CS 204 Discrete Mathematics

Homework 2 – Propositional Logic

Sample Solutions

4. Mathematicians say that "Statement P is a sufficient condition for statement Q" if P $\rightarrow$ Q is true. In other words, in order to know that Q is true, it is sufficient to know that P is true. Let x be an integer. Give a sufficient condition on x for x/2 to be an even integer.

Solution)

For x/2 to be an even integer, it is sufficient that x be divisible by 8. (Answers may vary.)

5. The NAND connective $\uparrow$ is defined by the following truth table.

p	q	p $\uparrow$ q
T	T	F
T	F	T
F	T	T
F	F	T

Use truth tables to show that p $\uparrow$ q is logically equivalent to $\neg$ (p $\wedge$ q). (This explains the name NAND: Not AND.)

Solution)

p	q	$p \wedge q$	$\neg(p \wedge q)$	$p \uparrow q$
T	T	T	F	F
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

6. The NAND connective is important because it is easy to build an electronic circuit that computes the NAND of two signals. Such a circuit is called a logic gate. Moreover, it is possible to build logic gates for the other logical connectives entirely out of NAND gates. Prove this fact by proving the following equivalences, using truth tables.

(a) $(p \uparrow q) \uparrow (p \uparrow q)$ is logically equivalent to $p \wedge q$.

(b) $(p \uparrow p) \uparrow (q \uparrow q)$ is logically equivalent to $p \vee q$

(c) $p \uparrow (q \uparrow q)$ is logically equivalent to $p \rightarrow q$.

Solution)

(a)

p	q	$p \uparrow q$	$(p \uparrow q) \uparrow (p \uparrow q)$	$p \wedge q$
T	T	F	T	T
T	F	T	F	F
F	T	T	F	F
F	F	T	F	F

(b)

p	q	$p \uparrow p$	$q \uparrow q$	$(p \uparrow p) \uparrow (q \uparrow q)$	$p \vee q$
T	T	F	F	T	T
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	F	F

(c)

p	q	$q \uparrow q$	$p \uparrow (q \uparrow q)$	$p \rightarrow q$
T	T	F	T	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

7. Fill in the reasons in the following proof sequence. Make sure you indicate which step(s) each derivation rule refers to.

Statement	Reasons
1. $p \wedge (q \vee r)$	given
2. $\neg(p \wedge q)$	given
3. $\neg p \vee \neg q$	
4. $\neg q \vee \neg p$	
5. $q \rightarrow \neg p$	
6. p	
7. $\neg(\neg p)$	
8. $\neg q$	
9. $(q \vee r) \wedge p$	
10. $q \vee r$	
11. $r \vee q$	
12. $\neg(\neg r) \vee q$	
13. $\neg r \rightarrow q$	
14. $\neg(\neg r)$	
15. r	
16. $p \wedge r$	

Solution)

Statements	Reasons
1. $p \wedge (q \vee r)$	given
2. $\neg(p \wedge q)$	given
3. $\neg p \vee \neg q$	De Morgan, 2
4. $\neg q \vee \neg p$	commutativity, 3
5. $q \rightarrow \neg p$	implication, 4
6. p	simplification, 1
7. $\neg(\neg p)$	double negation, 6
8. $\neg q$	*modus tollens*, 7, 5
9. $(q \vee r) \wedge p$	commutativity, 1
10. $q \vee r$	simplification, 9
11. $r \vee q$	commutativity, 10
12. $\neg(\neg r) \vee q$	double negation, 11
13. $\neg r \rightarrow q$	implication, 12
14. $\neg(\neg r)$	*modus tollens*, 8, 13
15. r	double negation, 14
16. $p \wedge r$	conjunction, 6, 15

9. Is $a \to \neg a$ a contradiction? Why or why not?

Solution)

It is not a contradiction. The truth table is not always false:

a	$\neg a$	$a \to \neg a$
T	F	F
F	T	T