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Homework-10: Finite probability space, events: Basic concept of probability theory

1) Total number of ways in which any 3 random people can be chosen from 100 is $\binom{100}{3}$. Now, in our favorable cases, participant wins the prize, so in all favorable cases participant is one of the 3 persons in the set which is chosen for winning. Since, other 2 persons can be chosen in $\binom{99}{2}$ ways, probability is

$$P=\frac{\binom{1}{1}\binom{99}{2}}{\binom{100}{3}}=\frac{\frac{99!}{\cancel{97!}2!}}{\frac{100!}{3!\cancel{97!}}}=\frac{\frac{99!}{2!}}{\frac{100!}{3!}}=\frac{\cancel{99!}\cdot 6}{2\cdot\cancel{99!}\cdot 100}=\boxed{\frac{3}{100}}$$

(Choose 1 (one) of the prizes for that participant, and from the remaining 99 contestants, choose the 2 other winners

$$\left(P=\frac{3P(99,2)}{P(100,3)}=\frac{3\cdot\cancel{99}\cdot\cancel{98}}{100\cdot\cancel{99}\cdot\cancel{98}}=\frac{3}{100}\ \checkmark\right)$$

(participant can win first prize in $\binom{99}{2}$ ways, since we have to include in the scenario of the results of other 2 picks, if we are going to divide by $\binom{100}{3}$. Participant can win 2^{nd} or 3^{rd} prize in the same number of ways. So, chances of winning becomes $\frac{\frac{3\cdot 99\cdot 98}{3!}}{\frac{100\cdot 99\cdot 98}{3!}}=\frac{3}{100}$ ✓)

2) Rolling a dice → Since $3+6=4+5=5+4=6+3=9$ and there is not "7 dots" in any dice, there are exactly 4 out of the 36 outcomes that result in a sum of 9

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

As it's seen from the table, possible pairs are $(3,6), (4,5), (5,4), (6,3)$, where total sample space $= 6 \times 6 = 36$.

The probability is the # of favorable outcomes divided by the # of possible outcomes

$$P(\text{sum of 9 with a dice}) = \frac{\#\text{ of favorable}}{\#\text{ of possible}} = \frac{4}{36} = \boxed{\frac{1}{9}}$$

Rolling three dice → We obtain a sum of 9 by 3 dice in the following ways:

126	216	315	414	513	612
135	225	324	423	522	621
144	234	333	432	531	
153	243	342	441		
162	252	351			
	261				

Thus, we note that there are $5+6+5+4+3+2=25$ ways to roll a sum of 9 with 3 dice. Since each dice has 6 possible outcomes, there are $6 \cdot 6 \cdot 6 = 216$ outcomes in total by the product rule. Hence, 25 of the 216 ways result in a sum of 9

$$\text{The probability} = \frac{\#\text{ of favorable outcomes}}{\#\text{ of possible outcomes}}$$

$P(\text{sum of 9 with 3 dice}) = \frac{\#\text{ of favorable}}{\#\text{ of possible}} = \boxed{\frac{25}{216}}$ $(24 \cdot 9 = 216)$

$\frac{1}{9} = \frac{24}{216} < \frac{25}{216} \Rightarrow$ $\boxed{P(\text{sum of 9 with 2 dice}) = \frac{1}{9} < \frac{25}{216} = P(\text{sum of 9 with 3 dice})}$

Hence, $\boxed{\text{probability is higher when rolling 3 dice}}$ Rolling a total of 9 when 3 dice are rolled

3) a) In roll of a die, there are 6 possible outcomes: 1, 2, 3, 4, 5, 6 $\Rightarrow$ As the dice is fair, probability of getting a six is $\frac{1}{6}$. Likewise, the probability of getting no six in one roll of a fair die $= \frac{5}{6}$.

