor-pole configurations. The most common AC motors have 4 poles, giving nonsynchronous *no-load speeds* of about 1725 rpm, which reflects slippage from the 60-Hz synchronous speed of 1800 rpm.

TABLE 2-6
AC Motor Speeds

Poles	Sync rpm	Async rpm
2	3600	3450
4	1800	1725
6	1200	1140
8	900	850
10	720	690
12	600	575



(a) Single phase (b) Three phase

FIGURE 2-42

Torque-speed curves for single- and three-phase AC motors

Figure 2-42 shows typical torque-speed curves for single-phase (1ϕ) and 3-phase (3ϕ) AC motors of various designs. The single-phase shaded pole and permanent split capacitor designs have a starting torque lower than their full-load torque. To boost the start torque, the split-phase and capacitor-start designs employ a separate starting circuit that is cut off by a centrifugal switch as the motor approaches operating speed. The broken curves indicate that the motor has switched from its starting circuit to its running circuit. The NEMA* three-phase motor designs B, C, and D in Figure 2-42 differ mainly in their starting torque and in speed sensitivity (slope) near the full-load point.

GEARMOTORS If different single (as opposed to variable) output speeds than the standard ones of Table 2-6 are needed, a gearbox speed reducer can be attached to the motor's output shaft, or a gearmotor can be purchased that has an integral gearbox. Gearmotors are commercially available in a large variety of output speeds and power ratings. The kinematics of gearbox design are covered in Chapter 9.

SERVOMOTORS These are fast-response, closed-loop-controlled motors capable of providing a programmed function of acceleration or velocity, providing position control, and of holding a fixed position against a load. **Closed loop** means that *sensors (typically shaft encoders) on the motor or the output device being moved feed back information on its position and velocity.* Circuitry in the motor controller responds to the fed back information by reducing or increasing (or reversing) the current flow (and/or its frequency) to the motor. Precise positioning of the output device is then possible, as is control of the speed and shape of the motor's response to changes in load or input commands. These are relatively expensive devices† that are commonly used in applications such as moving the flight control surfaces in aircraft and guided missiles, in numerically controlled machining centers, automated manufacturing machinery, and in controlling robots, for example.

Servomotors are made in both AC and DC configurations, with the AC type currently becoming more popular. These achieve speed control by the controller generating a variable frequency current that the synchronous AC motor locks onto. The controller first

* National Electrical Manufacturers Association.

† Costs of all electronic devices seem to continuously fall as technology advances and motor controllers are no exception.

rectifies the AC to DC and then "chops" it into the desired frequency, a common method being pulse-width modification. They have high torque capability and a flat torque-speed curve similar to Figure 2-41a (p. 75). Also, they will typically provide as much as three times their continuous rated torque for short periods such as under intermittent overloads. Other advantages of servomotors include their ability to do programmed "soft starts," hold any speed to a close tolerance in the face of variation in the load torque, and make a rapid emergency stop using dynamic braking.

STEPPER MOTORS These are brushless permanent magnet, variable reluctance, or hybrid-type motors designed to position an output device. Unlike servomotors, they typically run **open loop**, meaning they *receive no feedback as to whether the output device has responded as requested.* Thus, they can get out of phase with the desired program. They will, however, happily sit energized for an indefinite period, holding the output in one position (though they do get hot—100-150°F). Their internal construction consists of a number of magnetic strips arranged around the circumference of both the rotor and stator. When energized, the rotor will move one step, to the next magnet, for each pulse received. Thus, these are **intermittent motion** devices and do not provide continuous rotary motion like other motors. The number of magnetic strips and controller type determine their resolution (typically 200 steps/rev, but a microstepper drive can increase this to 2000 or more steps/rev). They are relatively small compared to AC/DC motors and have low drive torque capacity but have high holding torque. They are moderately expensive and require special controllers.

Air and Hydraulic Motors

These have more limited application than electric motors, simply because they require the availability of a compressed air or hydraulic source. Both of these devices are less energy efficient than the direct electrical to mechanical conversion of electric motors, because of the losses associated with the conversion of the energy first from chemical or electrical to fluid pressure and then to mechanical form. Every energy conversion involves some losses. Air motors find widest application in factories and shops, where high-pressure compressed air is available for other reasons. A common example is the air impact wrench used in automotive repair shops. Although individual air motors and air cylinders are relatively inexpensive, these pneumatic systems are quite expensive when the cost of all the ancillary equipment is included. Hydraulic motors are most often found within machines or systems such as construction equipment (cranes), aircraft, and ships, where high-pressure hydraulic fluid is provided for many purposes. Hydraulic systems are very expensive when the cost of all the ancillary equipment is included.

Air and Hydraulic Cylinders

These are linear actuators (piston in cylinder) that provide a limited stroke, straight-line output from a pressurized fluid flow input of either compressed air or hydraulic fluid (usually oil). They are the method of choice if you need a linear motion as the input. However, they share the same high cost, low efficiency, and complication factors as listed under their air and hydraulic motor equivalents above.

Another problem is that of control. Most motors, left to their own devices, will tend to run at a constant speed. A linear actuator, when subjected to a constant pressure fluid source, typical of most compressors, will respond with more nearly constant acceleration, which means its velocity will increase linearly with time. This can result in severe impact loads on the driven mechanism when the actuator comes to the end of its stroke at maximum velocity. Servovalve control of the fluid flow, to slow the actuator at the end of its stroke, is possible but is quite expensive.

The most common application of fluid power cylinders is in farm and construction equipment such as tractors and bulldozers, where open loop (nonservo) hydraulic cylinders actuate the bucket or blade through linkages. The cylinder and its piston become two of the links (slider and track) in a slider-crank mechanism. See Figure 1-1b (p. 7).

Solenoids

These are electromechanical (AC or DC) linear actuators that share some of the limitations of air cylinders, and they possess a few more of their own. They are *energy inefficient*, are limited to very short strokes (about 2 to 3 cm), develop a force that varies exponentially over the stroke, and deliver high impact loads. They are, however, inexpensive, reliable, and have very rapid response times. They cannot handle much power, and they are typically used as control or switching devices rather than as devices that do large amounts of work on a system.

