2021 Spring MAS 365: Homework 2

posted on Mar. 18; due by Mar. 25

1. [10 points] Let $\{p_n\}_{n=0}^{\infty}$ be a sequence generated by the Secant method. It can be shown that if $\{p_n\}_{n=0}^{\infty}$ converges to p, the solution to $f(x) = 0$, then a constant C exists with $|e_{n+1}| \approx C|e_n|\,|e_{n-1}|$ for sufficently large value of n, where $e_n := p_n - p$ for all n. Assuming that $\{p_n\}$ converges to p of order α, show that $\alpha = \frac{1+\sqrt{5}}{2} \approx 1.62$.

2. [5 points each]

   (a) Show that the sequence $p_n = 10^{-2^n}$ converges quadratically to 0.

   (b) Show that the sequence $p_n = 10^{-n^k}$ does not converge to 0 quadratically for any $k > 1$.

3. [5 points each] Let $P_n(x)$ be the nth Taylor polynomial for $f(x) = e^{-x}$ about $x_0 = 0$.

   (a) For fixed x, show that $p_n = P_n(x)$ satisfies

   $$\lim_{n\to\infty} \frac{p_{n+1} - p}{p_n - p} < 1,$$

   where p is a limit of $\{p_n\}_{n=0}^{\infty}$.

   (b) Let $x = 1$, and use Aitken's Δ^2 method to generate the sequence $\hat{p}_0, \ldots, \hat{p}_5$. Report five-digit rounding values. [Hint: Use MATLAB command `format long` for more digits.]

4. [10 points] Use Gaussian elimination with backward substitution and two-digit rounding arithmetic to solve the following linear system. Do not reorder the equations.

$$\begin{aligned} E_1 &: -x_1 + 4x_2 + x_3 = 8, \\ E_2 &: \frac{5}{3}x_1 + \frac{2}{3}x_2 + \frac{2}{3}x_3 = 1, \\ E_3 &: 2x_1 + x_2 + 4x_3 = 11. \end{aligned}$$

5. [10 points each]

   (1) Implement Steffensen's method via MATLAB grader.

   (2) Implement Müller's method via MATLAB grader. (See its pseudocode in the textbook.)

   (3) Implement Newton's method with Horner's method via MATLAB grader.