The probability of getting no six in four rolls is

$P(\text{no six in 4 rolls}) = \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} = \left(\frac{5}{6}\right)^4 = \frac{625}{1296} \approx 0.4822$

Thus, in 4 rolls of a fair die, probability of getting at least one six is

$P(\text{at least one six in four rolls}) = 1 - P(\text{no six in 4 rolls}) =$

$= 1 - \left(\frac{5}{6}\right)^4 = 1 - \frac{625}{1296} = \boxed{\frac{671}{1296}}$ ≈ 0.5177469

$\boxed{\text{The probability of getting at least one six in 4 rolls of a fair die is } \frac{671}{1296}}$

b) In a roll of a pair of dice, there are a total of 36 possible outcomes (six outcomes for the 1st die, and 6 outcomes for the 2nd dice, where product rule is used) Out of these 36 outcomes, Only one of them is a double six (i.e. (6,6)). Henceforth, the probability of getting a double six is $\frac{1}{36}$ when rolling a pair of dice $\Rightarrow$ Likewise, probability of not getting a double six is $\frac{35}{36}$

The probability of getting no double six in 24 rolls of a pair of dice is:

$$P(\text{no double six in 24 rolls}) = \left(\frac{35}{36}\right)^{24} \approx 0.5086$$

Thus, the probability of getting at least 1 double six in 24 rolls is:

$$P(\text{at least one double six in 24 rolls}) = 1 - P\left(\text{no double six in 24 rolls}\right)$$

$$= 1 - \left(\frac{35}{36}\right)^{24} \approx 1 - 0.5086 = 0.4914$$

Hence, $\boxed{P(\text{at least one double six in 24 rolls}) = 1 - \left(\frac{35}{36}\right)^{24}}$

Since $35 > 18 \Rightarrow \frac{35^{24}}{36^{24}} > \frac{18^{24}}{36^{24}}$ and $\frac{-35^{24}}{36^{24}} < \frac{-18^{24}}{36^{24}} \Rightarrow$

$1 - \frac{35^{24}}{36^{24}} < 1 - \frac{18^{24}}{36^{24}} = 1 - \left(\frac{18}{36}\right)^{24} = 1 - \left(\frac{1}{2}\right)^{24}$, where we find

$\left(\frac{35}{36}\right)^{24} \approx 0.508596$ and $1 - \left(\frac{35}{36}\right)^{24} \approx 0.49140387613 <$

Note

$< 0.5 = \frac{1}{2} \Rightarrow \boxed{P\left(\text{at least one double six in 24 rolls}\right) = 1 - \left(\frac{35}{36}\right)^{24} \approx 0.4914 < \frac{1}{2}}$

Less than $\frac{1}{2}$ ✓

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c) From part a) and part b), we found that

$P(\text{at least one six in four rolls}) = 1-\left(\frac{5}{6}\right)^4 = \frac{671}{1296}$ and

$P(\text{at least one double six in 24 rolls}) = 1-\left(\frac{35}{36}\right)^{24} \simeq 0.4914$

Since $35>30 \Rightarrow \frac{35}{36} > \frac{30}{36} = \frac{5}{6}$ and $\frac{671}{1296} \simeq 0.5177469$

$1-\left(\frac{5}{6}\right)^4 = \frac{671}{1296} \simeq 0.5177469 > \frac{1}{2} > 1-\left(\frac{35}{36}\right)^{24} \simeq 0.4914$, we

get $\boxed{P(\text{at least one six in 4 rolls}) > P(\text{at least one double six in 24 rolls})}$

Thus, it's more likely that a six comes up when a die is rolled 4 times ✓ ■

4) From the set $\{1,2,3,4\}$, we can select exactly $4\cdot3\cdot2\cdot1 = 4! = 24$ # of permutations (resulting from product rule)

a) We should count how many out of 24 have 1 preceding 4 $\Rightarrow$ There are $3! = 3\cdot2\cdot1 = 6$ permutations which start with 1 ($\boxed{1}\underbrace{\square\,\square\,\square}_{\text{any place for 4 ✓}}$ where 1 precedes 4 in each of 3! cases)

There are $2!\cdot3 = 6$ permutations when 1 is put in other places; i.e. When 1 is put on the second, 4 should be placed either on the third, or on the fourth place

$\underbrace{\square\,\boxed{1}\,\boxed{4}\,\square}$ $\quad$ $\square\,\boxed{1}\,\square\,\boxed{4}$ $\quad$ since $\{2,3\}$ can be put in any boxes for each of these cases

We get $2!+2!=4$ permutations when 1 is put on the second box (notice there are $2!=2\cdot 1$ permutations for $\{2,3\}$ in each these cases)
When 1 is put on the third, 4 should be placed immediately afterwards, and $\square\ \square\ \boxed{1}\ \boxed{4}$ remaining two cells for $\{2,3\}$ can be filled in $2!=2\cdot 1=2$ # of permutations.

