MAS241 ANALYSIS 1 QUIZ 5

Problem 1. *(21 points) Prove or disprove the following statements. You should write the proof or counterexample. If your answer is wrong, there will be -3 points deduction. Note that we always assume the Euclidean space with Euxlidean metric.*

(1) *If* $\{C_k\}$ *is a sequence of compact, nonempty subsets of* $\mathbb{R}^n$ *and satisfies* $C_k \supseteq C_{k+1}$ *for each* k*, then* $\bigcap_{n=1}^{\infty} C_k = \{x_0\}$ *for some point* $x_0 \in \mathbb{R}^n$*.*

(2) *Let* f *be continuous on* $[a, b] \subset \mathbb{R}$*. Define* $g(x)$ *on* $[a, b]$ *as follows:* $g(a) = f(a)$ *and* $g(x) = \inf\{f(y) : y \in [a, x]\}$ *for* $x \in (a, b]$*. Then,* g *is monotone decreasing and continuous on* $[a, b]$*.*

(3) *Let* S *be a compact subset of* $\mathbb{R}^n$ *and* $\{C_k\}$ *be a sequence of closed subsets of* $\mathbb{R}^n$ *which satisfies* $\bigcap_{n=1}^{\infty} C_k = \emptyset$*. Then, there exists a finite index set* $A \subset \mathbb{N}$ *which satisfies* $S \cap \bigcap_{\alpha \in A} C_\alpha = \emptyset$*.*

Problem 2. *(9 points) For* $x \in \mathbb{R}$*, let define function* $f(x) = x$ *if* x *is rational and* $f(x) = -x$ *if* x *is irrational. Show that* f *is continuous at only 1 point and discontinuous at others points.*

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