Ch 1. The Foundations: Logic and Proofs

Formal System

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Acknowledgement
- [Rosen 19] Kenneth H. Rosen, for Discrete Mathematics & Its Applications (8th Edition), Lecture slides
- [Hunter 11] David J. Hunter, Essentials of Discrete Mathematics, 2nd Edition, Jones & Bartlett Publishers, 2011, Lecture Slides

Ch 1. The Foundations: Logic and Proofs

Formal Logic

Formal Logic

Also known as "Symbolic Logic" and "Mathematical Logic"

Strictly speaking, "Formal" = "Symbolic".

In a wider sense of "Formal", "Formal" = "Rigorous"

Formalism (= Formal System = Formal Theory)

What is it? How does it work?

It is approach to problem solving that

Modeling

Formal notation

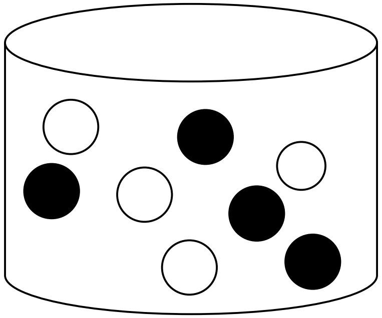
Translate a problem to notation, perform well-defined symbolic manipulations on that notation, then interpret the results to solve the problem.

Example 1 $101_2 + 11_2 = 1000_2$

Formal representations of 5, 3 and 8.

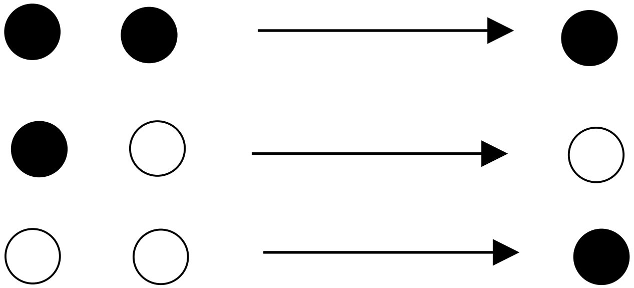
Example 2 - The Problem (1/4)

[Source: Prof. David Garlan]



A jar of balls

Rules



Example 2 - The Problem (2/4)

Question 1: Does the game stop?

Question 2: If so, can you say anything about the color of the last ball by knowing the original configuration?

Example 2 - The Problem (3/4)

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Example 2 - The Problem (4/4)

Question 2: If so, can you say anything about the color of the last ball by knowing the original configuration?

Example 2 - A Formal Model

b = black   w = white   f = transition function

$$
f(b, w) = \begin{cases} (b - 2 + 1, w) = (b - 1, w) \\ (b - 1, w - 1 + 1) = (b - 1, w) \\ (b + 1, w - 2) = (b + 1, w - 2) \end{cases}
$$

$$
\left.\begin{array}{l} f(0, 1) = (0, 1) \\ f(1, 0) = (1, 0) \end{array}\right\} \textbf{No change}
$$

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Is this a model of a problem or a model of a solution?

Example 2 - Reasoning

Theorem 1: The game "stops."

Theorem 2: f *(b, w) = if odd (w) then (0, 1)

else (1, 0).

(Question: How might you prove this?)

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Why use a formal model (or formalism)?

[1] Problem solving via Abstraction: lets us focus on essential things.

- Can take advantage of mathematical reasoning

[2] Computer programs are good at symbol processing.

☛ Computer programs can perform inferences.

We will see more cases of formalism as we move on.

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Quiz 01

[1] Suppose that in the game introduced in this lecture there are 12 white balls and 7 black balls in the jar. What would be the color of the ball remaining in the jar after the game stops?

(a) It is black.

(b) It is white.

(c) It can be both.

(d) We cannot tell.

[2] Which of the following will not be a good problem solving technique when applied suitably?

(a) Divide and conquer

(b) Abstraction

(c) Formal Modeling

(d) Focusing on all the details of the problem at once