A common application of solenoids is in camera shutters, where a small solenoid is used to pull the latch and trip the shutter action when you push the button to take the picture. Its nearly instantaneous response is an asset in this application, and very little work is being done in tripping a latch. Another application is in electric door or trunk locking systems in automobiles, where the click of their impact can be clearly heard when you turn the key (or press the button) to lock or unlock the mechanism.

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2.21 PROBLEMS

*2-1 Find three (or other number as assigned) of the following common devices. Sketch careful kinematic diagrams and find their total degrees of freedom.

a. An automobile hood hinge mechanism
b. An automobile hatchback lift mechanism
c. An electric can opener
d. A folding ironing board
e. A folding card table
f. A folding beach chair
g. A baby swing
h. A folding baby walker
i. A fancy corkscrew as shown in Figure P2-9 (p. 86)
j. A windshield wiper mechanism
k. A dump truck dump mechanism
l. A trash truck dumpster mechanism
m. A pickup truck tailgate mechanism
n. An automobile jack
o. A collapsible auto radio antenna

2-2 How many *DOF* do you have in your wrist and hand combined? Describe them.

*2-3 How many *DOF* do the following joints have?

a. Your knee
b. Your ankle
c. Your shoulder
d. Your hip
e. Your knuckle

*2-4 How many *DOF* do the following have in their normal environment?

a. A submerged submarine b. An earth-orbiting satellite
c. A surface ship d. A motorcycle
e. The print head in a 9-pin dot matrix computer printer
f. The pen in an XY plotter

*2-5 Are the joints in Problem 2-3 force closed or form closed?

TABLE P2-0
Topic/Problem Matrix

Topic	Problems
2.1 Degrees of Freedom	2-2, 2-3, 2-4
2.2 Types of Motion	2-6, 2-37
2.3 Links, Joints and Kinematic Chains	2-5, 2-17, 2-38, 2-39, 2-40, 2-41, 2-53, 2-54, 2-55
2.5 Mobility	2-1, 2-7, 2-20, 2-21, 2-24, 2-25, 2-26, 2-28, 2-44, 2-48 to 2-53
2.6 Mechanisms and Structures	2-8, 2-27
2.7 Number Synthesis	2-11
2.9 Isomers	2-12, 2-45, 2-46, 2-47
2.10 Linkage Transformation	2-9, 2-10, 2-13, 2-14, 2-30, 2-31, 2-34, 2-35, 2-36
2.13 The Grashof Condition	2-15, 2-22, 2-23, 2-29, 2-32, 2-42, 2-43
2.15 Springs as Links	2-18, 2-19
2.19 Motors and Drivers	2-16

* Answers in Appendix E

*2-6 Describe the motion of the following items as pure rotation, pure translation, or complex planar motion.

a. A windmill
b. A bicycle (in the vertical plane, not turning)
c. A conventional "double-hung" window
d. The keys on a computer keyboard
e. The hand of a clock
f. A hockey puck on the ice
g. The pen in an XY plotter
h. The print head in a computer printer
i. A "casement" window

*2-7 Calculate the mobility of the linkages assigned from Figure P2-1 part 1 and part 2.

*2-8 Identify the items in Figure P2-1 as mechanisms, structures, or preloaded structures.

2-9 Use linkage transformation on the linkage of Figure P2-1a to make it a 1-*DOF* mechanism.

2-10 Use linkage transformation on the linkage of Figure P2-1d to make it a 2-*DOF* mechanism.

2-11 Use number synthesis to find all the possible link combinations for 2-*DOF*, up to 9 links, to hexagonal order, using only revolute joints.

