Remark on elementary row operations.

For $i > j$, if we add a multiple of the ith row to the jth row of I_n, then the result is

$$\begin{array}{cc} & \begin{matrix} & j & & i & \end{matrix} \\ \begin{matrix} \\ j \\ \\ i \\ \\ \end{matrix} & \begin{bmatrix} 1 & & & & \\ & 1 & \cdots & c & \\ & & \ddots & & \\ & & & 1 & \\ & & & & \ddots \; 1 \end{bmatrix} \end{array},$$

and it equals the product

$$\begin{array}{cc} & \begin{matrix} & j & & i & \end{matrix} \\ \begin{matrix} \\ i \\ \\ j \\ \\ \end{matrix} & \begin{bmatrix} 1 & & & & \\ & 0 & & 1 & \\ & & \ddots & & \\ & 1 & & 0 & \\ & & & & \ddots \; 1 \end{bmatrix} \end{array} \begin{array}{cc} & \begin{matrix} & j & & i & \end{matrix} \\ \begin{matrix} \\ j \\ \\ i \\ \\ \end{matrix} & \begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & c & & 1 & \\ & & & & \ddots \; 1 \end{bmatrix} \end{array} \begin{array}{cc} & \begin{matrix} & j & & i & \end{matrix} \\ \begin{matrix} \\ i \\ \\ j \\ \\ \end{matrix} & \begin{bmatrix} 1 & & & & \\ & 0 & & 1 & \\ & & \ddots & & \\ & 1 & & 0 & \\ & & & & \ddots \; 1 \end{bmatrix} \end{array}.$$

Hence, when we define the second type of elementary row operation, we assume that $i < j$, as follows.

① Interchange the ith row and the jth row. $(1 \le i, j \le n)$

② Add a multiple of the ith row to the jth row. $(1 \le i < j \le n)$

③ Multiply a constant to the ith row. $(1 \le i \le n)$

THEREFORE, every elementary matrix corresponding to the second or the third kind of elementary row operation is lower triangular.