CS 204 Discrete Mathematics

Homework 10

Sample Solutions

Finite probability space, events; Basic concepts of probability theory

1. Suppose that a hundred people enter a contest and that different winners are selected at random for one first prize, one second prize, and one third prize. What is the probability that a participant of the contest wins one of these prizes?

Solution)

3/100

2. Which is more likely: rolling a total of 9 when two dice are rolled or rolling a total of 9 when three dice are rolled?

Solution)

Three dice

Guideline) there is no partial point.

4. What is the probability of the following events when we randomly select a permutation of {1, 2, 3, 4}?
   (a) 1 precedes 4.
   (b) 4 precedes 1.
   (c) 4 precedes 1 and 4 precedes 2.
   (d) 4 precedes 1, 4 precedes 2, and 4 precedes 3.
   (e) 4 precedes 3 and 2 precedes 1.

Solution)

(a) 1/2
(b) 1/2
(c) 1/3
(d) 1/4
(e) 1/4

Conditional probability, Bayes' theorem; Independence

1. What is the conditional proability that exactly four heads appear when a fair coin is flipped five times given that the first flip camp up heads?

Solution)

1/4

2. What is the probability that a family with five children does not have a boy, if the sexes of children are independent and if
   (a) a boy and a girl are equally likely.
   (b) the probability of a boy is 0.1
   (c) the probability that the i-th child is a boy is 0.51 – (i /100)

Solution)

(a) 1/32 = 0.03125
(b) $0.49^5 \approx 0.02825$
(c) 0.03795012

3. A family has two children. What is the conditional probability that both are boys given that at least one of them is a boy? Assume that the sample space S is given by S = {(b,b), (b,g),(g,b), (g,g)}, and all outcomes are equally likely. ((b,g) means for instance that the older child is a boy and the younger child is a girl.))

Solution)

Letting E denote the event that both children are boys, and F the event that at least one of them is a boy, then the desired probability is given by

$$
\begin{aligned}
P(E|F) &= P(E \,\&\, F) / P(F) \\
&= P(\{(b,b)\}) / P(\{(b,b), (b,g), (g,b)\}) \\
&= (1/4) / (3/4) \\
&= 1/3
\end{aligned}
$$

4. In answering a question on a multiple choice test, a student either knows the answer or he guesses. Let p be the probability that he knows the answer and 1-p be the probability that he guesses. Assume that a student who guesses at the answer will be correct with probability 1/m, where m is the number of multiple-choice alternatives. What is the conditional probability that a student knew the answer to a question given that he answered it correctly?

Solution)

Let C and K denote respectively the even that the student answers the question correctly and the event that he actually knows the answer. Now

$$
\begin{aligned}
P(K|C) &= P(K \,\&\, C) / P(C) \\
&= P(C|K)P(K) / \{P(C|K)P(K) + P(C|\sim K)\, P(\sim K) \\
&= p / \{p + (1/m)(1-p)\} \\
&= mp / \{1 + (m-1)p\}
\end{aligned}
$$

Thus, for example, if $m = 5$, $p = 1/2$, then the probability that a student knew the answer to a question he correctly answered is 5/6.

Guideline)

Get the right answer. (8pts)

Some calculations were wrong. (4pts)

All calculations were wrong. (0 point)