

**Homework 11- Probability**

November 28, 2020

**Integer random variables, Bernoulli trial; Expected value, Linearity of Expectation, Variance**

1. (10 pts) A natural number  $n$  is chosen at random from the set  $\{1, 2, 3, \dots, 99, 100\}$ . Let  $D$  be the number of digits that  $n$  has. (So, for example, if  $n = 100$  then  $D = 3$  and if  $n = 98$  then  $D = 2$ .) What is the expected value of  $D$ ?
2. (8 pts) Four fair coins are flipped. If the outcomes are assumed independent, what is the probability that two heads and two tails are obtained?
3. (7 pts) The final exam of a discrete mathematics course of a university consists of 50 true/false questions, each worth two points, and 25 multiple choice questions, each worth four points. (So the maximum score of the final exam is 200.) The probability that a student answers a true/false question correctly is 0.9 and the probability that a student answers a multiple-choice question correctly is 0.8. What is the expected score on the final?
4. (8 pts) Let  $X$  be the number appearing on the first die when two dice are rolled and let  $Y$  be the sum of the numbers appearing on the two dice. Show that  $E(X) \times E(Y) \neq E(XY)$ .
5. (15 pts) A fair coin is flipped until a tail appears for the first time, at which time no more flips are made.
  - (a) What is the probability that exactly five flips are made?
  - (b) What is the expected number of flips?

### Other Probability Problems

1. (10 pts) It is known that all items produced by a certain machine will be defective with probability 0.1, independently of each other. What is the probability that in a sample of three items, at most one will be defective?
  
2. (12 pts) Consider two bags. The first contains two white and seven black balls, and the second contains five white and six black balls. We flip a fair coin and then draw a ball from the first bag or the second bag depending upon whether the outcome was heads or tails. What is the conditional probability that the outcome to the toss was heads given that a white ball was selected?
  
3. (15 pts) Suppose that a town has the population of 100 and 10 of them are infected with coronavirus. Assume that currently I am not infected and that if I meet with a group of one or more people that includes a infected person then I will get infected.
  - (a) What is the probability that I will get infected when I meet with one person?
  - (b) What is the probability that I will get infected when I meet with two persons?
  - (c) What is the probability that I will get infected when I meet with three persons?