1 4 points

Prove that (a) $\iff$ (b).

(a) Archimedes' principle holds.

(b) For any $c > 0$, there exists some k in $\mathbb{N}$ such that $k - 1 \leq c < k$.

2 6 points

Let $S = \left\{ \sum_{k=1}^{n} \frac{1}{(k!)^2} \,\middle|\, n \in \mathbb{N} \right\}$.

(a) Explain why $\sup S$ exists in $\mathbb{R}$.

(b) Prove that $\sup S$ is a limit point of S.