Mathematica Homework #2

Email notebook to corbin@physics.ucla.edu
with a subject line: [Physics 105A]
by on or about Friday, 25 January

- In the first cell, enter your **name**, **student ID**, **email address** and the **assignment identifier** (eg. "HW 2") as text.

- 1) If there's one esoteric, weird construction that you would benefit right here, right now, by learning - it would be *rules* and *replacement*. It'll be a part of every interesting thing you do in Mathematica from here on out. Think of the pieces this way: the combination **/.** can be read '*where*' and combination $\rightarrow$ can be read '*goes to*', so that...

  $$x = t^2 /.\mathrm{t}\rightarrow 4$$

  would be read "*assign to the label x the value of t squared where t goes to 4*. The value 4 is never actually assigned to t, nevertheless, the value assigned to x would then be 16.

  Functions that **Solve** things in Mathematica always return the solution in *rule* ($\rightarrow$) form. You have to use *replacement* (/.) to assign these solutions to labels.

  In a single cell:

  - *i*) Solve the system of equations:

    $$\ddot{x} = a \qquad \dot{x}(0) = v_0 \qquad x(0) = x_0$$

    (where a, v_0 and x_0 are constants).

  - *ii*) Use the rules you obtained and replacement to construct a useable function $x(t)$.

  - *iii*) Assume the function you just obtained represents the position of a body in time. Pick some reasonable values for the constants and plot both the position and the velocity of the body as a function of time.

- 2) We're next going to solve the following for $x_1(t)$ and $x_2(t)$:

$$m\ddot{x_1} + 2kx_1 - kx_2 = 0 \qquad m\ddot{x_2} + 2kx_2 - kx_1 = 0$$

$$\dot{x_1}(0) = 0 \qquad \dot{x_2}(0) = 0 \qquad x_1(0) = 0 \qquad x_2(0) = L$$

  In a single cell:

  - *i*) Assign each of the equations to a unique label.
  - *ii*) Construct a list that contains all the equations.
  - *iii*) Construct a list that contains the functions you'd like to solve for.
  - *iv*) Use the lists within the appropriate function to obtain the solution for the system of equations.
  - *v*) Use reasonable values for the constants (k, m, L) and plot $x_1(t)$ and $x_2(t)$ against time.

- 3) Use **DSolve**[] to solve the differential equation $\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0$ subject to the initial conditions $x[0] = A_0$ and $\dot{x}[0] = 0$. Plot the resulting solution for each of the following cases and discuss:

  - *i*) $A = 5$ cm, $\omega_0 = 2\pi$ rad/s, $\beta = \frac{\pi}{7}$ rad/sec
  - *ii*) $A = 5$ cm, $\omega_0 = 2\pi$ rad/s, $\beta = \frac{5\pi}{2}$ rad/sec
  - *iii*) $A = 5$ cm, $\omega_0 = 2\pi$ rad/s, $\beta = 2\pi$ rad/sec

  (Case *iii* is, indeed, physical and you *should* be able to generate a meaningful plot. It may take you a while to figure out how. You may want to review functions and delayed assignments in Mathematica.)

  Once you've got this figured out, see if you can can make the plot within **Manipulate**[] and watch it evolve as β is increased from 0 to 4π. You'll probably need to use the **PlotRange** option to fix the axes.

- 4) An under-damped mass-spring system, initially at rest in equilibrium on a frictionless horizontal surface, is subjected to a constant driving force ($\frac{F}{m} = a$) for a length of time equal to some fraction (f) of the natural period of the system.

  - *i*) Solve for the equations of motion for the system during both the driven and subsequent undriven intervals.

- *ii*) Take $\tau_0 = 1$, $\beta = \frac{\omega_0}{3}$, $a = 12$, $f = 0.2$ and plot the motion of the system from $0 < t < 4$ (assuming the force is first applied at $t = 0$). It may help to color the two two intervals differently, say red and blue (PlotStyle).