CS 204 Discrete Mathematics Fall 2020

Homework 12

December 5, 2020

Graphs

1. (10 pts) An undirected graph is called *complete* if every vertex shares an edge with every other vertex. Draw a complete graph on five vertices. How many edges does it have?

2. (12 pts) Think of the Internet as one big graph, where each web page is a vertex and each link is an edge.

(a) Is this a directed graph? Why or why not?

(b) Is this graph connected? Why or why not?

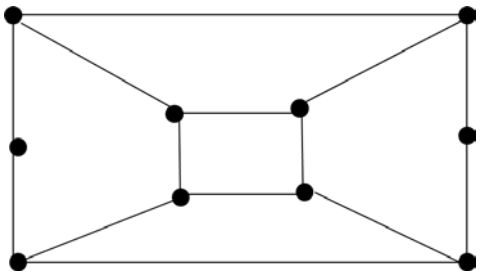
(c) Is this graph complete? Why or why not?

(d) Is this graph simple? Why or why not? (A graph is called simple if it has no multiple edges or loops.)

(e) For a given web page p, what does the outdegree of p represent?

(f) For a given web page p, what does the indegree of p represent?

3. (10 pts) Color the vertices of the following graph so that no vertices of the same color share an edge. Use as few colors as possible. Explain why the graph cannot be colored using fewer colors. Be specific.

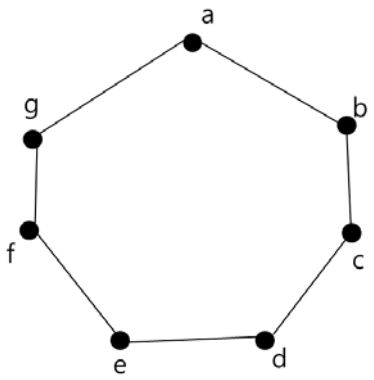


4. (12 pts) A round-robin tournament among four teams - Canadiens, Canucks, Flames, and Oilers - has the following results: Canucks defeat Canadiens; Canucks defeat Flames; Canucks defeat Oilers; Canadiens defeat Oilers; Flames defeat Canadiens; Oilers defeat Flames.

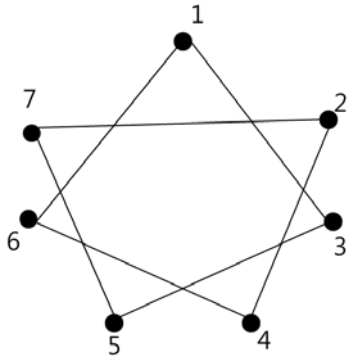
- Model these results with a directed graph, where ach vertex represents a team and each edge represents a game, pointing from the winner to the loser.
- Find a circuit in this graph.
- Explain why the existence of a circuit in such a graph makes it hard to rank the teams from best to worst.

5. (14 pts) Prove that the graph G1 and G2 are isomorphic.

G1:

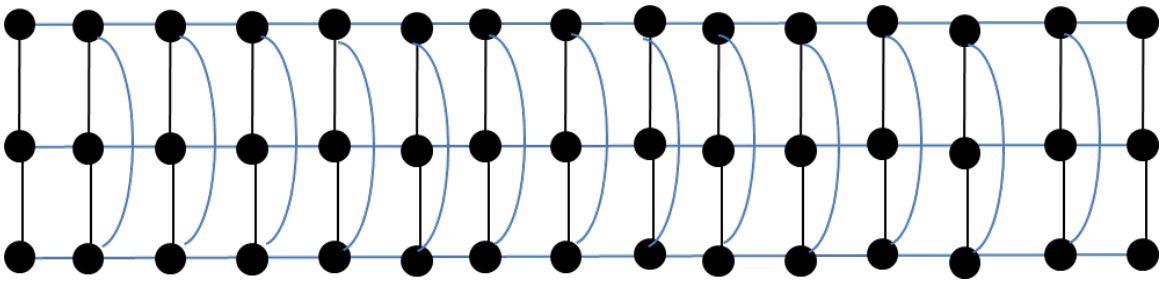


G2:



6. (15 pts)

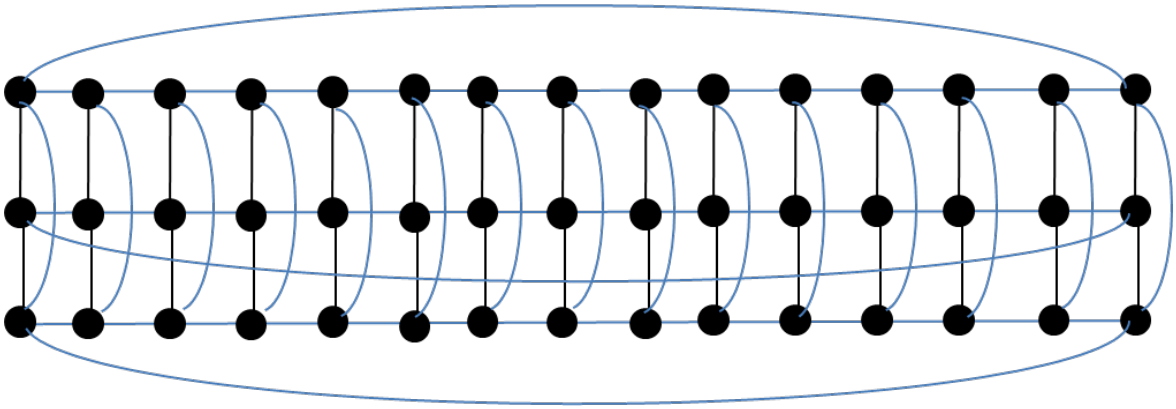
The following graph has 45 vertices.



(a) Does this graph have an Euler circuit? Why or why not?

(b) Does this graph have an Euler path? Why or why not?

(c) The graph below is a copy of the above graph, but with some additional edges added so that all of the vertices in the resulting graph have degree four.



How many edges does this new graph have?

7. (12 pts)

Draw a binary search tree constructed by inserting the words:

math, physics, geography , zoology, meteorology, psychology and chemistry

in that order.

Trees

1. (10 pts) We gave a recursive definition of a binary tree in class. Suppose we modify this definition by deleting part B1, so that an empty tree is not a binary tree. Let's call a tree satisfying this revised definition a *T binary tree*.
(a) Give an example of a T binary tree with five nodes.
(b) Give an example of a binary tree with five nodes that is not a T binary tree.

2. (12 pts) Consider the following list of numbers.

123, 684, 121, 511, 602, 50, 43

(a) Place the numbers, in the order given, into a binary search tree.
(b) The height of a binary search tree is the maximum number of edges you have to go through to reach the bottom of the tree, starting at the root. What is the height of the tree in part (a)?
(c) Reorder the numbers so that when they are put into a binary search tree, the height of the resulting tree is less than the height of the tree in part (a). Give both your new list and the search tree it produces.

3. (12 pts)
(a) Draw all non-isomorphic rooted trees (i.e. trees with their unique roots) having three vertices.
(b) Draw all non-isomorphic binary trees having four vertices.

4. (12 pts) Recall the definition of *full binary tree* discussed in class. Prove that if T is a full binary tree with i internal vertices (i.e. non-terminal nodes), then T has i+1 terminal vertices (i.e. leaf nodes) and 2i+1 total vertices.

5. (12 pts)

Put the following words

`Cheddar Swiss Brie Panela Stilton Mozzarella Gouda`

in that order into a binary search tree with the smallest height possible.