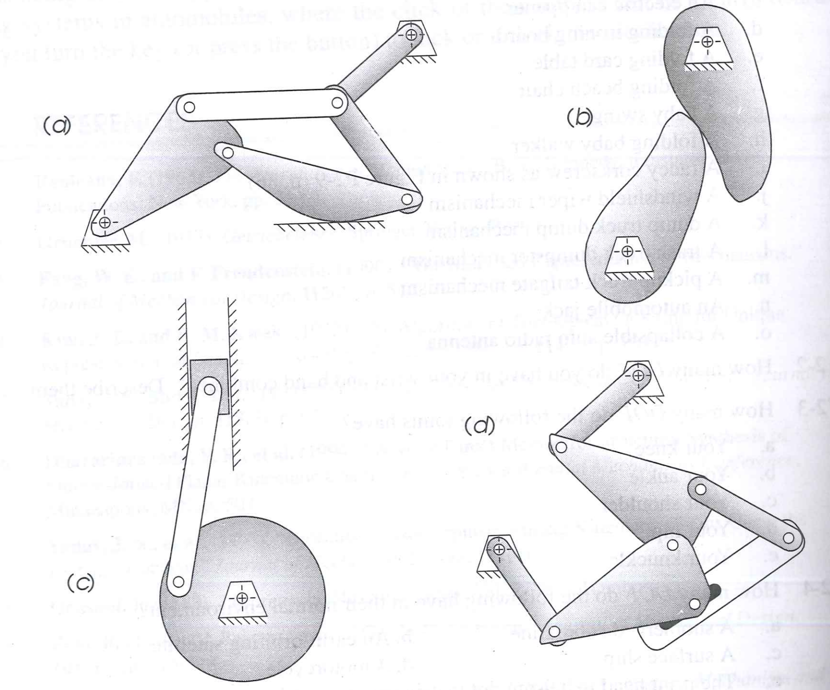


FIGURE P2-1 part 1
Linkages for Problems 2-7 to 2-10

* Answers in Appendix E

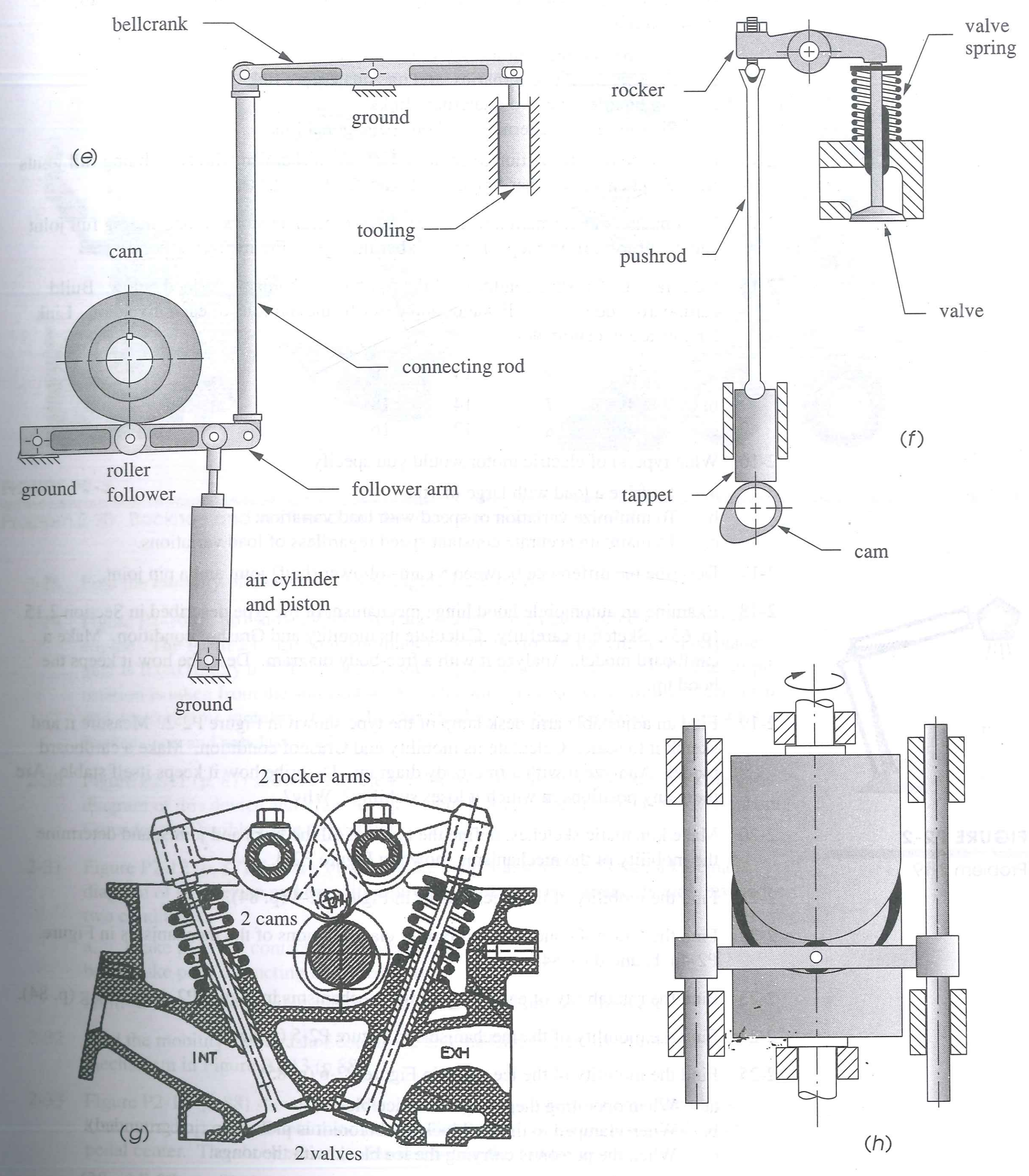


FIGURE P2-1 part 2
Linkages for Problems 2-7 to 2-8

2-12 Find all the valid isomers of the eightbar 1-*DOF* link combinations in Table 2-2 (p. 44) having:

a. Four binary and four ternary links
b. Five binaries, two ternaries, and one quaternary link
c. Six binaries and two quaternary links
d. Six binaries, one ternary, and one pentagonal link

2-13 Use linkage transformation to create a 1-*DOF* mechanism with two sliding full joints from Stephenson's sixbar linkage in Figure 2-16a (p. 54).

2-14 Use linkage transformation to create a 1-*DOF* mechanism with one sliding full joint and a half joint from Stephenson's sixbar linkage in Figure 2-16b (p. 54).

*2-15 Calculate the Grashof condition of the fourbar mechanisms defined below. Build cardboard models of the linkages and describe the motions of each inversion. Link lengths are in centimeters.

a.	4	9	14	18
b.	4	7	14	18
c.	4	8	12	16

2-16 What type(s) of electric motor would you specify

a. To drive a load with large inertia.
b. To minimize variation of speed with load variation.
c. To maintain accurate constant speed regardless of load variations.

2-17 Describe the difference between a cam-follower (half) joint and a pin joint.

2-18 Examine an automobile hood hinge mechanism of the type described in Section 2.15 (p. 63). Sketch it carefully. Calculate its mobility and Grashof condition. Make a cardboard model. Analyze it with a free-body diagram. Describe how it keeps the hood up.

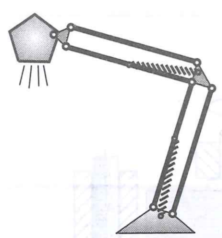


FIGURE P2-2
Problem 2-19

2-19 Find an adjustable arm desk lamp of the type shown in Figure P2-2. Measure it and sketch it to scale. Calculate its mobility and Grashof condition. Make a cardboard model. Analyze it with a free-body diagram. Describe how it keeps itself stable. Are there any positions in which it loses stability? Why?

2-20 Make kinematic sketches, define the types of all the links and joints, and determine the mobility of the mechanisms shown in Figure P2-3.

*2-21 Find the mobility of the mechanisms in Figure P2-4 (p. 84).

2-22 Find the Grashof condition and Barker classifications of the mechanisms in Figure P2-4a, b, and d (p. 84).

2-23 Find the rotatability of each loop of the mechanisms in Figure P2-4e, f, and g (p. 84).

*2-24 Find the mobility of the mechanisms in Figure P2-5 (p. 85).

2-25 Find the mobility of the ice tongs in Figure P2-6 (p. 85).

a. When operating them to grab the ice block.
b. When clamped to the ice block but before it is picked up (ice grounded).
c. When the person is carrying the ice block with the tongs.

*2-26 Find the mobility of the automotive throttle mechanism in Figure P2-7 (p. 85).

*2-27 Sketch a kinematic diagram of the scissors jack shown in Figure P2-8 (p. 86) and determine its mobility. Describe how it works.

