Anar Rzayev CS204 Midterm Exam 20190788

1) a) $S_1: (I \wedge S) \rightarrow (G \wedge P)$

$S_2: ((S \wedge \neg I) \rightarrow A) \wedge (A \rightarrow P)$

$S_3: I \rightarrow S$

$S_4: P$

b) Valid

c) ~~A true~~

A - false,
S - false
I - true

P - false $\Rightarrow$
In order for S_2 to be true, $A \rightarrow P$ should be true or A - false then $(S \wedge \neg I) \rightarrow A$ should be true, or $S \wedge \neg I \Rightarrow$ false

then $S =$ False, $I =$ True $\Big\} \; I \rightarrow S \cong$ True $\rightarrow$ False $=$

$=$ False

Hence, $\boxed{P\text{-true}}$ ✓

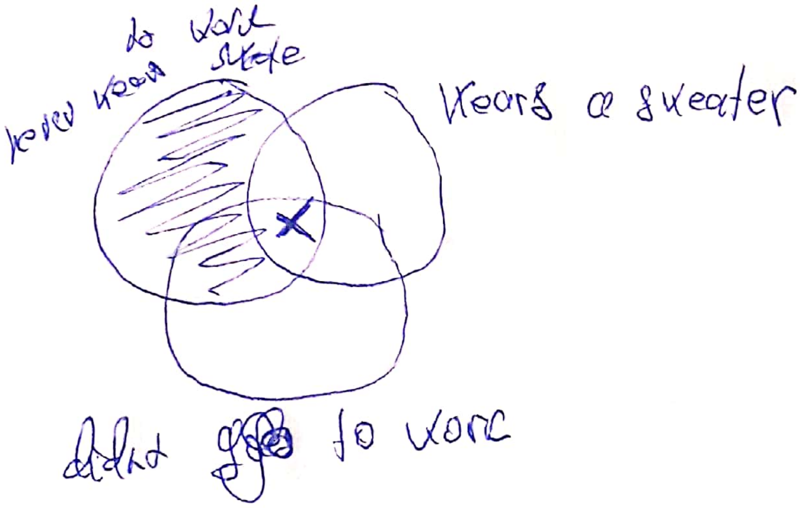
2) a) Biff wore a sweater

Biff never wears a sweater to work

Biff didn't go to work this morning

b) Subject S: Biff

Valid



4) a) $(\forall x)(Q(x) \to \neg A(x))$

$(\exists x)(A(x) \land G(x))$

$(\forall x)(Q(x) \to G(x))$

b) $(\forall x)(Q(x) \to A(x))$ or $\neg(\exists x)(Q(x) \land \neg A(x))$

$\neg(\exists x)(A(x) \land G(x))$ or $(\forall x)(A(x) \to \neg G(x))$

$(\forall x)(Q(x) \to \neg G(x))$

c) Valid

d)

5) a) From conditional-disjunction equivalence,

$P(x) \to Q(x) \equiv \neg P(x) \vee Q(x)$

$(\exists x)(\neg P(x) \vee Q(x)) \to ((\exists x) P(x) \to (\exists x) Q(x))$

$P(1) = False$
$Q(1) = False$
$Q(2) = False$
$P(2) = True$

$\boxed{(\exists x)(\neg P(x) \vee Q(x)) = True}$ by choosing $\underline{x=1} \Rightarrow \neg P(1) = True \Rightarrow$

$\neg P(1) \vee Q(1) = True$ ✓

$\boxed{(\exists x) P(x) = True}$ by choosing $\underline{x=2}$ ✓ $P(2) = True$ ✓

$\boxed{(\exists x) Q(x) = False}$ because $Q(1) = false$
$Q(2) = false$

there does not exist x such that $Q(x)$ is true
$x \in \{1, 2\}$

$$True \to \underbrace{(True \to False)}_{False} \equiv True \to False$$

$\equiv False$, since $True \to False \equiv False$

So, this assertion is not valid

b) NO

c) $P(1)$, $Q(1) \Rightarrow$ if $Q(1)$ was true, then $(\exists x) Q(x)$ would be true and $\left.\begin{array}{l} F \Rightarrow T \\ T \Rightarrow T \end{array}\right\}$ are valid (true)

statement will never be false

There are 4 cases: $P(1) = T, T, F, F$

$Q(1) = F, T, F, T$

If $Q(1)$ was true, $(\exists x)\, Q(x)$ would be true

$F \to T$, $T \to T$ are both true, $(\exists x) P(x) \to (\exists x) Q(x)$ becomes true, then

