2021 Spring MAS 365: Homework 3

posted on Mar 25; due by Apr 1

1. [10+5 points] The Frobenius norm (which is not a natural norm) is defined for an $n \times n$ matrix A by

$$||A||_F = \left(\sum_{i=1}^{n} \sum_{j=1}^{n} |a_{ij}|^2 \right)^{1/2}.$$

   (a) Show that $|| \cdot ||_F$ is a matrix norm.

   (b) For any matrix A, show that $||A||_2 \leq ||A||_F$.

2. [5+10 points]

   (a) Show that if A is symmetric, then $||A||_2 = \rho(A)$.

   (b) Show that if $|| \cdot ||$ is any natural norm, then $(||A^{-1}||)^{-1} \leq |\lambda| \leq ||A||$ for any eigenvalue λ of the nonsingular matrix A.

3. [5+5 points] The linear system

$$\begin{aligned} x_1 + 2x_2 - 2x_3 &= 7, \\ x_1 + x_2 + x_3 &= 2, \\ 2x_1 + 2x_2 + x_3 &= 5 \end{aligned}$$

   has the solution $(1, 2, -1)^t$.

   (a) Find T_j of Jacobi method, and compute $\rho(T_j)$. Report an approximation after two iterations of Jacobi method using $\boldsymbol{x}^{(0)} = \mathbf{0}$.

   (b) Find T_g of Gauss-Seidel method, and compute $\rho(T_g)$. Report an approximation after two iterations of Gauss-Seidel method using $\boldsymbol{x}^{(0)} = \mathbf{0}$.

4. [10+10+10 points]

   (a) Implement Gauss-Seidel methods via MATLAB grader.

   (b) Implement Gauss-Seidel methods via MATLAB grader.

   (c) Implement *Randomized* Gauss-Seidel methods via MATLAB grader.