CS 204: Discrete Mathematics
Midterm Exam

October 20 (Tuesday), 2020

1:00 pm ~ 3:00 pm

Part I	/ 75
Part II	/ 45
Total	/ 120

(Total number of pages including cover: 11)

Part I. Logic (75 pts)

1. (15 pts) Consider the following argument:

   If Smith is intelligent and studies hard, then she will get good grades and pass her courses. - (S1)
   If Smith studies hard but lacks intelligence, then her efforts will be appreciated; and if her efforts are appreciated, then she will pass her courses. - (S2)
   If Smith is intelligent, then she studies hard. - (S3)
   Therefore Smith will pass her courses. - (S4)

   (a) (7 pts) Translate the statements (S1), (S2), (S3) and (S4) using the following propositional letters:

   I: Smith is intelligent.
   S: Smith studies hard.
   G: Smith will get good grades.
   P: Smith will pass her courses.
   A: The efforts of Smith will be appreciated.

   (b) (2 pts) Is the argument above valid or invalid?

   (c) (6 pts) Justify your answer in (b) by providing a proof by Natural Deduction or by providing a counterexample..

2. (15 pts)

(a) (5 pts) Put the following syllogism into the standard form, which uses a horizontal line to separate premises and conclusion as shown in the lecture slides.

> Bill didn't go to work this morning because he wore a sweater, and he never wears a sweater to work.

(b) (10 pts) Tell whether it is valid or not by showing its Venn Diagram. (Hint: To indicate absence of elements use shade as in the example of the lecture slides. To indicate existence of an element belonging to a class, mark "x" inside the circle for the class.)

3. (10 pts) Prove the following inference using Natural Deduction.

$\exists x(F(x) \wedge G(x))$

$\neg\forall x(F(x) \rightarrow \neg G(x))$

4. (20 pts) For this problem, you should use the three predicates below.

   Q(x): x is a beauty queen.
   A(x): x is attractive.
   G(x): x is a ganster.

   Solve the problems (a), (b), (c) and (d) below:

(a) (3 pts) Translate the following sentences into predicate logic expressions.

   Beauty queens are unattractive.
   There are attractive gansters.
   Beauty queens are gansters.

(b) (5 pts) Translate the following inference into an inference using predicate logic expressions.

   No beauty queens are unattractive.
   There aren't any attractive gansters.

   Beauty queens are never gansters.

(c) (2 pts) Is the inference in (b) valid or invalid?

(d) (10 pts) If the inference in (b) is valid, prove the predicate logic inference using Natural Deduction. If the inference in (b) is invalid, explain why it is invalid.

5. (15 pts)

(a) (10 pts) Let the Universe of Discourse be {1,2}. (That is, the domain of x be {1,2}.) Construct a counterexample to show the following assertion is not valid:

$\exists x(P(x) \rightarrow Q(x)) \rightarrow (\exists x P(x) \rightarrow \exists x Q(x))$

(b) (2 pts) Let the Univers of Discourse be {1}. Is there an interpretation that makes the above formula false? Answer with Yes or No.

(c) (3 pts) If your answer to (b) is Yes, show the counterexample. Otherwise explain why there is no such interpretation.

Part II. Sets, Relations and Functions (45 pts)

1. (10 pts)

(a) Determine whether or not the following relation is an equivalence relation.

A = {(1,1), (2,2), (3,3), (4,4), (5,5), (1,5), (5,1), (3,5), (5,3), (1,3), (3,1)}

B = $\{(x,y) \mid x \in N^+ \wedge y \in N^+ \wedge 3$ divides $x + y\}$ where N^+ is the set of positive natural numbers.

(b) If the relation above is an equivalence relation, list the equivalence classes. If it is not an equivalence relation, prove that it is not an equivalence relation.

2. (10 pts)

(a) (6 pts) Let X = {1, 2, 3, 4, 5}. R1 and R2 below are relations on X. Determine for each of them whether it is an equivalence relation or not. If it is an equivalence relation, list the equivalence classes. If it is not an equivalence relation, explain why it is not an equivalence relation.

R1 = {(1,1), (2,2), (3,3), (4,4), (5,5), (1,5), (5,1), (3,5), (5,3),(1,3),(3,1)}

R2 = {(x,y) | 3 divides x + y}

(b) (4 pts) Let Y = {1, 2, 3, 4}. The following is a partition of an equivalence relation on Y. What is the equivalence relation? List all its elements.

{{1,2,3},{4}}

3. (15 pts)

Let X be the following set = {{b}, {b, e} {b, r}, {b, e, r}, {a, r}, {b, a, r},{b, e, a, r, s}}. Then X is a partially ordered set under the $\subseteq$ relation.

(a) (6 pts) Draw the Hasse diagram for this partial ordering.

(b) (2 pts) Name all minimal elements, if any exists.

(c) (7 pts) List all the pairs of incomparable elements, if any exists.

4. (10 pts)
   Let $A = \{0, 1, 2\}$. Find all functions $f: A \to A$ for which $(f \circ f)(x) = f(x)$.