Labor Market Dynamics with an Informal Sector

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First draft: May 15, 2025
This version: May 16, 2025
(Latest version)

1 Introduction

Labor market informality is a defining characteristic of developing economies, with informal employment accounting for a substantial share of total employment. While often viewed as a symptom of institutional and regulatory failures, the informal sector plays a complex dual role. Although avoiding regulations undermines government tax revenue and worker rights and protections, it also means being unburdened by formal sector rigidities that prevent the flexibility to adjust to changing economic conditions.

The state of the business cycle can crucially impact the decision process of firms and workers regarding informality, as both firms' incentives to create formal jobs and workers' valuation of informal employment opportunities vary with the aggregate conditions of the economy. This feature, however, is often overlooked in the literature. Despite the empirical evidence on the counter-cyclical behavior of informality and the differences in the patterns of the flows to and from unemployment of workers with distinct formality statuses, much of the previous work has focused on steady-state economies with no space for changing aggregate productivity.

This paper develops a quantitative framework to analyze how the presence of an informal sector shapes labor market dynamics over the business cycle. The model allows us to focus on two aspects: (1) the aggregate implications of informality for labor market outcomes during economic fluctuations, and (2) the distributional effects across workers with different skill levels.

Our framework displays heterogeneous workers who differ in their human capital and firms that can choose between creating formal or informal jobs. Hence, we focus on informality as informal wage work hired by firms rather than a frictionless self-employment state of the workers. Aggregate productivity shocks generate business cycle fluctuations, endogenous separations, and changes in the optimal formality sector choice of workers and firms.

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Our methodological contribution lies in developing a tractable framework while incorporating worker heterogeneity, search and matching frictions, and aggregate uncertainty. The key to achieving this is the model's block recursive structure (Menzio and Shi, 2010, 2011; Kaas, 2023). This feature means that agents' optimal decisions depend on the aggregate state only through the level of aggregate productivity, with no need to track the entire distribution of workers across states. Block recursivity emerges as a combination of (i) directed search, where firms post vacancies in specific submarkets and workers self-select into their preferred locations, (ii) free entry of firms, and (iii) a constant returns to scale matching function. These features make it possible to pin down each submarket's tightness, thus compressing all information necessary for the worker's choice. It contrasts with random search models, where firms and workers face uncertainty about their potential matches, and thus value functions depend on the full distribution of worker types and states.

We provide a numerical example with a preliminary calibration of the model to simulate the dynamic behavior of informality and employment through aggregate fluctuations and policy counterfactuals. We demonstrate that informal employment can serve as a crucial margin of adjustment during economic downturns, particularly for low-skilled workers. In the baseline economy, the presence of an informal sector significantly reduces unemployment and improves welfare for low-skilled workers, even though it has lower wages. High-skilled workers, operating exclusively in the formal sector, are unaffected in the alternative scenario of the absence of informality. These findings mark the importance of the distributional effects of the cyclical dynamic of formal and informal jobs.

In counterfactual policy experiments, we examine the impact of formal sector regulations. Not only do these margins impact the sectoral decisions of workers and firms, but they are also some of the feasible instruments in efforts to tame down informality. Hence, it is crucial for policymakers to know how efficient each particular intervention is, its aggregate impacts, its distributional consequences, and the relevance of the timing of the policy action along the business cycle. The potential trade-offs between less informality and more unemployment, particularly during downturns, add to the complexity of designing labor market policies in economies with substantial informal employment.

We build on a rich body of work analyzing labor market informality in steady-state environments. This literature has enhanced our understanding of the incentives faced by firms and workers and their implications on firm dynamics, productivity, and earnings, while also assessing the impacts of various policy and regulatory changes (Haanwinckel and Soares, 2021; Meghir, Narita, and Robin, 2015; Ulyssea, 2018, 2020). However, they abstract from the role of business cycle fluctuations.

The cyclical properties of informal labor markets are the focus of a relatively smaller literature. Shapiro (2014) studies the impact of having a large self-employed "sector" on the behavior of employment and output along the business cycle. Fiess, Fugazza, and Maloney (2010) also treat the informal sector as self-employment to understand its impact on the transmission of economic shocks. Fernández and Meza (2015), Leyva and Urrutia (2020), and Horvath and Yang (2022) build business cycle models with informal sectors and measure the impact of informality and labor regulation on macroeconomic volatility and the propagation of shocks. The first two also model informality as a frictionless self-employment state. Thus, they abstract from the informal margin of informality, through which formal firms hire workers off the books. Closer to our framework is Bosch and Esteban-Pretel (2012). However, as the previous papers mentioned, workers are ex-ante homogeneous. The block recursive setting of our framework allows us to study the distributional effects of the interaction of the business cycle and informality, as well as the impact of policy changes, by incorporating worker heterogeneity. On the firm's side, we endogenize both the job-finding and the separation rates, as well as the intensive margin of informality, by allowing firms to optimally choose when to create and terminate vacancies in both sectors.

In what follows, we start presenting a brief empirical overview of the cyclical behavior of labor market stocks and flows in Brazil (Section 2). Section 3 describes our model, while Section 4 presents the parametrization and the results of the numerical example. Section 5 concludes with the final remarks.

2 Empirical Evidence

In this section, we provide empirical evidence on the cyclical patterns of informality in developing economies using Brazil as our setting. We use microdata from two labor market surveys, PME and PNADC, which, taken together, span the period from 2002 to 2024.[1]

The *Pesquisa Mensal de Emprego* (PME) was a rotating household survey with monthly visits that covered urban areas of six large metropolitan regions. It is available for the period from 2002.03 to 2016.02. The *Pesquisa Nacional por Amostra de Domicilio Contínua* (PNADC), started in 2012, is a quarterly frequency survey that eventually replaced PME, updating its methodology and expanding its questionnaire and its covered area, with interviews conducted in all Brazilian states and national representation.

For both datasets, we restrict our sample to prime-age workers (18-55 years) in urban areas. We adopt the methodology implemented by Data Zoom (2025) to explore the panel aspect of the microdata and construct indicators of the labor market transitions experienced by each worker over subsequent interviews.

