CS 204 Discrete Mathematics Fall 2020

Homework 4 – Predicate Logic - Part II

Due date September 26, 2020

1. (10 pts) Let the following predicates be given in the domain of triangles.

R(x) = "x is a right triangle."

B(x) = "x has an obtuse angle."

Consider the following statements.

$S1 = \neg(\exists x)\ (R(x) \wedge B(x))$

$S2 = (\forall x)\ (R(x) \rightarrow \neg B(x))$

(a) Write a proof sequence to show that S1 $\Leftrightarrow$ S2.

(b) Write S1 in ordinary English.

(c) Write S2 in ordinary English.

2. (10 pts)

(a) Give an example interpretation of a pair of predicates P(x) and Q(x) in some domain to show that the $\exists$ quantifier does not distribute over the $\wedge$ connective. That is, give an example to show that the statements

$(\exists x)(P(x) \wedge Q(x))$ and $(\exists x)P(x) \wedge (\exists x)Q(x)$

are not logically equivalent.

(b) It is true, however, that $\exists$ distributes over $\vee$. That is,

$$(\exists x)(P(x) \vee Q(x)) \quad \Leftrightarrow \quad (\exists x)P(x) \vee (\exists x)Q(x)$$

is an equivalence rule for predicate logic. Verify that your example from part (a) satisfies this equivalence.

3. (10 pts)

Any equation or inequality with variables in it is a predicate in the domain of real numbers. For each of the following statements, tell whether the statement is true or false.

(a) $(\forall x)(x^2 > x)$

(b) $(\exists x)(x^2 -2 = 1)$

(c) $(\exists x)(x^2 + 2 = 1)$

(d) $(\forall x)(\exists y)(x^2 + y = 4)$

(e) $(\exists y)(\forall x)(x^2 + y = 4)$

4. (12 pts) The domain of the following predicates is all integers greater than 1.
   P(x) = "x is prime"
   Q(x,y) = "x divides y"

   Consider the following statement.

   For every x that is not prime, there is some prime y that divides it.

   (a) Write the statement in predicate logic.
   (b) Formally negate the statement
   (c) Write the English translation of your negated statement

5. (10 pts) Two common axioms for geometry are as follows. The undefined terms are "point", "line", and "is on".
   A. For every pair of points x and y, there is a unique line w such that x is on w and y is on w.
   B. Give a line w and a point x that is not on w, there is a unique line m such that x is on m and no point on w is also on m.

Recall that two lines w and m are *parallel* if there is no point on both w and m, in which case we write "w || m". Use this definition along with the above two axioms to prove the following.

Let w, m and n be distinct lines. If w || m and m || n, then w || n.

6. (12 pts) The domain for this problem is some unspecified collection of numbers.

   Consider the predicate

   P(x,y) = "x is greater than y"

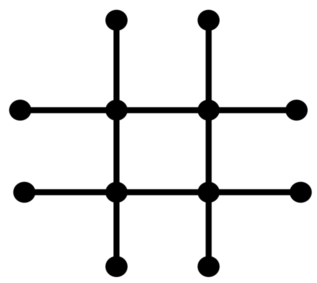
   (a) Translate the following statement into predicate logic.

   Every number has a number that is greater than it.

   (b) Negate your expression from part (a) and simplify it so that no quantifier or connective lies within the scope of a negation.
   (c) Translate your expression from part (b) into understandable English. Don't use variables in your English translation.

7. (10 pts) Suppose we would like to add to the Natural Deduction inference rules a set of inference rules for $\leftrightarrow$ using the fact that $A \leftrightarrow B$ is true if and only if $A \rightarrow B$ and $B \rightarrow A$ are true so that proofs related to $\leftrightarrow$ can take advantage of proofs related to $\rightarrow$. State a set of Natural Deduction style inference rules for $\leftrightarrow$.

8. (8 pts) In the Badda-Bing axiomatic system discussed in class, let a "badda" be a line segment, let a "bing" be a point, and say that a line segment "hits" a point if it passes through it. In the diagram below, there are 4 baddas and 12 bings. Is this a model for the system? Which of the axioms does this model satisfy? Explain.



9. (20 pts) For this problem, you should use the three prediates below.

H(x): x is a horse.
A(x): x is an animal.
IsTailOf(x,y): x is a tail of y.

Solve the problems (a), (b), (c) and (d) below:

(a) (3 pts) Translate the following sentence into a predicate logic expresson.

w is a tail of a horse.

(b) (5 pts) Translate the following inference into an inference using predicate logic expressions.

Horses are animals.

Horses' tails are tails of animals.

(c) (2 pts) Is the inference in (b) valid or invalid?

(d) (10 pts) If the inference in (b) is valid, prove the predicate logic inference using Natural Deduction. If the inference in (b) is invalid, explain why it is invalid.