2020 Spring CS300 Homework # 6

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Due: 5/28(Thu), 23:59:00 KST

· Total 100 points
· You **must** explain why your algorithm is correct.
· If you submit the code, please explain briefly about your code.
(Do not just submit the code)

Problem 1. [30 Points]

Given integers n and k, along with $p_1, \ldots, p_n \in [0, 1]$, you want to determine the probability of obtaining exactly k heads when n biased coins are tossed independently at random, where p_i is the probability that the ith coin comes up heads. Give an $O(n^2)$ algorithm for this task. Assume you can multiply and add two numbers in $[0, 1]$ in $O(1)$ time.

Problem 2. [30 Points]

Given an *unlimited* supply of coins of denominations $x_1, x_2, \cdots, x_n$, we wish to make change for a value v; that is, we wish to find a set of coins whose total value is v. This might not be possible: for instance, if the denominations are 5 and 10 then we can make change for 15 but not for 12. Give an $O(nv)$ dynmaic-programming alogrithm for the following problem.

Input: $x_1, \ldots, x_n; v$.
Question: Is it possible to make change for v using coins of denominations $x_1, \ldots, x_n$?

Problem 3. [40 Points]

You are going on a long trip. You start on the road at mile post 0. Along the way there are n hotels, at mile posts $a_1 < a_2 < \cdots < a_n$, where each a_i is measured from the starting point. The only places you are allowed to stop are at these hotels, but you can choose which of the hotels you stop at. You must stop at the final hotel (at distance a_n), which is your destination.

You'd ideally like to travel 200 miles a day, but this may not be possible (depending on the spacing of the hotels). If you travel x miles during a day, the *penalty* for that day is $(200 - x)^2$. You want to plan your trip so as to minimize the total penalty— that is, the sum, over all travel days, of the daily penalties.

Give an efficient alogirthm that determines the optimal sequence of hotels at which to stop.