* Answers in Appendix E

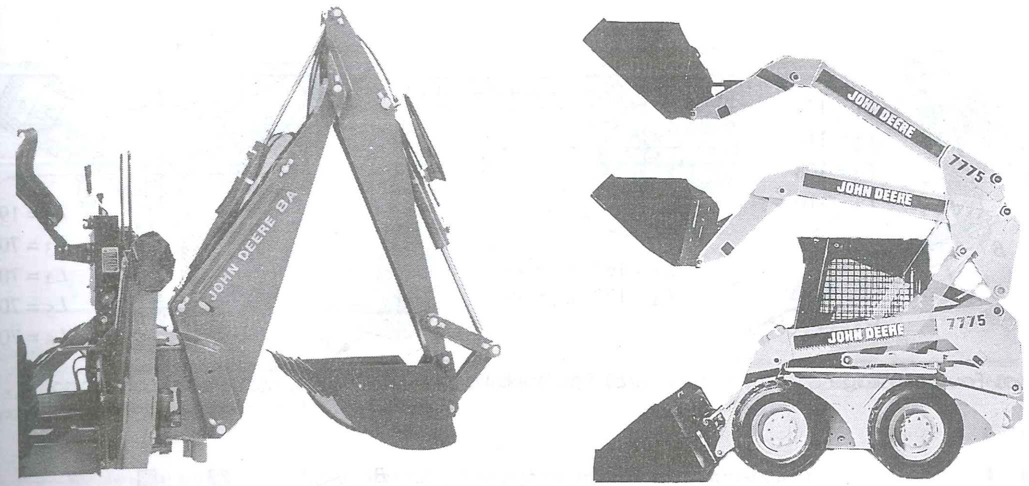


FIGURE P2-3
Problem 2-20 Backhoe and front-end loader *Courtesy of John Deere Co.*

2-28 Find the mobility of the corkscrew in Figure P2-9 (p. 86).

2-29 Figure P2-10 (p. 86) shows Watt's sun and planet drive that he used in his steam engine. The beam 2 is driven in oscillation by the piston of the engine. The planet gear is fixed rigidly to link 3 and its center is guided in the fixed track 1. The output rotation is taken from the sun gear 4. Sketch a kinematic diagram of this mechanism and determine its mobility. Can it be classified by the Barker scheme? If so, what Barker class and subclass is it?

2-30 Figure P2-11 (p. 87) shows a bicycle handbrake lever assembly. Sketch a kinematic diagram of this device and draw its equivalent linkage. Determine its mobility. Hint: Consider the flexible cable to be a link.

2-31 Figure P2-12 (p. 87) shows a bicycle brake caliper assembly. Sketch a kinematic diagram of this device and draw its equivalent linkage. Determine its mobility under two conditions.

a. Brake pads not contacting the wheel rim.
b. Brake pads contacting the wheel rim.

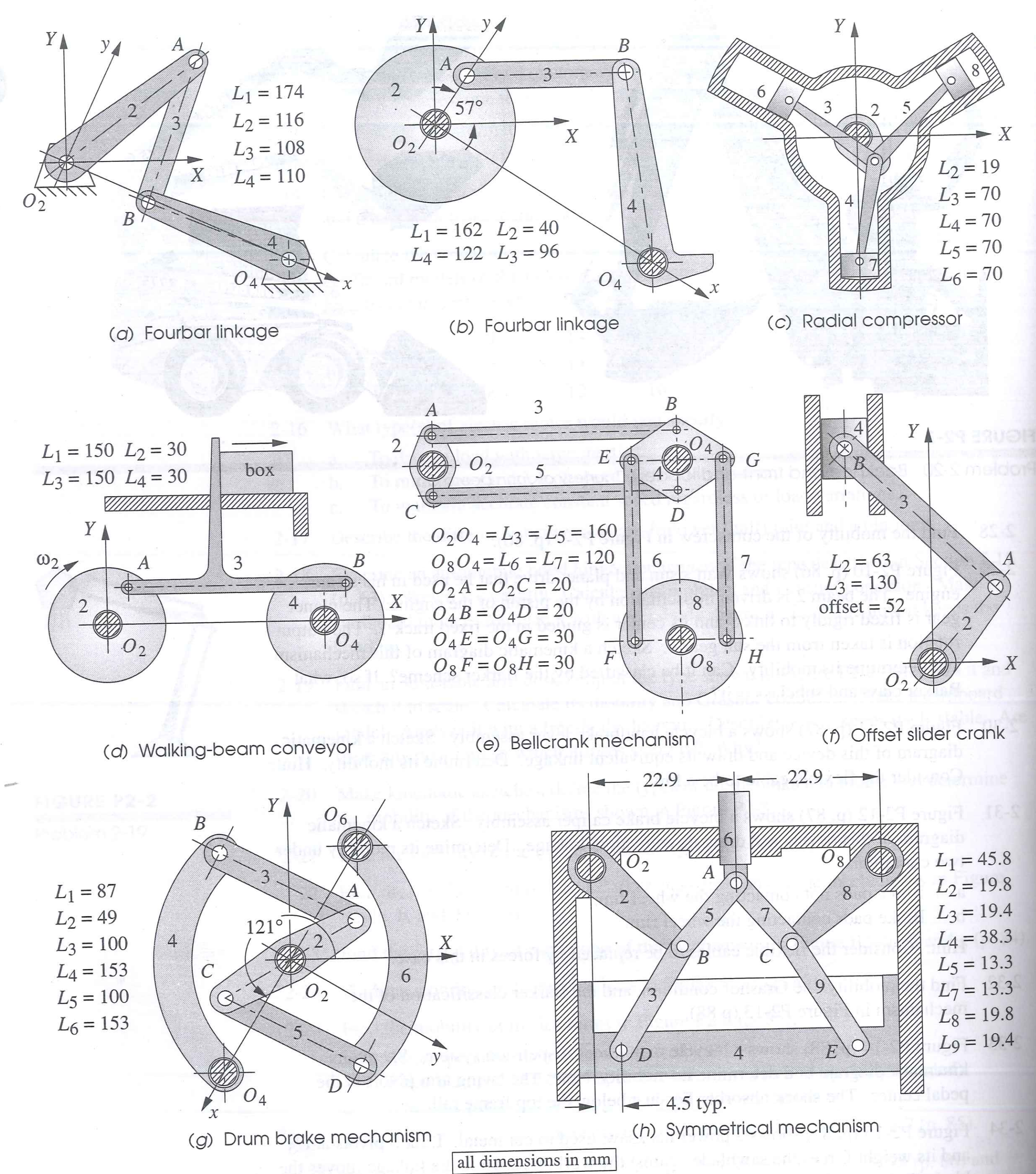
Hint: Consider the flexible cables to be replaced by forces in this case.

