1   Define a function f on $[0, 1]$ as

3+3
points

$$f(x) = \begin{cases} x^p & \text{if } x \text{ is rational,} \\ 0 & \text{elsewhere,} \end{cases}$$

where p is a positive constant and $p \neq 1$.

(a) Prove that f is continuous only at $x = 0$.

(b) Find all the points that f is differentiable, and fine the value of derivatives at those points.

2   Define a function f_n on $\mathbb{R}$ as

1+3
points

$$f_n(x) = \begin{cases} 0 & \text{if } x = 0, \\ x^n \sin \frac{1}{x} & \text{elsewhere,} \end{cases}$$

where n is a positive integer.

(a) Prove that f_n is continuous on $\mathbb{R}$ for all n.

(b) Is f_n is differentiable at $x = 0$? If so, find $f_n'(0)$.