3.7.13 Find an LDU-decomposition of A.

$$A = \begin{bmatrix} 2 & 1 & -1 \\ -2 & 0 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

Solution.

$$A = \begin{bmatrix} 2 & 1 & -1 \\ -2 & 0 & 2 \\ 2 & 2 & 1 \end{bmatrix} \qquad \begin{bmatrix} \bullet & 0 & 0 \\ \bullet & \bullet & 0 \\ \bullet & \bullet & \bullet \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ -2 & 0 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{matrix} \longleftarrow \text{ multiplier } = \frac{1}{2} \\ \\ \\ \end{matrix} \qquad \begin{bmatrix} 2 & 0 & 0 \\ \bullet & \bullet & 0 \\ \bullet & \bullet & \bullet \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{matrix} \\ \longleftarrow \text{ multiplier } = 2 \\ \longleftarrow \text{ multiplier } = -2 \end{matrix} \qquad \begin{bmatrix} 2 & 0 & 0 \\ -2 & \bullet & 0 \\ 2 & \bullet & \bullet \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{matrix} \\ \longleftarrow \text{ multiplier } = 1 \\ \\ \end{matrix} \qquad \begin{bmatrix} 2 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & \bullet & \bullet \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{matrix} \\ \\ \longleftarrow \text{ multiplier } = -1 \end{matrix} \qquad \begin{bmatrix} 2 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & 1 & \bullet \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{matrix} \\ \\ \longleftarrow \text{ multiplier } = 1 \end{matrix} \qquad L = \begin{bmatrix} 2 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix}$$

$$\begin{aligned} \therefore \quad A &= \begin{bmatrix} 2 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

$$\left(\text{Also, } A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \right)$$

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