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- No late submission will be accepted. Please submit before the submission deadline.
- Consider lg as logarithm with base 2.

1 Problem 1 (20 points)

Decide whether these statements are TRUE or FALSE. Consider $f(\cdot)$ and $g(\cdot)$ as positive functions.

(a) (5 points) For the functions n^k and c^n, when $k \geq 1$ and $c > 1$, n^k is $\Omega(c^n)$.

(b) (5 points) If $f(n) = O(g(n))$ and $g(n) = O(f(n))$, then $f(n) = g(n)$.

(c) (5 points) $f(n) + g(n) = \theta(max\{f(n), g(n)\})$.

(d) (5 points) $f(n) = \Theta(g(n))$ if and only if $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$

2 Problem 2 (60 points)

Solve the following recurrence relations to determine a good asymptotic tight bound for each of them. Include steps to your answer. (Specify case when applying Master Theorem.)

(a) (5 points) $T(n) = 3T(n/3) + n/2$

(b) (5 points) $T(n) = 4T(n/2) + n/lgn$

(c) (10 points) $T(n) = 2T(\sqrt{n}) + lg(n)$

(d) (10 points) $T(n) = 2T(n-1) + 1$

(e) (10 points) $T(n) = T(\sqrt{n}) + 1$

(f) (10 points) $T(n) = T(n/3) + T(2n/3) + n$

(g) (10 points) $T(n) = \sqrt{n}T(\sqrt{n}) + n$

3 Problem 3 (20 points)

Indicate, for each pair of expressions (A, B) in the table below, whether A is O, o, Ω, ω or Θ of B. Assume that $k \geq 1$ and $\epsilon > 0$ are constants. Your answer should be in the form of the table with "yes" or "no" written in each box. You don't need to prove each of them. Note that the statement A is O of B means $A = O(B)$.

(A,B)	O	o	Ω	ω	Θ
$(lg^k n, n^\epsilon)$					
$(n, lg^{lgn} n)$					
$(n!, n^{n/2})$					
$(\sqrt{n}, n^{cosn}$					