2-32 Find the mobility, the Grashof condition, and the Barker classification of the mechanism in Figure P2-13 (p 88).

2-33 Figure P2-14 (p. 88) shows a bicycle rear-wheel suspension system. Sketch its kinematic diagram and determine its mobility. Note: The swing arm pivots at the pedal center. The shock absorber lies just below the top frame rail.

2-34 Figure P2-15 (p. 89) shows a power hacksaw, used to cut metal. Link 5 pivots at O_5 and its weight forces the sawblade against the workpiece while the linkage moves the blade (link 4) back and forth within link 5 to cut the part. Sketch its kinematic diagram, determine its mobility and its type (i.e., is it a fourbar, a Watt sixbar, a

* Answers in Appendix E



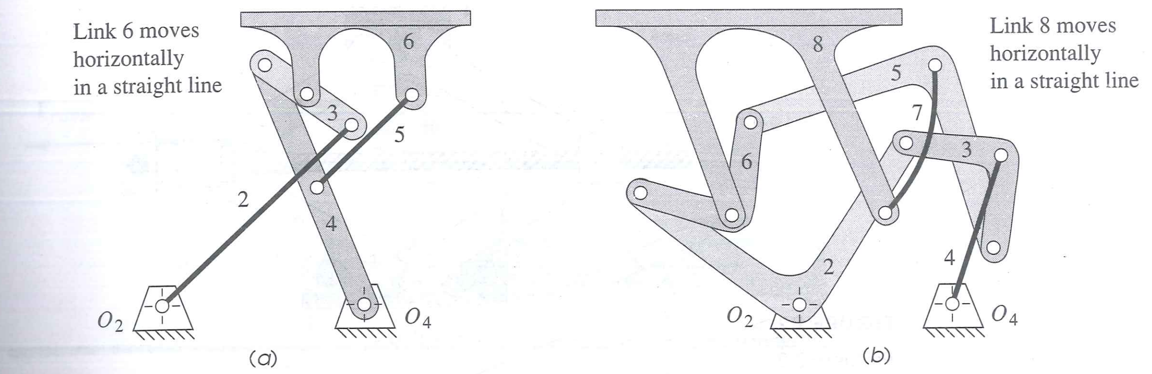
(a) Fourbar linkage (b) Fourbar linkage (c) Radial compressor

(d) Walking-beam conveyor (e) Bellcrank mechanism (f) Offset slider crank

(g) Drum brake mechanism (h) Symmetrical mechanism

FIGURE P2-4

Problems 2-21 to 2-23 *Adapted from P. H. Hill and W. P. Rule. (1960) Mechanisms: Analysis and Design, with permission*



(a) (b)

FIGURE P2-5

Problem 2-24 Chebyschev (a) and Sylvester-Kempe (b) straight-line mechanism *Adapted from Kempe, How to Draw a Straight Line, Macmillan: London, 1877*

Stephenson sixbar, an eightbar, or what?) Use reverse linkage transformation to determine its pure revolute-jointed equivalent linkage.

*2-35 Figure P2-16 (p. 89) shows a manual press used to compact powdered materials. Sketch its kinematic diagram, determine its mobility and its type (i.e., is it a fourbar, a Watt sixbar, a Stephenson sixbar, an eightbar, or what?) Use reverse linkage transformation to determine its pure revolute-jointed equivalent linkage.

2-36 Sketch the equivalent linkage for the cam and follower mechanism in Figure P2-17 (p. 89) in the position shown. Show that it has the same *DOF* as the original mechanism.

2-37 Describe the motion of the following rides, commonly found at an amusement park, as pure rotation, pure translation, or complex planar motion.

a. A Ferris wheel
b. A "bumper" car
c. A drag racer ride

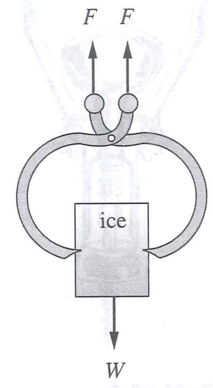


FIGURE P2-6

Problem 2-25

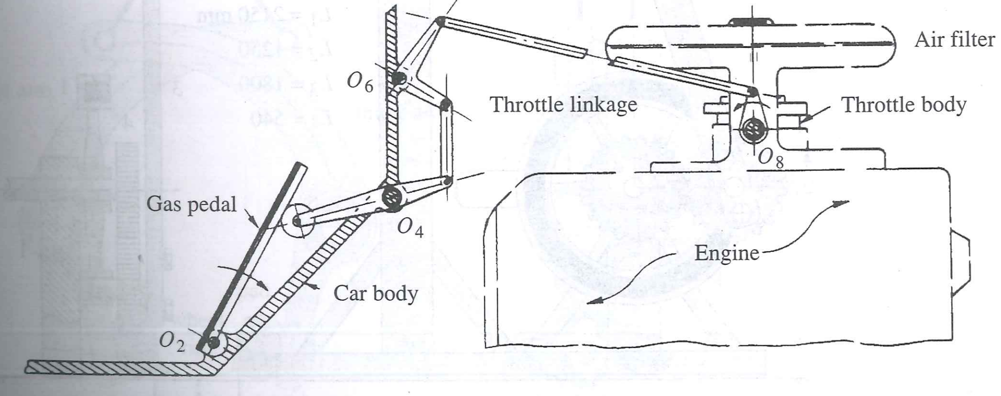


FIGURE P2-7

Problem 2-26. *Adapted from P. H. Hill and W. P Rule. (1960) Mechanisms: Analysis and Design, with permission*