Besides the fact that informality is a widespread phenomenon in the country, Brazil is an adequate setting for studying the topic because its institutional design provides a clear and widely understood notion of what constitutes a formal job: labor regulation requires that, when formally hiring a worker, the firm sign a document called *Carteira de Trabalho e Previdência Social* (CTPS). By doing so, the firm informs competent authorities of the hiring, committing itself to paying all mandatory benefits to the worker (such as compliance with the minimum wage and contributions to social security and a fund available upon layoff) and associated payroll taxes. Workers are aware of whether they were hired under a signed CTPS, and both PME and PNADC

[1] Given the uncommon behavior of labor markets during the COVID-19 pandemic, we restrict the sample up to the last quarter of 2019 when calculating the empirical moments.

surveys explicitly ask employed individuals if their work regime follows this formalized scheme or not. Thus, this provides a clear and effective way to classify the workers in the sample into the formal or the informal sector.

We define six employment statuses: employed in the formal sector (those with a CTPS contract, military, and statutory public workers), employed as informal wage workers (those working for someone else without a CTPS contract), self-employed, employers, unemployed, and out of the labor force. The informal sector comprises the combination of informal wage workers and the self-employed.

Table 1 presents the cyclical properties of key labor market stocks using data from the two household surveys. It reports the standard deviation and the correlation with economic activity of the logs of employment rates as a share of the working-age population, unemployment as a share of the labor force, informality as a share of total employment, and decompositions of employment by formality condition (formal, informal wage employment, and self-employment). All series are seasonally adjusted using the X-13 ARIMA-SEATS method and further smoothed using a centered moving average of adjacent periods to minimize high-frequency noise.

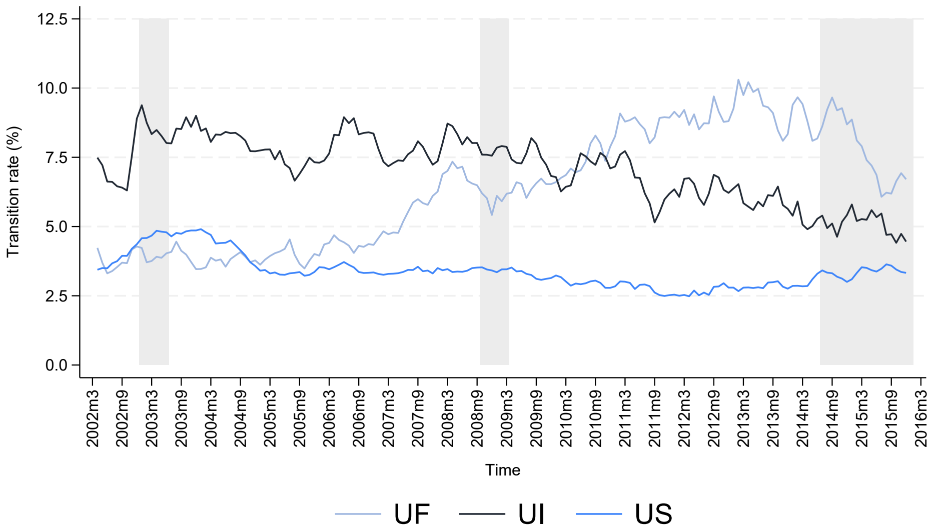
Table 1: Labor Market Stocks (in logs)

Variable	S.D	Corr. with activity
PME – Monthly, 2002-2016		
Employment (% work. age pop.)	0.039	0.115
Unemployment (% labor force)	0.289	-0.142
Informality (% employed)	0.123	-0.042
Formal employment (% work. age pop.)	0.115	0.062
Informal employment (% work. age pop.)	0.090	-0.013
Informal wage employment (% work. age pop.)	0.169	-0.026
Self-employment (% work. age pop.)	0.027	0.085
PNADC – Quarterly, 2012-2019		
Employment (% work. age pop.)	0.017	0.737
Unemployment (% labor force)	0.254	-0.601
Informality (% employed)	0.050	-0.226
Formal employment (% work. age pop.)	0.051	0.392
Informal employment (% work. age pop.)	0.040	0.023
Informal wage employment (% work. age pop.)	0.041	0.481
Self-employment (% work. age pop.)	0.057	-0.287

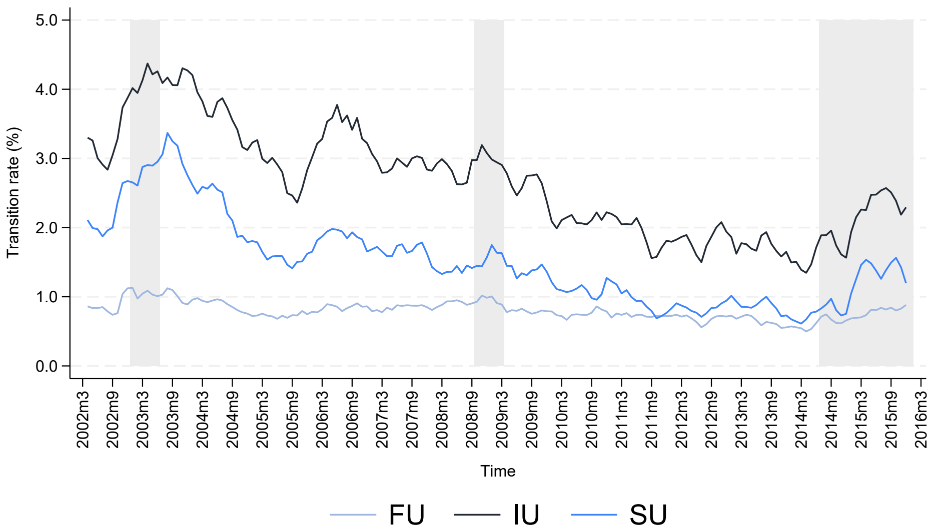
Notes: logs of seasonally adjusted stocks by the X-13 ARIMA-SEATS method and further smoothed by a centered-moving average using the two adjacent periods. Stocks calculated using the surveys' expansion weights. Monthly activity measure (for PME period): smoothed and seasonally adjusted IBC-BR index from Banco Central do Brasil (2025). Quarterly activity measure (for PNADC period): cycle component of the log of seasonally adjusted and smoothed GDP.

