8) <u>Proof by contradiction</u> Suppose graph G exists with at least 2 components and minimum degree $\delta(G) \geq \frac{n}{2}$ $\Rightarrow$ Choose any vertex u from one component and any vertex v from different component. u and v should have no common neighbor $\Rightarrow$ $N(u) \cap N(v) = \{\emptyset\}$ and observe that the size of the union of their neighbors

$|N(u) \cup N(v)| \leq n-2$ (since u, v - excluded)

By Inclusion/Exclusion $\Rightarrow$ $|N(u) \cup N(v)| =$

$= |N(u)| + |N(v)| - |N(u) \cap N(v)| \geq \frac{n}{2} + \frac{n}{2} = n$

whereas $|N(u) \cup N(v)| \leq n-2$ (X)

Hence, u, v - should have at least 2 common neighbors $\Rightarrow$ but this yields a contradiction and it means <u>G-connected</u> ✓

a) If it had a cycle $S_1, S_2, \ldots, S_k$, then $S_1 \cup S_2 \cup \ldots \cup S_k$ should be in the same strong connected component in G.