**1** 3+3+4 points

Indicate whether the following statements are true(**T**) or false(**F**). You do **not** need to justify your answer.

(a) Let $A = \begin{pmatrix} 19 & 22 & 24 \\ 0 & 20 & 23 \\ 0 & 0 & 21 \end{pmatrix}$. Then, the determinant of the matrix $A$ is 7980.

(b) If a square matrix has a $LU$-decomposition, then the matrix has the unique $LU$-decomposition.

(c) Let $A$ be a $3 \times 3$ matrix such that each entry of $A$ is either 0 or 1. Then $\det(A)$ is one of $-1$, 0, and 1.

*Solution.*

(a) True. The determinant of a triangular matrix equals to the multiplication of diagonal entries.

(b) False. $\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

(c) False. The determinant of the following matrix is $-2$.

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

**2** Find a $LU$-decomposition of the following matrix $A$ in which every diagonal entry of the lower triangular matrix is 1.

10 points

$$A = \begin{pmatrix} 1 & 5 & 7 \\ 3 & 2 & 4 \\ 7 & 9 & 1 \end{pmatrix}$$

*Solution.*

$$A = \begin{pmatrix} 1 & 5 & 7 \\ 3 & 2 & 4 \\ 7 & 9 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 7 & 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 5 & 7 \\ 0 & -13 & -17 \\ 0 & 0 & -14 \end{pmatrix}$$