Figure 1: Cyclicality of labor market flows

(a) Job-finding rate



(b) Job-separation rate



Notes: monthly flows from PME, seasonally adjusted by the X-13 ARIMA-SEATS method and further smoothed by a centered-moving average using the two adjacent periods. Flows calculated using the survey's expansion weights. Shaded areas indicate recession periods as defined by CODACE (2023). "U" stands for unemployment, "F" for formal employment, "I" for informal wage work, and "S" for self-employment.

The data is in line with previous findings of the literature on counter-cyclical behavior of the informality rate. While total and formal employment show pro-cyclical behavior, informal employment's cyclical relationship is weaker, with correlations near zero (-0.013 for PME and 0.023 for PNADC), suggesting its role as a buffer during economic fluctuations. Notably, the correlation between economic activity and the two types of informal work (wage employment versus self-employment) shows survey-dependent variations. During the PME period, informal wage work was slightly counter-cyclical while self-employment correlated positively with the business cycle. The opposite holds for the PNADC data.

In terms of variance over time, informal employment shows a more stable trajectory than formal employment in both surveys. Once more, however, the two components of informal work present opposite behavior depending on the survey, with informal wage work having lower variance than formal and self-employment during PME and higher variance than the other two for the PNADC pre-pandemic period.

We now move to the analysis of labor market flows. Figure 1 presents monthly transition rates between employment states using PME data (2002-2016), which provides a comprehensive view of labor market dynamics across three recession and expansion cycles.[2] Panel (a) shows job-finding rates from unemployment into formal employment (UF), informal wage employment (UI), and self-employment (US). During recessions, the formal sector job-finding rate exhibits substantial declines, while transitions into informal employment remain relatively stable. Panel (b) displays job-separation rates (FU, IU, and SU). The formal sector demonstrates consistently lower and more stable separation rates, with only modest increases during economic downturns.

In contrast, both types of informal employment show higher and more volatile separation rates, which increase substantially during recessions. These patterns suggest that the counter-cyclical nature of informality stems primarily from changes in job creation rather than job destruction. During economic downturns, while formal job opportunities become scarce, the informal sector continues to absorb workers despite its higher turnover rates. This pattern highlights the potential role of informal employment as a buffer against unemployment during adverse economic conditions, providing an alternative source of income when formal jobs are harder to secure.

3 The Model

The economy is populated by a continuum of workers of two skill types $h \in \{low, high\}$, with measures $\pi_l + \pi_h = 1$. There is a continuum of firms with positive measure. Each worker has an endowment of 1 (indivisible) unit of labor. Workers have linear instantaneous utility and all agents have discount factor β.[3]

There are two types of job relations ("sectors"), $s \in \{F, I\}$, where F stands for formal and I stands for informal. A job in the formal sector is subject to a minimum wage, a benefit for the worker in case of losing the job, and the application of costs: (i) wage taxes for the workers; (ii) payroll and profit taxes for the firms; and (iii) firing costs for the firms. An informal job is not subject to any of these labor market regulations. However, it is subject to a higher exogenous separation rate and uses less productive technology.

[2] We focus on the PME period as it spans a longer time frame and captures multiple business cycles, offering a broader view of the cyclical properties of labor market transitions. The PNADC series is shorter and restricted by the pandemic period, covering only one recession – the same that appears at the end of the PME sample. Still, the described behavior of the job-finding and job-separation rates in PME continues to hold for PNADC.

[3] Linear utility allows for a simple way to model unemployment insurance.

There is an aggregate productivity shock z. The productivity of a job depends on the skill level h of the worker and has an idiosyncratic time-fixed component, y, which is drawn after the match occurs from a distribution G. The production technology is sector-dependent, with $f(h, s, y; z) = \exp\{A_s + z + y\} \times h$, where $A_F > A_I$ captures the fact that formal firms have access to better institutions, such as credit markets, litigation services, etc. The vacancy posting cost in sector s is κ_s. Unemployed workers have home production $b(h)$.

Each worker searching for a job chooses a particular submarket indexed by $\eta = (h, s, w, \bar{y}; z)$, where h is their human capital level, s is the sector, w is the wage offered, $\bar{y}$ is the idiosyncratic productivity threshold for the match (more on that later), and z is the aggregate state of the economy. Note that, as skill level h and the aggregate state z are given, the choice component of a submarket is $(s, w, \bar{y})$. There is a constant return to scale matching function $M(u, v)$, where u is the number of unemployed workers and v is the number of vacancies. Each submarket has market tightness $\theta(\eta) = \frac{v(\eta)}{u(\eta)} \geq 0$, which is determined in equilibrium. The probability of a worker in a given market being selected to a match and the probability of a firm filling a vacancy posted in a market are, respectively:

$$p\big(\theta(\eta)\big) = \frac{M\big(u(\eta), v(\eta)\big)}{u(\eta)} = M\big(1, \theta(\eta)\big) \tag{1}$$

$$q\big(\theta(\eta)\big) = \frac{p\big(\theta(\eta)\big)}{\theta(\eta)}. \tag{2}$$

The timing of the economy is as follows:

i) The aggregate uncertainty is resolved.
ii) Separations happen exogenously and as an endogenous choice of the firms.
iii) Unemployed workers search, deciding which market to visit.
iv) Matches occur.
v) Productivity levels are drawn for the newly formed jobs. Only matches with productivity higher than the associated threshold continue.
vi) Production, consumption, and tax payments occur.

