MAS241 Analysis 1 Quiz 8

May 20, 2021, 13:45 – 14:10

Problem 1. (18 points) Let g be a nonnegative continuous function on $[a, b]$. Let f be the function on $[a, b]$ defined by

$$f(x) = \begin{cases} g(x) & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational.} \end{cases}$$

(1) (6 points) Find $L(f)$ and $U(f)$ in terms of g.

(2) (12 points) Prove that f is Riemann integrable on $[a, b]$ if and only if g is identically zero on $[a, b]$.

Problem 2. (12 points) Let f be a nonnegative Riemann integrable function on $[a, b]$. Prove that f^α is Riemann integrable on $[a, b]$ for all $\alpha > 1$. (Here, f^α is the function defined by $x \mapsto f(x)^\alpha$.)