CS 204 Discrete Mathematics Fall 2020

Homework 2 – Propositional Logic

Due date September 12, 2020

1. (20 pts)

(a) (14 pts) Construct truth tables for the following propositions:

$P \lor (P \land Q)$

$\neg P \lor (P \land (Q \rightarrow P))$

$(P \rightarrow Q) \rightarrow (\neg P \land Q)$

(b) (6 pts) Which of the above statements are:

Tautologies?

Contingencies ?

Contradictions ?

2. (10 pts) Show, using truth tables, that the following equivalence holds:

$$Q \rightarrow P \equiv \neg(Q \land \neg P)$$

3. (10 pts) Consider the statement $S = [\neg(p \rightarrow q)] \lor [\neg(p \lor q)]$.

(a) Construct a truth table for S

(b) Find a simpler expression that is logically equivalent to S.

4. (10 pts) Mathematicians say that "Statement P is a sufficient condition for statement Q" if $P \rightarrow Q$ is true. In other words, in order to know that Q is true, it is sufficient to know that P is true. Let x be an integer. Give a sufficient condition on x for x/2 to be an even integer.

5. (10 pts) The NAND connective $\uparrow$ is defined by the following truth table.

p	q	$p \uparrow q$
T	T	F
T	F	T
F	T	T
F	F	T

Use truth tables to show that $p \uparrow q$ is logically equivalent to $\neg (p \land q)$. (This explains the name NAND: Not AND.)

6. (12 pts) The NAND connective is important because it is easy to build an electronic circuit that computes the NAND of two signals. Such a circuit is called a logic gate. Moreover, it is possible to build logic gates for the other logical connectives entirely out of NAND gates. Prove this fact by proving the following equivalences, using truth tables.

(a) $(p \uparrow q) \uparrow (p \uparrow q)$ is logically equivalent to $p \wedge q$.

(b) $(p \uparrow p) \uparrow (q \uparrow q)$ is logically equivalent to $p \vee q$

(c) $p \uparrow (q \uparrow q)$ is logically equivalent to $p \rightarrow q$.

7. (14 pts) Fill in the reasons in the following proof sequence. Make sure you indicate which step(s) each derivation rule refers to.

Statement	Reasons
1. $p \wedge (q \vee r)$	given
2. $\neg(p \wedge q)$	given
3. $\neg p \vee \neg q$	
4. $\neg q \vee \neg p$	
5. $q \rightarrow \neg p$	
6. p	
7. $\neg(\neg p)$	
8. $\neg q$	
9. $(q \vee r) \wedge p$	
10. $q \vee r$	
11. $r \vee q$	
12. $\neg(\neg r) \vee q$	
13. $\neg r \rightarrow q$	
14. $\neg(\neg r)$	
15. r	
16. $p \wedge r$	

8. (12 pts) Write a proof sequence for the following inference. Justify each step.

$P \rightarrow \neg Q$

$\neg Q \rightarrow \neg R$

$P \vee \neg R$

$\neg R$

9. (10 pts) Is $a \rightarrow \neg a$ a contradiction? Why or why not?