Anar Rzayev MAS 365: HW submission ID: 20190788

1) a) By Taylor's expansion, we obtain

$$y(t_{i+1}) = y(t_i) + h y'(t_i) + \frac{h^2}{2} y''(t_i) + \frac{h^3}{6} y^{(3)}(t_i) + \dots = y(t_i) + \sum_{n=1}^{\infty} \frac{h^n}{n!} y^{(n)}(t_i)$$

$$y(t_{i-1}) = y(t_i) - h y'(t_i) + \frac{h^2}{2} y''(t_i) - \frac{h^3}{6} y^{(3)}(t_i) + \dots = y(t_i) + \sum_{n=1}^{\infty} \frac{(-h)^n}{n!} y^{(n)}(t_i)$$

$$y(t_{i-2}) = y(t_i) - (2h) y'(t_i) + \frac{(2h)^2}{2} y''(t_i) - \frac{(2h)^3}{6} y^{(3)}(t_i) + \dots =$$

$$= y(t_i) + \sum_{n=1}^{\infty} \frac{(-2h)^n}{n!} y^{(n)}(t_i) = y(t_i) + \sum_{n=1}^{\infty} \frac{(-2)^n h^n}{n!} y^{(n)}(t_i)$$

plugging the expressions back to equation, one can observe

$$3f(t_i, y_i) = \frac{y(t_{i+1}) + \frac{3}{2} y(t_i) - 3 y(t_{i-1}) + \frac{1}{2} y(t_{i-2})}{h} =$$

$$= \Bigg( \left[ y(t_i) + \frac{3}{2} y(t_i) - 3 y(t_i) + \frac{1}{2} y(t_i) \right] + h \left[ y'(t_i) + 3 y'(t_i) - y'(t_i) \right]$$

$$+ y''(t_i) \left[ \frac{h^2}{2} - \frac{6h^2}{4} + h^2 \right] + y^{(3)}(t_i) \left[ \frac{h^3}{6} + \frac{h^3}{2} - \frac{2h^3}{3} \right] +$$

$$+ \sum_{n=4}^{\infty} y^{(n)}(t_i) \left[ \frac{h^n}{n!} + \frac{(-3)(-h)^n}{n!} + \frac{1}{2} \frac{(-2)^n h^n}{n!} \right] \Bigg) \Big/ h =$$

$$= \left( 3h y'(t_i) + \sum_{n=4}^{\infty} y^{(n)}(t_i) \left[ \frac{h^n}{n!} \left( 1 + (-3)(-1)^n + \frac{1}{2} (-2)^n \right) \right] \right) \Big/ h =$$

$$= 3 y'(t_i) + \sum_{n=4}^{\infty} \frac{y^{(n)}(t_i) h^{n-1}}{n!} \left[ 1 + (-3)(-1)^n + (-1)^n 2^{n-1} \right] =$$

$$= 3 y'(t_i) + \frac{h^3}{24} y^{(4)}(t_i) (-2 + 8) + \sum_{n=5}^{\infty} \frac{y^{(n)}(t_i) h^{n-1}}{n!} \left[ 1 + (-1)^n (2^{n-1} - 3) \right]$$

Hence, $3f(t_i, y_i) = 3y'(t_i) + \frac{h^3}{4} y^{(4)}(t_i) + \sum_{n=5}^{\infty} \frac{y^{(n)}(t_i) h^{n-1}}{n!}$.

$\cdot \left[ 1 + (-1)^n (2^{n-1} - 3) \right] = 3y'(t_i) + \frac{h^3}{4} y^{(4)}(t_i) + O(h^4)$

Therefore, for some $t_{i-2} < \xi_i < t_i \Rightarrow 3f(t_i, y_i) = 3y'(t_i) + \frac{h^3}{4} y^{(4)}(\xi_i)$ Henceforth,

$3f(t_i, y_i) - 3y'(t_i) \rightsquigarrow$ answer

the local truncation error $\Rightarrow \tau_i(h) = \frac{h^3}{4} y^{(4)}(\xi_i)$ for some $t_{i-2} < \xi_i < t_i$

Note: When we wanted to identify the local truncation error term, it was associated with the given multistep method: $w_{i+1} = -\frac{3}{2} w_i + 3 w_{i-1} - \frac{1}{2} w_{i-2} + 3h f(t_i, w_i)$ where

this was desired ?

