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HW 1 - Propositional Logic

1) a) $(q \wedge p) \longrightarrow (\neg r)$

b) If the car will start, then the head gasket is not blown nor is there water in the cylinders

2) a) $\neg r \longrightarrow \neg (p \vee q)$

b) If you are in Kwangju, then you are in South Korea and you are not in Seoul

3) We'll prove the following logical equivalence to translate our formulas into natural English:

$p \to q$ is logically equivalent to $\neg p \vee q$

Proof: The truth table for the conditional statement

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Hence, $p \to q$ is false $\Leftrightarrow$ $\Leftrightarrow$ p is True, q is False

p	q	$p \vee q$
T	T	T
F	T	T
T	F	T
F	F	F

The truth table for disjunction of two propositions

$\neg p \vee q$ is false $\Leftrightarrow$ $\neg p$ is false, q is false $\Leftrightarrow$ $\Leftrightarrow$ p is True, q is False $\Leftrightarrow$ $p \to q$ is false

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Hence, we proved logical equivalence of these 2 expressions

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a) Using the previous formula, we obtain that $P \to (Q \to R)$ is logically equivalent to $P \to (\neg Q \vee R)$. Therefore, translating into English

If logic is enjoyable, then Kim will not pass or Kim concentrates

b) Using previous property, $(R \to Q) \to P$ is logically equivalent to $(\neg R \vee Q) \to P$, and translating it

If Kim does not concentrate or Kim will pass, then logic is enjoyable

c) Using previous formula, $S \to (P \to (\neg Q \to \neg R))$ is logically equivalent to $S \to (P \to (Q \vee \neg R))$ since $\neg(\neg Q)$ is same as Q. Similarly, the latter is logically equivalent to $S \to (\neg P \vee (Q \vee \neg R))$ or just $S \to (\neg P \vee Q \vee (\neg R))$. Translating into English

If the text is readable, then either logic is not enjoyable, or Kim will pass, or Kim does not concentrate.

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HW-1: Propositional Logic

4) a) $q \to \neg p$

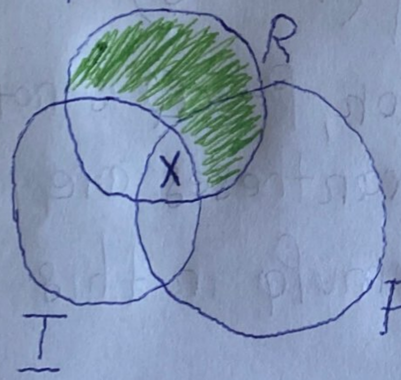
b) $q \wedge p$

c) $(\neg q) \vee (\neg p)$

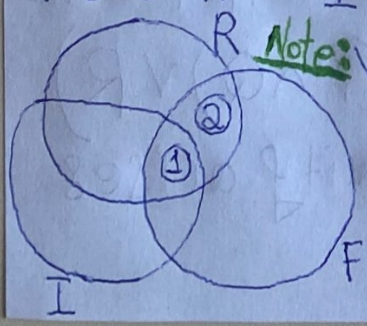
d) $q \longleftrightarrow p$

6) reformers $\to$ R
idealists $\to$ I
fanatics $\to$ F

All R are I	A
Some R are F	I
Some I are F	I



Using green shade to indicate the absence of elements from premise "All R are I" and implementing that result to put "X" for indicating the existence of elements from "Some R are F", we showed that given conclusion is valid. It is true since we can observe from Venn Diagram that "X" states there are some I elements which are also F



Note: when we use "Some R are F", it means we should put "X" either on 1 or 2. As region 2 was shaded in green, this indicates the absence of that region and forces "x" to be put on region 1

5) Let's first write the BNF grammar of all atomic propositional formulas (denoted by $\mathcal{A}$):

$$\mathcal{A} ::= A \mid B \mid C \mid \ldots$$

where $A, B, C, \ldots$ are defined in the original problem. Note that this definition does not require any form of induction, apart from the one used in definition of integer in the original problem statement.

Then, the BNF grammar of all propositional formulas (denoted by $F, G, \ldots$) is the following:

$$F, G ::= \mathcal{A} \mid \neg F \mid (F \wedge G) \mid (F \vee G) \mid (F \rightarrow G)$$

Note that, according to this definition, $P \rightarrow \neg Q$ is not a propositional formula, because parentheses are missing. The correct propositional formula in this case is $(P \rightarrow \neg Q)$.

This massive use of parentheses is required to avoid ambiguous expressions such as $P \wedge Q \vee R$ can be considered as propositional formulas. Indeed, in $P \wedge Q \vee R$, it's not clear what is the principal connective. Correct propositional formulas are $((P \wedge Q) \vee R)$ and $(P \wedge (Q \vee R))$, where no ambiguity arises

Note: An *atomic proposition* is a statement that must be true or false. *Propositional formulas* are constructed from atomic propositions by using logical connectives.

For well-formed propositional formulas, we can use BNF

<formula> ::= Atomic Proposition
| $\neg$<formula>
| <formula> $\wedge$ <formula>
| <formula> $\vee$ <formula>
| <formula> $\rightarrow$ <formula>
| <formula> $\Leftrightarrow$ <formula>
| (<formula>)

Note: Sentence $\longrightarrow$ AtomicSentence | ComplexSentence
AtomicSentence $\longrightarrow$ True | False | P | Q | R | ...
ComplexSentence $\longrightarrow$ (Sentence)
| Sentence Connective Sentence
| $\neg$ Sentence
Connective $\longrightarrow$ $\wedge$ | $\vee$ | $\Rightarrow$ | $\Leftrightarrow$

Ambiguities are resolved through precedence
$\neg \wedge \vee \Rightarrow \Leftrightarrow$ or parantheses