2020 Fall CS300 Homework # 4

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Due: 11/11(Wed), 18:00:00 KST

· Total 100 points

Problem 1. [3 × 10 = 30 Points]

We use Huffman's algorithm to obtain an encoding of alphabet $\{a, b, c, d\}$ with frequencies f_a, f_b, f_c, f_d. In each of the following cases, either give an example of frequencies (f_a, f_b, f_c, f_d) that would yield the specified code, or explain why the code cannot possibly be obtained (no matter what the frequencies are).

(a) Code: $\{0, 10, 110, 111\}$

(b) Code: $\{1, 00, 01, 110\}$

(c) Code: $\{00, 01, 10, 11\}$

Problem 2. [40 Points]

Let G be a graph with n vertices. Suppose there are two vertices $x, y \in V(G)$ such that the every simple path between x and y has length strictly greater than $n/2$ (at least one path exists between x and y). Prove that there exists some vertex $z(\neq x, y)$ such that deleting z from G destroys all x-y paths (in other words, no path from x to y).

Problem 3. [2 × 15 = 30 Points]

For each of the following statements, decide whether it is true or false. If it is true, give a short explanation. If it is false, give a counterexample.

(a) Suppose we are given an instance of the Minimum Spanning Tree Problem on a graph G, with edge costs that are all positive. Let T be a minimum spanning tree for this instance. Suppose we replace each edge cost c_e by adding 1, $c_e + 1$, thereby creating a new instance of the problem with the same graph but different costs.

True or false? "T must still be a minimum spanning tree for this new instance."

(b) Suppose we are given an instance of the Shortest s-t Path Problem on a directed graph G. We assume that all edge costs are positive. Let P be a minimum-cost s-t path for this instance. Suppose we replace each edge cost c_e by adding 1, $c_e + 1$, thereby creating a new instance of the problem with the same graph but different costs.

True or false? "P must still be a minimum-cost s-t path for this new instance.