3.1 Bellman Equations

Unemployed worker

An unemployed worker with skill level h enjoys utility from their home production level $b(h)$ and chooses the sector s, wage w, and productivity threshold $\bar{y}$ that characterize a submarket $\eta = (h, s, w, \bar{y}; z)$ to search for a job, given aggregate productivity level z. They are matched to a vacancy with probability $p(\theta(\eta))$. If the match happens, the productivity y is drawn from G. If the value of y is smaller than $\bar{y}$, the match is immediately destroyed and the worker remains unemployed. Otherwise, they become an employed worker with value function $W(h, s, w, y; z')$ next period. We denote by $m^U(h; z) \in (S \times W \times Y)$ the policy function of the optimal submarket choice of the unemployed worker. The value function for an unemployed worker is

$$U(h;z) = b(h) + \beta\, \mathbb{E}_{z'}\left[S^U(h;z') \,\middle|\, z\right], \tag{3}$$

where $S^U(h;z)$ is the value of searching while unemployed for a worker of skill level h when the aggregate state is z, given by

$$\begin{aligned} S^U(h;z) = \max_{s,w,\bar{y}} \Big\{ & U(h;z) \; + \; p\big(\theta(h,s,w,\bar{y};z)\big)\big(1-G(\bar{y})\big) \times \\ & \bigg(\mathbb{E}_y\Big[W(h,s,w,y;z) \,\Big|\, y > \bar{y}\Big] - U(h;z)\bigg)\Big\}. \end{aligned} \tag{4}$$

We can simplify notation by defining the consolidated probability of getting a job in submarket $\eta = (h,s,w,\bar{y};z)$ as

$$\tilde{p}\big(\theta(\eta)\big) = p\big(\theta(\eta)\big)\big(1-G(\bar{y})\big). \tag{5}$$

Employed worker

Denote by $x = (h,s,w,y;z)$ the state of a worker with human capital h employed in sector s, with wage w, and idiosyncratic productivity y when the aggregate productivity level is z. Their value function is $W(x)$. Over their wage w, the worker is subject to a tax rate $\tau^w(s)$ given by

$$\tau^w(s) = \begin{cases} \tau^w > 0 & \text{if } s = F \\ 0 & \text{if } s = I, \end{cases} \tag{6}$$

that is, there is income tax only in the formal sector.

An ongoing match is subject to endogenous separation by the firm. If the firm wants to keep the match, there is still an exogenous separation rate $\delta(s)$. Let $d(x)$ represent this consolidated separation rate, which we define more precisely later in this section. If the match ends due to the firm choice or by exogenous reasons, the worker is entitled to receive benefits $B(s) \times w$, where the rate $B(s)$ is given by

$$B(s) = \begin{cases} B > 0 & \text{if } s = F \\ 0 & \text{if } s = I. \end{cases} \tag{7}$$

This captures the fact that only formal jobs are associated with (temporary) unemployment insurance, which we model as a one-time payment. A laid-off worker can search immediately as an unemployed worker.

Let $x' = (h,s,w,y;z')$. The value of an ongoing match for the worker with state x is

$$W(x) \;=\; \big(1-\tau^w(s)\big)w \;+\; \beta\mathbb{E}_{z'}\bigg[W(x') + d(x')\Big(B(s)w \;+\; S^U(h;z') - W(x')\Big) \,\bigg|\, z\bigg]. \tag{8}$$

Note that the only state variable that changes between periods for an employed worker is the aggregate productivity level z. This change might trigger a separation decision by the firm.

Value of a vacancy

A vacancy posted by a firm in the submarket $\eta = (h, s, w, \bar{y}; z)$ costs κ_s to be maintained open and has an associated value of

$$V(\eta) = -\kappa_s + q\big(\theta(\eta)\big)\mathbb{E}_y\left[\tilde{J}(\eta, y)\right], \tag{9}$$

where

$$\tilde{J}(\eta, y) = \begin{cases} J(h, s, w, y; z) & \text{if } y \geq \bar{y} \\ 0 & \text{otherwise.} \end{cases} \tag{10}$$

Equation 10 captures that, after the match is formed, its idiosyncratic productivity y is drawn from G, but the resulting level might be too low for the agreed threshold $\bar{y}$. In that case, the match is immediately terminated and the firm gains no value. If the drawn value of y is high enough for the match to go forward, the firm enjoys value $J(h, s, w, y; z)$.

With free entry, it must be the case that

$$\kappa_s \geq q\big(\theta(\eta)\big)\mathbb{E}_y\left[\tilde{J}(\eta, y)\right] \tag{11}$$

and $\theta(\eta) \geq 0$ with complementary slackness. That means that (11) holds with equality ($V(\eta) = 0$) in every submarket where firms post vacancies (markets where $\theta(\eta) > 0$). Hence, the equilibrium condition for vacancy posting can be rewritten as

$$\theta(\eta) = q^{-1}\left(\frac{\kappa_s}{\mathbb{E}_y\left[\tilde{J}(\eta, y)\right]}\right) \quad \text{if } \theta(\eta) > 0. \tag{12}$$

Value of a match to the firm

When a match of type $x = (h, s, w, y; z)$ is maintained, the firm enjoys a profit flow $\pi(x)$ given by

$$\pi\big(x\big) = \big(1 - \tau^p(s)\big)\Big(e^{A_s+z+y} \times h \; - \; \big(1 + \tau^f(s)\big)w\Big) \tag{13}$$

The firm is subject to sector-specific payroll taxes and profit taxes at rates given by

$$\tau^f(s) = \begin{cases} \tau^f & \text{if } s = F \\ 0 & \text{if } s = I \end{cases} \tag{14}$$

and

$$\tau^p(s) = \begin{cases} \tau^p & \text{if } s = F \\ 0 & \text{if } s = I. \end{cases} \tag{15}$$

After the aggregate uncertainty is resolved, the firm can choose to terminate a no longer profitable match. The match is terminated with probability one if the continuation value for the firm is inferior to the cost of separation, $C(s)$, with

$$C(s) = \begin{cases} C > 0 & \text{if } s = F \\ 0 & \text{if } s = I. \end{cases} \tag{16}$$

