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CS 204: Discrete Mathematics
Midterm II

May 30, 2018 (13:00 pm ~ 14:15 pm)

Part I	30	/ 30
Part II	35	/ 45
Part III	25	/ 25
Total	95	/ 100

Instructions:

- Try to put your answers on these pages. For overflow you may use the back side of the page on which the question appears.
- **Be sure to write your name and student Id on** EVERY **page; we will be disassembling exams during grading.**

(Total number of pages including cover: 6)

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Part I. (Graphs, Recursive Data Structures - 30 pts)

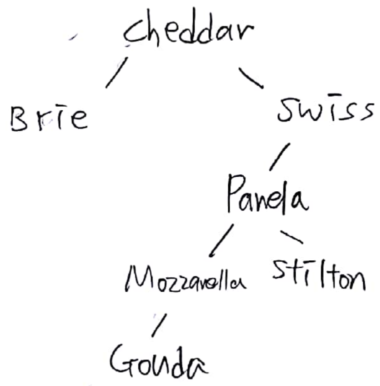
1. (10 pts) 10

Put the following words

ABCDEFGHIJKLMNOPQRSTUVW

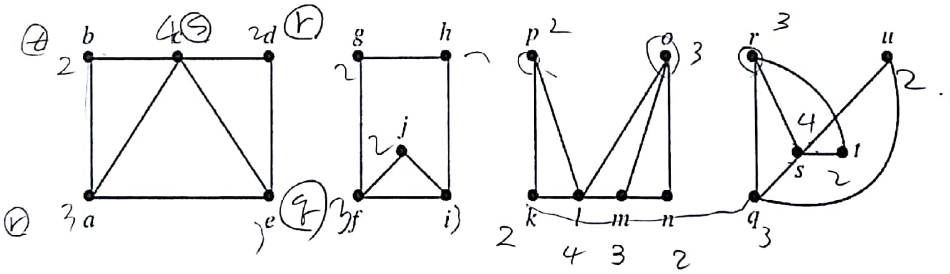
`Cheddar  Swiss  Brie  Panela  Stilton  Mozzarella  Gouda`

in that order into a binary search tree with the smallest height possible.



2. (10 pts) 10

Find a pair, G and H, of isomorphic graphs among the four graphs below, and give the one-to-one correpondence α: $V_G \rightarrow V_H$ of vertices. (For each vertext of G, tell to which vertex of H it corresponds.)



1st Graph and 4th Graph are Isomorphic.

a→r, b→t, c→s, d→u, e→q

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3. (10 pts)

Recall the definition of SList below. 10

Definition

An *SList* is

B. x where $x \in \mathbf{R}$, the real numbers.

R. (X, Y) where X and Y are SLists having the same number of elements, and the last number in X is less than the first number in Y.

Let L be an SList. Define a recursive function Flip as follows:

B. If L = x, i.e. a single element, then Flip(L) = x.

R. If L = (X, Y), then Flip(L) = (Flip(Y), Flip(X)).

Compute Flip([((2,3), (7,9))]), showing all steps.

flip(((2,3), (7,9)))

L_1 L_2

Flip(((2,3),(7,9))) = (Flip((7,9)), Flip((2,3))) ← (L = ((2,3), (7,9))) by R.

= ((Flip(9), Flip(7)) (Flip(3), Flip(2))) ← (L_1 = (7,9) , L_2 = (2,3)) by R.

= ((9,7), (3,2)) ← all the flip element is single. by B.

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Part II. (Counting - 45 pts)

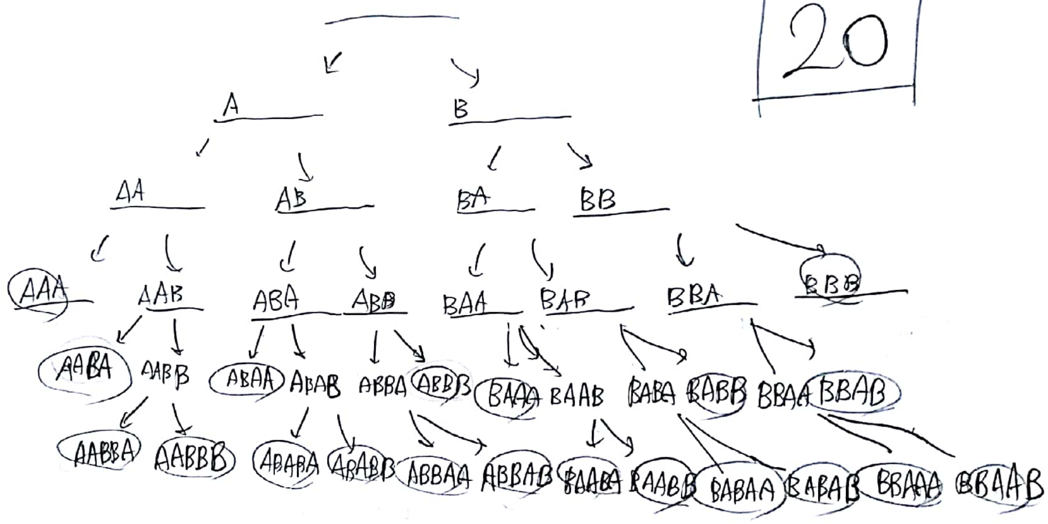
1. (10 pts)

10

Two teams (A and B) play a best-of-five match. The match ends when one team wins three games. (It is assumed that after each game there is always a winner and a loser.) How many different win or loss scenarios are possible? (Use a decision tree.)