Thus, # of favorable outcomes $=6+4+2=12$ and # of possible outcomes $=4!=$ $=24$

$\Rightarrow \boxed{P(1 \text{ precedes } 4)=\frac{12}{24}=\frac{1}{2}}$ (Half of the permutations have 1 before 4, and half have 1 after the 4)

B) There are no permutations which start with 1, 2 permutations which start with 2 ($\boxed{2}\boxed{4}\boxed{1}\boxed{3}$, $\boxed{2}\boxed{4}\boxed{3}\boxed{1}$, $\boxed{2}\boxed{3}\boxed{4}\boxed{1}$), 3 permutations which start with 3 ($\boxed{3}\boxed{4}\boxed{1}\boxed{2}$, $\boxed{3}\boxed{4}\boxed{2}\boxed{1}$, $\boxed{3}\boxed{2}\boxed{4}\boxed{1}$), and 6 permutations which start with 4 ($\boxed{4}\ \underbrace{\square\ \square\ \square}_{\text{any for } 1}$ since 1 can be placed in any 3 boxes, it's equivalent to any permutation for $\{1,2,3\}$, or just $3\cdot 2\cdot 1=3!=6$ where 4 precedes 1 $\Rightarrow$ Thus, $\boxed{\# \text{ of favorable} = 0+3+3+6 = 12}$

$|\text{sample space}|=4!=24 \Rightarrow \boxed{P(4 \text{ precedes } 1)=\frac{12}{24}=\frac{1}{2}}$

It's equivalent problem with part a), where you just swap 4 and 1, and solve problem likewisely same

c) Since 4 should come before 1, and 4 must come before 2, we will consider seperate cases one-by-one: There are no permutations which start with 1, no permutations which start with 2, 2 permutations which start with 3 ($\boxed{3}\boxed{4}\boxed{1}\boxed{2}$, $\boxed{3}\boxed{4}\boxed{2}\boxed{1}$ since 4 should come before 1 and 2, it must be placed on the second box, and there are remaining $2! = 2\cdot 1$ # of permutations for the 3^{rd} and 4^{th} box to fill with $\{1,2\}$) and 6 permutations which start with 4 (since 4 is placed on the first cell, $\boxed{4}\underbrace{\square\square\square}_{\text{any for } 1,2}$ ✓ 1 and 2 can be placed on any of these 3 cells, and consequently, these boxes are any permutations for $\{1,2,3\} \Rightarrow 3! = 3\cdot 2\cdot 1 = 6$ # of permutations are assigned to this case) where 4 precedes both 1 and $2 \Rightarrow$ $\underbrace{\text{\# of favorable} = 0+0+2+6 = 8}$ and $\underbrace{|\text{sample space}| = 4! = 24}$ $\Rightarrow$ $\boxed{P\left(\begin{array}{l}4 \text{ precedes } 1, \text{and} \\ 4 \text{ precedes } 2\end{array}\right) = \frac{8}{24} = \frac{1}{3}}$ ✓

d) Since 4 precedes 1, 4 precedes 2, and 4 precedes $3 \Rightarrow$ it's obvious that $\boxed{4}\square\square\square$ first box should be assigned to 4, (otherwise, it does not satisfy given condition) and remaining set $\{1,2,3\}$ is permuted arbitrarily in those 3 cells. Clearly, arbitrary such permutation satisfies the condition and therefore, it's enough

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to count as $3! = 3\cdot 2\cdot 1 = 6$ ways to place the other three

and that gives $\boxed{\#\text{ of favorable} = 6}$ As there are exactly

$|\text{sample space}| = 4! = 24$ ways to permute $\{1,2,3,4\}$ arbitrarily

We get (no permutations which start with 1,2,3) $\boxed{P\left(\begin{array}{c}4 \text{ precedes } 1,\ 4 \text{ precedes } 2,\\ 4 \text{ precedes } 3\end{array}\right) = \frac{6}{24} = \frac{1}{4}}$ $\boxed{A}$ ✓

c) 4 precedes 3, and 2 precedes 1 $\Rightarrow$ Because 2 comes before 1, there are no permutations which start with 1. Similarly, there are no permutations which start with 3, because 4 should come before 3. There are exactly 3 permutations which start with 2, where the following outcomes are the only possible ones:

$\boxed{2}\ \boxed{}\ \boxed{}\ \boxed{} \longrightarrow \boxed{2}\ \boxed{4}\ \boxed{3}\ \boxed{1}$

$\boxed{2}\ \boxed{4}\ \boxed{1}\ \boxed{3}$

$\boxed{2}\ \boxed{1}\ \boxed{4}\ \boxed{3}$

"1" can be placed in any of remaining 3 boxes, and "4,3" is put on the exactly 1 such way that 4 comes before 3, thus 1 possible outcome for each of these 3 sub-cases ✓ (total $= 1+1+1 = 3$ becomes valid)

Likewise, there are 3 permutations which start with 4 where constructions will be similar to previous arrangements, with 4 starting on the first box:

$\boxed{4}\ \boxed{\ }\ \boxed{\ }\ \boxed{\ } \longrightarrow$

$$\left.\begin{array}{cccc} \boxed{4} & \boxed{3} & \boxed{2} & \boxed{1} \\ \boxed{4} & \boxed{2} & \boxed{3} & \boxed{1} \\ \boxed{4} & \boxed{2} & \boxed{1} & \boxed{3} \end{array}\right\}$$

"3" can be placed in any of these 3 boxes, and remaining 2 cells can be constructed in one way such that 2 comes before 1

Thus, there exist exactly 1 way in each of these 3 cases, giving us a total of 3 possible arrangements starting with 4 $\Rightarrow$ # of outcomes where 4 precedes 3, and 2 precedes 1 $= 0+0+3+3=6$

$\boxed{\text{\# of favorable outcomes} = 6}$ and $\boxed{|\text{sample space}| = 4! = 24}$

Hence, $\boxed{P(4 \text{ precedes } 3,\ 2 \text{ precedes } 1) = \frac{6}{24} = \frac{1}{4}}$ ✓

Conditional probability, Bayes' Theorem; Independence

1) Let A = first flip is heads

B = exactly 4 heads appear in the 5 flips

1 out of the 2 possible outcomes of each flip is heads

Since, probability is the $\dfrac{\text{\# of favorable outcomes}}{\text{\# of possible outcomes}} \Rightarrow$

$\boxed{P(A) = \frac{1}{2}}$ $A \cap B$ represents the event that first flip is heads, and remaining 4 flips contain 3 more heads. The order of the flips is not crucial, thus we use a combination (Totally, there exist $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5 = 32$ ways from the product rule)

$P(A \cap B) = \frac{\text{\# of favorable}}{\text{\# of possible}} = \frac{\binom{4}{3}\binom{1}{1}}{2^5} = \frac{\frac{4!}{3!1!}}{32} = \frac{4}{32} = \frac{1}{8}$

Using the definition of conditional probability, we get

$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{8}}{\frac{1}{2}} = \frac{2}{8} = \frac{1}{4} \Rightarrow \boxed{P(B|A) = \frac{1}{4}}$ ■

(Note that A represents the event that first flip is head; then from the product rule, $P(A) = \frac{1 \cdot 2^4}{2^5} = \frac{1}{2}$, since first flip is already defined)

2) Before starting the sub-problems, we'll define some terms and prove related properties to solve the questions.

- Suppose that an experiment can have only 2 possible outcomes. For instance, when coin is flipped, possible outcomes are heads and tails. Each performance of an experiment with 2 possible outcomes is called a Bernoulli trial. In general, a possible outcome of a Bernoulli trial is called a success or a failure. If p - probability of a success, and q - probability of a failure, it follows that $p + q = 1$