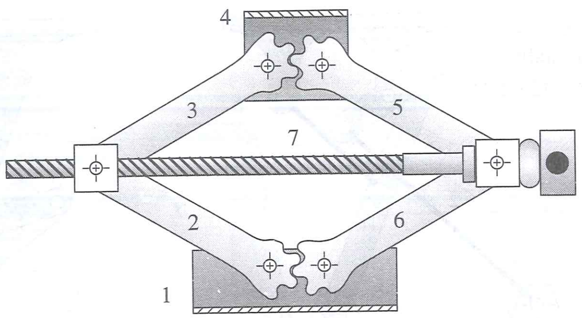


FIGURE P2-8
Problem 2-27

d. A roller coaster whose foundation is laid out in a straight line
e. A boat ride through a maze
f. A pendulum ride
g. A train ride

2-38 For the mechanism in Figure P2-1a (p. 80), number the links, starting with 1. (Don't forget the "ground" link.) Letter the joints alphabetically, starting with point *A*.
a. Using your link numbers, describe each link as binary, ternary, etc.
b. Using your joint letters, determine each joint's order.
c. Using your joint letters, determine whether each is a half or full joint.

2-39 Repeat Problem 2-38 for Figure P2-1b (p. 80).

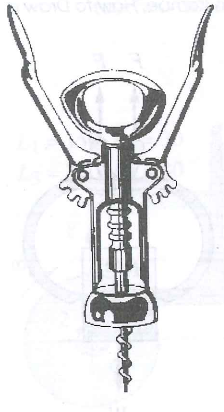


FIGURE P2-9
Problem 2-28

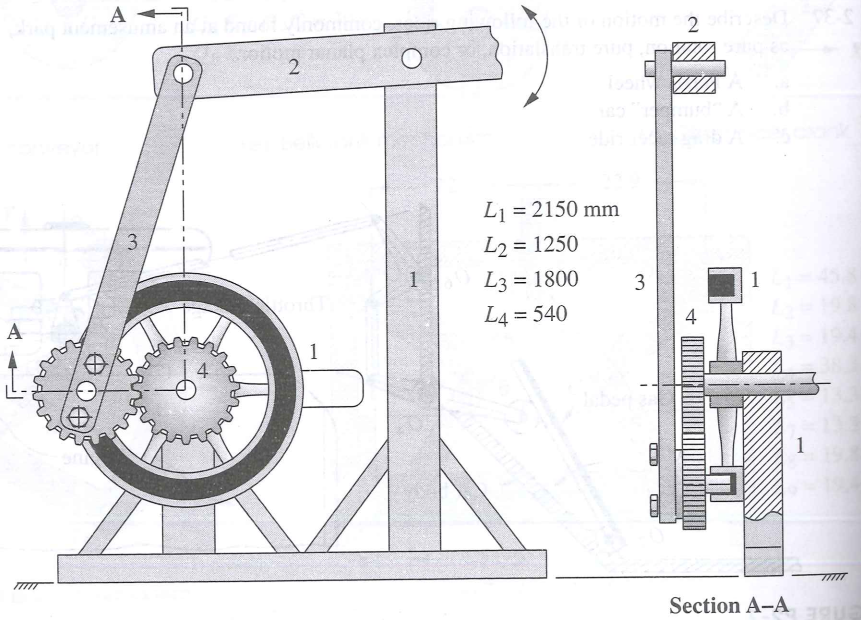


FIGURE P2-10
Problem 2-29 James Watt's sun and planet drive

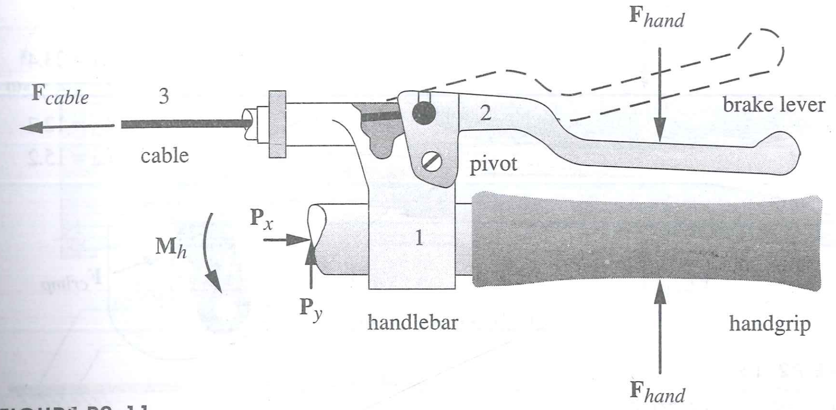


FIGURE P2-11
Problem 2-30 Bicycle hand brake lever assembly

2-40 Repeat Problem 2-38 for Figure P2-1c (p. 80).

2-41 Repeat Problem 2-38 for Figure P2-1d (p. 80).

2-42 Find the mobility, the Grashof condition, and the Barker classification of the oil field pump shown in Figure P2-18 (p. 90).

2-43 Find the mobility, the Grashof condition, and the Barker classification of the aircraft overhead bin shown in Figure P2-19 (p. 90). Make a model and investigate its motions.

