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HW-8: Predicate Logic

a) a) $\exists x(E(x) \land G(x))$

b) $\forall x(E(x) \rightarrow G(x))$

c) $\forall x((A(x) \land G(x)) \rightarrow E(x))$

d) $\neg\exists x((A(x) \land G(x)) \land E(x))$ or $\forall x((A(x) \land G(x)) \rightarrow \neg E(x))$

e) There exists exactly one green elephant:

$\exists x(G(x) \land E(x) \land \forall y((G(y) \land E(y)) \rightarrow y = x))$

All green elephants are named James:

$\forall x((G(x) \land E(x)) \rightarrow N(x, James))$

There's exactly 1 green elephant, and his name's James:

$$\boxed{\exists x(G(x) \land E(x) \land \forall y((G(y) \land E(y)) \rightarrow y = x)) \land \forall x((G(x) \land E(x)) \rightarrow N(x, James))}$$

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5) a) $(\exists x)(P(x))$

b) $\neg(\exists x)(P(x) \wedge Q(x))$

c) $(\exists x)(\neg P(x) \wedge \neg Q(x))$

6) a) $(\forall x)(\exists y)(N(x) \to P(x,y))$

b) By successfully applying De Morgan's laws for quantifiers, we can move the negation in $\neg(\forall x)(\exists y)(\ldots)$ inside all the quantifiers. We find that

$\neg(\forall x)(\exists y)(N(x) \to P(x,y)) \equiv (\exists x)(\neg(\exists y)(N(x) \to P(x,y)))$

$\equiv (\exists x)(\forall y)(\neg(N(x) \to P(x,y)))$ where applying implication rule yields $N(x) \to P(x,y) \equiv \neg N(x) \vee P(x,y)$ and $\neg(N(x) \to P(x,y)) \equiv \neg(\neg N(x) \vee P(x,y)) \equiv$

$\equiv N(x) \wedge (\neg P(x,y)) \Rightarrow \boxed{(\exists x)(\forall y)(N(x) \wedge (\neg P(x,y)))}$

c) There exist some integer x such that, for all integers y, $x \neq 0$ and $xy \neq 1$

d) (a) is false in the domain of integers, since we can choose $x=2$, and $x \neq 0 \Rightarrow$ there should be some integer y such that $2y=1$, which is definitely false. (b) is true in the domain of integers, because we can pick $x=2$, and for all $y \in \mathbb{Z}$, $2 \neq 0$ and $2y \neq 1$, otherwise $y = \frac{1}{2} \notin \mathbb{Z} \Rightarrow \boxed{(b)}$

7) a) Every trader who works at the Korea Stock Exchange earns more money than some other trader at the Korea Stock Exchange

b) Some trader at the Korea Stock Exchange earns more than any other traders at the Korea Stock Exchange

c) (a) is impossible in this context, since there should be a trader at the KSE who earns the least among all traders. As there are finite number of traders, there should be one who earns the least in this group. If (a) was possible, then that person needed to make more money than some other trader, which is contradiction due to the fact that he/she earns the least amount of money. (b) is possible in this context, since trader who earns the most in the group exists and he/she earns more money than any other trader in that group. (considering $x \neq y$)

8) Domain → set of real numbers

The domain of the following predicate is the set of all real numbers:

$P(x) =$ "x is rational"

$(\forall x)(\forall y)([P(x) \wedge (\neg P(y))] \rightarrow (\neg P(x+y)))$

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This formula states that for all real numbers x and y if x is rational and y is irrational, then $(x+y)$ is irrational. To negate it, we'll apply some techniques as in 6(b) to find $\neg(\forall x)(\forall y)((P(x)\wedge(\neg P(y)))\to(\neg P(x+y)))$ (we'll apply De Morgan's law for quantifiers and implication rule) $\neg(\forall x)(\forall y)((P(x)\wedge(\neg P(y)))\to(\neg P(x+y)))$

$$\equiv(\exists x)\left(\neg(\forall y)((P(x)\wedge(\neg P(y)))\to(\neg P(x+y)))\right)\equiv$$