I will write a winning team.

$\boxed{20}$



2. (10 pts)

10

Use the Binomial Theorem to expand $(2x + 7)^5$.

$(2x+7)^5$

$= {}_5C_0\cdot(2x)^5(7)^0 + {}_5C_1(2x)^4\cdot 7^1 + {}_5C_2\cdot(2x)^3\cdot 7^2 + {}_5C_3(2x)^2\cdot 7^3 + {}_5C_4(2x)^1\cdot 7^4 + {}_5C_5(2x)^0\cdot 7^5$

$= ({}_5C_0\cdot 2^5)x^5 + ({}_5C_1\cdot 2^4\cdot 7)x^4 + ({}_5C_2\cdot 2^3\cdot 7^2)x^3 + ({}_5C_3\cdot 2^2\cdot 7^3)x^2 + ({}_5C_4\cdot 2\cdot 7^4)x + {}_5C_5\cdot 7^5$

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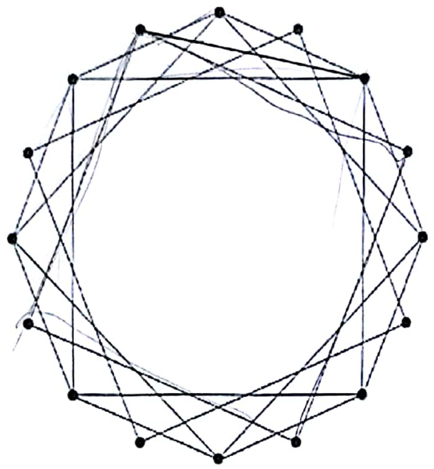
3. (15 pts)

15

In the following graph, let S be the set of all squares formed by edges in the graph, and let V be the set of all sets of four vertices in the graph.



Define a function f: S →V as follows: For any square x ∈S, let f(x) be the set containing the four vertices of the square x.

(a) Is f one-to-one? Explain

—unique (−4)

3 Yes, If we make square by 4 edges, then 4 vertices are selected by itself,

Then that set of 4 vertices must be in V, since V has all the set of all sets of four vertices in the graph.

(b) Is f onto? Explain.

No. the edges don't connect all the vertices, so If we select 4 vertices, the case that we can't make square because there is no edges to connect that vertices can happen. So f is not onto.

4. (10 pts)

$\lceil \frac{n}{m} \rceil$

How many ways are there to put n identical objects into m distinct containers so that no containers is empty?

4

If $n<m$, There is no way to make no containers is empty. (+4)

If $n=m$, There is $P(n,m)$ ways. $= m!$ (−1)

If $n>m$. at least 1 objects must be in container. → $P(n,m)$

left $n-m$ objects can go anywhere → $(m)^{n-m}$

So, $P(n,m) \times (m)^{n-m}$ ${}_{n-1}C_{m-1}$ (−5)

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Part III. (Discrete Probability - 25 pts)

5

1. (5 pts)

Which is more likely: rolling a total of 9 when two dice are rolled or rolling a total of 9 when three dice are rolled?

two dice rolled → total : 36 / make total of 9 : (3.6) (4.5) (5.4) (6.3) → $\frac{4}{36} = \frac{1}{9}$

three dice rolled → total : 216 / make total of 9 (1.2.6) (1.3.5) (1.4.4) (1.5.3) (1.6.2)
(2.1.6) (2.2.5) (2.3.4) (2.4.3) (2.5.2) (2.6.1)
(3.1.5) (3.2.4) (3.3.3) (3.4.2) (3.5.1)
(4.1.4) (4.2.3) (4.3.2) (4.4.1)
(5.1.3) (5.2.2) (5.3.1)
(6.2.1) (6.1.2) → $\frac{25}{216}$

so. three dice rolling is more likely

10

2. (10 pts)

A natual number n is chosen at random from the set {1, 2, 3, …, 99, 100}. Let D be the number of digits that n has. (So, for example, if n = 100 then D = 3 and if n = 98 then D = 2.) What is the expected value of D?

$1 \sim 9 : D=1 \rightarrow P(D=1) = \frac{9}{100}$

$10 \sim 99 : D=2 \rightarrow P(D=2) = \frac{90}{100}$

$100 : D=3 \rightarrow P(D=3) = \frac{1}{100}$

$E(D) = \frac{9}{100} \times 1 + \frac{90}{100} \times 2 + \frac{1}{100} \times 3 = \frac{192}{100} = 1.92$

10

3. (10 pts)

What is the conditional probability that exactly four heads appear when a fair coin is flipped five times, given that the first flip camp up heads?

A = four heads appear when a fair coin is flipped five times

B = first flip came up head.

$P(A|B) = P(A \cap B) / P(B) = \frac{\frac{1}{8}}{\frac{1}{2}} = \frac{1}{4}$

$\boxed{P(A|B) = \frac{1}{4}}$