**1** 4 points

Prove that

$$X = \{0\} \cup \bigcup_{k=1}^{\infty} \left[ \frac{1}{2k} \, , \, \frac{1}{2k-1} \right]$$

is a compact subset of $\mathbb{R}$.

**2** 6 points

Prove that

$$Y = \{0\} \cup \left\{ \frac{\cos k}{k^3} \;\middle|\; k \in \mathbb{N} \right\}$$

is a compact subset of $\mathbb{R}$.