Ch 1. The Foundations: Logic and Proofs

Propositional Logic-2

Truth Table

Sungwon Kang

Acknowledgement
- [Rosen 19] Kenneth H. Rosen, for Discrete Mathematics & Its Applications (8th Edition), Lecture slides
- [Hunter 11] David J. Hunter, Essentials of Discrete Mathematics, 2nd Edition, Jones & Bartlett Publishers, 2011, Lecture Slides

Propositional Logic: Semantics

Any statement has two possible truth values: true(T) or false(F).

☛ A connective can be viewed as a "truth value function".

Ch 1. The Foundations: Logic and Proofs

1.1 Propositional Logic
1.2 Applications of Propositional Logic
1.3 Propositional Equivalences
1.4 Predicates and Quantifiers
1.5 Nested Quantifiers
1.6 Rules of Inference
1.7 Introduction to Proofs
1.8 Proof Methods and Strategy

Truth tables for $\neg$ [redacted]

p	$\neg p$
T	F
F	T

not p.

Truth tables for $\wedge$ [redacted]

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p and q.

Truth tables for ∨ [redacted]

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p or q.

"Or" can be inclusive or exclusive.

Example

"Soup or Salad?"

Which one is this?

Truth tables for $\rightarrow$ [redacted]

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

p implies q.
If p then q.
p only if q.

Why T when p is F?
1) p $\rightarrow$q is not false.
2) If it is F, then $\rightarrow$
becomes the same as $\wedge$.

Truth tables for $\leftrightarrow$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

p if and only if q.
p iff q.

$\leftrightarrow$: if and only if
bi-conditional
bi-implication

Logical Equivalences

Definition

Two statements are *logically equivalent* if they have the same T/F values for all cases, that is, if they have the same truth tables.

Example

$p \wedge q \equiv q \wedge p$

$p \vee q \equiv q \vee p$

Note) "≡" is not a logical connective.
It is a "meta" language symbol
that asserts truth value
equivalence of two logical expressions.

Compare the following statements:

$p \to q$

If a quadrilateral has a pair of parallel sides, then it has a pair of supplementary angles.

$\neg q \to \neg p$ (contrapositive of "$p \to q$")

If a quadrilateral does not have a pair of supplementary angles, then it does not have a pair of parallel sides.

Equivalent to "$p \to q$" ?

$q \to p$ (converse of "$p \to q$")

If a quadrilateral has a pair of supplementary angles, then it has a pair of parallel sides.

Equivalent to "$p \to q$" ?

That is $p \to q \equiv \neg q \to \neg p$ $p \to q \not\equiv q \to p$

How can you prove these observations?

Contrapositive and Converse

$p \to q$ is logically equivalent to $\neg q \to \neg p$: contrapositive

p	q
T	T
T	F
F	T
F	F

Contrapositive and Converse

$p \to q$ is logically equivalent to $\neg q \to \neg p$: contrapositive

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

$\neg q \to \neg p$
?

Contrapositive and Converse

$p \to q$ is logically equivalent to $\neg q \to \neg p$: contrapositive

p	q	$p \to q$	$\neg q$	$\neg p$	$\neg q \to \neg p$
T	T	T	F	F	
T	F	F	T	F	?
F	T	T	F	T	
F	F	T	T	T	

Contrapositive and Converse

$p \to q$ is logically equivalent to $\neg q \to \neg p$: contrapositive

p	q	$p \to q$	$\neg q$	$\neg p$	$\neg q \to \neg p$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Contrapositive and Converse

$p \to q$ is logically equivalent to $\neg q \to \neg p$: contrapositive

p	q	$p \to q$	$\neg q$	$\neg p$	$\neg q \to \neg p$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

$q \to p$ is *not* logically equivalent to $p \to q$: converse

p	q	$q \to p$
T	T	T
T	F	T
F	T	F
F	F	T

A harder example

If Aaron is late then Bill is late, and, if both Aaron and Bill are late, then class is boring. Suppose that class is not boring. What can you conclude about Aaron?

A harder example

p = "Aaron is late."
q = "Bill is late."
r = "Class is boring."
$S = (p \rightarrow q) \wedge ((p \wedge q) \rightarrow r)$

S = "If Aaron is late then Bill is late, and, if both Aaron and Bill are late, then class is boring." Suppose that class is not boring. What can you conclude about Aaron?

A harder example

p = "Aaron is late."
q = "Bill is late."
r = "Class is boring."
$S = (p \rightarrow q) \wedge ((p \wedge q) \rightarrow r)$

S = "If Aaron is late then Bill is late, and, if both Aaron and Bill are late, then class is boring." Suppose that class is not boring. What can you conclude about Aaron?

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

A harder example

p = "Aaron is late."
q = "Bill is late."
r = "Class is boring."
S = $(p \rightarrow q) \wedge ((p \wedge q) \rightarrow r)$

S = "If Aaron is late then Bill is late, and, if both Aaron and Bill are late, then class is boring." Suppose that class is not boring. What can you conclude about Aaron?

p	q	r	$p \rightarrow q$
T	T	T	T
T	T	F	T
T	F	T	F
T	F	F	F
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

$(p \wedge q) \rightarrow r$
?

A harder example

p = "Aaron is late."
q = "Bill is late."
r = "Class is boring."
S = $(p \to q) \wedge ((p \wedge q) \to r)$

S = "If Aaron is late then Bill is late, and, if both Aaron and Bill are late, then class is boring." Suppose that class is not boring. What can you conclude about Aaron?

p	q	r	$p \to q$	$p \wedge q$	$(p \wedge q) \to r$	S
T	T	T	T	T	T	
T	T	F	T	T	F	
T	F	T	F	F	T	
T	F	F	F	F	T	?
F	T	T	T	F	T	
F	T	F	T	F	T	
F	F	T	T	F	T	
F	F	F	T	F	T	

A harder example

p = "Aaron is late."
q = "Bill is late."
r = "Class is boring."
$S = (p \to q) \land ((p \land q) \to r)$

S = "If Aaron is late then Bill is late, and, if both Aaron and Bill are late, then class is boring." Suppose that class is not boring. What can you conclude about Aaron?

p	q	r	$p \to q$	$p \land q$	$(p \land q) \to r$	S
T	T	T	T	T	T	T
T	T	F	T	T	F	F
T	F	T	F	F	T	F
T	F	F	F	F	T	F
F	T	T	T	F	T	T
F	T	F	T	F	T	T
F	F	T	T	F	T	T
F	F	F	T	F	T	T

A harder example

p = "Aaron is late."
q = "Bill is late."
r = "Class is boring."
S = (p → q) ∧ ((p∧ q) →r)

S = "If Aaron is late then Bill is late, and, if both Aaron and Bill are late, then class is boring." Suppose that class is not boring. What can you conclude about Aaron?

p	q	r	$p \to q$	$p \wedge q$	$(p \wedge q) \to r$	S
T	T	T	T	T	T	T
T	T	F	T	T	F	F
T	F	T	F	F	T	F
T	F	F	F	F	T	F
F	T	T	T	F	T	T
F	T	F	T	F	T	T
F	F	T	T	F	T	T
F	F	F	T	F	T	T

Quiz 03-1

For each pair of propositions P and Q below, state whether or not P is logically equivalent to Q.

(1) P: $p \wedge q$ Q: $\neg p \vee \neg q$ ______________

(2) P: $p \rightarrow q$ Q: $\neg p \vee q$ ______________

(3) P: $p \wedge (\neg q \vee r)$ Q: $p \vee (q \wedge \neg r)$ ______________