$y(t_{i+1}) = -\frac{3}{2} y(t_i) + 3y(t_{i-1}) - \frac{1}{2} y(t_{i-2}) + 3h y'(t_i) + \tau_i(h)$

b) Since $\tau_i(h) = \frac{h^3}{4} y^{(4)}(\xi_i)$ for some $t_{i-2} < \xi_i < t_i$ was found, and taking into account that $f \rightsquigarrow$ smooth function for the interval where indeed we have to find the solution, $y^{(4)} \rightsquigarrow$ bounded, and one finds $\lim_{h \to 0} \max_{1 \le i \le N} |\tau_i(h)| =$

$= \lim_{h \to 0} \frac{h^3}{4} \max_{1 \le i \le N} \left| y^{(4)}(\xi_i) \right| = 0 \Rightarrow$ So, $\boxed{\text{system is consistent}}$

Suppose we take the characteristics polynomial such as $\lambda^{i+1} = -\frac{3}{2} \lambda^i + 3 \lambda^{i-1} - \frac{1}{2} \lambda^{i-2} + 3h f(t_i, \lambda^i)$ where $h \to 0$

This recursive property will reveal for us the following:

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$$\boxed{\lambda^3 + \frac{3}{2}\lambda^2 - 3\lambda - \frac{1}{2} = 0}$$

We can check that the roots are approx equal to $\longrightarrow$ $\boxed{\begin{array}{l}\lambda_1 \approx -2.60 \\ \lambda_2 \approx -0.16 \\ \lambda_3 \approx 1.24\end{array}}$

As $|\lambda_1| \approx 2.6 > 1$, it means that this will not satisfy the root condition

$\Rightarrow$ $\boxed{\text{system is not stable}}$ In the past, system has been shown to be consistent, and unstable $\Rightarrow$ In the conclusion, we deduce $\boxed{\text{system is not convergent}}$

$$\boxed{\begin{array}{ll}\text{system is} & \text{consistent} \\ & \text{unstable} \\ & \text{not convergent}\end{array}}$$

$\boxed{v}$ . $(+)$

2) The Runge-Kutta algorithm for order 4 in order to solve IVP $y'=f(t,y)$, $a\le t\le b$, $y(a)=\alpha$, is by

$$w_0=\alpha$$
$$k_1=hf(t_i,w_i)$$
$$k_2=hf(t_i+h/2,\, w_i+k_1/2)$$
$$k_3=hf\left(t_i+\frac{h}{2},\, w_i+\frac{k_2}{2}\right)$$
$$k_4=hf(t_i+h,\, w_i+k_3)$$

$$w_{i+1}=w_i+\frac{1}{6}(k_1+2k_2+2k_3+k_4)$$

for $i=1,2,\dots,N-1$

When applied to the DE $y'=f(t,y)=\lambda y$, we can rewrite the given form into the format

$$k_1=hf(t_i,w_i)=h\lambda w_i$$

$$k_2=hf\left(t_i+\frac{h}{2}, w_i+\frac{k_1}{2}\right)=h\lambda\left(w_i+\frac{k_1}{2}\right)=w_i\left(h\lambda+\frac{1}{2}(h\lambda)^2\right)$$

$$k_3=hf\left(t_i+\frac{h}{2}, w_i+\frac{k_2}{2}\right)=h\lambda\left(w_i+\frac{k_2}{2}\right)=$$
$$=w_i\left(h\lambda+\frac{1}{2}(h\lambda)^2+\frac{1}{4}(h\lambda)^3\right)$$

$$k_4=hf(t_i+h, w_i+k_3)=h\lambda(w_i+k_3)=$$
$$=w_i\left(h\lambda+(h\lambda)^2+\frac{1}{2}(h\lambda)^3+\frac{1}{4}(h\lambda)^4\right)$$

Since we know that $w_{i+1}=w_i+\frac{1}{6}(k_1+2k_2+2k_3+k_4)$ holds true, plugging in the previous results will reveal

$$k_1+2k_2+2k_3+k_4=h\lambda w_i+2w_i\left(h\lambda+\frac{1}{2}(h\lambda)^2\right)+$$
$$+2w_i\left(h\lambda+\frac{1}{2}(h\lambda)^2+\frac{1}{4}(h\lambda)^3\right)+w_i\left(h\lambda+(h\lambda)^2+\frac{1}{2}(h\lambda)^3+\frac{1}{4}(h\lambda)^4\right)$$

