MAS241 Analysis 1 Quiz 9

May 27, 2021, 13:45 – 14:10

Problem 1. (18 points) For each of the following sentences, we assume f is Riemann integrable on $[a,b]$ and F is the function on $[a,b]$ defined by

$$F(x) = \int_a^x f(t)\,\mathrm{d}t.$$

Mark **True** or **False**. Then **justify your answers in one or two sentences**. (There are no extra minus points if your answer is opposite to the correct one.)

(a) (6 points) The function $x \mapsto f(x)F(x)$ is Riemann integrable on $[a,b]$.

(b) (6 points) If F is differentiable at $c \in (a,b)$, then f is continuous at c.

(c) (6 points) There exists $c \in [a,b]$ such that

$$\frac{F(b)-F(a)}{b-a} = f(c).$$

Problem 2. (12 points) Let f be a bounded function on $\mathbb{R}$, which is continuous on $\mathbb{R} \setminus S$ for some finite set S. Prove that f is Riemann integrable on every closed interval. (Hint: Consider a small neighborhood near each point in S.)