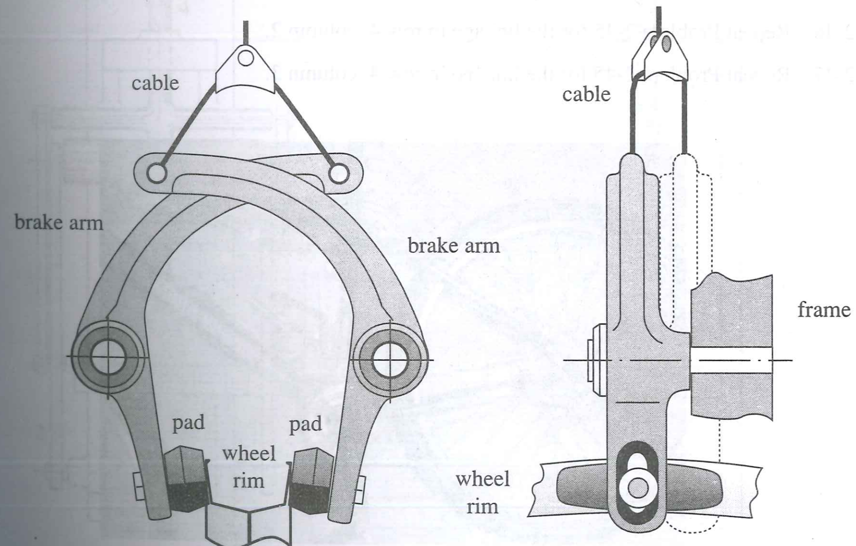


FIGURE P2-12
Problem 2-31 Bicycle brake caliper assembly

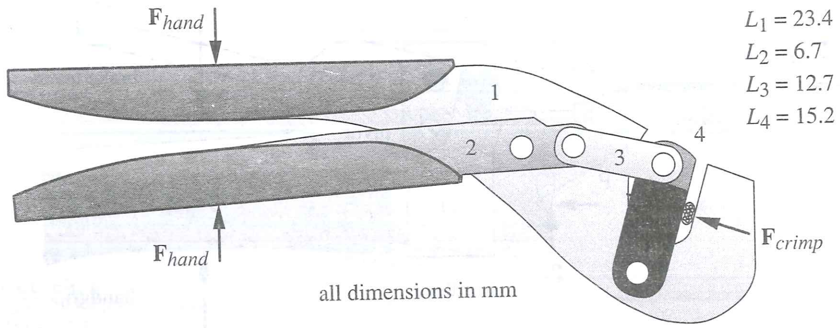


FIGURE P2-13
Problem 2-32 Crimping tool

2-44 Figure P2-20(p. 91) shows a "Rube Goldberg" mechanism that turns a light switch on when a room door is opened and off when the door is closed. The pivot at O_2 goes through the wall. There are two spring-loaded piston-in cylinder devices in the assembly. An arrangement of ropes and pulleys inside the room (not shown) transfers the door swing into a rotation of link 2. Door opening rotates link 2 CW, pushing the switch up as shown in the figure, and door closing rotates link 2 CCW, pulling the switch down. Consider the spring-loaded cylinder at the switch to be effectively a single variable-length binary link. Find the mobility of the linkage.

2-45 All the eightbar linkages in Figure 2-11 part 2 (p. 48) have eight possible inversions. Some of these will give motions similar to others. Those that have distinct motions are called *distinct inversions.* How many distinct inversions does the linkage in row 4, column 1 have?

2-46 Repeat Problem 2-45 for the linkage in row 4, column 2.

2-47 Repeat Problem 2-45 for the linkage in row 4, column 3.

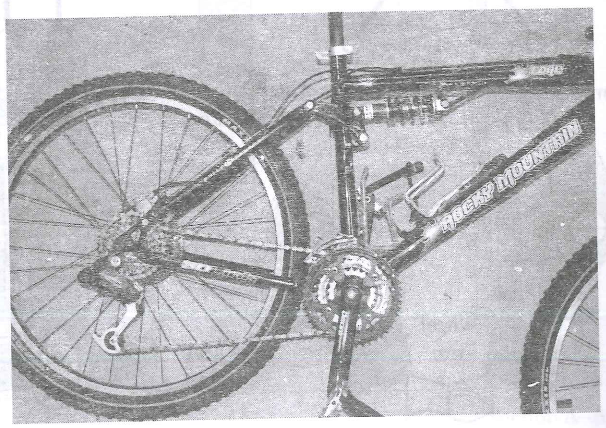


FIGURE P2-14
Problem 2-33 Mountain bicycle suspension system

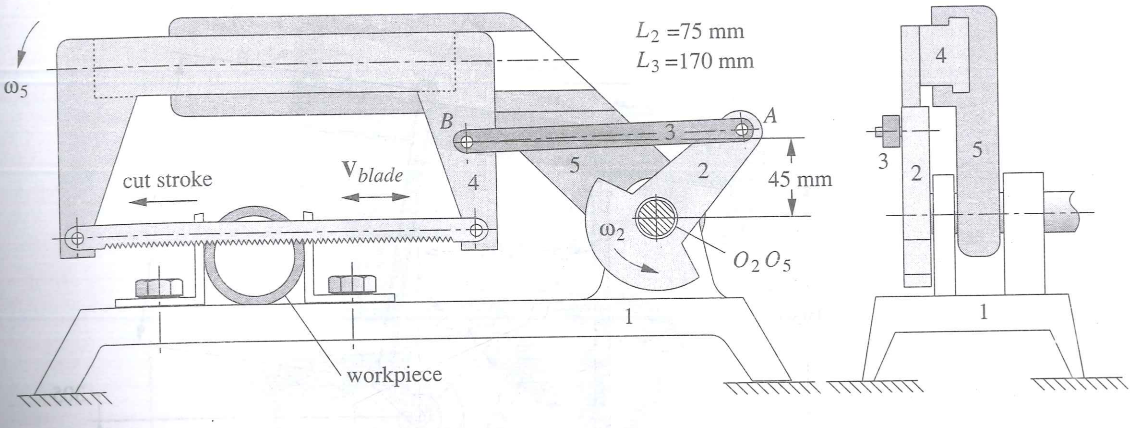


FIGURE P2-15
Problem 2-34 Power hacksaw *Adapted from P. H. Hill and W. P. Rule. (1960). Mechanisms: Analysis and Design, with permission*

2-48 Find the mobility of the mechanism shown in Figure 3-33 (p. 149).

2-49 Find the mobility of the mechanism shown in Figure 3-34 (p. 150).

2-50 Find the mobility of the mechanism shown in Figure 3-35 (p. 150).

2-51 Find the mobility of the mechanism shown in Figure 3-36 (p. 151).

2-52 Find the mobility of the mechanism shown in Figure 3-37b (p. 151).