- Many problems can be solved by determining probability of k successes when an experiment consists of n mutually independent Bernoulli trials (Bernoulli trials are mutually independent if the conditional probability of success on any given trial is p, given any info whatsoever about the outcomes of other trials)

a) There are $2^5 = 32$ possible outcomes when there are five children (since each child is either boy, or girl, $2\cdot2\cdot2\cdot2\cdot2=2^5$ # of outcomes are valid from product rule) The number of ways that family with 5 children does not have a boy, it means such possible outcomes $= C(5,0)$. Because the genders of these 5 children are independent, the proba-bility of each of these outcomes (0 boy, 5 girls) is $\left(\frac{1}{2}\right)^0 \cdot \left(1-\frac{1}{2}\right)^5 = \frac{1}{2^5}$, since boy/girl is equally likely. Thus, probability that exactly 0 boys appear is

$$C(5,0)\cdot\left(\frac{1}{2}\right)^0\cdot\left(1-\frac{1}{2}\right)^5 = \frac{5!}{0!\,5!}\cdot 1\cdot \frac{1}{32} = \boxed{\frac{1}{32}} \qquad \boxed{0.03125}$$

b) Similar to part a), # of ways 0 boys appear out of 5 children is $C(5,0)$. Because genders of the children are independent, probability of each of these outcomes (0 boys, 5 girls) is $(0.51)^0\cdot(1-0.51)^5$, where we found this from the product rule, and given condition that probability of a boy $= 0.51 \Rightarrow$ Thus, probability that exactly 0 boys appear is

$$C(5,0)\cdot(0.51)^0\cdot(1-0.51)^5 = \frac{5!}{0!\,5!}\cdot 1\cdot 0.49^5 = \boxed{0.49^5}$$

$$\boxed{0.0282475249}$$

c) Before solving this problem, I will define some terms so that it could be easily understood

-Many problems are concerned with a numerical value associated with the outcome of an experiment. To study problems of this type, we introduce the concept of a random variable. A <u>random variable</u> is a function from the sample space of an experiment to the set of real numbers. That is, random variable assigns a real number to each possible outcome

-The <u>distribution</u> of a random variable X on a sample space S is the set of pairs $(r, p(X=r))$ for all $r \in X(S)$, where $p(X=r)$ is the probability that X takes the value r. (The set of pairs in this distribution is determined by the probabilities $p(X=r)$ for $r \in X(S)$)

$X_i=1$ denotes that the i^{th} child is a boy, and likewise $X_i=0$ defines the i^{th} child to be a girl. From the given condition, $P(X_i=1)=0.51-\frac{i}{100}$ for each $i=1,2,3,4,5$

The probability for each of these Bernoulli trials is independent, thus we can use the <u>multiplication rule for independent events</u> $(P(A \cap B)=P(A \text{ and } B)=P(A) \cdot P(B))$

Since it's asked to evaluate probability that family with 5 children does not have a boy, it means all of that children should be girls, and we have to find probability for

Since $P(X_i=0)+P(X_i=1)=1$, for each $i=1,2,3,4,5$, and we know $X_i=0$ denotes that i^{th} child is a girl,

$$P(X=0)=P(X_1=0)\cdot P(X_2=0)\cdot P(X_3=0)\cdot P(X_4=0)\cdot P(X_5=0)$$

$$=(1-P(X_1=1))(1-P(X_2=1))(1-P(X_3=1))(1-P(X_4=1))(1-P(X_5=1))$$

$$=\left(1-0.51+\frac{1}{100}\right)\left(1-0.51+\frac{2}{100}\right)\left(1-0.51+\frac{3}{100}\right)\left(1-0.51+\frac{4}{100}\right)\cdot$$

$$\cdot\left(1-0.51+\frac{5}{100}\right)=(0.49+0.01)(0.49+0.02)(0.49+0.03)\cdot$$

$$\cdot(0.49+0.04)(0.49+0.05)=(0.50)(0.51)(0.52)(0.53)(0.54)$$

Hence, $\boxed{P(X=0)=(0.50)(0.51)(0.52)(0.53)(0.54)}$ ✱ where

$X=0$ denotes that family with 5 children does not have a boy, i.e. there are 0 boys in the family. Thus,

$\boxed{P(X=0)=0.0379501}$ ✱ ✓

3) We need to find the probability that both the children are boys, given that at least one of them is a boy ⇒ F - at least one of the children is a boy

E - Both the children are boys

Question asks to evaluate $P(E|F)$, where sample space S is given by $S=\{(b,b),(b,g),(g,b),(g,g)\}$ and all outcomes are equally likely

