1 3+3+4 points

Indicate whether the following statements are true(**T**) or false(**F**). You do **not** need to justify your answer.

(a) If E is an elementary matrix, then $\det(E)$ is the product of the entries on the main diagonal.

(b) Every square matrix has a LU-decomposition.

(c) For the following two matrices A and B, $\det(A) \cdot \det(B)$ is positive.

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 4 & 5 & 6 \\ 7 & 0 & 9 \end{pmatrix}, \ B = \begin{pmatrix} 1 & 2 & -1 \\ 0 & -4 & 5 \\ 0 & 0 & 6 \end{pmatrix}$$

Solution.

(a) False. A matrix $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ is an elementary matrix, but has the determinant as -1.

(b) False. $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ has no LU-decomposition.

(c) False. Since $\det(A) = 57$ and $\det(B) = -24$, $\det(A) \cdot \det(B)$ is negative.

2 10 points Find a LU-decomposition of the following matrix A in which every diagonal entry of the upper triangular matrix is 1.

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 5 & 1 & 1 \\ 3 & 4 & 1 \end{pmatrix}$$

Solution.

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 5 & 1 & 1 \\ 3 & 4 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \\ 5 & -\frac{3}{2} & 0 \\ 3 & \frac{5}{2} & -\frac{43}{3} \end{pmatrix} \cdot \begin{pmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ 0 & 1 & \frac{13}{3} \\ 0 & 0 & 1 \end{pmatrix}$$