So, $k_1 + 2k_2 + 2k_3 + k_4 = w_i\left(6\lambda h + 3(\lambda h)^2 + (\lambda h)^3 + \frac{1}{4}(h\lambda)^4\right)$

$$w_{i+1} = w_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = w_i\left(1 + \lambda h + \frac{1}{2}(\lambda h)^2 + \frac{1}{6}(\lambda h)^3 + \frac{1}{24}(h\lambda)^4\right)$$

$$\Rightarrow \boxed{w_{i+1} = w_i\left(1 + \lambda h + \frac{1}{2}(\lambda h)^2 + \frac{1}{6}(\lambda h)^3 + \frac{1}{24}(\lambda h)^4\right)}$$

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3) a) If $\phi_0(x), \phi_1(x), \phi_2(x), \ldots, \phi_n(x)$ are orthogonal polynomials on $[a,b] \Rightarrow \phi_0(x) = 1$

$\phi_1(x) = \underline{1} - \beta_{\underline{1}}$ where we have

$$\beta_{\underline{1}} = \frac{\int_a^b x w(x) [\phi_0(x)]^2 dx}{\int_a^b w(x) [\phi_0(x)]^2 dx}$$

Moreover, the desired poly-nomials for $k \geq 2$ will be revealed as follows:

$\phi_k(x) = (x - \beta_k)\phi_{k-1}(x) - C_k \phi_{k-2}(x)$ in which

$$\beta_k = \frac{\int_a^b x w(x) [\phi_{k-1}(x)]^2 dx}{\int_a^b w(x) [\phi_{k-1}(x)]^2 dx}$$ and

$$C_k = \frac{\int_a^b x w(x) \phi_{k-1}(x) \phi_{k-2}(x) dx}{\int_a^b w(x) [\phi_{k-2}(x)]^2 dx}$$

weight functions
$\rightarrow L_0(x) = 1$
$w(x) = e^{-x}$

From the famous Gram-Schmidt procedure,

$L_1(x) = (x - \beta_{\underline{1}}) L_0(x)$

$L_0(x) = e^{-x}$

$$\beta_{\underline{1}} = \frac{\int_0^\infty x e^{-x} dx}{\int_0^\infty e^{-x} dx}$$

where the interval is taken as $(0, \infty)$

Applying integration by parts, $\beta_{\underline{1}} = \dfrac{-xe^{-x}\Big]_0^\infty + \int_0^\infty e^{-x} dx}{-e^{-x}\Big]_0^\infty}$

$\Rightarrow \beta_{\underline{1}} = \dfrac{-e^{-x}\Big]_0^\infty}{1} = \underline{1}$ and thus, $\boxed{L_1(x) = (x-1)}$ *

According to Gram-Schmidt procedure, $L_2(x) = (x - \beta_2)\phi_1(x) - C_2\phi_0(x)$

$$\beta_2 = \frac{\int_0^\infty x e^{-x}(x-1)^2 dx}{\int_0^\infty e^{-x}(x-1)^2 dx} = \frac{\int_0^\infty (x^3-2x^2+x)e^{-x}dx}{\int_0^\infty (x^2-2x+1)e^{-x}dx} =$$

$$= \frac{-(x^3-2x^2+x)e^{-x}\Big]_0^\infty + \int_0^\infty (3x^2-4x+1)e^{-x}dx}{-(x^2-2x+1)e^{-x}\Big]_0^\infty + \int_0^\infty (2x-2)e^{-x}dx} =$$

$$= \frac{\int_0^\infty (3x^2-4x+1)e^{-x}dx}{1+\int_0^\infty (2x-2)e^{-x}dx} = \frac{-(3x^2-4x+1)e^{-x}\Big]_0^\infty + \int_0^\infty e^{-x}(6x-4)dx}{1+ -(2x-2)e^{-x}\Big]_0^\infty + 2\int_0^\infty e^{-x}dx}$$

$$= \frac{1+ -(6x-4)e^{-x}\Big]_0^\infty + \int_0^\infty e^{-x}dx \cdot 6}{1+(-2)+2\cdot -e^{-x}\Big]_0^\infty} = \frac{1-4+6\cdot -e^{-x}\Big]_0^\infty}{1-2+2\cdot 1} =$$

$$= \frac{-3+6}{1} = 3 \Rightarrow \boxed{\beta_2 = 3} \qquad C_2 = \frac{\int_0^\infty x e^{-x}(x-1)\cdot 1\, dx}{\int_0^\infty e^{-x}\cdot 1^2 dx} =$$

$$= \frac{\int_0^\infty x e^{-x}(x-1)dx}{\int_0^\infty e^{-x}dx} = \frac{-e^{-x}(x^2-x)\Big]_0^\infty + \int_0^\infty e^{-x}(2x-1)dx}{-e^{-x}\Big]_0^\infty} =$$

$$= \frac{-e^{-x}(2x-1)\Big]_0^\infty + \int_0^\infty e^{-x}\cdot 2\,dx}{1} = (-1)+2\cdot -e^{-x}\Big]_0^\infty = -1+2$$