Otherwise, the match ends accordingly to the exogenous separation rate $\delta(s)$ – which might differ by sector:

$$\delta(s) = \begin{cases} \delta_F \geq 0 & \text{if } s = F \\ \delta_I > 0 & \text{if } s = I. \end{cases} \tag{17}$$

The value for the firm of an ongoing match $x = (h, s, w, y; z)$ is thus given by

$$\begin{aligned} J(x) &= \pi(x) + \beta \mathbb{E}_{z'} \left[\max_d \left\{ d\big(-C(s)\big) + \big(1-d\big) J\big(x'\big) \right\} \middle| z \right] \\ &\text{s.t.} \quad \delta(s) \leq d \leq 1. \end{aligned} \tag{18}$$

3.2 Distribution of workers across states

Let $\mu_z(a)$ denote the measure of employed workers in a match of a given type $a = (h, s, w, y)$ when the aggregate state is z and $u_z(h)$ be the measure of unemployed workers over state $(h; z)$, both at the production stage.

In a given instant, the inflow of people into unemployment is formed by the employed workers who were laid off and were unsuccessful in their search attempt:

$$\sum_a \left[\mu_z(a) \times d(a; z)\big(1 - \tilde{p}\big(\theta(h, m^*; z')\big)\right], \tag{19}$$

where $m^* = m^U(h; z')$.

On the other hand, the outflow from unemployment is composed of the individuals who were matched to a firm and drew high enough productivity to sustain the match in their submarket choice:

$$\sum_h \Big(u_z(h) \times \tilde{p}\big(\theta(h, m^*; z')\big) \Big). \tag{20}$$

3.3 The government

The government has four sources of revenue:

i) Income tax at rate τ^w charged only of workers in formal matches:

$$\tau^w \times \sum_{h,w,y} \Big(w \times \mu_z(h, F, w, y) \Big)$$

ii) Payroll tax at rate τ^f charged only of firms in formal matches:

$$\tau^f \times \sum_{h,w,y} \Big(w \times \mu_z(h, F, w, y) \Big)$$

iii) Profit tax at rate τ^p charged only of firms in formal matches:

$$\tau^p \times \sum_{h,w,y} \Big[\Big(e^{A_s + z + + y} \times h - \big(1 + \tau^f(s)\big) w \Big) \times \mu_z(h, F, w, y) \Big]$$

iv) Firing costs C charged of formal matches subject to separation:

$$C \times \sum_{h,w,y} \Big(\mu_z(h, F, w, y) \times d\big(h, F, w, y; z'\big) \Big),$$

where the timing should be noted: $\mu_z(x)$ is the measure of workers in matches x at the production stage when the aggregate state is z. Next period, the aggregate state is z', which leads to firing decisions ruled by $d(x; z')$. Hence, at tomorrow's firing stage, the measure of workers with state x is still $\mu_z(x)$.

On the expenditure side, the government provides unemployment insurance payments B to workers being exogenously separated from formal matches. The amount paid by the government in benefits (in the production stage next period) is

$$B \times \sum_{h,w,y} \Big(w \times \mu_z(h, F, w, y) \times d\big(h, F, w, y; z'\big) \Big)$$

3.4 Equilibrium

Let $x = (h, s, w, y; z)$ represent a match state and $\eta = (h, s, w, \bar{y}; z)$ represent a submarket. A recursive equilibrium in this economy is a set of individual policy functions for unemployed search $m^U(h; z)$, firing by the firms $d(x)$, market tightness functions $\theta(\eta)$, and distributions $u_z(h)$ and $\mu_z(h, s, w, y)$ for the unemployed and employed workers over their states, respectively, such that:

1. Agents' decision rules are optimal.
2. The labor market tightness satisfies the free entry condition given in Equation 12.
3. The distribution of workers across states is consistent with individual policy functions.

We relax the necessity of a balanced government budget to have Conditional Block Recursivity in the model: the objects in conditions (1) and (2) will depend on the aggregate state only through z and not through the distributions of workers.

4 A Numerical Example

In this section, we discuss the parametrization of a numerical example of the model's application. We start by defining the specific functional forms for the functions presented in the model in the previous section and then discuss the baseline values set for the parameters. This is a preliminary exercise, as the complete estimation of the model is still forthcoming, but it is useful to illustrate the role of the informal sector and of the labor market regulation throughout the business cycle.

4.1 Parametrization

Table 2 summarizes the six functions in the model and the selected functional forms specified for each of them. Employed and unemployed workers enjoy linear utility from their income. The home production function is set as two discrete values, one for each human capital, with $b_h \geq b_l$. Production is a function of the human capital of the worker, the productivity of sector in which they work, the idiosyncratic productivity of the match, and the aggregate productivity level. Sector, idiosyncratic, and aggregate components jointly impact the productivity of the worker through an exponential term.

Table 2: Functional forms of the model

Function	Description	Functional Form	
$u(w)$	Utility function	Linear	$u(w) = w$
$b(h)$	Home production function	Discrete	$b(h) = b_h$
$f(\cdot)$	Production function	Exponential	$f(h, s, y, z) = e^{A_s + y + z} \times h$
$M(u, v)$	Matching function	From Menzio and Shi (2010)	$M(u, v) = v\left(1 + (\frac{v}{u})^{\alpha}\right)^{\frac{-1}{\alpha}}$
$G(y)$	Distribution of y	Normal	$y \sim N(\mu_y, \sigma_y^2)$
$\Phi(z)$	Law of motion of z	AR(1)	$z_t = \rho l z_{t-1} + \epsilon_t$, where $\epsilon_t \sim N(0, \sigma_\epsilon^2)$

The constant returns to scale matching function probability follows Menzio and Shi (2010), and it is such that $p(\theta) = \theta\left(1 + \theta^{\alpha}\right)^{-\frac{1}{\alpha}}$. We assume the idiosyncratic productivity y is drawn from a normal distribution with mean μ_y and standard deviation σ_y. For the aggregate productivity level, we assume it follows a mean-zero AR(1), which we discretize using Rouwenhorst method as a three state grid: $Z = [z_1, z_2, z_3]$, where $z_1 < 0$ is a bad state, $z_2 = 0$ is a neutral state, and $z_3 > 0$ is a good state.

