CS300 Homework #3

TA: Seungchan Jeong

winner_sc@kaist.ac.kr

Total 100 points Due : 2020-10-16 18:00:00 KST

- Write in English or Korean
- Make sure your writing is readable

1. Black-box (25 pts)

Suppose we have mysterious machine which return median m of given set S and $S\backslash\{m\}$ in 0 second. Prove that we can sort any list of n elements in linear time using such machine.

Caution

1. The machine can return 'm' and simultaneously delete 'm' from the set in 0 second.

2. If there are an even number of elements in a set, the median is a smaller middle number. (not an average of two middle numbers)

2. Graph Coloring (25 pts)

The graph-coloring problem is usually stated as the vertex-coloring problem: assign the smallest number of colors to vertices of a given graph so that no two adjacent vertices are the same color. Consider the edge-coloring problem: assign the smallest number of colors possible to edges of a given graph so that no two edges with the same endpoint are the same color. Explain how the edge-coloring problem can be reduced to a vertex-coloring problem. *(Hint: create a new graph)*

3. Bridge and biconnected components (25 pts)

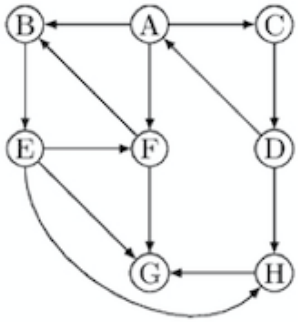
Let $G = (V, E)$ be connected undirected graph. A bridge of G is $b \in E$ s.t. removal of b disconnects G. Biconnected components of G are maximal sets of edges s.t. any two edges in the set lie on common simple cycle.

a. Prove that an edge of G is a bridge if and only if it does not lie on any simple cycle of G.

b. Prove that every edge which is not a bridge is in exactly one of the biconnected components of G.

4. DFS (25 pts)

Start the traversal at vertex A and resolve ties by the vertex alphabetical order.



Traverse the following graph by depth-first search and construct the corresponding depth-first search tree. Give the order in which the vertices were reached for the first time and the order in which the vertices became dead-ends.