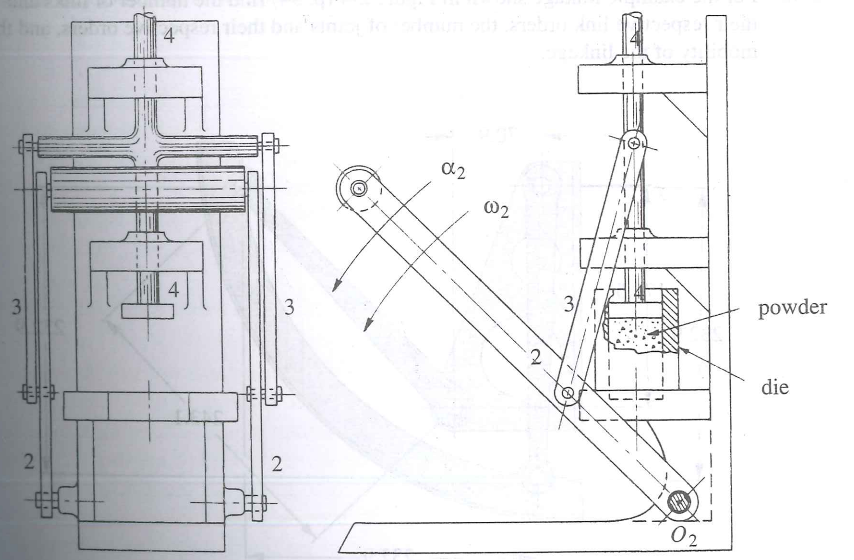


FIGURE P2-16
Problem 2-35 Powder compacting press *Adapted from P. H. Hill and W. P. Rule. (1960). Mechanisms: Analysis and Design, with permission*

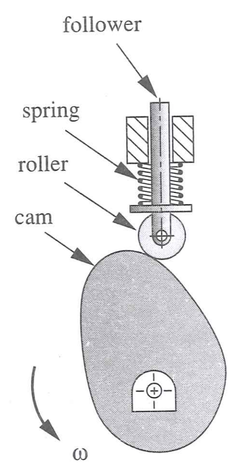


FIGURE P2-17
Problem 2-36

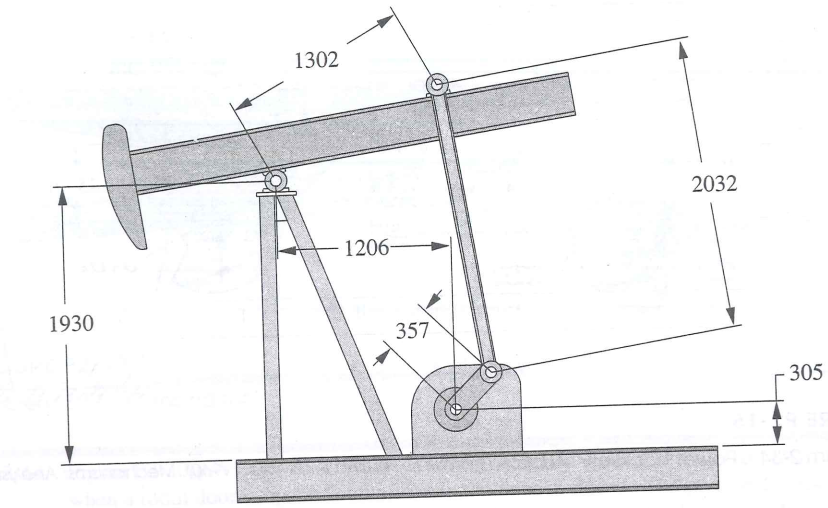


FIGURE P2-18
Problem 2-42 An oil field pump - dimensions in mm

2-53 Repeat Problem 2-38 for Figure P2-1e (p. 80)

2-54 Repeat Problem 2-38 for Figure P2-1f (p. 80)

2-55 Repeat Problem 2-38 for Figure P2-1g (p. 80)

2-56 For the example linkage shown in Figure 2-4 (p. 34) find the number of links and their respective link orders, the number of joints and their respective orders, and the mobility of the linkage.

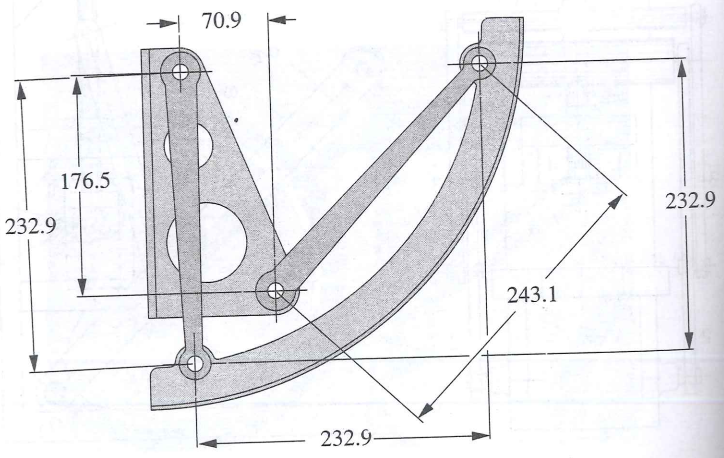


FIGURE P2-19
Problem 2-43 An aircraft overhead bin mechanism - dimensions in mm

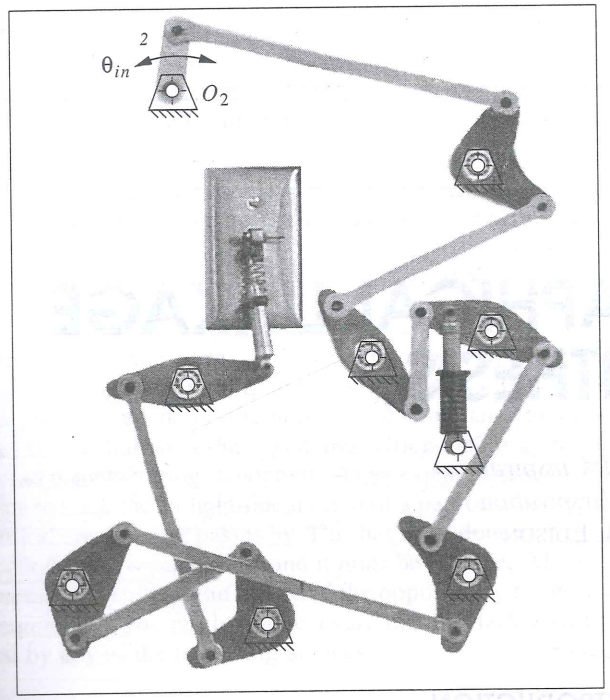


FIGURE P2-20
A "Rube Goldberg" light switch actuating mechanism *(Courtesy of Robert Taylor, WPI)*