Quiz 23-1 Solutions Sketch

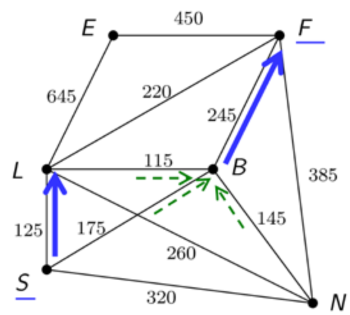
Approach 1)

Since a simple path is a path whose edges are all distinct, there is the start edge that starts from node S and the end edge that ends at node F in the graph of our problem. There are 3 start edges (from S) and 4 end edges (to F). Since any simple path should begin with one of the start nodes and end with one of the end nodes, there are 3 x 4 = 12 different groups of simple paths. Note that these groups are mutually exclusive. These twelve cases are as below. (Below I am going to use the distance between two cities as the label of the edge connecting the two cities.)

(1) Start 125; End 450
(2) Start 125; End 220
(3) Start 125; End 245
(4) Start 125; End 385
(5) Start 175; End 450
(6) Start 175; End 220
(7) Start 175; End 245
(8) Start 175; End 385
(9) Start 320; End 450
(10) Start 320; End 220
(11) Start 320; End 245
(12) Start 320; End 385

Manually analyzing the number of different simple paths for each of the above twelve cases and by adding them up, we get the total number of simple paths. Let me show how to do this just for the case "(3) Start 125; End 245".

(3) Start 125; End 245



The start edge is 125 and the end edge is 245 (the blue arrows). Any simple path from S to F that begins with 125 and ends with 245 should traverse the rest of the edges at most once, the subpaths of which should begin at L and arrive at B.
To calculate the number of simple paths from L to B, we consider the three mutually exclusive cases: the last edge taken to arrive at B is 115, 175 or 145 (the green arrows).
Let's conduct case analysis for these 3 cases:

Case 115)
To take 115 as the last edge to B, the following paths from L are possible:
null (i.e. no additional edge need to be taken)
645-450-220
220-450-645
645-450-385-260
220-385-260
645-450-385-145-175-320-260
220-385-145-175-320-260
645-450-385-320-175-145-260
220-385-320-175-145-260
260-385-220
260-385-450-645

Case 145)

To take 145 as the last edge to B, the following paths from L are possible:

260

115-175-320

645-450-385

220-385

645-450-385-260-115-175-320

220-385-260-115-175-320

645-450-385-320-175-115-260

220-385-320-175-115-260

No other simple paths are possible. So there are 8 such paths.

Case 175)

To take 175 as the last edge to B, the following paths from L are possible:

260-320

115-145-320

645-450-385-320

220-385-320

645-450-385-260-115-145-320

220-385-260-115-145-320

645-450-385-145-115-260-320

220-385-145-115-260-320

No other simple paths are possible. So there are 8 such paths.

Approach 2)

You can write a computer program based on the following algorithm:

Algorithm *Find all simple paths and count them.*

Step 1) Generate all permutations of 11 edges

/* There are 11! such sequences and all edges are distinct. */

Step 2) Eliminate sequences that are not paths

/* To be a path, two neighboring edges should be adjacent, i.e.
the end node of the preceding edge should be the same node as
the start node of the succeeding edge. */

Step 3) Count the number of remaining sequences.

Approach 3)

The algorithm in Approach 2 is known as the "generate-and-test" solution. It is simple to design but the performance is not good.

You can write a smarter algorithm with better performance. This is the subject of the algorithms course and beyond the scope of this course.