Ch 1. The Foundations: Logic and Proofs

Propositional Logic-1

Language

Sungwon Kang

Acknowledgement

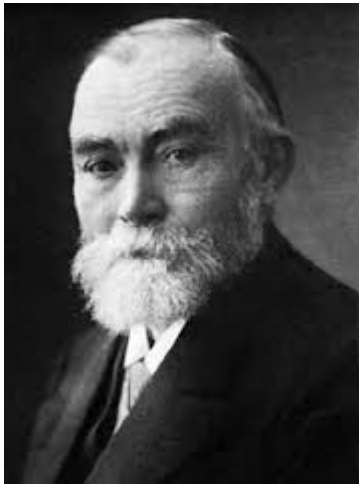
- [Rosen 19] Kenneth H. Rosen, for Discrete Mathematics & Its Applications (8th Edition), Lecture slides
- [Hunter 11] David J. Hunter, Essentials of Discrete Mathematics, 2nd Edition, Jones & Bartlett Publishers, 2011, Lecture Slides

Modern Logic

- Modern Logic = Symbolic Logic = Mathematical Logic
- Modern symbolic logic consists of propositional logic and predicate logic and has made logic mathematical and formal.

Propositional logic:
Logic that treats the whole sentences without considering the internal structure of sentences.

Predicate logic: considers the internal structure of propositions and arguments.



Gottlob Frege
(1848 –1925)

Concept-Script: A Formal Language for Pure Thought Modeled on that of Arithmetic
Note) Frege's work was discovered by B. Russell around 1900.

BEGRIFFSSCHRIFT,

EINE DER ARITHMETISCHEN NACHGEBILDETE

FORMELSPRACHE

DES REINEN DENKENS.

VON

Dr. GOTTLOB FREGE,

PRIVATDOCENTEN DER MATHEMATIK AN DER UNIVERSITÄT JENA.

HALLE a/S.
VERLAG VON LOUIS NEBERT.
1879.

(Published in 1879)

Propositional Logic: Syntax

- **Propositional letters:** represent statements

  p, q, r, s, ... , P, Q, R, S, ...

Example
p: The moon is made of green cheese.
q: The Earth is flat.
r: I will move to Mars.

The five logical connectives

Compound statement: a statement that is made up of component statements, using logical connectives.

Name	Symbol	Compound statement
and	$\wedge$	$p \wedge q$
or	$\vee$	$p \vee q$
not	$\neg$	$\neg p$
implies (if . . . then)	$\rightarrow$	$p \rightarrow q$
if and only if	$\leftrightarrow$	$p \leftrightarrow q$

Compound Statements: Terms

Compound Statement	Names	Component Statement	Names
$p \wedge q$	conjunction	p, q	conjunct
$p \vee q$	disjunction	p, q	disjunct
$\neg p$	negation		
$p \rightarrow q$	conditional implication	p	antecedent hypothesis premise
		q	consequent consequence conclusion
$p \leftrightarrow q$	bi-conditional bi-implication		

Conventions (1/3)

- We usually drop the parentheses when the resulting sentence is unambiguous

  ☛ A left parenthesis extends to the first unmatched right parenthesis or the end of the expression, skipping over "holes"

- Precedence rules are (from highest to lowest), and

  $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$

Conventions (2/3)

- We usually drop the parentheses when the resulting sentence is unambiguous

  ☞ A left parenthesis extends to the first unmatched right parenthesis or the end of the expression, skipping over "holes"

- Precedence rules are (from highest to lowest), and

  $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$

Conventions (3/3)

Examples

$P \wedge Q \rightarrow R$ is the same as $((P \wedge Q) \rightarrow R)$

$P \vee Q \wedge \neg R \rightarrow \neg Q$ is the same as $((P \vee (Q \wedge (\neg R))) \rightarrow (\neg Q))$

Can we omit parentheses in "$P \rightarrow (Q \rightarrow R)$"?

Translation to natural language sentences

Example

p: the statement "you are wearing shoes"
q: the statement "you can't cut your toenails"

Natural language translation:
$\neg q$: ?
$p \wedge q$: ?
$p \rightarrow q$: ?

Translation from natural language sentences

Example 1

p: The moon is made of green cheese.
q: The Earth is flat.
r: I will move to Mars.

The moon is made of green cheese and the Earth is flat.

☛

If the moon is made of green cheese, then the Earth is flat.

☛

If the Earth is not flat, then the moon is not made of green cheese.

☛

Translation from natural language sentences

Example 1
p: The moon is made of green cheese.
q: The Earth is flat.
r: I will move to Mars.

The moon is made of green cheese and the Earth is flat.
☛ $p \wedge q$

If the moon is made of green cheese, then the Earth is flat.
☛ $p \rightarrow q$

If the Earth is not flat, then the moon is not made of green cheese.
☛ $\neg q \rightarrow \neg p$

Example 2
$p \vee q \rightarrow r$
☛ If the moon is made of green cheese or the Earth is flat, then I will move to Mars.

Quiz 02-1

[1] Given the following propositions:

p: the statement "you are wearing shoes"
q: the statement "you can't cut your toenails“
r: the statement “you should take off shoes”

how would you translate

$p \wedge q \rightarrow r$

to a natural language expression?