Ch 10. Graphs

Introduction to Graphs

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Acknowledgement

- [Rosen 19] Kenneth H. Rosen, for Discrete Mathematics & Its Applications (8th Edition), Lecture slides
- [Hunter 11] David J. Hunter, Essentials of Discrete Mathematics, 2nd Edition, Jones & Bartlett Publishers, 2011, Lecture Slides

Ch 10. Graphs

10.1 Graphs and Graph Models

10.2 Graph Terminology and Special Types of Graphs

10.3 Representing Graphs and Graph Isomorphism

10.4 Connectivity

10.5 Euler and Hamiltonian Graphs

- **Data structure**
  - "A data structure is a (structured) collection of data, organized so that items can be stored and retrieved by some fixed technique."

  Note) “SCRUD” for data: Search, Create, Read, Update, Delete

  ☛ To be useful, a data structure should have operations (= functions or methods) associated with it

Example Array

It allows individual items to be stored and retrieved based on an index([0],[1],[2],...) that is assigned to each item.

Allows random access: A[i] returns the i-th element.

- **My conjecture**: “*All useful computations involve data storing*.”

- **Problem**: Implement a telephone directory that stores a person's name and the person's phone number.
  - Would it be a good idea to implement the dictionary in two arrays? One for names and the other for phone numbers.

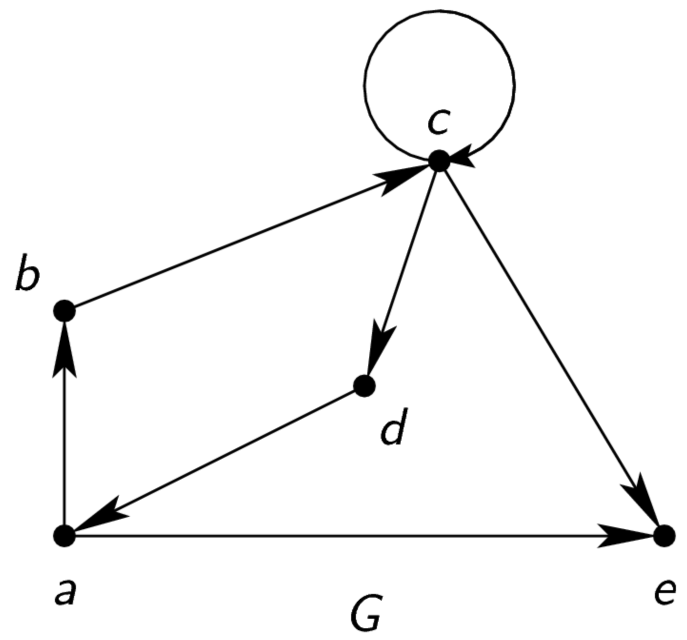
Introduction to Graphs

1. Edges and Vertices
2. Terminology
3. Modeling Relationships with Graphs

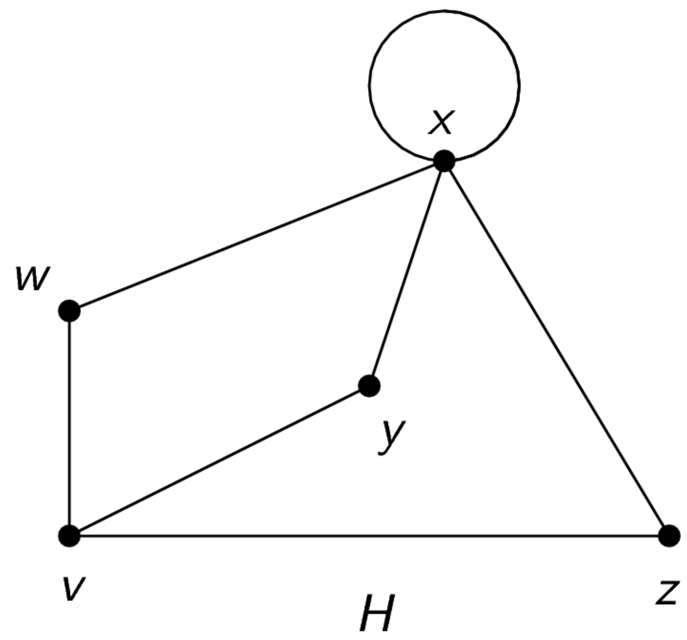
1. Edges and Vertices

A graph is a set of nodes together with a set of links connecting them.
(= vertices) (= edges)

A graph is a diagram of dots, called *vertices*, connected by lines or curves, called *edges*. The edges may be directed or undirected.