The model parameters are set based on the features of the Brazilian labor market as its reference or standard values from the literature. The exceptions are the processes for the idiosyncratic and the aggregate productivity levels, which, for now, are merely illustrative. All baseline levels are shown in Table 3.

Worker Parameters. The discount factor β is set to 0.99175, corresponding to a quarterly real interest rate of 3.4% per annum – a value close to the observed real rate for Brazil in the period between 2012-2024. Worker skill levels are normalized with low-skilled workers having human capital $h_L = 1.0$, while high-skilled workers are twice as productive with $h_H = 2.0$. Home production values are set proportionally to human capital levels, with both types receiving 40% of their respective human capital ($b_L = 0.4h_L$ and $b_H = 0.4h_H$).

Firm Parameters. The productivity parameter of the informal sector is set at $A_I = 0.0$ so that it has a neutral effect on the worker's productivity. The parameter for the formal sector is $A_F = ln(1.5)$, making it 1.5 times more productive than the informal sector. The vacancy posting cost in the formal sector is set to $\kappa_F = 1.0$, which corresponds to approximately a fourth of the production of a low-skilled worker in the intermediate productivity levels for y and z. The vacancy posting cost in the informal sector is set as being five times smaller, so $\kappa_I = 0.2$. This captures the fact that vacancies are more expensive in the formal sector due to labor regulations.

Ambient Parameters. Following Menzio and Shi (2010), the elasticity of the matching function α is set to 0.2. The labor force is equally divided between skill types ($\pi_L = \pi_H = 0.5$). Separation rates are calibrated to match average quarterly data for workers of 18-55 years from the PNADC survey for the period 2012-2019, with the formal sector experiencing lower separations ($\delta_F = 0.023$) compared to the informal sector ($\delta_I = 0.065$).

Shock Parameters. The aggregate shock process features high persistence ($\rho_z = 0.9$) with innovation standard deviation $\sigma_\epsilon = 0.25$. The idiosyncratic shock is centered at $\mu_y = 1.0$ with standard deviation $\sigma_y = 0.10$.

Policy Parameters. The minimum wage $\bar{w}_m = 3.0$ is set to approximately half of the average formal sector production at the intermediary values of y and z. Tax rates reflect Brazilian institutional features, with an 11% income tax rate based on maximum effective tax rates from Sindifisco Nacional (2023), and a 37% payroll tax following Bosch and Esteban-Pretel (2012). The profit tax rate τ^p is set to zero. Unemployment insurance provides an additional full wage at layoff ($B = 1.0$), while separation costs are set at five times the human capital of low-skilled workers ($C = 5h_L$).

Table 3: Baseline levels of parameters

Parameter	Description	Default Value	Obs.
Parameters of the worker			
β	Discount factor	0.99175	Quarterly (avg. real $r = 3,4\%$ p.a.)
h_L	Human capital level of low-skilled workers	1.0	Normalized to 1
h_H	Human capital level of high-skilled workers	2.0	Twice more productive than L
b_L	Home production of low-skilled workers	$0.4h_L$	
b_H	Home production of high-skilled workers	$0.4h_H$	
Parameters of the firm			
A_I	Productivity of the informal sector	0.0	
A_F	Productivity of the formal sector	$ln(1.5)$	1.5 times more productive than I
κ_F	Vacancy posting cost in the formal sector	1.0	Approx. 25% of average $f(L, F, \cdot, \cdot)$
κ_I	Vacancy posting cost in the informal sector	0.2	A fifth of the formal sector cost
Parameters of the matching function:			
α	Elasticity of the matching function	0.2	Menzio and Shi (2010)
Parameters of the ambient			
π_L	Share of low-skilled workers	0.5	
π_H	Share of high-skilled workers	0.5	
δ_F	Exogenous separation rate in the formal sector	0.023	Avg. quarterly FU rate (PNADC)
δ_I	Exogenous separation rate in the informal sec.	0.065	Avg. quarterly IU rate (PNADC)
Parameters of the aggregate shock			
ρ_z	Persistence	0.9	
σ_ϵ	Std. deviation of the innovation	0.25	
Parameters of the idiosyncratic shock			
μ_y	Mean	1.0	
σ_y	Standard deviation	0.10	
Policy parameters			
$\bar{w}_m$	Minimum wage	3.0	Approx. half average $f(\cdot, F, \cdot, \cdot)$
τ^w	Income tax rate	0.11	Max. eff. tax rate (Sindifisco Nacional, 2023)
τ^f	Payroll tax rate	0.37	Bosch and Esteban-Pretel (2012)
τ^p	Profits tax rate	0.0	
B	UI benefits multiplier	1.0	Additional full wage at layoff
C	Separation cost	$5h_L$	
Computational parameters			
N_z	Size of the aggregate shock grid	3	
N_y	Size of the idiosyncratic shock grid	5	
N_w	Size of the wage grid	200	
w_{min}	Min value of wage grid	10^{-6}	> 0
w_{max}	Max value of wage grid	30	> largest feasible production
ζ	Curvature of expanding grid	2.0	
ν	Dampening factor for VFI	0.30	
tol	Tolerance for convergence	10^{-7}	

Computational Parameters. The numerical implementation uses discrete grids with 3 points for aggregate shocks (N_z), 5 points for idiosyncratic shocks (N_y), and 200 points for wages (N_w). The wage grid spans from near zero ($w_{min} = 10^{-6}$) to 30, exceeding the maximum possible production given this economy's technology. Instead of being uniformly spaced, the wage grid is defined with a curvature parameter $\zeta = 2.0$, which puts more points near its lower bound. The update in each iteration of the value function is dampened by a factor $\nu = 0.30$, and convergence is defined with a tolerance of 10^{-7}.