$= 1 \Rightarrow \boxed{C_2 = 1}$ where integration by parts had been applied iteratively

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$L_2(x) = (x-3)(x-1) - 1 = x^2 - 4x + 3 - 1 = x^2 - 4x + 2 \Rightarrow$ Therefore,

$\boxed{L_2(x) = x^2 - 4x + 2}$ $\boxed{V}$

b) Considering the function $f(x) = x^3 = a_0 L_0(x) + a_1 L_1(x) + a_2 L_2(x)$ and using the previous results, we can obtain that

$$a_0 = \frac{\int_0^\infty e^{-x} x^3 dx}{\int_0^\infty e^{-x} 1^2 dx} = \frac{-x^3 e^{-x}\Big]_0^\infty + \int_0^\infty e^{-x} \cdot 3x^2 dx}{-e^{-x}\Big]_0^\infty} = \frac{3\int_0^\infty e^{-x} x^2 dx}{1} =$$

$$= 3\left[-e^{-x}x^2\Big]_0^\infty + 2\int_0^\infty e^{-x} x dx\right] = 6\int_0^\infty e^{-x} x dx = 6\left[-e^{-x}x\Big]_0^\infty + \int_0^\infty e^{-x} dx\right]$$

$= 6 \Rightarrow \boxed{a_0 = 6}$

$$a_1 = \frac{\int_0^\infty e^{-x} x^3 (x-1) dx}{\int_0^\infty e^{-x} (x-1)^2 dx} = \frac{\int_0^\infty e^{-x} \cdot x^4 - \int_0^\infty e^{-x} x^3 dx}{\int_0^\infty e^{-x} \cdot x^2 - 2\int_0^\infty e^{-x} \cdot x + \int_0^\infty e^{-x} 1 dx} =$$

$$= \frac{-e^{-x}x^4\Big]_0^\infty + 3\int_0^\infty e^{-x} x^3 - \int_0^\infty e^{-x} x^3 dx}{2 - 2 \cdot 1 + 1} = \frac{3\int_0^\infty e^{-x} x^3 dx}{1} = 3 \times 6 = 18$$

as we previously computed that $\int_0^\infty e^{-x} x^2 dx = 2$, $\int_0^\infty e^{-x} x dx = 1$, $\int_0^\infty e^{-x} dx = 1$ and $\int_0^\infty e^{-x} x^3 dx = 6 \Rightarrow \boxed{a_1 = 18}$

$$a_2 = \frac{\int_0^\infty e^{-x} x^3 (x^2 - 4x + 2) dx}{\int_0^\infty e^{-x} (x^2 - 4x + 2)^2 dx} = \frac{\int_0^\infty e^{-x} \cdot x^5 dx - 4\int_0^\infty e^{-x} x^4 dx + 2 \times 6}{\int_0^\infty e^{-x} (x^4 + 16x^2 + 4 - 8x^3 + 4x^2 - 16x) dx}$$

Using previous values and integration by parts for $\int_0^\infty e^{-x}x^5dx$

and $\int_0^\infty e^{-x}x^4dx \Rightarrow \int_0^\infty e^{-x}x^4 = -e^{-x}x^4\Big]_0^\infty + 4\int_0^\infty e^{-x}x^3 = 4\times 6 = 24$

and $\int_0^\infty e^{-x}x^5dx = -e^{-x}x^5\Big]_0^\infty + 5\int_0^\infty e^{-x}x^4dx = 5\times 24 = 120 \Rightarrow$

$$a_2 = \frac{120 - 4\times 24 + 12}{24 + 16\times 2 + 4 - 8\times 6 + 4\times 2 - 16\times 1} = \frac{36}{56+4-48-8} = \frac{36}{4} = 9,80$$

