# MATLAB assignment 5 solution

**Introduction to Linear Algebra (Week 5)** **Fall, 2019**

1. *Symbolic Computation in* MATLAB (continued)

   (a) In this problem, we are doing several calculus problems by symbolic computation.

      i. Using the MATLAB commands `diff`, find $f_{xx}$ which is the second derivative of the function $f$ where

      $$f(x, y) = \sin(x^2 y) + \cos(xy^2).$$

      ii. From the result of (i), using the MATLAB command `subs`, find $f_{xx}(1, 2)$. In order to change a symbolic answer to a numeric one, use the MATLAB command `double` which stands for double precision.

      iii. Using the MATLAB command `limit`, find the following limit

      $$\lim_{h \to 0} \frac{\cos(x + h) - \cos(x)}{h}.$$

   *Solution.*

```
% Assignment 5_1 (a)-i
syms x y;                % Set x, y to be symbols.
f = sin(x^2 * y);
fxx = diff(f, x, 2);     % Differentiate f with respect to x twice.
disp('The 2nd derivative of f with respect to x is'); disp(fxx);
```

```
% Assignment 5_1 (a)-ii
syms x y;                % Set x, y to be symbols.
f = sin(x^2 * y);
fxx = diff(f,x,2);       % Differentiate f with respect to x twice.
sfxx = simplify(fxx);    % Simplify the expression fxx.
% Substitute x=1, y=2 into sfxx.
value = subs(sfxx, [x,y], [1,2]); format rat;
res = double(value);     % Get a number from a symbol.
disp('fxx(1,2)='); disp(res);
```

```
% Assignment 5_1 (a)-iii
syms x h;            % Set x,h as symbols.
% Find the limit of the function as h->0.
res = limit((cos(x+h)-cos(x))/h, h, 0);
disp('The result of the limit is');
pretty(res);         % Produce a typeset type display.
```

(b) In this problem, we plot the Taylor series expansion of the following function:

$$f(x) = \frac{1}{5 + \cos x}.$$

i. Use the MATLAB command `taylor` to find the Taylor series expansion for $f(x)$ centered at $x = 2$ up to the 10th order terms.
[Remember that a constant term is 0th order term.]

ii. Use the MATLAB commands `fplot` and `plot`(and also `hold on`) to plot the resulting polynomial of (i) for $1 \leq x \leq 3$ together with the given function $f(x)$ in the same figure.

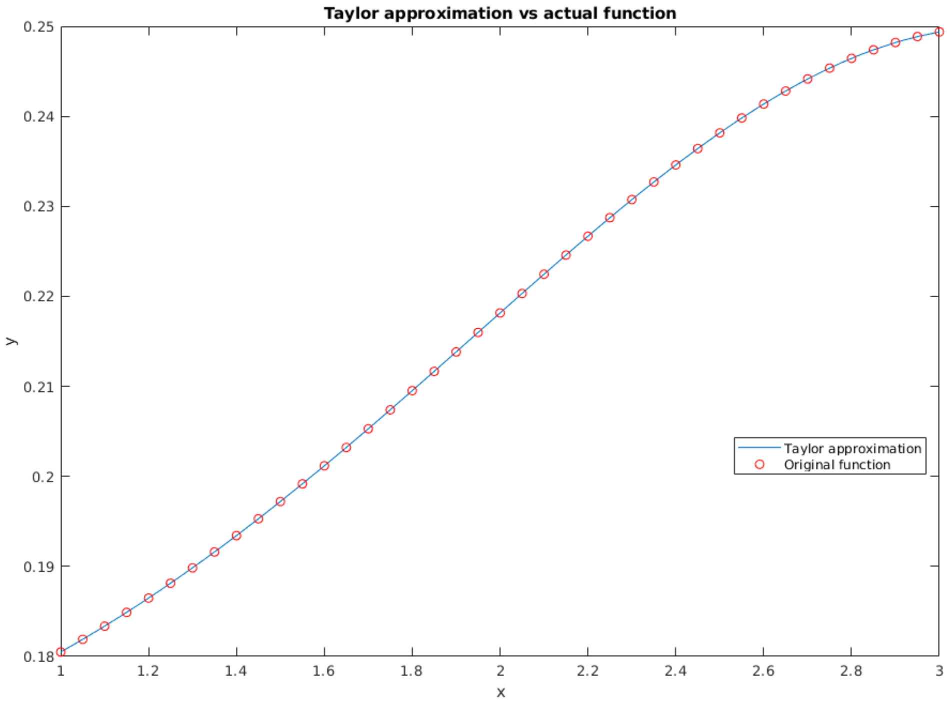
*Solution.*

```
% Assignment 5_1 (b)

syms x;      % Regard x as a symbol.
f = 1/(5 + (cos(x)));

% (i). Find the Taylor series expansion of f at x=2 up to the 10th order terms.
T = taylor(f, x, 2, 'order', 11);

% (ii). Draw the resulting polynomial (i) and the original function f(x).
fplot(T, [1, 3]); hold on;
xd = 1:0.05:3;       yd = subs(f, x, xd);
plot(xd, yd, 'or');
title('Taylor approximation vs actual function');
xlabel('x'); ylabel('y');
legend('Taylor approximation', 'Original function', 'Location', 'best');
```



Taylor approximation vs actual function
0.25
0.24
0.23
0.22
0.21
0.2
0.19
0.18
y
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
3
x
Taylor approximation
Original function

(c) In this problem, we plot the graph and find local maxima, minima, and inflection points of the following function:

$$f(x) = \frac{3x^2 + 6x - 1}{x^2 + x - 3}.$$

i. Using the MATLAB command `ezplot`, plot the graph of $f$.
   [Actually, there is an command `fplot` which is a upgraded version of `plot`. But let's use the `ezplot` to follow the intent of this problem.]

ii. Using the MATLAB commands `limit` and `plot`, on the figure from (i), draw the horizontal and vertical asymptotes of $f$.

iii. Using the MATLAB commands `diff` and `solve`, find local maxima, minima and inflection points of $f$.
   [Can you expect what the 'integration command' is?.]

iv. Using the MATLAB commands `plot`, on the figure from (ii), indicate all critical and inflection points of $f$.

