CS300 Homework #6

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Total 100 points Due: June 4, 2019, 10 AM

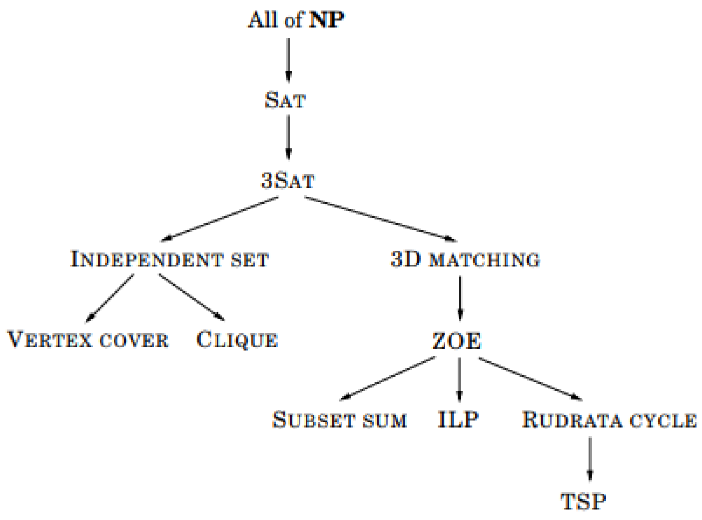
The submission box is in front of the elevator at 3rd floor, E3-1.

1. (60 pts) Below are the descriptions of **3-COLOR** problem and **EXACT-COVER** problem. Show that **3-COLOR** problem and **EXACT-COVER** problem are NP-complete by reduction. You can use known NP-complete problems shown in the figure below.

3-COLOR Given a graph, is there a way to color the vertices with 3 colors so that no adjacent vertices have the same color?

EXACT-COVER Given a finite set X and a family of subsets of X, S ($2^X \supseteq S$), is there a subset $S' \subseteq S$ such that every element of X lies in exactly one element of S'?

Figure 8.7 Reductions between search problems.



2. (40 pts) A boolean formula is in *disjunctive normal form* (or *DNF*) if it consists of a *disjunction* (OR) or several *terms*, each of which is the conjunction (AND) of one or more literals. For example, the formula

$$(\overline{x} \wedge y \wedge \overline{z}) \vee (y \wedge z) \vee (x \wedge \overline{y} \wedge \overline{z})$$

is in disjunctive normal form. DNF-SAT asks, given a boolean formula in disjunctive normal form, whether that formula is satisfiable.

   (a) Describe a polynomial-time algorithm to solve DNF-SAT.

   (b) What is the error in the following argument that P=NP?

Suppose we are given a boolean formula in conjunctive normal form with at most three literals per clause, and we want to know if it is satisfiable. We can use the distributive law to construct an equivalent formula in disjunctive normal form. For example,

$$(x \vee y \vee \overline{z}) \wedge (\overline{x} \vee \overline{y}) \iff (x \wedge \overline{y}) \vee (y \wedge \overline{x}) \vee (\overline{z} \wedge \overline{x}) \vee (\overline{z} \wedge \overline{y})$$

Now we can use the algorithm from part (a) to determine, in polynomial time, whether the resulting DNF formula is satisfiable. We have just solved 3SAT in polynomial time. Since 3SAT is NP-hard, we must conclude that P=NP!