$(\exists x)(P(x) \to Q(x)) \to$ True becomes true, where we substituted $(\exists x) P(x) \to (\exists x) Q(x)$ with True $\Rightarrow$ then our statement would be valid

So, $Q(1) = $ False $\boxed{(\exists x)\, Q(x) = \text{False}}$ Since there's no x for which $Q(x)$ is true

$P(1) = $ False $\Rightarrow (\exists x) P(x) = $ False, since there is no x such that $P(x)$ is True $\Rightarrow (\exists x) P(x) \to (\exists x) Q(x) \equiv$

$\equiv$ False $\to$ False $\equiv$ True, so it becomes

$(\exists x)(P(x) \to Q(x)) \to$ True which's always valid

So, $P(1) = $ True $\Rightarrow$ $P(1) = $ True $\to Q(1) = $ False $\equiv$

$\equiv$ False, or $(\exists x)(\neg P(x) \vee Q(x))$ is false, since

$\neg P(1) = $ False, $Q(1) = $ False $\Rightarrow$ there's no such x to make T

$\Rightarrow$ False $\to (\exists x P(x) \to \exists x Q(x)) \equiv$ ~~False~~ True here

statement is always valid on $\{1\}$

Sets, Relations, Functions

1) a) A is an equivalence relation, because it satisfies all three conditions: reflexivity, transitivity, symmetry

1R1	1R5	5R1	1R5 5R3 $\rightarrow$ 1R3
2R2	3R5	5R3	3R5 5R3 $\rightarrow$ 3R3
3R3	1R3	3R1	5R1 1R3 $\rightarrow$ 5R3
4R4			3R1 1R5 $\rightarrow$ 3R5
5R5			

$aRb \Leftrightarrow bRa$

$aRb, bRc \Rightarrow aRc$

Yes

A $\rightarrow$ Yes

~~B is not an equiv~~

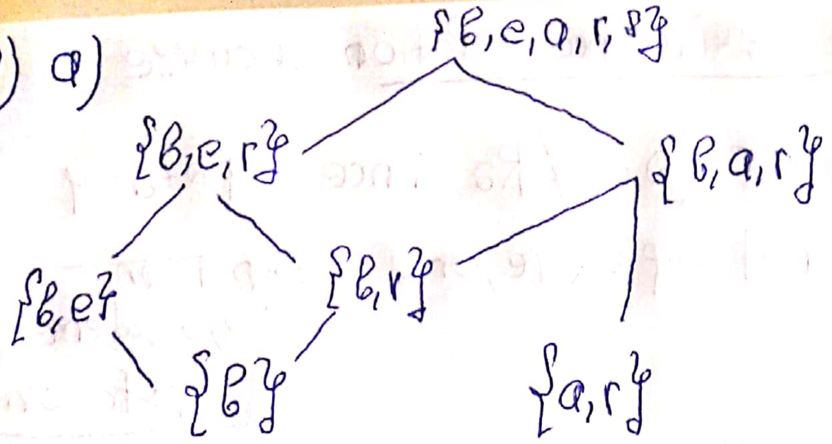
B $\rightarrow$ NO

b) $R_1 = \{1, 3, 5\} = R_3 = R_5$
$R_2 = \{2\}, R_4 = \{4\}$ $\Big\}$ A

In B, (2,2) is not included

so not equivalence

3) a)



b) An element $m \in X$ is minimal if there is no $x \in X$ $(x \neq m)$ such that $x \subseteq m$

In this case, $\boxed{\text{minimal elements: } \{b\}, \{a,r\}}$

c) In a poset, there may be to have elements a, b such that neither $a \subseteq b$ nor $b \subseteq a$. Such elements are incomparable $\Rightarrow$ {b} and {a,r}

{a,r} and {b,r}

{a,r} and {b,e,r}

{a,r} and {b,e}

{b,e} and {b,r}

{b,e} and {b,a,r}

{b,e,r} and {b,a,r}

2) a) Ra is not an equivalence relation, because

$2 \in X$, but $2 \cancel{R} 2$ or $(2,2) \notin Ra$, since $3 \nmid 2+2=4$

This means Ra is not reflexive, and so $\Rightarrow$ $\boxed{\text{not an equivalence relation}}$

