MAS242 Analysis I Quiz 7

May 13, 2021

Problem 1. (6 points) Define a function f on $[0,2]$ as follows:

$$f(x) := \begin{cases} 0 & \text{if } x = 0 \\ x + 1 & \text{if } 0 < x < 1 \\ \frac{5}{2} & \text{if } x = 1 \\ (x-1)^2 + 3 & \text{if } 1 < x < 2 \\ \frac{17}{4} & \text{if } x = 2 \end{cases}$$

Sketch the graphs of f, of the saltus function s_f associated with f, and of $f_c = f - s_f$.
(You do not need to justify them.)

Problem 2.

(a) (12 points) Let $f : [a,b] \to \mathbb{R}$ be a function. Prove that if there is a constant M such that

$$|f(x) - f(y)| \leq M|x - y|$$

for all $x, y \in [a,b]$, then $f \in BV(a,b)$.

(b) (12 points) Let $\alpha, \beta \in \mathbb{R}$ with $\alpha \geq \beta + 1$ and $\beta > 0$. Define $f : \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \begin{cases} x^\alpha \sin\left(1/x^\beta\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0. \end{cases}$$

For $a, b \in \mathbb{R}$, show that $f \in BV(a,b)$.
(Hint : Use (a) and the Mean Value Theorem.)