2021 Spring MAS 365: Homework 7

posted on May 13; due by May 20

1. [10+10 points]

   (a) Let $f \in C^2[a,b]$, and let the nodes $a = x_0 < x_1 < \cdots < x_n = b$ be given. Derive an error estimate similar to that in Theorem 3.13 in the textbook for the piecewise linear interpolating function F.

   (b) A clamped cubic spline s for a function f is defined by

   $$s(x) = \begin{cases} s_0(x) = 1 + Bx + 2x^2 - 2x^3, & \text{if } 0 \le x < 1, \\ s_1(x) = 1 + b(x-1) - 4(x-1)^2 + 7(x-1)^3, & \text{if } 1 \le x \le 2. \end{cases}$$

   Find $f'(0)$ and $f'(2)$.

2. [10+10 points]

   (a) Use the most accurate three-point formula to determine each missing entry in the following table.

x	$f(x)$	$f'(x)$
-0.3	-0.27652	
-0.2	-0.25074	
-0.1	-0.16134	
0	0	

   (b) The data were taken from the function $f(x) = e^{2x} - \cos 2x$. Compute the actual error, and find error bounds using the error formulas. Note that the $f^{(3)}(x)$ is nonincreasing function on $[-0.3, 0]$.

3. [10 points] Suppose that $N(h)$ is an approximation to M for every $h > 0$ and that

   $$M = N(h) + K_1 h^2 + K_2 h^4 + K_3 h^6 + \cdots,$$

   for some constants $K_1, K_2, K_3, \ldots$. Use the values $N(h)$, $N\left(\frac{h}{3}\right)$, and $N\left(\frac{h}{9}\right)$ to produce and $O(h^6)$ approximation to M.

4. [5+5 points]

   (a) The quadrature formula $\int_0^2 f(x)dx = c_0 f(0) + c_1 f(1) + c_2 f(2)$ is exact for all polynomials of degree less than or equal to two. Determine c_0, c_1 and c_2.

   (b) Find the constants c_0, c_1, and x_1 so that the quadrature formula

   $$\int_0^1 f(x)dx = c_0 f(0) + c_1 f(x_1)$$

   has degree of precision 2.

5. [10+10 points]

   (a) Implement Newton-Cotes formulas via MATLAB grader.

   (b) Impement composite numerical intergration methods via MATLAB grader.