$\boxed{a_2 = 9}$ and we conclude $\boxed{P(x) = 6L_0(x) + 18L_1(x) + 9L_2(x)}$

$P(x) = 6 + 18(x-1) + 9(x^2 - 4x + 2) = 9x^2 - 18x + 6$

4) a) $f(x)=x^3-x$, approximating by $P_2(x)=a_2x^2+a_1x+a_0$ that minimizes $\int_0^1 [f(x)-P_2(x)]^2dx \Rightarrow P_2(x)=?$

For such instance, the 3-linear normal equations must be solved for 3 unknowns $a_j$ in order to identify $P_2(x)$:

$$\sum_{k=0}^{n} a_k\int_a^b x^{j+k}dx=\int_a^b x^j f(x)dx \quad \text{for each } j=0,1,\dots,n$$

since $n=2$, $a=0$, $b=1$, $f(x)=x^3-x$ $\Rightarrow a_0\int_0^1 x^0dx+a_1\int_0^1 x^1dx+a_2\int_0^1 x^2dx=$

$$=a_0+a_1\cdot \frac{x^2}{2}\Big]_0^1+a_2\cdot\frac{x^3}{3}\Big]_0^1=a_0+\frac{1}{2}a_1+\frac{1}{3}a_2=\int_0^1(x^3-x)dx=$$

$$=\frac{x^4}{4}-\frac{x^2}{2}\Big]_0^1=\frac{1}{4}-\frac{1}{2}=-\frac{1}{4}\Rightarrow \boxed{a_0+\frac{1}{2}a_1+\frac{1}{3}a_2=-\frac{1}{4}} \text{ for } j=0$$

$$j=1\Rightarrow a_0\int_0^1 x^1dx+a_1\int_0^1 x^2dx+a_2\int_0^1 x^3dx=a_0\cdot\frac{x^2}{2}\Big]_0^1+$$

$$+a_1\cdot\frac{x^3}{3}\Big]_0^1+a_2\cdot\frac{x^4}{4}\Big]_0^1=\frac{1}{2}a_0+\frac{1}{3}a_1+\frac{1}{4}a_2=\int_0^1 x(x^3-x)dx$$

$$=\int_0^1(x^4-x^2)dx=\frac{x^5}{5}-\frac{x^3}{3}\Big]_0^1=\frac{1}{5}-\frac{1}{3}=\frac{3}{15}-\frac{5}{15}=-\frac{2}{15}\Rightarrow$$

$$\boxed{\frac{1}{2}a_0+\frac{1}{3}a_1+\frac{1}{4}a_2=-\frac{2}{15}} \quad j=2\Rightarrow a_0\int_0^1 x^2dx+a_1\int_0^1 x^3dx+a_2\int_0^1 x^4dx$$

$$=a_0\cdot\frac{x^3}{3}\Big]_0^1+a_1\cdot\frac{x^4}{4}\Big]_0^1+a_2\cdot\frac{x^5}{5}\Big]_0^1=\frac{a_0}{3}+\frac{a_1}{4}+\frac{a_2}{5}=\int_0^1 x^2(x^3-x)dx$$

$$\int_0^1(x^5-x^3)dx=\frac{x^6}{6}-\frac{x^4}{4}\Big]_0^1=\frac{1}{6}-\frac{1}{4}=\frac{2}{12}-\frac{3}{12}=-\frac{1}{12}\Rightarrow$$

$$\boxed{\frac{1}{3}a_0+\frac{1}{4}a_1+\frac{1}{5}a_2=-\frac{1}{12}}$$

Combining these resultant equations, we obtain that

$$a_0+\frac{1}{2}a_1+\frac{1}{3}a_2=-\frac{1}{4}$$
$$\frac{1}{2}a_0+\frac{1}{3}a_1+\frac{1}{4}a_2=-\frac{2}{15}$$
$$\frac{1}{3}a_0+\frac{1}{4}a_1+\frac{1}{5}a_2=-\frac{1}{12}$$

$$a_0+\frac{1}{2}a_1+\frac{1}{3}a_2=-\frac{1}{4}$$
$$a_0+\frac{2}{3}a_1+\frac{1}{2}a_2=-\frac{4}{15}$$

$$\frac{2}{3}a_1+\frac{1}{2}a_2-\frac{1}{2}a_1-\frac{1}{3}a_2=-\frac{4}{15}+\frac{1}{4}$$

$$\frac{4}{6}a_1-\frac{3}{6}a_1+\frac{3}{6}a_2-\frac{2}{6}a_2=\frac{1}{6}a_1+\frac{1}{6}a_2=-\frac{16}{60}+\frac{15}{60}=-\frac{1}{60}$$

