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HW 2 - Propositional Logic

1) a)

P	Q	$P \land Q$	$P \lor (P \land Q)$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

P	Q	$\neg P$	$Q \to P$	$P \land (Q \to P)$	$\neg P \lor (P \land (Q \to P))$
T	T	F	T	T	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	T	F	T

P	Q	$\neg P$	$P \to Q$	$\neg P \land Q$	$(P \to Q) \to (\neg P \land Q)$
T	T	F	T	F	F
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	F	F

b) First statement is contingency, since $P \lor (P \land Q)$ obtains both T and F (neither tautology nor contradiction). The output consists of both T and F.

Second statement is tautology, because output consists of only T => all statements are true.

Third statement is contingency, as $(P \to Q) \to (\neg P \land Q)$ achieves both T and F (neither tautology nor contradiction)

2)

P	Q	$Q \to P$	$\neg P$	$Q \wedge \neg P$	$\neg(Q \wedge \neg P)$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	F	T	T	F
F	F	T	T	F	T

As we see from the truth table, for each statement P and Q (for each truth value of P and Q) $\Rightarrow$ $Q \to P$ and $\neg(Q \wedge \neg P)$ output the same truth values. Therefore,

$Q \to P \equiv \neg(Q \wedge \neg P)$

3) a)

P	Q	$P \to Q$	$\neg(P \to Q)$	$P \vee Q$	$\neg(P \vee Q)$	S
T	T	T	F	T	F	F
T	F	F	T	T	F	T
F	T	T	F	T	F	F
F	F	T	F	F	T	T

b) By looking at the truth table, we see that for each truth value of P and Q, $\neg Q$ and S have same truth values

Q	$\neg Q$	S
T	F	F
F	T	T
T	F	F
F	T	T

P	Q	S	$\neg Q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	T

As we observe, the truth values for $\neg Q$ and S are same

$\boxed{\neg Q \equiv S}$

From implication, $p \to q \equiv \neg p \vee q \Rightarrow \neg(p \to q) \equiv \neg(\neg p \vee q)$

De Morgan's Law and double negation $\Rightarrow \neg(\neg p \vee q) \equiv p \wedge \neg q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q \Rightarrow S \equiv (p \wedge \neg q) \vee (\neg p \wedge \neg q)$. Then, from commutativity and distributivity, we find

$(p \wedge \neg q) \vee (\neg p \wedge \neg q) \equiv (\neg q \wedge p) \vee (\neg q \wedge \neg p) \equiv \neg q \wedge (p \vee \neg p)$

Since $p \vee \neg p \equiv T$ for each truth value of p (negation law)

$(p = T \Rightarrow T \vee F \equiv T;\ p = F \Rightarrow F \vee T \equiv T) \Rightarrow S \equiv \neg q \wedge (p \vee \neg p) \equiv$

$\equiv \neg q \wedge T \equiv \neg q$ from identity law, $\boxed{S \equiv \neg q}$ ☒

4) $\boxed{\text{X should be a multiple of 4}}$ Assume that

statement P: X is a multiple of 4

statement Q: $\frac{X}{2}$ is an even integer

If P is true, then x is a multiple of $4 \Rightarrow X = 4q$ for some $q \in \mathbb{Z}$. $\frac{X}{2} = 2q$ - even integer, meaning statement Q is true $\Rightarrow$ $\boxed{\text{If P is True, then Q is also True}}$ We should construct truth table for $P \rightarrow Q$ and observe whether $P \rightarrow Q$ is always True

P	Q	$P \rightarrow Q$
T	T	T
F	T	T
F	F	T

regardless of truth value for Q, if we have "P is False", then "$P \rightarrow Q$ is True". From truth table, we can conclude that $\boxed{P \rightarrow Q \text{ is always true}}$ and therefore,

sufficient condition on X: $\boxed{\text{X is a multiple of 4}}$ ☒ In fact, if P is false, and Q is true $\Rightarrow$ x is not a multiple of 4, $\frac{X}{2}$ is an even integer, $\frac{X}{2} = 2q$ for some $q \in \mathbb{Z} \Rightarrow X = 4q$ or x is a multiple of 4, meaning P is true ☒ $\boxed{\begin{matrix} P - \text{false} \\ Q - \text{false} \end{matrix}}$

