CS 204 Discrete Mathematics Fall 2020

Homework 3 – Predicate Logic - Part I

Due date September 19, 2020

1. (12 pts) Which occurrences of the underlined variables x and y are free and which are bound in each of the following? (HINT: Recall that a variable may be both bound and free in the same formula) Assume the following predicate definition:

   N(x): x is a natural number.

   (a) $\exists y(N(x) \rightarrow \underline{y} > 2) \wedge \forall x(N(x) \rightarrow \cdot\ \underline{x} + 1 > \underline{x})$
   (b) $\underline{x} = 2 * \underline{y}$
   (c) $\exists y(N(y) \rightarrow \underline{y} > 2) \wedge \forall x(N(x) \rightarrow \underline{x} > \underline{y})$
   (d) $\forall x\ (N(x) \rightarrow \exists y\ (N(y) \rightarrow \underline{y} > \underline{x}) \wedge \underline{x} = 2 * \underline{y})$

2. (15 pts) Translate the following sentences into predicate logic, using the translation key provided.
   (Hint: For this problem, you may freely use the binary predicate “___ = _ _ _” for equality.)

   G(x): x is green
   A(x): x is an animal
   E(x): x is an elephant
   N(x, y): the name of x is y

(a) Some elephant is green.
(b) All elephants are green
(c) If an animal is green, it is an elephant.
(d) No green animal is an elephant
(e) There is exactly one green elephant, and his name is James
    (NOTE: For this problem, use only the standard universal and existential quantifiers ($\forall$, $\exists$), do not use the unique existential quantifier ($\exists$!).)

3. (10 pts) Let P be a predicate over elements of type T. Assume you can test elements for equality. Write predicates to capture the following English sentences.

There is exactly one element of T that satisfies P.

There are at least two elements of T that satisfy P.

4. (10 pts) Give English translations for each of the following assertions and explain how the two are related, assuming P is some predicate.

(a) $(\forall x)(\exists y)P(x,y)$

(b) $(\exists y)(\forall x)P(x,y)$

4. (9 pts) The domain of the following predicates is the set of all plants.

P(x) = "x is poisonous."

Q(x) = "Jeff has eaten x."

Translate the following statements into predicate logic.

(a) Some plants are poisonous.

(b) Jeff has never eaten a poisonous plant.

(c) There are some nonpoisonous plants that Jeff has never eaten.

6. (12 pts) In the domain of integers, consider the following predicates: Let N(x) be the statement "$x \neq 0$". Let P(x,y) be the statement that "xy = 1."

(a) Translate the following statement into the symbols of predicate logic.

For all integers x, there is some integer y such that if $x \neq 0$, then xy =1.

(b) Write the negation of your answer to part (a) in the symbols of predicate logic, Simplify your answer so that it uses the $\wedge$ connective.

(c) Translate your answer from part (b) into an English sentence.

(d) Which statement, (a) or (b), is true in the domain of integers? Explain.

7. (9 pts) The domain of the following predicates is the set of all traders who work at the Korea Stock Exchange.

$$P(x,y) = \text{"x makes more money than y."}$$

$$Q(x,y) = \text{"}x \neq y\text{."}$$

Translate the following predicate logic statements into ordinary, everyday English. (Don't simply give a word-for-word translation; try to write sentences that make sense.)

(a) $(\forall x)\ (\exists y)\ P(x,y)$

(b) $(\exists x)\ (\forall y)\ (Q(x,y) \rightarrow P(x,y))$

(c) Which statement is impossible in this context? Why?

8. (8 pts) Write the following statement in predicate logic, and negate it. Say what your predicates are, along with the domains.

> Let x and y be real numbers. If x is rational and y is irrational, then x + y is irrational.