Homework 1 – Propositional Logic

Due date September 5, 2020

1. (8 pts) Let the following statements be given.

p = "There is water in the cylinders."

q = "The head gasket is blown."

r = "The car will start."

(a) Translate the following statement into symbols of formal logic.

If the head gasket is blown and there's water in the cylinders, then the car won't start.

(b) Translate the following statement into everyday English.

$$r \rightarrow \neg (q \lor p)$$

2. (8 pts) Let the following statements be given.

p = "You are in Seoul."

q = "You are in Kwangju."

r = "You are in South Korea."

(a) Translate the following statement into symbols of formal logic.

If you are not in South Korea, then you are not in Seoul or Kwangju.

(b) Translate the following statement into everyday English.

$$q \rightarrow (r \land \neg p)$$

3. (12 pts) On the basis of the following abbreviation

P: logic is enjoyable

Q: Kim will pass

R: Kim concentrates

S: the text is readable

T: Kim will secure graduation

U: Kim will be employed

V: the lectures are exciting

translate the following symbolic sentences into natural English:

(a)
$$P \rightarrow (Q \rightarrow R)$$

(b)
$$(R \rightarrow Q) \rightarrow P$$

(c)
$$S \rightarrow (P \rightarrow (\neg Q \rightarrow \neg R))$$

4. (12 pts) Let

p: "I will do every exercise in this book"

q: "I will get an "A" in this course."

Express each of the following using p and q:

- a) I will get an "A" in this course only if I do every exercise in this book.
- b) I will get an "A" in this course and I will do every exercise in this book.
- c) Either I will not get an "A" in this course or I will not do every exercise in this book.
- d) For me to get an "A" in this course it is necessary and sufficient that I do every exercise in this book.
- 5. (12 pts) Given the alphabet of $\{P, P1, ..., Q, Q1, ..., R, R1, ..., ..., \neg, \land, \lor, \rightarrow, (,) \}$, write a BNF grammar that generates all legal propositional formulas. You can start with the following BNF grammar rules:

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\begin{aligned} &\text{digit} ::= \text{``0''} \mid \text{``1''} \mid \text{``2''} \mid \text{``3''} \mid \dots \mid \text{``8''} \mid \text{``9''} \\ &\text{integer} ::= &\text{digit} \mid \text{digit, integer} \\ &\text{A} ::= &P \mid P \text{, integer} \\ &\text{B} ::= &Q \mid Q \text{, integer} \\ &\text{C} ::= &R \mid R \text{, integer} \end{aligned} \ // \text{generates } Q, Q1, \dots \end{aligned}
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(Hint: You do not need to consider priorities of connectives. It suffices to generate fully parenthesized formulas that have no omission of parentheses. You may use "..." to indicate omission as in the above BNF grammar.)

6. (12 pts) Put the following syllogism into the standard form, which uses a horizontal line to separate premises and conclusion as shown in the lecture slides, and tell whether it is valid or not by showing its Venn Diagram. (Hint: To indicate absence of elements use shade as in the example of the lecture slides. To indicate existence of an element belonging to a class, mark "x" inside the circle for the class.)

Some reformers are fanatics, so some idealists are fanatics, because all reformers are idealists.