5)

P	Q	$P \land Q$	$\neg(P \land Q)$	$P \uparrow Q$
T	T	T	F	F
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

$p \uparrow q \equiv \neg(p \land q)$

As we see from truth table, for each truth value of P and Q, $\neg(P \land Q)$ and $P \uparrow Q$ give the ⇒same⇒ ⇒results of True & False

6) a)

P	Q	$P \uparrow Q$	$(P \uparrow Q) \uparrow (P \uparrow Q)$	$P \land Q$
T	T	F	T	T
T	F	T	F	F
F	T	T	F	F
F	F	T	F	F

As seen from truth table, for each truth value for P and Q, $(P \uparrow Q) \uparrow (P \uparrow Q)$ and $P \land Q$ output the same T & F values, correspondingly. Therefore ⇒

$\boxed{(P \uparrow Q) \uparrow (P \uparrow Q) \equiv P \land Q}$ ☒

b)

P	Q	$P \uparrow P$	$Q \uparrow Q$	$(P \uparrow P) \uparrow (Q \uparrow Q)$	$P \lor Q$
T	T	F	F	T	T
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	F	F

According to truth table, T & F values for $P \lor Q$ and $(P \uparrow P) \uparrow (Q \uparrow Q)$ are, correspondingly, equal for each values of P and Q ⇒ $\boxed{(P \uparrow P) \uparrow (Q \uparrow Q) \equiv P \lor Q}$

☒

c)

P	Q	Q↑Q	P↑(Q↑Q)	P→Q
T	T	F	T	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

As we can see from truth table, T & F values for $P \to Q$ and $P \uparrow (Q \uparrow Q)$ are correspondingly equivalent for each values of P and Q $\Rightarrow$ $\boxed{P \uparrow (Q \uparrow Q) \equiv P \to Q}$

7)

	Statement	Reasons
1)	$p \wedge (q \vee r)$	given
2)	$\neg(p \wedge q)$	given
3)	$\neg p \vee \neg q$	De Morgan's Law, 2
4)	$\neg q \vee \neg p$	Commutativity, 3
5)	$q \to \neg p$	Implication, 4
6)	p	Simplification, 1
7)	$\neg(\neg p)$	Double negation, 6
8)	$\neg q$	Modus tollens, 5, 7
9)	$(q \vee r) \wedge p$	Commutativity, 1
10)	$q \vee r$	Simplification, 9
11)	$r \vee q$	Commutativity, 10
12)	$\neg(\neg r) \vee q$	Double negation, 11
13)	$\neg r \to q$	Implication, 12
14)	$\neg(\neg r)$	Modus tollens, 13, 8
15)	r	Double negation, 14
16)	$p \wedge r$	Conjunction, 6, 15

9) Assume $q \to \neg q$ is a contradiction. By the definition of contradiction, for each truth value of q, output will always be False. However, let q = False $\Rightarrow$ $F \to T$ outputs True value (Because of truth table for conditional statement) But this is impossible ☒ since $q \to \neg q$ was contradiction

Therefore, our assumption was wrong $\Rightarrow$ $\boxed{q \to \neg q \text{ is not contradiction}}$

Note: If $q = F \Rightarrow \neg q = T$ and because $F \to T$ is true, $q \to \neg q$ becomes True

q	$\neg q$	$q \to \neg q$
T	F	F
F	T	T

$\boxed{\text{not contradiction}}$

8)

Statement	Reasons
1) $P \to \neg Q$	given
2) $\neg Q \to \neg R$	given
3) $P \vee \neg R$	given
4) $\neg(\neg Q) \vee \neg R$	Implication, 2
5) $Q \vee \neg R$	Double negation, 4
6) $\neg R \vee Q$	Commutativity, 5
7) $\neg R \vee P$	Commutativity, 3
8) $(\neg R \vee P) \wedge (\neg R \vee Q)$	Conjunction, 7, 6
9) $\neg R \vee (P \wedge Q)$	Distributivity, 8
10) $R \to (P \wedge Q)$	Implication, 9
11) $\neg P \vee \neg Q$	Implication, 1
12) $\neg(P \wedge Q)$	De Morgan's Law, 11
13) $\neg R$	Modus tollens, 10, 12

This table provides a proof sequence for the inference