R1 is an equivalence relation $\rightarrow$ reflexive

$\rightarrow$ symmetric

$\rightarrow$ transitive

$\forall x \in X \Rightarrow (x,x) \in R$

for $x = 1,2,3,4,5$ ✓ (reflexive)

if $(x,y) \in R_1$, then $(y,x) \in R_1$

$(1,5) \in R_1 \Leftrightarrow (5,1) \in R_1$ (symmetry) ✓

$(3,5) \in R_1 \Leftrightarrow (5,3) \in R_1$

$(1,3) \in R_1 \Leftrightarrow (3,1) \in R_1$

✓transitive $\rightarrow$ $(1,5) \in R_1$, $(5,5) \in R_1$ $\Rightarrow (1,5) \in R_1$

$(1,3) \in R_1, (3,5) \in R_1 \Rightarrow (1,5) \in R_1$ ✓

$Rx = \{a \in X \mid x R a\}$ $\rightarrow$ equivalence class

$R1 = \{1,3,5\}$

$R2 = \{2\}$

$R3 = \{1,3,5\}$

$R4 = \{4\}$

$R5 = \{1,3,5\}$

8) $R_x = \{a \in Y \mid xRa\}$

R-equivalence relation $\Rightarrow \forall a \in Y,\ aRa$

$R_4 = \{4\}$

$R_3 = R_2 = R_1 = \{1, 2, 3\}$

$$\boxed{R = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3), (4,4)\}}$$

1)

4) a) Suppose $f(x)=y$, then $f^2(x)=f(f(x))=f(y)=f(x)=y$
$\Rightarrow f(y)=y$. In other words, f must map any element in the image of f to itself

We now have 3 cases: the image of f can contain 1 element, 2 elements, or be the whole of A

- There are 3 functions where the image of f contains 1 element. Each such f certainly maps the single element in its image to itself
- There are 18 functions where the image of f contains two elements. But in only 6 of these is each element in the image of f mapped to itself
- There are 6 functions where the image of f is the whole of A. But there's only one such function that maps each element of A to itself

So, we find $\underbrace{3+6+1}=\underbrace{10} \Rightarrow$ $\boxed{\text{There are 10 such functions}}$

b)
1. $f(0)=0,\ f(1)=0,\ f(2)=0$
2. $f(0)=1,\ f(1)=1,\ f(2)=1$
3. $f(0)=2,\ f(1)=2,\ f(2)=2$
4. $f(0)=0,\ f(1)=0,\ f(2)=2$
5. $f(0)=0,\ f(1)=1,\ f(2)=0$
6. $f(0)=0,\ f(1)=1,\ f(2)=1$
7. $f(0)=0,\ f(1)=2,\ f(2)=2$
8. $f(0)=1,\ f(1)=1,\ f(2)=2$
9. $f(0)=2,\ f(1)=1,\ f(2)=2$
10. $f(0)=0,\ f(1)=1,\ f(2)=2$

We listed all 10 possible cases

(1) $f(0)=0,\ f(1)=0,\ f(2)=0 \Rightarrow f^2(0)=f(0)=0,\ f^2(1)=f(0)=$
$=f(1)=0,\ f^2(2)=f(0)=f(2)=0$ ✓

(2) $f^2(0)=f(1)=1=f(0),\ f^2(1)=f(1),\ f^2(2)=f(1)=1=f(2)$ ✓

(3) $f^2(0)=f(2)=2=f(0),\ f^2(1)=f(2)=2=f(1),\ f^2(2)=f(2)$ ✓

(4) $f(0)=0,\ f(1)=0,\ f(2)=2 \Rightarrow f^2(0)=f(0),\ f^2(1)=f(0)=0=$
$=f(1),\ f^2(2)=f(2)$ ✓

(5) $f^2(0)=f(0),\ f^2(1)=f(1),\ f^2(2)=f(0)=0=f(2)$ ✓

(6) $f^2(0)=f(0),\ f^2(1)=f(1),\ f^2(2)=f(1)=1=f(2)$ ✓

(7) $f^2(0)=f(0),\ f^2(1)=f(2)=2=f(1),\ f^2(2)=f(2)$ ✓

(8) $f^2(0)=f(1)=1=f(0),\ f^2(1)=f(1),\ f^2(2)=f(2)$ ✓

(9) $f^2(0)=f(2)=2=f(0),\ f^2(1)=f(1),\ f^2(2)=f(2)$ ✓

(10) $f^2(0)=f(0),\ f^2(1)=f(1),\ f^2(2)=f(2)$ ✓