4.2 Simulation

To measure the dynamic role of the informal sector during changing economic conditions, we simulate an economy with $N = 1000$ individuals, respecting the distribution of skill types, for 500 periods.[4] We obtain randomly generated trajectories for the aggregate productivity shock, the draws of the idiosyncratic productivity levels, the exogenous separation shocks, and the realization of the matching probabilities.

We identify recession periods as moments when the aggregate productivity level reaches its lowest value, z_1, and "normal times" when it assumes the intermediary or the highest level. The baseline model successfully reproduces the counter-cyclical behavior of the unemployment and informality rates observed in the data.[5] Recessions lead to an increase in the endogenous separations by firms, as previously profitable matches are no longer sustainable given the worsened aggregate state. Relatedly, unemployed workers switch their choice of submarket to search towards the informal sector, as it presents higher matching probabilities during downturns. It's worth highlighting that such probabilities are not exogenous, but equilibrium objects resulting from the optimal decisions of workers and firms engaging in the search and match effort.

Panels (a) and (b) of Figure 2 show the trajectories of the unemployment and the informality rates for the baseline scenario as blue lines. To highlight the role of the informal sector, we compare the baseline results to an alternative version of the model where the informal sector is shut down. More specifically, we prohibit matches in the informal sector by setting their probabilities to zero. The counterfactual trajectories for this alternative setting in Figure 2 are the red lines. In a world without informality, the unemployment rate would be systematically higher, particularly during recessions. As aggregate conditions deteriorate, the regulation in the formal sector becomes too burdensome for less productive matches, as they may no longer produce enough to sustain the costs associated with payroll taxes and the minimum wage.

Panel (c) shows the average wage among employed workers in the two economies. The world without an informal sector has higher wages: the formal sector offers better pay, as it is more productive and bound to pay at least the minimum wage. However, this higher average wage is also due to composition effects among the employed population: workers who would be receiving less in the informal sector are, instead, unemployed. This phenomenon is so strong that it leads to contrary cyclical behavior of the average wage between the two economies. In the baseline model, labor income falls during recessions and increases in normal times. The opposite is true for the setting without informality, because when less productive matches end during recessions, with the formal sector inaccessible, workers become unemployed. Only the

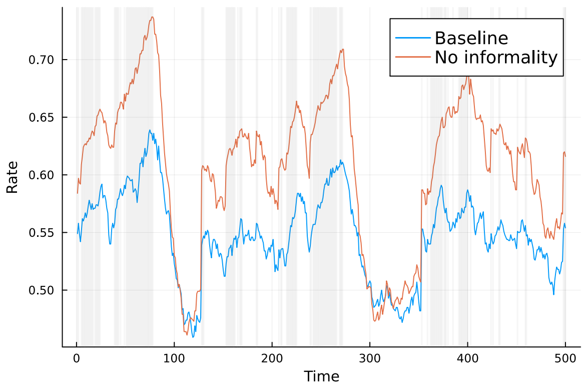
[4]More precisely, we simulate the model for 600 periods but drop the initial 100.

[5]This preliminary calibration generates exceedingly high levels of unemployment, despite the exogenous separation rates and the endogenous job-finding rates being in line with what we observe in the data. The fraction of the time workers in the model experience unemployment in too high, likely due to the high variance of the aggregate shock process set in this illustrative example.

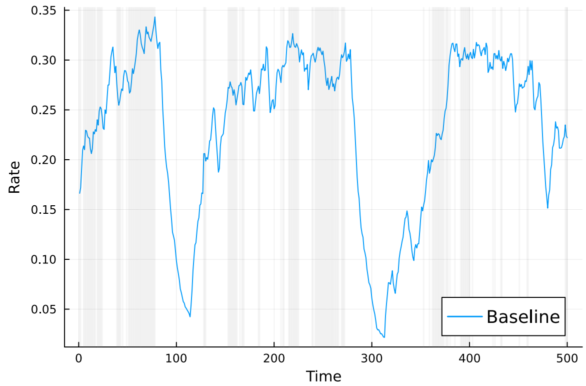
most productive individuals and matches, associated with the higher wages, remain active. When economic conditions improve, less productive workers return to the workforce, bringing down the average wage. Compared to the baseline scenario, the overall welfare effect is negative (see Panel d).

Figure 2: Labor Market Outcomes: Baseline vs No Informality

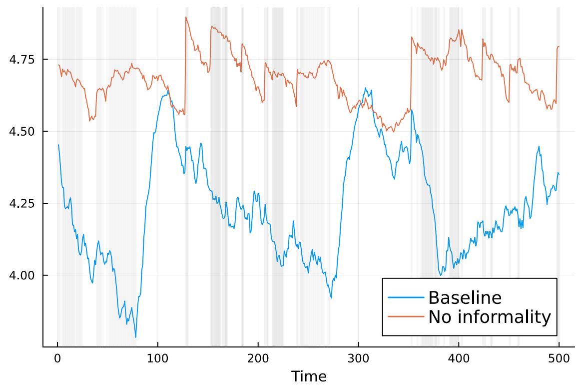
(a) Unemployment Rate



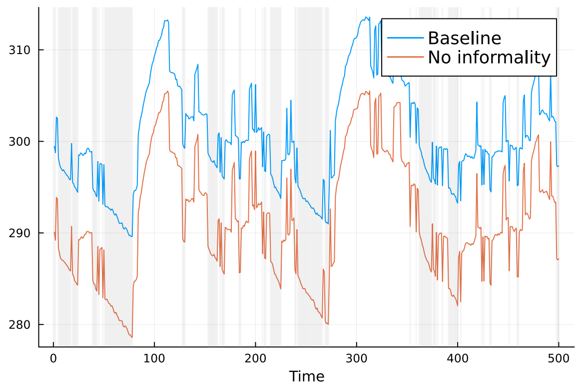
(b) Informality Rate



(c) Average Wage



(d) Average Welfare



Note: The figure shows the evolution of key labor market outcomes comparing the baseline model (with informality) to a counterfactual model without informality. Gray shaded areas indicate recession periods.

