

Let $f(n) = 8k$ and let $n^2 > k$

$f(n) = O(n^{\log_2^{16} - \epsilon})$ is true for $\epsilon = 2 > 0$

choosing $n^2 > k \Rightarrow n^{4-\epsilon} = n^2$ and $O(n^2) = f(n) = 8k$

since $n^2 > k \Rightarrow 8k \leq c \cdot k < c n^2 = O(n^2)$ where
we choose sufficiently large c ($c > 8$)

$T(n) = 16T\left(\frac{n}{2}\right) + 8k = 16T\left(\frac{n}{2}\right) + f(n)$ for $n^2 > k$

and $f(n) = O(n^{\log_2^{16} - \epsilon})$ with $\epsilon = 2 > 0$

$$f(n) = 8k \leq c \cdot k < c \cdot n^2 = O(n^2) = O(n^{\log_2^{16} - \epsilon}) = O(n^4)$$

so, by master's theorem for case 1,

$$T(n) = \Theta(n^{\log_2^{16}}) = \Theta(n^4)$$

$$T(n) = \Theta(n^4) \quad \checkmark$$



$T(n) = k$ for $n^2 \leq k$ is just
smaller cases, we should not bother
about it at all