2021 Spring MAS 365: Homework 8

posted on May 27; due by June 3

1. [10+5 points]

   (a) Determine constants a, b, c and d that will produce a quadrature formula

   $$\int_{-1}^{1} f(x)dx = af(-1) + bf(1) + cf'(-1) + df'(1)$$

   that has degree of precision three.

   (b) Show that the formula $\sum_{i=1}^{n} c_i P(x_i)$ cannot have degree of precision greater than $2n-1$, regardless of the choice of $c_1, \ldots, c_n$ and $x_1, \ldots, x_n$. [Hint: Construct a polynomial that has a double root at each of the x_j's.]

2. [10 points] Show that the initial-value problem

   $$y' = t^2 y + 1, \quad 0 \le t \le 1, \quad y(0) = 1$$

   is well-posed.

3. [5+10 points]

   (a) Use Euler's method with $h = 0.5$ to approximate the solution to

   $$y' = \frac{1+t}{1+y}, \quad 1 \le t \le 2, \quad y(1) = 2.$$

   (b) Given that the solution to the above problem is $y(t) = \sqrt{t^2 + 2t + 6} - 1$, find the bound for the approximation error. [Hint: Consider a set $D = \{(t, y) \mid 1 \le t \le 2 \text{ and } 2 \le y < \infty\}$ for the Lipschitz condition in the variable y.]

4. [10+5 points]

   (a) Derive the Adams-Bashforth Two-Step method by using the Lagrange form of the interpolating polynomial.

   (b) Derive Milne's method in page 313 of the textbook by applying the open Newton-Cotes formula to the integral, and report its local truncation error.

   $$y(t_{i+1}) - y(t_{i-3}) = \int_{t_{i-3}}^{t_{i+1}} f(t, y(t))dt.$$

5. [10+10 points]

   (a) Implement Taylor's method of order two with cubic Hermite interpolation via MATLAB grader.

   (b) Implement Adams fourth-order predictor-corrector method via MATLAB grader.