

Homework 10

November 21, 2020

Finite probability space, events; Basic concepts of probability theory

1. (10 pts) Suppose that a hundred people enter a contest and that different winners are selected at random for one first prize, one second prize, and one third prize. What is the probability that a participant of the contest wins one of these prizes?
2. (10 pts) Which is more likely: rolling a total of 9 when two dice are rolled or rolling a total of 9 when three dice are rolled?
3. (12 pts) Answer the following questions:
 - (a) What is the probability of rolling a six when a die is rolled four times.
 - (b) What is the probability that a double six comes up when a pair of dice is rolled 24 times? Is it greater than $1/2$?
 - (c) Is it more likely that a six comes up when a die is rolled four times or that a double six comes up when a pair of dice is rolled 24 times?
4. (10 pts) What is the probability of the following events when we randomly select a permutation of $\{1, 2, 3, 4\}$?
 - (a) 1 precedes 4.
 - (b) 4 precedes 1.
 - (c) 4 precedes 1 and 4 precedes 2.
 - (d) 4 precedes 1, 4 precedes 2, and 4 precedes 3.
 - (e) 4 precedes 3 and 2 precedes 1.

Conditional probability, Bayes' theorem; Independence

1. (10 pts) What is the conditional probability that exactly four heads appear when a fair coin is flipped five times given that the first flip came up heads?
2. (12 pts) What is the probability that a family with five children does not have a boy, if the sexes of children are independent and if
 - (a) a boy and a girl are equally likely.
 - (b) the probability of a boy is 0.51
 - (c) the probability that the i -th child is a boy is $0.51 - (i/100)$
3. (10 pts) A family has two children. What is the conditional probability that both are boys given that at least one of them is a boy? Assume that the sample space S is given by $S = \{(b,b), (b,g), (g,b), (g,g)\}$, and all outcomes are equally likely. ((b,g) means for instance that the older child is a boy and the younger child is a girl.))
4. (10 pts) In answering a question on a multiple choice test, a student either knows the answer or he guesses. Let p be the probability that he knows the answer and $1-p$ be the probability that he guesses. Assume that a student who guesses at the answer will be correct with probability $1/m$, where m is the number of multiple-choice alternatives. What is the conditional probability that a student knew the answer to a question given that he answered it correctly?
5. (10 pts) In a laboratory blood test is 95% effective in detecting a certain disease when it is, in fact, present. However, the test also yields a "false positive" result for 1 % of the healthy persons tested. (That is, if a healthy person is tested, then, with probability 0.01, the test result will imply he has the disease.) If 0.5% of the population actually has the disease, what is the probability a person has the disease given that his test result is positive?