**1** 5 points

For $\mathbf{x} = (x_1, x_2)$ in $\mathbb{R}^2$, define

$$||\mathbf{x}||_\infty = \max\{|x_1|, |x_2|\}$$

and

$$d_\infty(\mathbf{x}, \mathbf{y}) = ||\mathbf{x} - \mathbf{y}||_\infty.$$

Prove that $d_\infty(\mathbf{x}, \mathbf{y})$ is a metric on $\mathbb{R}^2$.

**2** 5 points

Define the set $D$ on $\mathbb{R}^2$ as

$$D = \{(x, y) \in \mathbb{R}^2 | x^2 + y^2 \leq 1, y > 0\}.$$

Is $D$ an open set or a closed set on $\mathbb{R}^2$? Explain your answer.