Ch 1. The Foundations: Logic and Proofs

Formal Logic – Classical Logic

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Statements (i.e., propositions)

To study formal logic, let's begin with statements.

> **Definition**
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> A *statement* (also known as a *proposition*) is a declarative sentence that is either true or false, but not both.

Examples of statements:

- 7 is odd.
- $1 + 1 = 4$
- If it is raining, then the ground is wet.
- Our professor is from Mars.

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What about "How beautiful this flower is!" or "Please close the door"?

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How can a declarative sentence **fail** to be a statement?

(1) has a unspecified term
   Example "x is even."

(2) is self-referential
   Example "This sentence is false."

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   Example "This sentence is false." -- (S)

Suppose S is true.
Then it must be false. →←
So it cannot be true and it must be false.
If false, it must be true,
which contradicts the assumption
that it is false. →←
Therefore S is neither true nor false.
However, the sentence
"This sentence is true" is not a contradiction.
For if it is false, it is a false sentence.
If it is true, then there is no contradiction. 5

Classical Logic (as opposed to Modern Logic) (1/3)

Aristotle
384–
322 BC

- Classical Logic = Aristotelian Logic
  - focuses on relations of "classes of things"

Example

All swans are mammals.
All black swans are swans.
_____
All black swans are mammals.

Is this argument
(logically) valid?

Classical Logic (as opposed to Modern Logic) (2/3)

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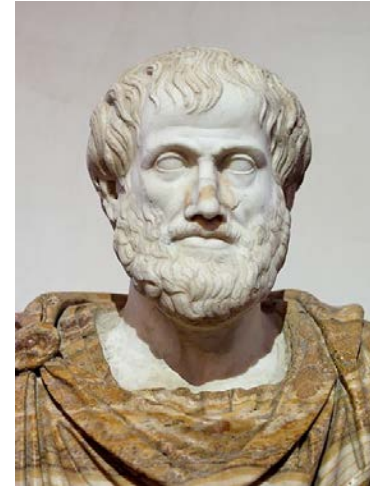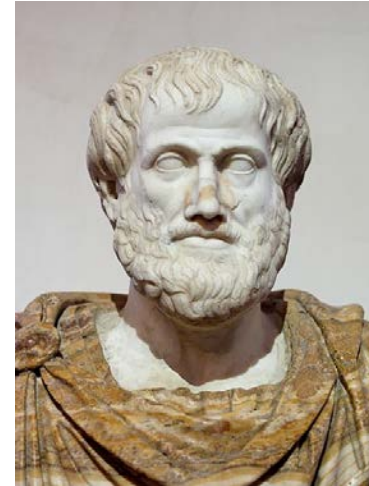 - focuses on relations of "classes of things"
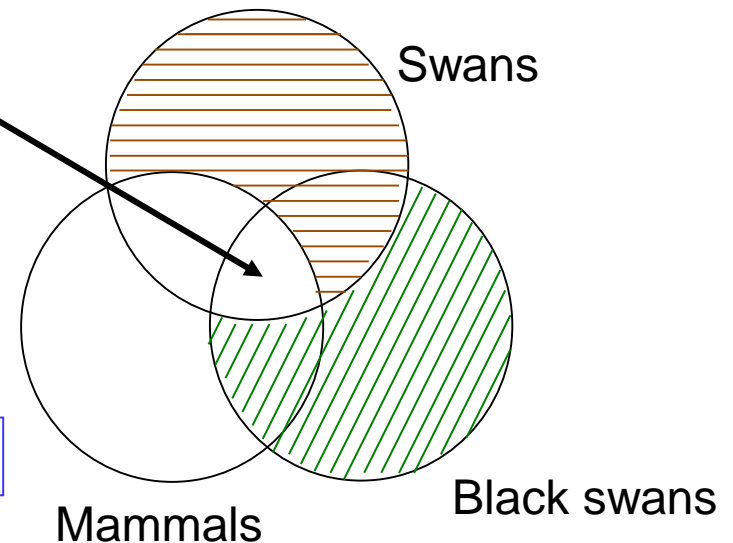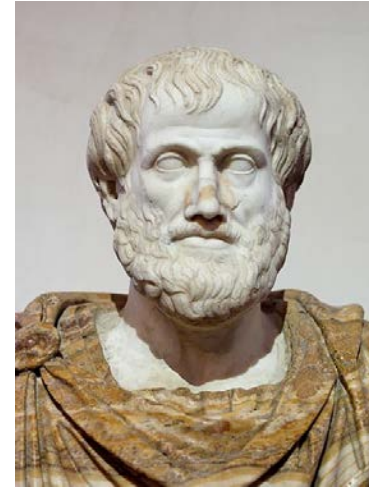
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A Venn Diagram

Swans

Mammals

Black swans

Classical Logic (as opposed to Modern Logic) (3/3)

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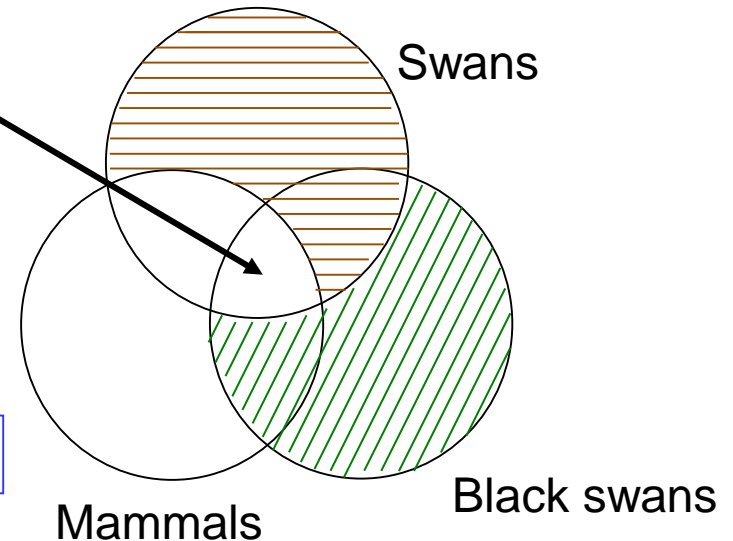 - focuses on relations of "classes of things"

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Categorical Propositions

Class (= Category) : A collection of all objects that have some characteristic in common

Classes can be related in one of the following three ways:

1. All of one class may be completely included in another class.

2. Some, but not all, of the member of one class may be included in another class.

3. Two classes may have no members in common.

Categorical Proposition: The propositions that state the relations between one category and some other category.

Standard-Form Categorical Propositions

Among categorical propositions, there are four standard form.

Form	Proposition
Form	**Proposition**
A (Universal affirmative)	All S is P.
E (Universal negative)	No S is P.
I (Particular affirmative)	Some S is P.
O (Particular negative)	Some S is not P.

S: Subject term
P: predicate term

Example

All bats can fly.

Socrates is a bat.
_____
Socrates can fly.

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Categorical Syllogism

Syllogism: A deductive argument in which a conclusion is inferred from two premises.

Categorical Syllogism: A deductive argument consisting of three categorical propositions that together contain exactly three terms,

each of which occurs in exactly two of the constituent propositions.

Example

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Quiz 02-1

[1] Which of the following is true about the inference below ?

    (a) It is valid.

    (b) It is invalid.

    (c) It can be either.

    (d) We cannot know.

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