CS 204 Discrete Mathematics Fall 2020

Homework 6 - Relations and Functions

October 17, 2020

RELATIONS

1. (8 pts) Let R1 = {(1, 2), (2, 3), (3, 4)} and R2 = {(1, 1), (1, 2),(2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4)} be relations from {1, 2, 3} to {1, 2, 3, 4}. Find

a) $R1 \cup R2$

b) $R1 \cap R2$

c) $R1 - R2$

d) $R2 - R1$

2. (15 pts) Determine whether the relation R on the set of all real numbers is reflexive, symmetric, antisymmetric, and/or transitive, where $(x, y) \in R$ if and only if

a) $x + y = 0$

b) $x = \pm y$

c) $x - y$ is a rational number.

d) $x = 2y$

e) $xy \geq 0$

3. (12 pts)

(a) Let R be a relation from a set A to a set B and S a relation from B to a set C. The composite of R and S is the relation consisting of ordered pairs (a, c), where $a \in A$, $c \in C$, and for which there exists an element $b \in B$ such that $(a, b) \in R$ and $(b, c) \in S$. We denote the composite of R and S by $S \circ R$. Now R be the relation {(1, 2), (1, 3), (2, 3), (2, 4), (3, 1)}, and let S be the relation {(2, 1), (3, 1), (3, 2), (4, 2)}. Find $S \circ R$.

(b) Let R be a relation on the set A. The powers R^n, n = 1, 2, 3, . . . , are defined recursively by $R^1 = R$ and $R^{n+1} = R^n \circ R$. Now let R be the relation on the set {1, 2, 3, 4, 5} containing the ordered pairs (1, 1), (1, 2), (1, 3), (2, 3), (2, 4), (3, 1), (3, 4), (3, 5), (4, 2), (4, 5), (5, 1), (5, 2), and (5, 4). Find R^2, R^3, R^4 and R^5.

4. (15 pts) Suppose that R and S are reflexive relations on a set A. Prove or disprove each of these statements. (Note that R1 ⊕ R2 consists of all ordered pairs (a, b), where student a has taken course b but does not need it to graduate or needs course b to graduate but has not taken it.)

a) R ∪ S is reflexive.

b) R ∩ S is reflexive.

c) R ⊕ S is irreflexive.

d) R − S is irreflexive.

e) S ∘ R is reflexive.

ORDERED RELATIONS

1. (24 pts) Fill in the following table describing the characteristics of the given ordered sets. Answer with T for True or F for False. (Note that "⊂" indicates proper containment, that is, for two sets A and B, $A \subset B$ if and only if $A \subseteq B \wedge A \neq B$.)

	Partial Order	Total Order	Well Order
$<\mathbb{N}, <>$			
$<\mathbb{N}, \leq>$			
$<\mathbb{Z}, \leq>$			
$<\mathbb{R}, \leq>$			
$<\mathbb{P}(\mathbb{N}), \subset>$			
$<\mathbb{P}(\mathbb{N}), \subseteq>$			
$<\mathbb{P}(\{a\}), \subseteq>$			
$<\mathbb{P}(\varnothing), \subseteq>$			

2. (12 pts) Let A = {*a*, *b*, *c*}. Use Hasse diagrams to describe all partial orderings on A for which *a* is a minimal element.

EQUIVALENCE RELATIONS

1. (5 pts) Give a specific reason why the following set R does not define an equivalence relation on the set {1, 2, 3, 4}.

$$R = \{(1,1), (2,2), (3,3), (4,4), (2,3), (3,2), (2,4), (4,2)\}$$

2. (5 pts) The following set R defines an equivalence relation on the set {1, 2, 3}, where aRb means that $(a,b) \in R$.

$$R = \{(1,1), (2,2), (3,3), (2,3), (3,2)\}$$

What are the equivalence classes?

3. (8 pts) Let X be a finite set. For subsets, $A, B \in \mathcal{P}(X)$, let $A \, R \, B$ if $|A| = |B|$. This is an equivalence relation on $\mathcal{P}(X)$. If X = {1, 2, 3}, list the equivalence classes.

4. (12 pts)

Define a relation on $\mathbb{Z}$ by a R b if $a^2 = b^2$.

(a) Prove that R is an equivalence relation

(b) Describe the equivalence classes (with respect to the relation R above).

FUNCTIONS

1. (10 pts) Let A = {0, 1, 2} Find all total functions f: $A \to A$ for which $f^2(x) = f(x)$. (Note: $f^2 = f \circ f$)

(a) How many such functions are there?

(b) List all such functions.

2. (10 pts) Let P be a set of people, and let Q be a set of occupations. Define a function f: $P \to Q$ by setting f(p) equal to p's occupation. What must be true about the people in P for f to be a total function?

3. (10 pts) Let S = {0, 1, 2, 3, 4, 5}, and let $\mathcal{P}(S)^*$ be the set of all nonempty subsets of S. Define a function m: $\mathcal{P}(S)^* \to S$ by

$$m(H) = \text{the largest element in } H$$

for any nonempty subset $H \subseteq S$.

(a) Is *m* one-to-one? Why or why not?

(b) Does *m* map $\mathcal{P}(S)^*$ onto S? Why or why not?

4. (12 pts)

Define a function $f: \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ by $f(x) = (2x + 3, x-4)$

(a) Is f one-to-one? Prove or disprove.

(b) Does f map $\mathbb{Z}$ onto $\mathbb{Z} \times \mathbb{Z}$? Prove or disprove.