Fall, 2019

1. A common problem in experimental work is to find a curve y = f(x) of a specified form corresponding to experimentally determined values of x and y, say

$$(x_1, y_1), (x_2, y_2), \cdots, (x_n, y_n).$$

The followings are the three important models in applications.

- Linear line model (y = ax + b)
- Exponential model $(y = ae^{bx})$
- Logarithmic model $(y = a + b \ln x)$

A function file LS_solver.m fits given experimental data to the proper mathematical model using least squares method.

Problem.

(a) Download the function file LS_solver_guide.m on KLMS and complete the missing parts. Save completed file as LS_solver.m. (10 points) solution.

```
1 function [a, b]=LS_solver(x, y, opt)
2 % ----- function file LS_Solver ----- %
3 % input data: x, y and opt
       if opt=1, linear model (y=a*x+b)
       if opt=2, exponential model (y=a*exp(b*x))
       if opt=3, logarithmic model (y=a+b*ln(x))
  [m1, n1]=size(x); [m2, n2]=size(y);
                                             % Size of the input data.
   xx=linspace(min(x), max(x), 100);
                                             % xx will be used to plot
      the fitting curve.
10
   if (m1~=1) | | (m2~=1) | | (n1~=n2)
                                             % If the input data size is
11
       not proper,
       fprintf('Error: Improper input data.\n');
12
13
       if (opt==1) | |(opt==2) | |(opt==3)
                                             % option = 1, 2, 3.
14
           figure; plot(x, y, 'o');
                                                 % Plot the given data
15
               points.
           hold on;
                                                 % Ready to draw the
16
               next graph.
           switch opt
17
               case 1
                                                 % Linear model
18
                    fprintf('Linear model\n'); % Print out the 'Linear
19
                        model'.
                    A=[x' ones(n1,1)]; Y=y';
                                                 % Set the least squares
20
                        problem.
                                                 % Find the least
                    sol=A\setminus Y;
21
                       squares solution.
                    a=sol(1); b=sol(2);
                                                 % Fitting constants a
22
                       and b.
```

```
23
                    plot(xx, a*xx+b);
                                                  % Plot the fitting
                        curve with a and b.
                    title('Linear model (y=a*x+b)');
                case 2
                                                  % Exponential model
25
                    fprintf('Exponential model\n'); % Print out the '
26
                        Exponential model'.
                    A=[ones(n1,1) x']; Y=log(y'); % Set the least
27
                        squares problem.
                    sol=A \setminus Y;
                                                  % Find the least
28
                        squares solution.
                    a=exp(sol(1)); b=sol(2);
                                                  % Fitting constants a
29
                        and b.
30
                    plot(xx, a*exp(b*xx));
                                                  % Plot the fitting
                        curve with a and b.
31
                    title('Exponential model (y=a*exp(b*x))')
                                                  % Logarithmic model
32
                case 3
                    fprintf('Logarithmic model\n'); % Print out the '
33
                        Logarithmic model'.
                    A=[ones(n1,1) log(x)']; Y=y'; % Set the least
34
                       squares problem.
                    sol=A \setminus Y;
                                                  % Find the least
35
                        squares solution.
                    a=sol(1); b=sol(2);
                                                  % Fitting constants a
36
                        and b.
                    plot(xx, a+b*log(xx));
                                                  % Plot the fitting
37
                        curve with a and b.
38
                    title('Logarithmic model (y=a+b*ln(x))');
39
            end
           hold off;
                                                  % No more graph.
40
41
                                                  % for invalid [opt]
       else
42
            fprintf('Error: Improper option value.\n'); % error message
43
           return;
                                                  % Return the process.
44
       end
45
46
   end
```

(b) Replace the switch-case command of the LS_solver.m with the if-else if-else command. (5 points) solution.

```
function [a, b]=LS_solver_sol(x, y, opt)
2 % ----- function file LS_Solver ----- %
3 % input data: x, y and opt
    if opt=1, linear model (y=a*x+b)
4 %
      if opt=2, exponential model (y=a*exp(b*x))
5 %
      if opt=3, logarithmic model (y=a+b*ln(x))
  [m1, n1] = size(x); [m2, n2] = size(y);
                                      % Sizes of the input datas.
  xx=linspace(min(x), max(x), 100);
                                      % xx will be used to plot
     the fitting curve.
10
11 if (m1~=1) | | (m2~=1) | | (n1~=n2)
                                      % If the input data size is
      not proper,
```

```
message.
13 else
       figure; plot(x, y, 'o');
                                              % Plot the given data
14
           points.
                                              % Ready to draw the next
       hold on;
15
           graph.
       if opt == 1
                                              % Linear model
16
           fprintf('Linear model\n');
                                              % Print out the 'Linear
17
               model'.
            A=[x' ones(n1,1)]; Y=y';
                                              % Set the least squares
18
               problem.
            sol=A \setminus Y;
                                              % Find the least squares
19
               solution.
20
            a=sol(1); b=sol(2);
                                              % Fitting constants a and b
            plot(xx, a*xx+b);
                                              % Plot the fitting curve
21
               with a and b.
            title('Linear model (y=a*x+b)');
22
       else
23
            if opt == 2
                                              % Exponential model
24
                fprintf('Exponential model\n'); % Print out the '
25
                    Exponential model'.
                A=[ones(n1,1) x']; Y=log(y'); % Set the least squares
26
                   problem.
                sol=A \setminus Y;
                                              \% Find the least squares
27
                    solution.
28
                a=exp(sol(1)); b=sol(2);
                                              % Fitting constants a and b
                plot(xx, a*exp(b*xx));
                                              % Plot the fitting curve
29
                    with a and b.
                title('Exponential model (y=a*exp(b*x))')
30
            else
31
                if opt == 3
                                              % Logarithmic model
32
                    fprintf('Logarithmic model\n'); % Print out the '
33
                        Logarithmic model'.
                    A=[ones(n1,1) log(x)']; Y=y'; % Set the least
34
                        squares problem.
                                                   % Find the least
                    sol=A \setminus Y;
35
                        squares solution.
                    a=sol(1); b=sol(2);
                                                   % Fitting constants a
36
                        and b.
                    plot(xx, a+b*log(xx));
                                                   % Plot the fitting
37
                        curve with a and b.
                    title('Logarithmic model (y=a+b*ln(x))');
38
                else
                                                   % for invalid [opt]
39
                    fprintf('Error: Improper option value.\n'); % error
40
                         message.
                    return
                                                   % Return the process.
41
                end
42
            end
43
       end
44
       hold off;
                                                   % No more graph.
45
```

46 end