1 (4 points) Define a sequence of real numbers (x_n) by

$$x_0 = 1, \qquad x_{n+1} = \frac{1}{2 + x_n} \quad \text{for} \quad n \geq 0.$$

Show that (x_n) converges, and evaluate its limit.

2 (6 points) Let (a_n) and (ϵ_n) be sequences of positive numbers. Assume that $\lim_{n\to\infty} \epsilon_n = 0$ and that there is a number k in $(0, 1)$ such that $a_{n+1} \leq k a_n + \epsilon_n$ for every n. Prove that $\lim_{n\to\infty} a_n = 0$
(Hint : If you are stuck, start like this. Fix $\delta > 0$, and choose n_0 such that $\epsilon_n < \delta$ for all $n \geq n_0$. Then $a_{n_0+1} \leq k a_{n_0} + \epsilon_{n_0} < k a_{n_0} + \delta$)