2021 Spring MAS 365: Homework 5

posted on Apr 8; due by Apr 15

1. [10+10 points] The nonlinear system

$$x_1^2 - 10x_1 + x_2^2 + 8 = 0, \quad x_1x_2^2 + x_1 - 10x_2 + 8 = 0$$

can be transformed into the fixed-point problem

$$x_1 = g_1(x_1, x_2) = \frac{x_1^2 + x_2^2 + 8}{10}, \quad x_2 = g_2(x_1, x_2) = \frac{x_1x_2^2 + x_1 + 8}{10}.$$

   (a) Use Theorem 10.6 in the textbook to show that $G = (g_1, g_2)^t$ mapping $D \subset \mathbb{R}^2$ to $\mathbb{R}^2$ has a unique fixed point in

$$D = \{(x_1, x_2)^t \mid 0 \le x_1, x_2 \le 1.5\}.$$

   (b) Use Theorem 10.6 to estimate the number of iterations required for the fixed-point iteration to achieve 10^{-3} accuracy of $||\boldsymbol{x}^{(k)} - \boldsymbol{p}||_\infty$ with $\boldsymbol{x}^{(0)} = (0, 0)^t$, where $\boldsymbol{p}$ is the unique fixed point.

2. [10 points] What does Newton's method reduce to for the linear system $A\boldsymbol{x} = \boldsymbol{b}$ given by

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1, \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2, \\ &\vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n &= b_n, \end{aligned}$$

   where A is a nonsingular matrix?

3. [10+10+10 points]

   (a) Implement the conjugate gradient method via MATLAB grader.

   (b) Implement the preconditioned conjugate gradient method via MATLAB grader.

   (c) Implement Newton's method via MATLAB grader.