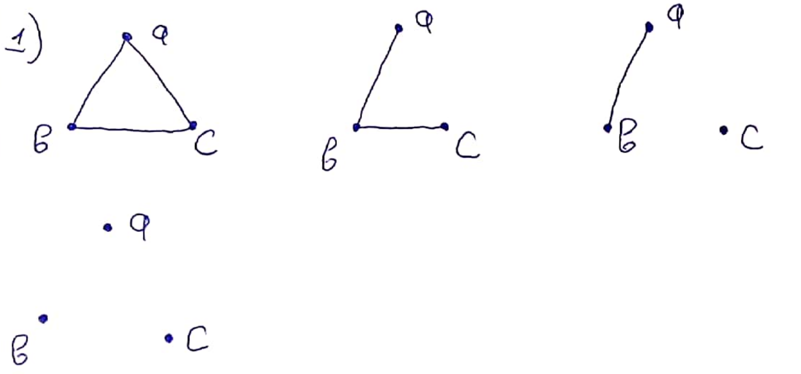
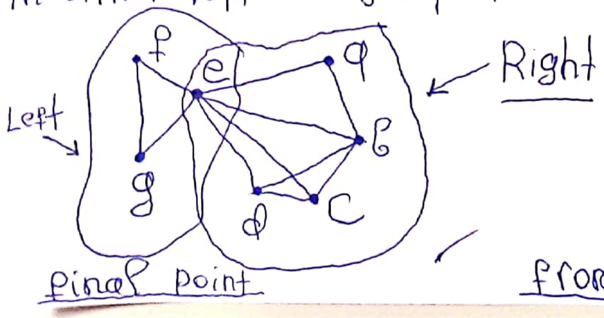
Anar Rzayev   Part V

1)



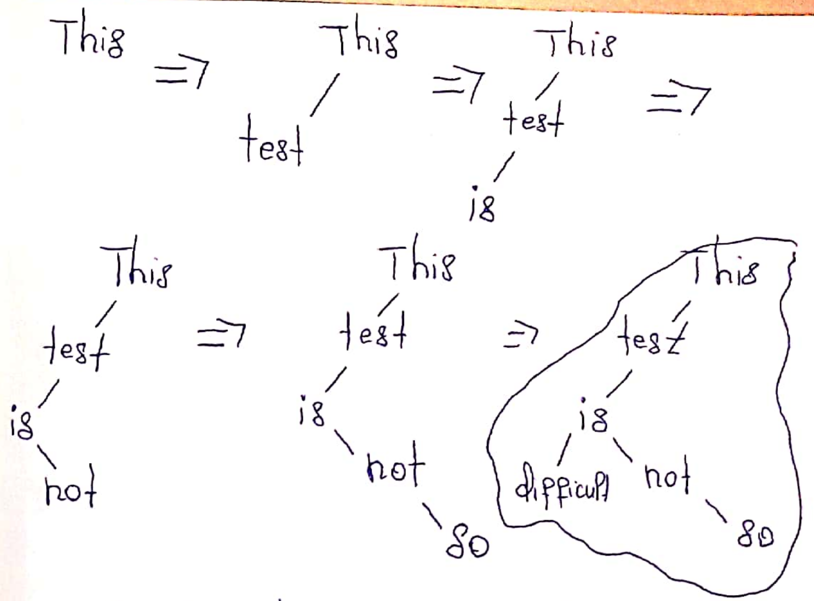
2) a) Yes ⇒ e-g-f-e-a-b-c-d-a-c-e-b-d-e

b) No ⇒ If it doesn't start at vertex e, since it should go to start vertex, so it will pass through e twice. Therefore, e should be starting vertex. But, in this case, it will not be in either left or right part



If starts at e, goes Left (or Right), then for passing Right (or Left), we pass from e again, not as

3)



Because "test" < "This" since "e" < "h"

"is" < "This", with i < T and is < test ⇒ i < t

not < This, not < test, not > is

so < This, so < test, so > is, so > not

difficult < This, difficult < test

difficult < is

Part 4

1) a) If at i^{th} time, objects move up assign 1, and if at i^{th} time it goes 1 unit right, assign 0 since table is 8x24 (8 rows, 24 columns)

For coming to B from A, we only go up and right if we want to do it in 32 moves

It means that all paths can be representable by set Y uniquely, and all strings in Y is one of the unique path in set X

b) $|X| = |Y| = \binom{32}{8} = 10518300$

Because of one-to-one relation, cardinality must be same

2)

