CS 204 Discrete Mathematics Fall 2020

Homework 9

November 14, 2020

Basic Counting Techniques

1. (5 pts) Let A and B be finite sets, |A| = m and |B| = n. How many binary relations are there from A to B?

2. (5 pts) How many ways are there to rearrange the letters in INANENESS?

3. (5 pts) How many distinct ways are there to encode the decimal digits 0-9 as binary sequences of length 4? Consider only codes which represent different digits by different sequences.

4. (8 pts) A string in {0,1}* has even parity if the symbol 1 occurs in the word an even number of times; otherwise, it has odd parity.
   (a) How many words of length n have even parity?
   (b) How many words of length n have odd parity?

5. (8 pts) Suppose you have eight squares of stained glass, all of different colors, and you would like to make a rectangular stained glass window in the shape of a 2×4 grid.

How many different ways can you do this, taking symmetry into account?
(Note that any pattern may be rotated 180 degrees, flipped vertically, or flipped horizontally. You should count all the possible resulting patterns as the same window.)

6. (8 pts) Explain why any set of three natural numbers must contain a pair of numbers whose sum is even.

Selections and Arrangements

1. (9 pts) If you flip a coin 5 times, how many different ways can you get exactly 1 head? 2 heads? Find a formula for the number of ways of obtaining r heads with n flips of a coin.

2. (10 pts) Prove the following equality for $n \geq 0$.

$$C(n+1, r) = C(n, r-1) + C(n, r)$$

3. (10 pts) Possible grades for a class are A, B, C, D, and F. (No +/-'s.)
   (a) How many ways are there to assign grades to a class of seven students?
   (b) How many ways are there to assign grades to a class of seven students if nobody receives an F and exactly one person receives an A?

4. (14 pts) Show that if n is a positive integer, then
   $C(2n,2) = 2 \times C(n,2) + n^2$
   a) using a combinatorial argument. (A combinatorial argument, or combinatorial proof, is an argument that involves counting.)
   b) by algebraic manipulation.

Counting & Counting with Functions

1. (8 pts) A small college offers 250 different classes. No two classes can meet at the same time in the same room, of course. There are twelve different time slots at which classes can occur. What is the minimum number of classrooms needed to accommodate all the classes?

2. (8 pts) Suppose that 100 lottery tickets are given out in sequence to the first 100 guests to arrive at a party. Of these 100 tickets, only 12 are winning tickets. The generalized pigeonhole principle guarantees that there must be a streak of at least k losing tickets in a row. Find k.

3. (15 pts)

(a) How many total functions are there from a set with three elements to a set with four elements?

(b) How many are one-to-one?

(c) How many are onto?

4. (12 pts) How many solutions are there to the equation $x1 + x2 + x3 = 17$, where x1, x2, and x3 are nonnegative integers with

a) $x1 > 1$, $x2 > 3$, and $x3 > 3$?

b) $x1 < 6$ and $x3 > 5$?

c) $x1 < 4$, $x2 < 3$, and $x3 > 5$

5. (15 pts) How many subsets of a set with ten elements

a) have fewer than five elements?

b) have more than seven elements?

c) have an odd number of elements?

6. (6 pts) Suppose you have four squares of stained glass, all of different colors, and you wish to make a 2 x 2 square stained glass window. How many different windows are possible? (Beware: a square has more symmetries than a rectangle.)