CS300 Homework #3

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due : April 9th 10AM

1. Black-box (20 pts)

Suppose we have mysterious machine which return median m of given set S and $S \setminus \{m\}$ in 0 second. Prove that we can sort any list of n elements in linear time using such machine.

2. Kahn's algorithm (30 pts)

a) Prove that directed acyclic graph G has at least one vertex with in-degree 0 and one vertex with out-degree 0.

b) Prove that following algorithm can find the topological sorting of given directed acyclic graph G.

Algorithm 1 Kahn's algorithm

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L ← empty List
S ← Queue of all vertex with in-degree 0
while S is not empty do
   n = S.dequeue()
   L.add(n)
   for every vertex m with e = (n, m) ∈ E do
      delete(e)
      if in-degree of m = 0 then
         S.enqueue(m)
      end if
   end for
end while
return L
```

3. Bridge and biconnected components (20 pts)

Let $G = (V, E)$ be connected undirected graph. A bridge of G is $b \in E$ s.t. removal of b disconnects G. Biconnected components of G are maximal sets of edges s.t. any two edges in the set lie on common simple cycle.

a) Prove that an edge of G is a bridge if and only if it does not lie on any simple cycle of G.

b) Prove that every edge which is not a bridge is in exactly one of the biconnected components of G.

4. One-line expression (30 pts)

Suppose you have n distinct values $x_1, \ldots, x_n$. You can express the fact that they are all different by using the symbol '$\neq$'. 'One-line expression' of n values is the shortest equation which can express $x_1, \ldots, x_n$ are all different using '$\neq$' and denoted by by $OL(n)$. (Note that 'One-line expression' is not unique.)

For example, the equation $< x_1 \neq x_2 \neq x_3 \neq x_1 >$ means that x_1, x_2, x_3 are all different. So such equation is $OL(3)$. However, $< x_1 \neq x_2 \neq x_3 >$ is not $OL(3)$ because x_1 can be equal to x_3. Also, $< x_1 \neq x_2 \neq x_3 \neq x_1 \neq x_2 \neq x_3 >$ is not $OL(3)$ because there exists shorter expression to express 3 distinct values.

a) Let $n \geq 3$ be odd integer. How many symbol $\neq$ are in $OL(n)$?

b) Derive an algorithm which prints $OL(n)$ with odd integer n as input. (Hint : Euler tour)