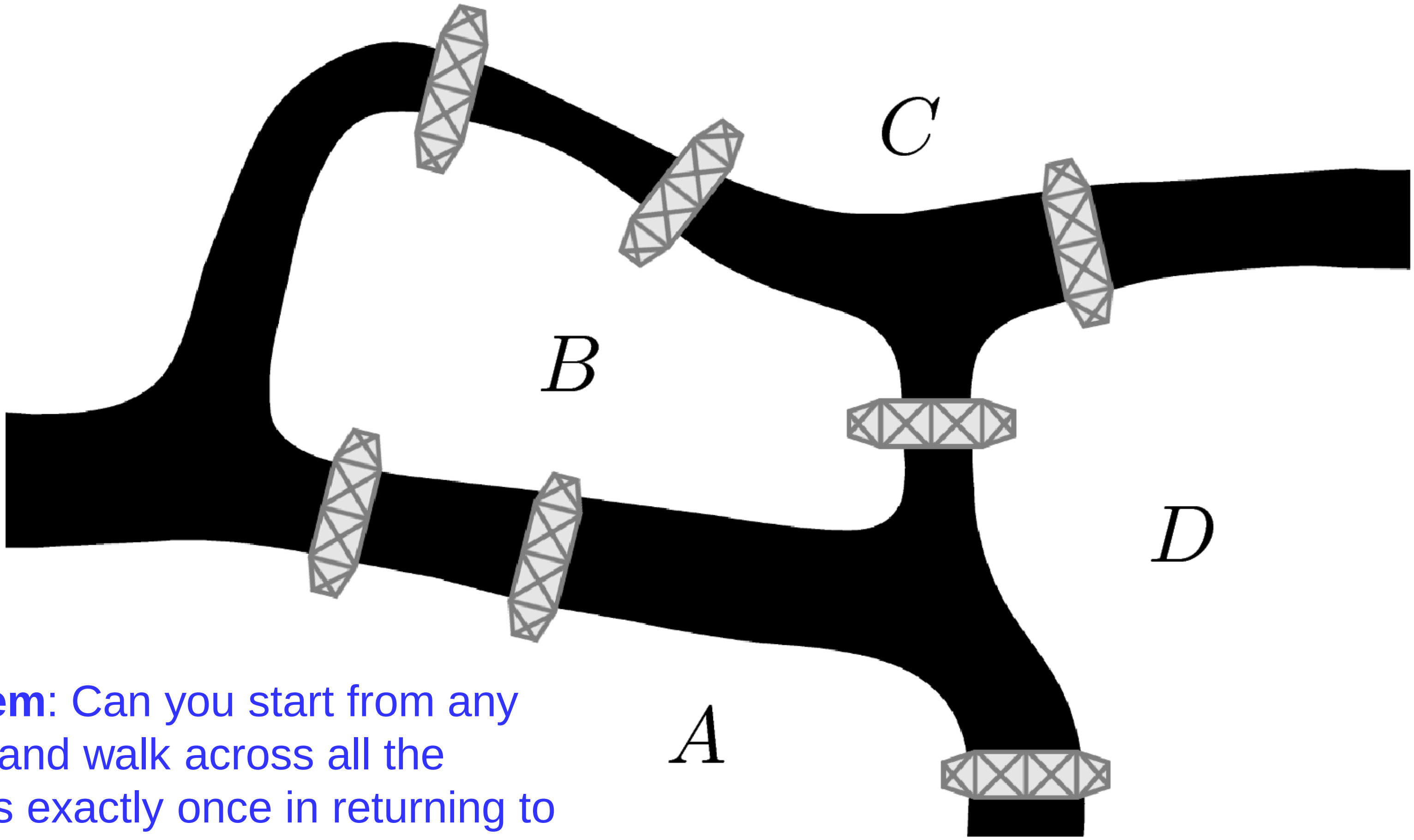


Directed graph



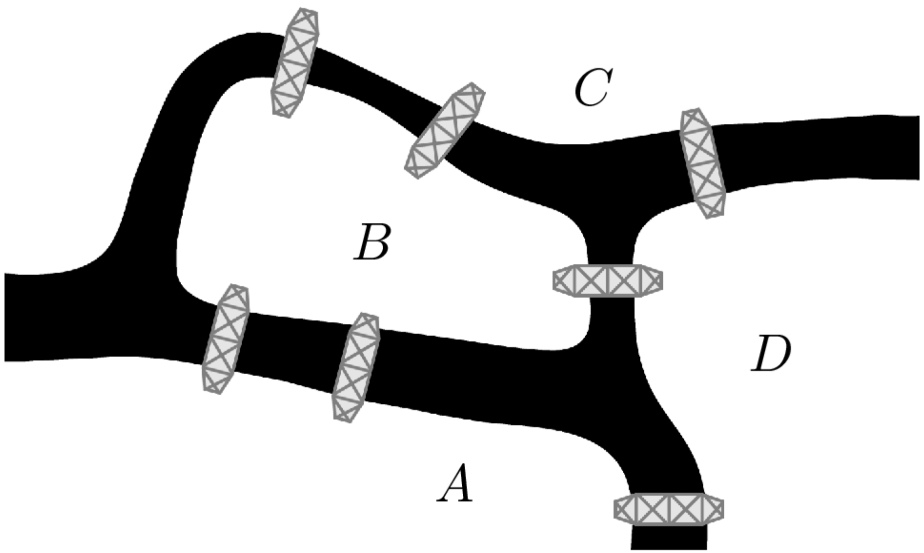
Undirected graph

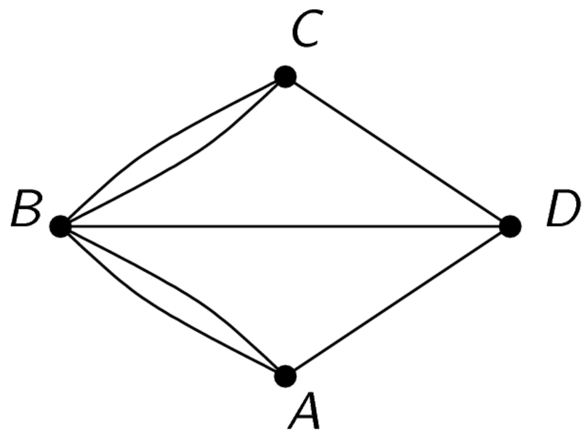
The bridges of Königsberg (Euler, 1736)



Problem: Can you start from any island and walk across all the bridges exactly once in returning to the starting island?

