2-(a) Let $\pi = \{a = x_0 < x_1 < \cdots < x_p = b\}$ be a partition of $[a, b]$. Since $x_i \in [a, b]$, we have from the given condition that

$$\sum_{i=1}^{p} |f(x_i) - f(x_{i-1})| \leq \sum_{i=1}^{p} M|x_i - x_{i-1}| = M(b-a).$$

Since $M(b-a)$ does not depend on choice of partition, we conclude $f \in BV(a, b)$.

2-(b) For $x \neq 0$, we have

$$f'(x) = \alpha x^{\alpha-1} \sin \frac{1}{x^\beta} - \beta x^{\alpha-\beta-1} \cos \frac{1}{x^\beta}.$$

For $x = 0$, we have

$$f'(0) = \lim_{x \to 0} \frac{x^\alpha \sin 1/x^\beta}{x} = \lim_{x \to 0} x^{\alpha-1} \sin \frac{1}{x^\beta} = 0$$

since $\alpha > 1$. Thus $f$ is differentiable on $\mathbb{R}$ and by the Mean Value Theorem, for each $x, y \in [a, b]$, there is $c \in (x, y)$ such that

$$|f(x) - f(y)| = |f'(c)||x - y|.$$

Note that

$$|f'(c)| \leq \alpha M^{\alpha-1} + \beta M^{\alpha-\beta-1} =: M'$$

where $M = \max\{|a|, |b|\}$ since $\alpha - \beta - 1 \geq 0$. Therefore, for any $x, y \in [a, b]$, we have

$$|f(x) - f(y)| \leq M'|x - y|.$$

Since $M'$ does not depend on $x, y \in [a, b]$, we conclude from (a) that $f$ is in $BV(a, b)$.