CS 204 Discrete Mathematics

Homework 3 – Predicate Logic Part I

Sample Solutions

5. The domain of the following predicates is the set of all plants.

$P(x)$ = "x is poisonous."

$Q(x)$ = "Jeff has eaten x."

Translate the following statements into predicate logic.

1. Some plants are poisonous.
2. Jeff has never eaten a poisonous plant.
3. There are some nonpoisonous plants that Jeff has never eaten.

Solution)

(a) $(\exists x)P(x)$

(b) $(\forall x)(P(x) \to \neg Q(x))$

(c) $(\exists x)(\neg P(x) \wedge \neg Q(x))$

6. In the domain of integers, consider the following predicates: Let N(x) be the statement "$x \neq 0$". Let P(x,y) be the statement that "$xy = 1$."

(a) Translate the following statement into the symbols of predicate logic.

For all integers x, there is some integer y such that if $x \neq 0$, then $xy = 1$.

(b) Write the negation of your answer to part (a) in the symbols of predicate logic, Simplify your answer so that it uses the $\wedge$ connective.

(c) Translate your answer from part (b) into an English sentence.

(d) Which statement, (a) or (b), is true in the domain of integers? Explain.

Solution)

(a) $(\forall x)(\exists y)(N(x) \to P(x, y))$

(b) $(\exists x)(\forall y)(N(x) \wedge \neg P(x, y))$

(c) There is a nonzero integer x such that $xy \neq 1$ for all integers y.

(d) Statement (b) is true; for example, $x = 2$ works.

7. The domain of the following predicates is the set of all traders who work at the Korea Stock Exchange.

$$P(x,y) = \text{"x makes more money than y."}$$

$$Q(x,y) = \text{"} x \neq y \text{."}$$

Translate the following predicate logic statements into ordinary, everyday English. (Don't simply give a word-for-word translation; try to write sentences that make sense.)

(a) $(\forall x)\ (\exists y)\ P(x,y)$

(b) $(\exists x)\ (\forall y)\ (Q(x,y) \rightarrow P(x,y))$

(c) Which statement is impossible in this context? Why?

Solution)

(a) For every trader, there is a trader that makes less money.

(b) There is some trader that makes more money than every other trader.

(c) Statement (a) is impossible, because there must be some trader that makes the smallest amount of money.

8. Write the following statement in predicate logic, and negate it. Say what your predicates are, along with the domains.

> Let x and y be real numbers. If x is rational and y is irrational, then x + y is irrational.

Solution)

Let $P(x)$ be the statement "x is rational" in the domain of real numbers. The statement given in the problem translates as

$$(\forall x)(\forall y)((P(x) \wedge \neg P(y)) \rightarrow \neg P(x+y)).$$

The negation is

$$(\exists x)(\exists y)(P(x) \wedge \neg P(y) \wedge P(x+y)),$$

i.e., "there is a rational x and irrational y such that $x + y$ is rational."

From TA.

Since there were many students who misused quantifier $\forall$ and $\in$ when they were used together, I announce the correct answer for

3. There is exactly one element of T that satisfies P.

Answer : $\exists x \in T\ (P(x) \wedge \forall y \in T\ (P(y) \rightarrow y=x\ ))$

Also, many students made some mistakes when they were using parentheses.

$\exists x\ (P(x) \wedge \forall y\ (P(y) \rightarrow y=x\ ))$ ... (a)

$\exists x\ P(x) \wedge \forall y\ (P(y) \rightarrow y=x\ )$ ... (b)

Two predicate logics above are different since second x in (b) is not bounded and a free variable. Please keep in mind the correct usage of parentheses through the difference between the two sentences above.