A graph model



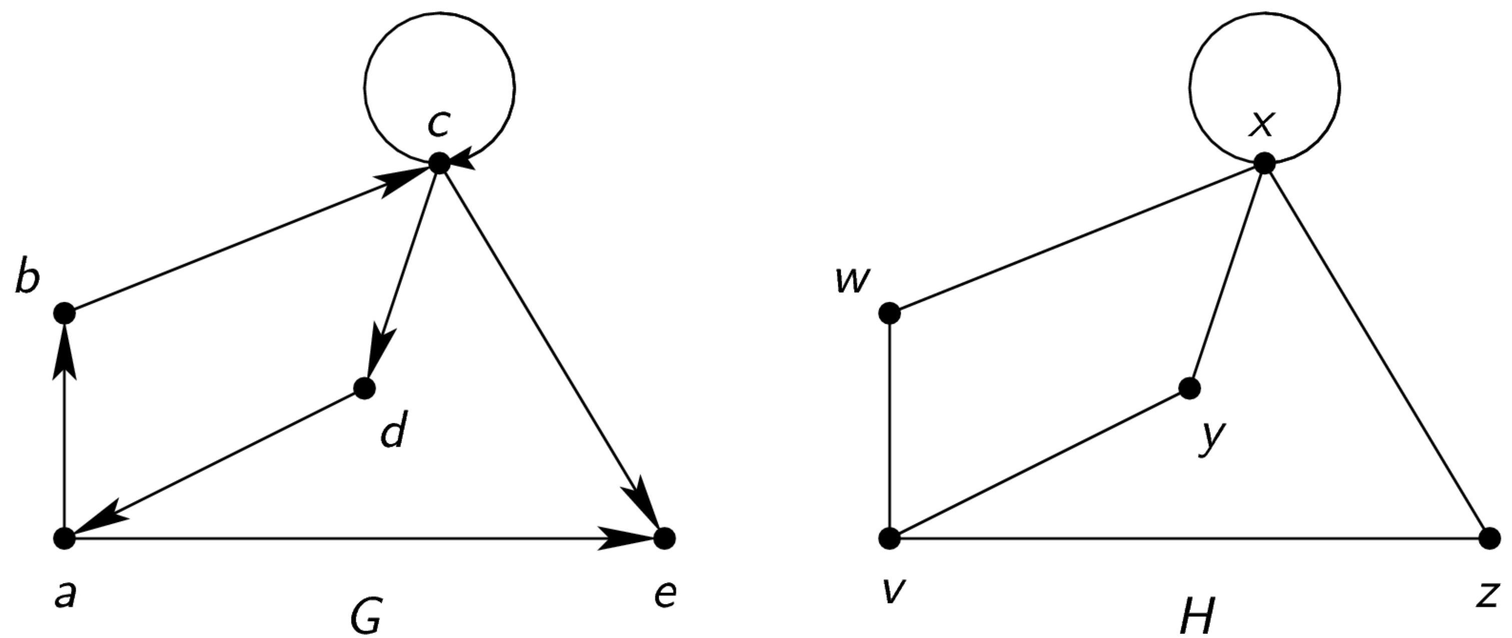


Problem: Can you start from any island and walk across all the bridges exactly once in returning to the starting island?

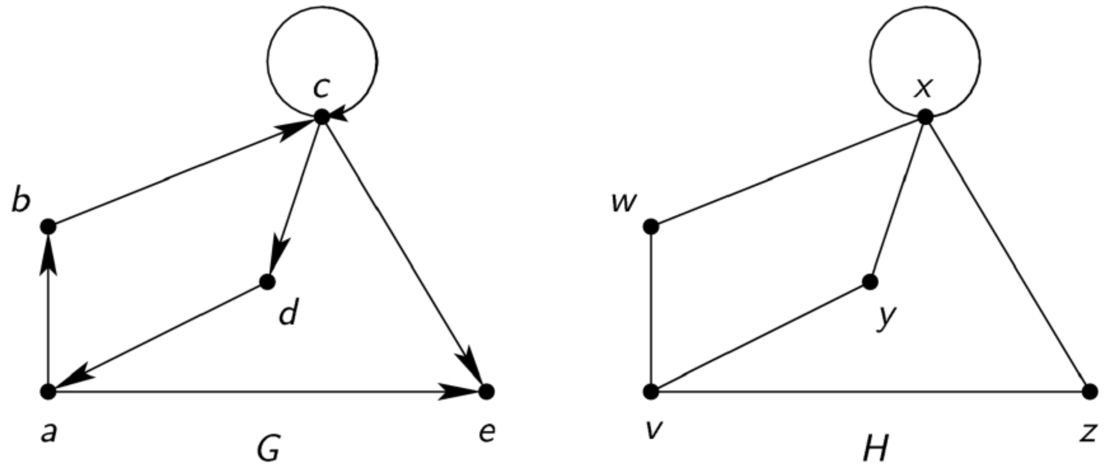
Euler Circuit: A path starting from any node, going through each edge exactly once and terminating at the start node

2. Terminology

- degree (indegree and outdegree)
- loops
- paths and circuits (Euler paths and circuits) circuit = cycle
- connected (= There is a path from any node to any node.)