Then, $\boxed{a_1+a_2=-\frac{1}{10}}$

$$a_0+\frac{1}{2}a_1+\frac{1}{3}a_2=-\frac{1}{4}$$
$$a_0+\frac{3}{4}a_1+\frac{3}{5}a_2=-\frac{1}{4}$$

$\frac{1}{2}a_1+\frac{1}{3}a_2=\frac{3}{4}a_1+\frac{3}{5}a_2$, $\frac{1}{2}a_1-\frac{3}{4}a_1=\frac{3}{5}a_2-\frac{1}{3}a_2$ and

$$\frac{2}{4}a_1-\frac{3}{4}a_1=-\frac{1}{4}a_1=\frac{9}{15}a_2-\frac{5}{15}a_2=\frac{4a_2}{15}=-\frac{a_1}{4}\Rightarrow \boxed{16a_2=-15a_1}$$

$$16a_1+16a_2=-\frac{16}{10}=-15a_1+16a_1=\boxed{a_1=-\frac{8}{5}} \text{ Similarly,}$$

$$16a_2=-15\times\left(\frac{-8}{5}\right)=\frac{120}{5}=24,\quad a_2=\frac{24}{16}=\frac{3}{2},\quad \boxed{a_2=\frac{3}{2}}$$

$$a_0+\frac{1}{2}a_1+\frac{1}{3}a_2=a_0+\frac{1}{2}\left(-\frac{8}{5}\right)+\frac{1}{3}\cdot\frac{3}{2}=a_0-\frac{4}{5}+\frac{1}{2}=-\frac{1}{4}$$

$$a_0-\frac{16}{20}+\frac{10}{20}=a_0-\frac{6}{20}=-\frac{5}{20}\Rightarrow a_0=-\frac{5}{20}+\frac{6}{20}=\frac{1}{20},\ \boxed{a_0=\frac{1}{20}}$$

In the end, $\boxed{P_2(x) = a_0 + a_1 x + a_2 x^2 = \frac{1}{20} - \frac{8}{5}x + \frac{3}{2}x^2}$ V

б) $P_2(x) = \frac{3}{2}x^2 - \frac{8}{5}x + \frac{1}{20}$ ~~$P(x) + \frac{3}{8} =$~~

$\max\limits_{x\in[0,1]} |P_2(x) - P(x)| = \frac{a}{2}|x^2 - x|$ where we can plug in some value $x$ to get $\frac{a}{2} \Rightarrow \max\limits_{x\in[0,1]} |P_2(x) - P(x)| = \frac{a_2}{2} =$

$= \frac{3}{4}$, if we just modify secant, the result will be as follows: $\max\limits_{x\in[0,1]} |P_2(x) - P(x)| = \max\limits_{x\in[0,1]} \left| a(x^2 - x) + \frac{a}{4} \right|$

$\Rightarrow$ $P_2(0) = \frac{1}{20}$, $P_2(1) = \frac{1}{20} - \frac{8}{5} + \frac{3}{2} = \frac{1 - 32 + 30}{20} = \frac{-1}{20}$ $= \frac{a}{4}$

Our construction: $P_1(x) = -\frac{1}{10}x + \frac{1}{20} + \frac{\frac{3}{2}}{4} =$

$P_2(0) = \frac{1}{20}$ $= -\frac{1}{10}x + \frac{1}{20} + \frac{3}{8}$ $\boxed{Q(x) = -\frac{1}{10}x + \frac{1}{20}}$

$P_2(1) = \frac{1}{20} - \frac{8}{5} + \frac{3}{2} = \frac{1 - 32 + 30}{20} = -\frac{1}{20}$ $\frac{2}{40} + \frac{15}{40} = \frac{17}{40}$

$\boxed{Q(x) = -\frac{1}{10}x + \frac{1}{20}}$ $\boxed{P_1(x) = -\frac{1}{10}x + \frac{17}{40}}$

$|P_2(x) - P_1(x)| = \frac{\frac{3}{2}}{4} = \frac{3}{8} \Rightarrow$ From Cheby ~~schev's~~ Theorem $\Rightarrow P_1(x)$ is best estimation

$(x^2 - x)' = 2x - 1 \Rightarrow x = \frac{1}{2}$

$\max |P_2(x) - Q(x)| = \frac{3}{2} \max\limits_{x\in[0,1]} |x^2 - x| \to \frac{3}{2} \cdot \frac{1}{4} =$

$= \frac{3}{8}$ $\to 3/8$

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