MATLAB assignment 9

Introduction to Linear Algebra (Week 9) **Fall, 2019**

1. Let $\mathcal{B}_1$ and $\mathcal{B}_2$ be bases for $\mathbb{R}^n$. Write a function code `TransMatrix` which produces two transition matrices $P_{\mathcal{B}_1 \to \mathcal{B}_2}$ and $P_{\mathcal{B}_2 \to \mathcal{B}_1}$ as outputs when it takes two matrices U and V whose column vectors are members of $\mathcal{B}_1$ and $\mathcal{B}_2$, respectively. In other words, if

$$\mathcal{B}_1 := \{\mathbf{u}_1, \cdots, \mathbf{u}_n\} \quad \text{and} \quad \mathcal{B}_2 := \{\mathbf{v}_1, \cdots \mathbf{v}_n\}$$

then

$$U = [\mathbf{u}_1 \,|\, \cdots \,|\, \mathbf{u}_n] \quad \text{and} \quad V = [\mathbf{v}_1 \,|\, \cdots \,|\, \mathbf{v}_n]$$

and the line defining the function `TransMatrix` is :

```
function [P_B12, P_B21] = TransMatrix(U, V)
```

where the `P_B12` and `P_B21` are variables for transition matrices $P_{\mathcal{B}_1 \to \mathcal{B}_2}$ and $P_{\mathcal{B}_2 \to \mathcal{B}_1}$, respectively.

Problem.

(a) Write script file which finds the transition matrices $P_{\mathcal{B}_1 \to \mathcal{B}_2}$ and $P_{\mathcal{B}_2 \to \mathcal{B}_1}$, where the $\mathcal{B}_1$ and $\mathcal{B}_2$ are bases for $\mathbb{R}^5$ and

$$\begin{aligned}
\mathbf{u}_1 &= (3,\ 1,\ 3,\ 2,\ 6) & \mathbf{v}_1 &= (2,\ 6,\ 3,\ 4,\ 2) \\
\mathbf{u}_2 &= (4,\ 5,\ 7,\ 2,\ 4) & \mathbf{v}_2 &= (3,\ 1,\ 5,\ 8,\ 3) \\
\mathbf{u}_3 &= (3,\ 2,\ 1,\ 5,\ 4) & \mathbf{v}_3 &= (5,\ 1,\ 2,\ 6,\ 7) \\
\mathbf{u}_4 &= (2,\ 9,\ 1,\ 4,\ 4) & \mathbf{v}_4 &= (8,\ 4,\ 3,\ 2,\ 6) \\
\mathbf{u}_5 &= (3,\ 3,\ 6,\ 6,\ 7) & \mathbf{v}_5 &= (5,\ 5,\ 6,\ 3,\ 4)\,.
\end{aligned}$$

Find the coordinate of $\mathbf{w} = (1,\ 1,\ 1,\ 1,\ 1)$ with respect to $\mathcal{B}_1$ and $\mathcal{B}_2$ and check your answers using $P_{\mathcal{B}_1 \to \mathcal{B}_2}$ and $P_{\mathcal{B}_2 \to \mathcal{B}_1}$.

solution.

```matlab
%% Looking for and validate the transition matrices.
format short;
%- Find the transition matrices
% construct the matrices for bases B1 and B2
u1 = [3 1 3 2 6]'; v1 = [2 6 3 4 2]';
u2 = [4 5 7 2 4]'; v2 = [3 1 5 8 3]';
u3 = [3 2 1 5 4]'; v3 = [5 1 2 6 7]';
u4 = [2 9 1 4 4]'; v4 = [8 4 3 2 6]';
u5 = [3 3 6 6 7]'; v5 = [5 5 6 3 4]';

U = [u1,u2,u3,u4,u5];V = [v1,v2,v3,v4,v5];
% P_B12 means the transition matrix from B1 to B2.
% P_B21 means the transition matrix from B2 to B1.
[P_B12, P_B21] = TransMatrix(U,V);
disp('The transition matrix from B1 to B2 is'); disp(P_B12);
disp('The transition matrix from B2 to B1 is'); disp(P_B21);

%- Validation
w = [1 1 1 1 1]';
w_B1 = U\w;    % Find the coordinate matrix of w with respect to B1
w_B2 = V\w;    % Find the coordinate matrix of w with respect to B2

% calculate P_B12 * [w]_B1. We want to be v = [w]_B2.
v = P_B12 * w_B1;

% TOL for validation
tol = 10^-6;
if abs(v-w_B2) < tol
    disp('[w]_B2 = '); disp(w_B2');
    disp('P_(B1->B2) * [w]_B1 = '); disp(v');
    disp('They are same thus the equation ''P_B12 * [w]_B1 = [w]_B2'' holds');
else
    disp('Wrong transition matrix!!!!!!!!!');
end
%% Make a function 'TransMatrix'
function [P_B12, P_B21] = TransMatrix(U, V)
    [m,n]=size(U);      [k,l]=size(V);
    if (n~=l)||(m~=n)||(m~=k)
        fprintf('Mismatch of the dimension\n');
        return
    else
    P_B12 = V\U;
    P_B21 = U\V;
    end
end
```