In Table 4, we compare the outcomes of the baseline and the no informality simulations for workers of each of the two human capital levels. Given the preliminary calibration, a dual market emerges with the complete segmentation of the informal sector, where only low-skilled workers participate. For this group, the presence of informality substantially reduces unemployment rates, not only during recessions, but also in normal times. While these workers earn lower wages in the baseline model with informality, their welfare is markedly higher, suggesting that the flexibility and employment opportunities provided by the informal sector outweigh the wage penalty. In contrast, high-skilled workers show no difference in outcomes between the two scenarios, as they operate exclusively in the formal sector. This contrast in effects across skill levels emphasizes that informality primarily serves as a buffer for vulnerable workers, particularly during economic downturns. These findings illustrate the complexity of policy decisions regarding informal labor markets, as measures to reduce informality without proper adjustment of social safety nets could disproportionately affect the most vulnerable workers.

Table 4: Labor Market Outcomes by Skill Level and Economic Conditions

	Baseline (A)		No Informality (B)		Difference (B-A)	
	Recession	Normal	Recession	Normal	Recession	Normal
Low-Skilled Workers:						
Unemployment	0.707	0.675	0.868	0.773	0.161	0.098
Welfare	151.023	158.435	130.621	141.184	-20.402	-17.251
Wage	54.484	58.672	64.651	65.211	10.167	6.539
High-Skilled Workers:						
Unemployment	0.439	0.395	0.439	0.395	0.000	0.000
Welfare	438.445	448.489	438.445	448.489	0.000	0.000
Wage	83.172	83.998	83.172	83.998	0.000	0.000

Note: This table shows average labor market outcomes by worker skill level and economic conditions. The last two columns show the difference between the baseline model with informality (A) and the model without informality (B).

While the calibration remains preliminary, the model provides a valuable framework for analyzing policy interventions over the instruments governing the formal labor market regulation. To illustrate this point, we observe the labor market outcomes of conducting counterfactual exercises on a few key policy dimensions: (1) the minimum wage policy ($\bar{w}_m$), comparing scenarios with 20% lower and 20% higher wage floors relative to baseline; (2) firing costs (C), comparing scenarios where it is increased or decreased by 50%; and (3) payroll taxes (τ^f), analyzing the effects of 50% reductions and increases in the formal sector tax rate. For each counterfactual, we simulate the economy using the same sequence of aggregate and idiosyncratic shocks as in the baseline model, allowing us to isolate the pure policy effects. In Table 5, we show how each counterfactual policy instrument affects unemployment, informality rates, and welfare across worker skill levels, providing insights into the aggregate and distributional impacts of labor market regulations.

Table 5: Policy Counterfactuals: Impact on Labor Market Outcomes

	Low-Skilled			High-Skilled		
Policy	Unemp.	Inform.	Welfare	Unemp.	Inform.	Welfare
Baseline ($\bar{w}_m = 3.00, C = 5.00, \tau^f = 0.37$)	0.685	0.660	156.049	0.409	0.000	445.255
Minimum wage						
Lower: $\bar{w}_m = 2.40$	-0.120	-0.660	31.230	0.000	0.000	0.000
Higher: $\bar{w}_m = 3.60$	0.026	0.336	-6.167	0.000	0.000	0.000
Firing cost						
Lower: $C = 2.50$	-0.004	-0.012	1.213	-0.004	0.000	1.757
Higher: $C = 7.50$	-0.002	-0.006	-1.069	0.007	0.000	0.206
Payroll tax						
Lower: $\tau^f = 0.19$	-0.079	-0.660	47.390	-0.000	0.000	66.018
Higher: $\tau^f = 0.55$	0.026	0.336	-6.167	0.058	0.119	-52.622

Note: First row shows baseline levels. All other entries show differences relative to baseline. Results correspond to entire simulation period, including both normal and recession periods.

The minimum wage counterfactual exercises reveal heterogeneity in policy impacts across worker types. In this calibrated setting, a 20% reduction in the minimum wage generates improvements for low-skilled workers: both unemployment and informality fall, generating a welfare increase relative to the baseline scenario. Conversely, raising the minimum wage by 20% increases unemployment and informality while reducing welfare. High-skilled workers remain unaffected by minimum wage changes, suggesting they operate well above this threshold.

Changes in firing costs produce smaller effects. A 50% reduction in C generates modest improvements: unemployment falls slightly for both skill groups, while welfare increases. As for payroll tax changes, cutting the tax rate decreases both unemployment and informality, with benefits for both skill groups. Notably, increasing the payroll tax pushes even the high-skilled workers into informality in some scenarios.[6]

5 Concluding Remarks

We've proposed a model capable of incorporating aggregate uncertainty into a search and match framework with informality in order to understand its dynamic role along the business cycle. In a numerical example with a preliminary calibration, we've shown that the model reproduces the main cyclical patterns of the informality and the unemployment rate observed in the data, with distributional implications as workers of different skill levels are affected differently. Naturally, without an adequate estimation of the model, no quantitative conclusions can be drawn. Therefore, the estimation step is a required and ongoing effort in this project, so

[6]We abstract from welfare analysis for the payroll tax case as this is not a balanced-budget exercise.

we can later focus on providing insights on the distributional consequences of informality and the impacts of different policies counterfactuals.

In terms of next steps, the model also calls for some fine-tuning. The idiosyncratic productivity process, aimed at enabling wage and sector choice dispersion between the two skill types, isn't quite fulfilling its intended role. The trade-off between choosing a higher productivity cut-off (which could provide a higher wage) and it being less likely to turn into a match is not valuable to the workers, as optimal choices in the numerical example were to guarantee a match at the lowest cut-off level in all scenarios. This could be a result of workers wanting to leave unemployment at all costs. Hence, and related to the lack of sector dispersion, the addition of on-the-job search is a natural extension to the current framework.

Finally, this framework can be of particular novelty in the analysis of the timing of labor market policy interventions. Are there persistent aggregate or distributional impacts of changing regulations during a specific phase of the business cycle rather than at another moment?

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