CS300 Homework 1

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Due : 4/7(Tue) 23:59:00 / **Total 100 points**

No late submission will be accepted

※ lg means logarithm with base 2

1. Consider the recurrence $\mathrm{T(n)} = 4T(n/2) + n^2 lgn$, $\mathrm{T}(1) = \Theta(1)$ and answer the following questions. Give an asymptotic tight bound for this recurrence.

(10pt, partial credit)

2. Solve the following recurrence relations to determine a good Θ bound for each of them. (assume $\mathrm{T}(1) = \Theta(1)$)

(Total 50pt, partial credit)

(a) $\mathrm{T}(n) = 3\mathrm{T}(n/2) + 1$ **(5pt)**

(b) $\mathrm{T}(n) = 4\mathrm{T}(n/4) + n$ **(5pt)**

(c) $\mathrm{T}(n) = 8\mathrm{T}(n/5) + n^2$ **(5pt)**

(d) $\mathrm{T}(n) = 2\mathrm{T}(n-1) + 1$ **(5pt)**

(e) $\mathrm{T}(n) = \mathrm{T}(\sqrt{n}) + 1$ **(10pt)**

(f) $\mathrm{T}(n) = 2\mathrm{T}(\sqrt{n}) + lg(n)$ **(10pt)**

(g) $\mathrm{T}(n) = \mathrm{T}(n/3) + \mathrm{T}(2n/3) + n$ **(10pt)**

3. Indicate, for each pair of expressions (A, B) in the table below, whether A is O, o, Ω, ω or Θ of B. Assume that $k \geq 1$ and $\varepsilon > 0$ are constants. Your answer should be in the form of the table with "yes" or "no" written in each box. You don't need to prove each of them.

(Total 25pt, each box gives 1pt)

(A, B)	O	o	Ω	ω	Θ
$(n^k, 2^n)$					
$(lg^k n,\ n^\varepsilon)$					
$(n,\ lg^{lgn} n)$					
$(n!,\ n^{n/2})$					
$(\sqrt{n},\ n^{cosn})$					

4. Prove the following theorem.

$f(n) = \Theta(g(n))$ if and only if $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$

(Total 15pt, partial credit)