1 5 points

Suppose f is continuous on $[0,1]$. Prove that

$$\int_0^\pi x f(\sin x)dx = \frac{\pi}{2}\int_0^\pi f(\sin x)dx.$$

2 5 points

Let $\{f_k\}$ be a sequence of continuously differentiable functions on $[a,b]$ such that

(i) $\lim_{k\to\infty} f_k = f_0$ pointwise on $[a,b]$, and

(ii) $\lim_{k\to\infty} f_k' = g$ uniformly on $[a,b]$.

Prove that f_0 is differentiable on $[a,b]$ and $f_0' = g$.