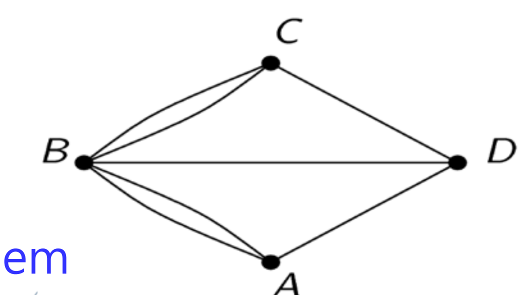


Euler's observations



1. In any graph, the sum of the degrees of the vertices equals twice the number of edges, because each edge contributes 2 to the sum of the degrees.
2. If a graph has more than two vertices of odd degree, it does not have an Euler path.
3. If a connected graph has exactly two vertices v and w of odd degree, then there is an Euler path from v to w.
4. If all the vertices of a connected graph have even degree, then the graph has an Euler circuit.

Graph model for the Bridges of Königsberg problem



Using graph models: scheduling

The following pairs of classes always have students in common, so they can't be scheduled in the same time slot:

Physics and Computer Science
Physics and Chemistry
Calculus and Chemistry
Calculus and Physics
Calculus and Computer Science
Calculus and Discrete Math
Calculus and Biology
Discrete Math and Computer Science
Discrete Math and Biology
Psychology and Biology
Psychology and Chemistry

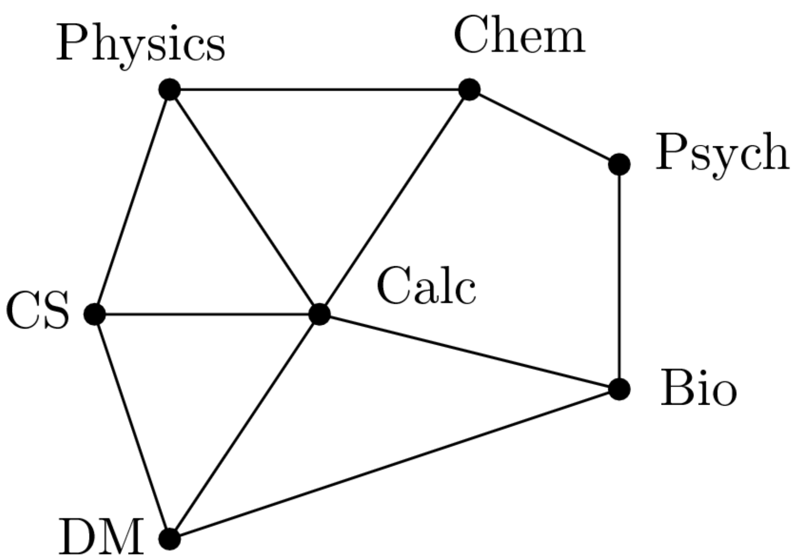
How would you solve this problem?

What is the fewest number of time slots needed?

Graph coloring

Color the vertices of the graph:

- Make sure no edge connects two vertices of the same color.
- Use as few colors as possible.



What is the fewest number of colors that are needed?

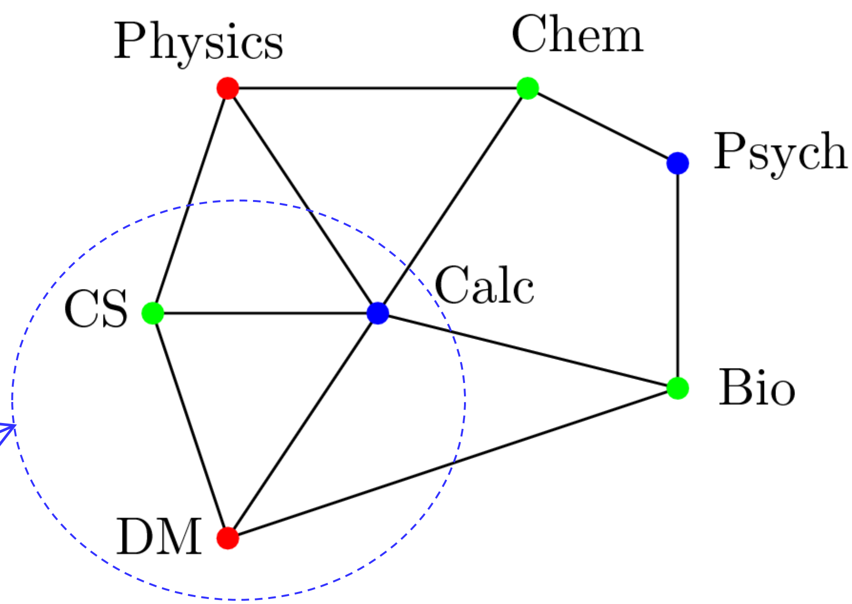
[redacted]

Color the vertices of the graph:

- Make sure no edge connects two vertices of the same color.
- Use as few colors as possible.

