CS 204 Discrete Mathematics

Homework 5

Sample Solutions

PROOF

4. Consider the following theorem.

Theorem. Let x be a wamel. If x has been schlumpfed, then x is a borfin.

Answer the following questions.

(a) Give the converse of this theorem.

(b) Give the contrapositive of this theorem.

(c) Which statement, (a) or (b), is logically equivalent to the Theorem?

Solution)

(a) Let x be a wamel. If x is a borfin, then x has been schlumpfed.

(b) Let x be a wamel. If x is not a borfin, then x has not been schlumpfed.

(c) (b).

5. In the four-point geometry discussed in class, do triangles exist? In other words, is it possible to have three distinct points, not on the same line, such that a line passes through each pair of points? Why or why not?

Solution)

Yes. There are four points (Axiom 3), and three can't be on the same line (Axiom 4). Every pair of distinct points determines a line (Axiom 1), so any three points and the three lines they determine will form a triangle.

6. Give a direct proof.

Let a, b, and c be integers. If a | b and b | c, then a | c.

Solution)

Proof. Suppose a, b, c are integers with $a \mid b$ and $b \mid c$. Then $b = k_1 a$ and $c = k_2 b$ for some integers k_1 and k_2. Therefore $c = k_2(k_1 a) = (k_2 k_1)a$, so $a \mid c$. □

7. Prove that the rational numbers are closed under addition. That is, prove that, if a and b are rational numbers, then a + b is a rational number.

Solution)

Proof. Let a and b be rational numbers. Then $a = p/q$ and $b = r/s$ for some integers p, q, r, s. Therefore,

$$a + b = \frac{p}{q} + \frac{r}{s} = \frac{ps + rq}{qs},$$

which is the ratio of two integers $ps + rq$ and qs, using Axiom 1.1. So $a + b$ is rational. □

SETS

1. Let A = {2, 3, 4}, B = {3, 4, 5, 6}, and suppose the universal set if U = {1, 2, 3, 4, 5, 6, 7, 8, 9}. List all the elements in the following sets.

(a) $(A \cup B)'$

(b) $(A \cap B) \times A$

(c) $\mathcal{P}(B \backslash A)$

Solution)

(a) $\{1, 7, 8, 9\}$

(b) $\{(3, 2), (3, 3), (3, 4), (4, 2), (4, 3), (4, 4)\}$

(c) $\{\emptyset, \{5\}, \{6\}, \{5, 6\}\}$

2. Let the following sets be given. The universal set for this problem is the set of all students at some university.

F = the set of all freshmen.

S= the set of all seniors.

M = the set of all math majors.

C = the set of all CS majors.

(a) Using only the symbols F, S, M, C, | |, $\cap$, $\cup$, $'$, and >, translate the following statement into the language of set theory.

There are more freshmen who aren't math majors than there are senior CS majors.

(b) Translate the following statement in set theory into everyday English.

$(F \cap M) \subseteq C$

Solution)

(a) $|F \cap M'| > |C \cap S|$

(b) All freshmen math majors are CS majors.

3. Let X be a finite set with |x| > 1. What is the difference between P1 = X $\times$ X and P2 = {S $\in \mathcal{P}$(X) | |S| = 2}? Which set, P1 or P2, has more elements?

Solution)

The set P_1 is the set of all ordered pairs of elements of X, and the set P_2 is the set of all unordered pairs. Since there are two ordered pairs for every unordered pair, $|P_1| > |P_2|$.