# MAS242 Analysis I Quiz 6

May 6, 2021

**Problem 1.**

(a) (6 points) Find $\lim_{x\to\infty} x^{\frac{1}{x}}$.

(b) (12 points) Define $f : (-1, 1) \to \mathbb{R}$ by

$$f(x) = \frac{1}{1-x}$$

for $x \in (-1, 1)$. Find the $k$-th degree Taylor polynomial $p_k$ of $f$ at $x = 0$ and prove that $\lim_{k\to\infty} p_k = f$ [uniformly] on any closed interval $[a, b] \subset (-1, 1)$.

**Problem 2.** For partitions $\pi_1$ and $\pi_2$ of $[a, b] \subset \mathbb{R}$, define a partition $\pi_1 \wedge \pi_2$ of $[a, b]$ by taking the intersection of the partition points in $\pi_1$ and $\pi_2$.

(a) (4 points) Prove that, if $\pi \preceq \pi_1$ and $\pi \preceq \pi_2$, then $\pi \preceq \pi_1 \wedge \pi_2$.

(b) (4 points) Prove that, if $\pi_1 \wedge \pi_2 = \pi_1$, then $\pi_1 \preceq \pi_2$.

(c) (4 points) Prove that, $\pi_1 \vee \pi_2 = \pi_1 \wedge \pi_2$ if and only if $\pi_1 = \pi_2$.