MAS242 ANALYSIS I QUIZ 3

Problem 1. *(15 points) For* $\mathbf{x} = (x_1, x_2)$ *and* $\mathbf{y} = (y_1, y_2)$ *in* $\mathbb{R}^2$, *define*

$$d_1(\mathbf{x}, \mathbf{y}) = |x_1 - y_1| + |x_2 - y_2|$$

(1) Show that d_1 is a metric on $\mathbb{R}^2$.(5pt)

(2) Define a d_1 neighborhood $N_1(\mathbf{x}; s)$ of $\mathbf{x} = (x_1, x_2)$ to be $N_1(\mathbf{x}; s) = \{\mathbf{y} \in \mathbb{R}^2 : d_1(\mathbf{x}, \mathbf{y}) < s\}$. Let $N(\mathbf{x}; r)$ be any Euclidean neighborhood of $\mathbf{x}$. Show that there exist positive r_1 and r_2 such that

$$N_1(\mathbf{x}; r_1) \subset N(\mathbf{x}; r) \subset N_1(\mathbf{x}; r_2)$$

(10pt)

Problem 2. *(15 points) Prove that, if* $\{\mathbf{x}_k\}$ *is a bounded sequence in* $\mathbb{R}^n$ *and* $\mathbf{y}_1, \ldots, \mathbf{y}_M$ *are cluster points of* $\{\mathbf{x}_k\}$, *then* $S = \{\mathbf{x}_k : k \in \mathbb{N}\} \cup \{\mathbf{y}_1, \ldots, \mathbf{y}_M\}$ *is closed*

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