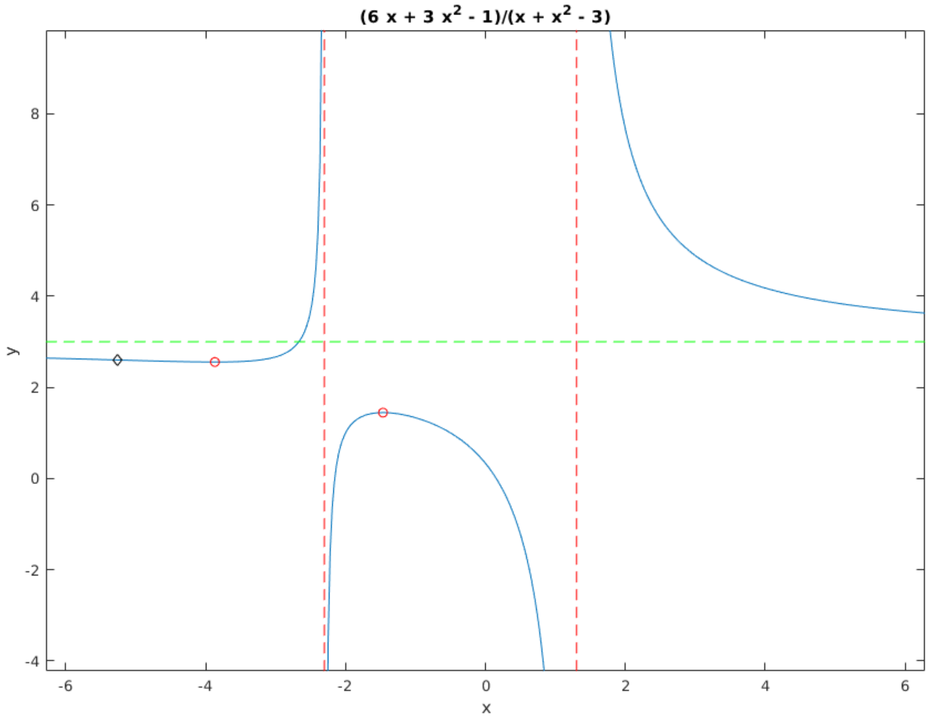
*Solution.*

```
% Assignment 5_1 (c)
% (i). Plot the given function.
syms x;                 % Regard x as a symbol.
num = 3*x^2 + 6*x - 1;
denom = x^2 + x - 3;
f = num / denom;
ezplot(f);              % Plot f(x).
hold on;

% (ii). Draw asymptotes of the given function.
hori_asym = limit(f, inf);  % Horizontal asymptote.
roots = solve(denom);       % Vertical asymptote.
plot([-10 10], double(hori_asym)*[1, 1], '--g');    % Draw horizontal asymptote.
plot(double(roots(1))*[1 1], [-100, 100], '--r');   % Draw vertical asymptote.
plot(double(roots(2))*[1 1], [-100, 100], '--r');   % Draw vertical asymptote.

% (iii). Find local maxima, minima, and inflection points.
crit_pts = solve(diff(f, x, 1));    % Solve the equation f'=0 in terms of x.
double(subs(f, crit_pts));  % The local maximum and minimum values of f.
% Solve the equation f''=0 in terms of x.
inflec_pt = solve(diff(f, x, 2), 'real', true);
double(inflec_pt);      % Get a number from a symbol.

% (iv). Indicate critical and inflection points.
plot(double(crit_pts), double(subs(f, crit_pts)), 'ro');
plot(double(inflec_pt), double(subs(f, inflec_pt)), 'kd');
xlabel('x'); ylabel('y');
```


(6 x + 3 x^2 - 1)/(x + x^2 - 3)
y
x
8
6
4
2
0
-2
-4
-6
-4
-2
0
2
4
6

2. Let $f$ and $g$ be two functions given by

$$f(x,\, y) = \mathrm{e}^{\sin(xy^2)} + \mathrm{e}^{-\cos(x^2 y)},$$
$$g(z) = \arctan(z).$$

Referring to the problem 1 compute values of $f_{xy}\,(2,\,-3)$ and $\int_{-1}^{1} \left|g(z) - T_g^9(z)\right|^2 \mathrm{d}z$ where the $T_g^9(z)$ denotes 9th order Maclaurin expansion for $g(z)$. *Solution.*

```
% assignment 5_2

syms x y z  % Declare symbolic variables x, y, z

% Make functions f and g using the variables
f = exp(sin(x * y^2)) + exp(-cos(x^2 * y));
g = atan(z);

fxy = diff(f, x, y);    % Calculate exact form of 'd^2f/dxdy'
Vf = double(subs(fxy, [x, y], [2, -3]));  % find the value (d^2f/dxdy)(2, -3)

% Compute Taylor expansion of g to the 9th order term
T = taylor(g, z, 'Order', 10);

h = abs(g - T)^2;   % The squared difference

% Find the value of integration which is given in the assignment
Vg = double(int(h, [-1, 1]));

% Display
fprintf('The value of f_xy(2,-3) is %f\n', Vf);
fprintf('The L2 error between g and T on [-1, 1] is %f\n', Vg);
```