2-(b) If you did not clarify and show what is the k-th derivative of f, then there is (-3 points). (You must use mathematical induction on k.) Most of you used theorme 4.5.3 to show uniform convergence. But it does not work in this case. Since $f^{(k)}(x) = \frac{k!}{(1-x)^{k+1}}$, we have $\|f^{(k)}\|_\infty^{1/k} \geq (k!)^{1/k}$ if $b \geq 0$. But

$$\log(k!)^{1/k} = \frac{\log 1 + \cdots + \log k}{k} \geq \frac{(k-m)\log m}{k-m}\frac{k-m}{k} \to \log m$$

as $k \to \infty$ for any $m = 1, 2, \cdots$. Thus $(k!)^{1/k}$ diverges as $k \to \infty$ and theorem 4.5.3 can not be used. To show the uniform convergence, you must note that

$$|f(x) - p_k(x)| = \left|\frac{x^{k+1}}{1-x}\right| \leq \frac{c^{k+1}}{1-b} \to 0$$

as $k \to \infty$ where $c = \max\{|a|, |b|\}$. Since c does not depend on $x \in [a, b]$, this is a direct proof of the uniform convergence.

Also, using the remainder R_k in Taylor's theorem 4.5.2 is incorrect. In the theorem, the constant c depends on the degree k, x and x_0. Thus, since k tends to ∞ and we must consider all of $x \in [a, b]$, it does not work well.