$$\equiv(\exists x)(\exists y)\left(\neg((P(x)\wedge(\neg P(y)))\to(\neg P(x+y)))\right)\equiv$$

$$\equiv(\exists x)(\exists y)\left(\neg\left(\neg(P(x)\wedge\neg P(y))\vee\neg P(x+y)\right)\right)\equiv$$

$$\equiv(\exists x)(\exists y)(P(x)\wedge\neg P(y)\wedge P(x+y))$$

$\Rightarrow$ Hence, we're

$\boxed{\text{negation}\Rightarrow(\exists x)(\exists y)(P(x)\wedge\neg P(y)\wedge P(x+y))}$ By translating it, there exist some real numbers x, y such that x is rational, y is irrational, and $(x+y)$ is rational.

4) a) For all x, there exist y such that $P(x,y)$ is true

b) There exist y such that, for all x, $P(x,y)$ is true

The order of quantifiers matters. In both of these statements, a claim is made that $P(x,y)$ is true. In the first statement, you are first given an arbitrary x, then the claim is that it's possible to find some y such that predicate P is true. However, the second statement claims there is some y, such that, given any other x, predicate P will be true.

In the second, we should decide on what y is before we pick x. In the first, we pick x first, then we can think about y. Therefore, knowing the domain and predicate P will enable us to find the truth value of $(\forall x)(\exists y) P(x,y)$ and $(\exists y)(\forall x) P(x,y)$ correctly.

3) Consider given domain where elements are comparable, and let's first analyze what does "exactly 1 element" mean. One method is to recognize that "exactly 1" is equivalent to "at least 1 and at most 1". Now, to do "at most 1" is to deny "at least 2". Therefore, considering at least 2 p's: $\exists x \exists y (P(x) \wedge P(y) \wedge (x \neq y))$

at most 1 P: $\neg \exists x \exists y (P(x) \wedge P(y) \wedge (x \neq y))$ or

Bringing $\neg$ inside and using implication rule $\Rightarrow$

$\neg ((P(x) \wedge P(y)) \wedge (x \neq y)) \equiv \neg (P(x) \wedge P(y)) \vee (\neg (x \neq y))$

$\equiv \neg (P(x) \wedge P(y)) \vee (x = y) \equiv P(x) \wedge P(y) \longrightarrow (x = y)$

at most 1 P: $\forall x \forall y ((P(x) \wedge P(y)) \longrightarrow (x = y))$

exactly 1 P: $\exists x P(x) \wedge \forall x \forall y ((P(x) \wedge P(y)) \rightarrow x = y)$

Considering our domain from problem statement and the possibility for testing elements in terms of equality,

There is exactly 1 element of T that satisfies P →

$\longrightarrow (\exists x)P(x) \wedge (\forall x)(\forall y)\Big(\big(P(x) \wedge P(y)\big) \longrightarrow x=y\Big)$

There are at least 2 elements of T that satisfy P →

$\longrightarrow \exists x \exists y \big(P(x) \wedge P(y) \wedge (x \neq y)\big)$

1) a) $\exists y(N(x) \longrightarrow \underbrace{y}_{\text{bound}} > 2) \wedge \forall x(N(x) \longrightarrow \underbrace{x}_{\text{bound}} + 1 > \underbrace{x}_{\text{bound}})$

b) $\underbrace{x}_{\text{free}} = 2\underbrace{y}_{\text{free}}$

c) $\exists y(N(y) \longrightarrow \underbrace{y}_{\text{bound}} > 2) \wedge \forall x(N(x) \longrightarrow \underbrace{x}_{\text{bound}} > \underbrace{y}_{\text{free}})$

d) $\forall x\big(N(x) \longrightarrow \exists y(N(y) \longrightarrow \underbrace{y}_{\text{bound}} > \underbrace{x}_{\text{bound}}) \wedge (\underbrace{x}_{\text{bound}} = 2\underbrace{y}_{\text{free}})\big)$