	Red	Blue	Total
1^{st}	6	3	9
2^{nd}	5	8	13

$P(1) = P(2) = \frac{1}{2}$

$$P(1|R) = \frac{P(R|1)\,P(1)}{P(R|1)\,P(1) + P(R|2)\,P(2)}$$

$$= \frac{P(R|1)\cdot\frac{1}{2}}{P(R|1)\cdot\frac{1}{2} + P(R|2)\cdot\frac{1}{2}} = \frac{P(R|1)}{P(R|1)+P(R|2)} =$$

$$= \frac{\frac{6}{9}}{\frac{6}{9}+\frac{5}{13}} = \frac{\frac{2}{3}}{\frac{2\cdot 13 + 5\cdot 3}{3\cdot 13}} = \frac{26}{41}$$

since $P(R|1) = \frac{6}{9}$

$P(R|2) = \frac{5}{13}$

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Part 3

1) Let $P(n)$ be statement $P(ns) = nP(s)$

Basis $n=1 \Rightarrow P(ns) = P(s) = 1 \cdot P(s)$, thus $P(1)$ is true

Inductive Let $P(k-1)$ be true

$$P((k-1)s) = (k-1)P(s)$$

We need to prove that $P(k)$ is true

$P(ks) = P((k-1)ss) = P((k-1)s) + P(s) =$

$= (k-1)P(s) + P(s) = kP(s)$, since $P(k-1)$ is true

Thus, $P(k)$ is true

By the principle of math induction $\Rightarrow$ $\boxed{P(n) \text{ is true for all } n \in \mathbb{N}}$

for all positive integers n

2) a) Let S - nonempty strings of decimal digits (set of)

Basis Smallest digit in a string of 1 decimal digit is the digit itself $\Rightarrow m(d) = d$ whenever $d \in \{0, 1, \dots, 9\}$

Recursive Let ds represent the string with digit d added to the front of string s. Smallest digit in the string ds is then minimum of digit d and smallest digit in the string $s \Rightarrow m(ds) = \min(d, m(s))$ whenever $d \in \{0, 1, \ldots, 9\}$ and $s \in S$

b) Basis Let s and t both be a digit

$m(st) = \min(s, m(t)) = \min(m(s), m(t))$, thus property is true for basis step

Recursive step $m(ds) = \min(d, m(s)) = \min(m(d), m(s))$

Conclusion By the principle of structural induction $m(st) = \min(m(s), m(t))$

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Part 2

1) a) True

b) We must show that $(x,x) \in R_1 \circ R_2$ for all $x \in A$. Let $x \in A \Rightarrow$ Since R_1, R_2 - reflexive, $(x,x) \in R_1$ and $(x,x) \in R_2$. Therefore, $(x,x) \in R_1 \circ R_2$ and $R_1 \circ R_2$ is reflexive ✓

c) False

d) If R_1 and R_2 are irreflexive, then $R_1 \circ R_2$ need not be irreflexive

$R_1 = \{(1,2)\}$, $R_2 = \{(2,1)\}$

$\Rightarrow$ since they are not reflexive

R_1, R_2 - irreflexive, but

$R_1 \circ R_2 = \{(1,1)\}$ which is

reflexive ✓

By the def of composition, $\langle a,a \rangle \in R_1 \circ R_2 \Leftrightarrow$ there exist $x \in A$ such that $\langle a,x \rangle \in R_1$ and $\langle x,a \rangle \in R_2$ ✓

2) a) max elements are values in the top row
maximal elements of partial order: 7

b) least element exists when there is only 1 minimal element => But, in this case, all a, b, c, d - are minimal elements => does not exist
(at the very bottom, they are all minimal)

c) Greatest Lower Bound does not exist, because there is no vertex below a, b, c such that, there is a path from that node to all these 3 points

Anar Rzayev

Part 1

1) a) $\forall x\big(C(m,x) \supset C(h,x)\big)$

b) $\exists x\big(C(x,m)\big) \supset C(h,m)\big)$

c) $\forall x\big(C(x,h) \supset C(x,m)\big)$

d) $\forall x \exists y\big(C(x,y) \wedge \neg C(y,m)\big)$

Note that $F \supset G$ is true iff either F is not true or G is true $\Rightarrow$ $\neg F \vee G$

a) $\forall x\big(\neg C(m,x) \vee C(h,x)\big)$

b) $\exists x\big(\neg C(x,m) \vee C(h,m)\big)$

c) $\forall x\big(\neg C(x,h) \vee C(x,m)\big)$

2) a) No

b)