1 4 points

Let f be a continuous function on $[0,1]$. Evaluate the limit

$$\lim_{n\to\infty} \int_0^1 e^{-nx} f(x)\, dx.$$

2 6 points

Let S be a closed, unbounded set in $\mathbb{R}$ and let f be a function in $C_\infty(S)$. Suppose that $\lim_{x\to-\infty} f(x)$ and $\lim_{x\to\infty} f(x)$ exist. Prove that f is uniformly continuous on S.