

CS300 Homework #5

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Total 100 points

Due: 2020-05-22 23:59:00 KST

- Write in English or Korean.

1. (30 points)

Let $G = (V, E)$ be an undirected graph. Prove that if all its edge weights are distinct, then it has a unique minimum spanning tree.

2. (20 points)

Consider the assertion that an edge-weighted graph has a unique MST *only* if its edge weights are distinct. Give a proof or a counterexample.

3. (20 points)

Give a counterexample that shows why the following strategy does not necessarily find the MST: ‘Start with any vertex as a single-vertex MST, then add $V - 1$ edges to it, always taking next a min-weight edge incident to the vertex most recently added to the MST.’

4. (3×10 points = 30 points)

We use Huffman’s algorithm to obtain an encoding of alphabet $\{a, b, c\}$ with frequencies f_a, f_b, f_c . In each of the following cases, either give an example of frequencies (f_a, f_b, f_c) that would yield the specified code, or explain why the code cannot possibly be obtained (no matter what the frequencies are).

(a) Code: $\{0, 10, 11\}$

(b) Code: $\{0, 1, 00\}$

(c) Code: $\{10, 01, 00\}$