How can you prove that 2-coloring is impossible?

Consider these three nodes for example. Can you 2-color them?

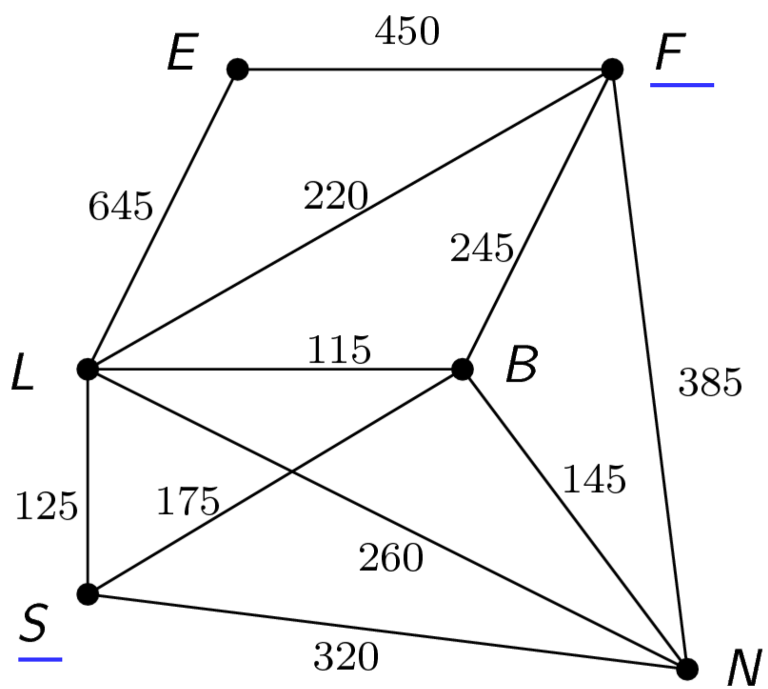


Networks: graphs with weighted edges

Pairwise driving distances (in miles) on selected routes between some California cities:

	B	E	F	L	N	S
Barstow						
Eureka						
Fresno	245	450				
Los Angeles	115	645	220			
Needles	145		385	260		
San Diego	175			125	320	

Is there a direct route between San Diego and Fresno?
What is the distance of the road that connects Fresno and Barstow?



How far is it from San Diego to Fresno?

How many simple paths are there?

Quiz 23-1

In the problem on graphs with weighted edges,

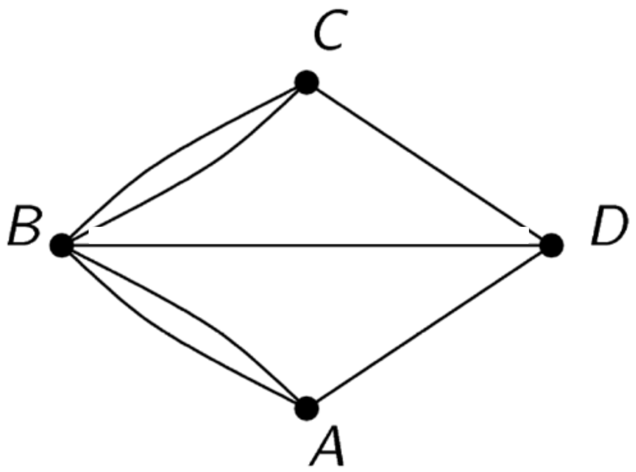
[1] How many simple paths are there from San Diego to Fresno?

[2] What is the shortest distance from San Diego to Fresno?

Concerning the graph below,

[3] Does it have an Euler circuit?

[4] Does it have an Euler path?



(3) Start 125; End 245