E: Both children are boys

$E=\{(b,b)\}$, and $|E|=1$, $|S|=4$. Using the definition of probability, $\boxed{P(E)=\frac{1}{4}}$

F: at least one of the children is a boy

$F=\{(g,b),(b,g),(b,b)\}$, and $|F|=3$, $|S|=4$. According to the definition of probability, $\boxed{P(F)=\frac{3}{4}}$

Moreover, $E\cap F=\{(b,b)\}$ from the previous cases, and it's logical since ("at least 1 boy" and "2 boys") is equivalent to "2 boys". Since $|E\cap F|=1$ and $|S|=4 \Rightarrow \boxed{P(E\cap F)=\frac{1}{4}}$

From the definition of conditional probability (Bayes' Theorem)

$$P(E|F)=\frac{P(E \text{ and } F)}{P(F)}=\frac{P(E\cap F)}{P(F)}=\frac{1/4}{3/4}=\frac{1}{3} \Rightarrow \text{Hence,}$$

$$\boxed{P\left(\text{Both children are boys} \,\middle|\, \begin{array}{c}\text{at least one of the}\\ \text{children is a boy}\end{array}\right)=P(E|F)=\frac{1}{3}}$$

4) Let A - student knows the answer

B - student guesses

C - student answers correctly

We need to find the conditional probability that a student knew the answer to a question, given that he answered correctly

Let C and K denote, respectively, the events that the student answers the question correctly and the event that he actually knows the answer. Thus, we have to compute $P(K|C)$, where $P(K)=p$. Note that if he knows the answer, then probability student will answer correctly is 1. That's, probability that student will answer question correctly, given that he actually knows the answer is $P(C|K)=1$ ⟹ From Bayes' Theorem and the definition of conditional probability, notice that

$P(K \text{ and } C)=P(K)\,P(C|K)=p\cdot 1=p \Rightarrow \boxed{P(K\cap C)=p}$ ★

To compute the probability that the student answers correctly, we condition on whether or not he knows the answer. That is,

$P(C)=P(C|K)\,P(K)+P(C|\sim K)\,P(\sim K)$ where we're

$\boxed{\begin{array}{l} P(K)=p \\ P(\sim K)=1-p \end{array}}$ $\boxed{P(C|K)\,P(K)=P(C \text{ and } K)=p}$ From the given definition a student either knows the answer or he guesses

Thus, $\sim K$ means student guesses, and according to statement, student who guesses at the answer will be correct with probability $\frac{1}{m} \Rightarrow \boxed{P(C|\sim K)=\frac{1}{m}}$ putting values back

$P(C) = p + \frac{1}{m}(1-p) = p + \frac{1-p}{m} = \frac{mp - p + 1}{m} \Rightarrow \boxed{P(C) = \frac{p(m-1)+1}{m}}$

Since we have to compute $P(K|C)$, we'll use conditional probability

$P(K|C) = \frac{P(K \text{ and } C)}{P(C)} = \frac{p}{\frac{p(m-1)+1}{m}} = \frac{mp}{p(m-1)+1}$, hence $\Rightarrow$

$$\boxed{P(K|C) = \frac{mp}{1+(m-1)p}}$$

5) Let D be the event that the tested person has the disease, and E be the event that his test result is positive. Then, we have to find the probability $P(D|E)$ where $P(D) = 0.005$, $P\left(\text{test result is positive} \,\middle|\, \text{tested person does not have disease}\right) = P(E|\sim D) = 0.01$, $P(\sim D) = 0.995$, and likewisely, $P\left(\text{test result is positive} \,\middle|\, \text{tested person has the disease}\right) = P(E|D) = 0.95$

Using the definition of conditional probability and Bayes' Theorem

$$P(D|E) = \frac{P(D \text{ and } E)}{P(E)} = \frac{P(E|D)\,P(D)}{P(E|D)\,P(D) + P(E|\sim D)\,P(\sim D)} =$$

$$= \frac{0.95 \cdot 0.005}{0.95 \cdot 0.005 + 0.01 \cdot 0.995} = \frac{0.00475}{0.00475 + 0.00995} = \frac{0.00475}{0.0147} \approx \boxed{0.32313} \quad \boxed{P(D|E) = \frac{475}{1470}}$$