2020 Spring MAS 365: Midterm Exam

Write the following Honor Pledge and sign your name under it.

> *"I have neither given nor received aid on this examination, nor have I concealed a violation of the Honor Code."*

1. [20 (10+10) points]

   (a) Perform two iterations of the Bisection method (that outputs p_2) to find an approximation to $\sqrt{11}$ in $[3, 3.7]$, using two-digit rounding arithmetic.

   (b) Determine the rate of convergence of the function $\frac{\sin h}{h}$ as $h \to 0$, where $\lim_{h\to 0} \frac{\sin h}{h} = 1$, using a form $O(h^p)$.

2. [20 (5+5+5+5) points] State whether the iterations $\{p_n\}_{n=0}^{\infty}$ converge to $p = 1$ for an initial approximation $p_0 \in [p-\delta, p+\delta]$, where $0 < \delta < 0.5$ is *sufficiently* small. If it converges, find the order of convergence. Justify your answer.

   (a) $p_{k+1} = -1 + 2p_k + p_k^2$

   (b) Newton's method for $f(x) = (x-1)^2$

   (c) $p_{k+1} = -1 + p_k^2 + \frac{1}{p_k^2}$

   (d) Newton's method for $f(x) = \ln x$

3. [30 (10+10+10) points]

x	$f(x)$
0	-5
0.5	-2
1	13

   (a) Use the Newton backward-difference formula to construct interpolating polynomial for the above data.

   (b) Construct the clamped cubic spline for the above data with additional information $f'(0) = 6$ and $f'(1) = 74$.

   (c) Use $O(h)$ and $O(h^2)$ formulas to approximate $f'(0)$.

4. [15 points] For every $n \geq 2$, there is a C_n such that $n! = C_n n^{n+\frac{1}{2}} e^{-n}$. Use the composite trapezoidal rule for $\int_1^n \ln x dx = n \ln n - n + 1$ to approximate $\ln(n!)$ with error term. Find an upper bound on C_n.

5. [30 (10+10+10) points]

   (a) Use the Hermite interpolating polynomial of f at distinct nodes $x_0, \ldots, x_n$ to approximate the integral $\int_{-1}^{1} f(x)dx$.

(b) Based on (a), thus using the Hermite interpolating polynomial, describe how to choose the coefficients $c_0, \ldots, c_n$ and the nodes $x_0, \ldots, x_n$ in the approximation

$$\int_{-1}^{1} f(x)dx \approx \sum_{j=0}^{n} c_j f(x_j),$$

so that it is exact for any f that is a polynomial of degree less than $2n + 1$.

(c) Use the quadrature in (b) with $n = 1$ to approximate $\int_{-0.5}^{0.5} x^2 \cos\big(4\sqrt{3}\pi x\big)\, dx$.