Ch 1. The Foundations: Logic and Proofs

Propositional Logic-3

Tautology, Contradiction, Satisfiability

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Acknowledgement

- [Rosen 19] Kenneth H. Rosen, for Discrete Mathematics & Its Applications (8th Edition), Lecture slides
- [Hunter 11] David J. Hunter, Essentials of Discrete Mathematics, 2nd Edition, Jones & Bartlett Publishers, 2011, Lecture Slides

Propositional Logic

Special Kinds of Propositions

- Tautologies
- Contradictions
- Contingencies

Property of Propositions

- Propositional Satisfiability

Tautologies

Statements that are always true, no matter what the truth values of the component statements take.

Example: $(p \wedge q) \rightarrow p$ is always true:

Proof?

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p	q	$p \wedge q$	$(p \wedge q) \rightarrow p$
T	T		
T	F		
F	T		
F	F		

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T	F	F	
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T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Contradictions

Statements that are never true are called *contradictions*. For example:

p	$\neg p$	$p \wedge \neg p$
T	F	
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T	F	F
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Contingencies

A *contingency* is a proposition which is neither a tautology nor a contradiction.

Example Which of the following are contingencies?

P	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

Propositional Satisfiability (1/2)

A compound proposition is *satisfiable* if there is an assignment of truth values to its variables that make it true. When no such assignments exist, the compound proposition is *unsatisfiable*.

Theorem A compound proposition is *unsatisfiable* if and only if its negation is a tautology.

Propositional Satisfiability (2/2)

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Propositional Satisfiability

Example: Determine the satisfiability of the following compound propositions:

(1) $\left(p \vee \neg q \right) \wedge \left(q \vee \neg r \right) \wedge \left(r \vee \neg p \right)$

Propositional Satisfiability

Example: Determine the satisfiability of the following compound propositions:

(1) $$(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$$

Solution: Satisfiable. Assign **T** to *p*, *q*, and *r*.

(2) $$(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$$

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Example: Determine the satisfiability of the following compound propositions:

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(2) $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$

Solution: Satisfiable. Assign **T** to *p* and ***F*** to *q*.

(3) $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p) \wedge (p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$

Propositional Satisfiability

Example: Determine the satisfiability of the following compound propositions:

(1) $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$

Solution: Satisfiable. Assign **T** to *p*, *q*, and *r*.

(2) $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$

Solution: Satisfiable. Assign **T** to *p* and ***F*** to *q*.

(3) $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p) \wedge (p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$

Solution: Not satisfiable. Check each possible assignment of truth values to the propositional variables and none will make the proposition true.

Quiz 03-2

Answer whether each of the following assertions is true or false.

[1] The negation of a satisfiable proposition is unsatisfiable. [ ]

[2] The negation of a tautology is a contradiction. [ ]

[3] The negation of a contingency is a contingency. [ ]