2021 Spring MAS 365: Homework 6

posted on May 6; due by May 13

1. [10+10 points]

   (a) Construct the second Lagrange interpolating polynomial for the function

   $$f(x) = x^{-3}, \quad \text{and the nodes } x_0 = 1, x_1 = 2, x_2 = 3.$$

   You don't have to write the polynomial in a compact form.

   (b) Find a bound for the corresponding absolute error on the interval $[x_0, x_2]$.

2. [10 points] Prove Taylor's Theorem 1.14 in the textbook by following the procedure in the proof of Theorem 3.3 in the textbook. [Hint: Let $g(t) = f(t) - P(t) - [f(x) - P(x)]\frac{(t-x_0)^{n+1}}{(x-x_0)^{n+1}}$, where P is the nth Taylor polynomial, and use the Generalized Rolle's Theorem and $g(x_0) = g'(x_0) = g''(x_0) = \cdots = g^{(n)}(x_0) = 0$.]

3. [10 points] Neville's method is used to approximate $f(0.4)$, giving the following table.

$x_0 = 0$	$P_0 = 1$			
$x_1 = 0.25$	$P_1 = 2$	$P_{0,1} = 2.6$		
$x_2 = 0.5$	P_2	$P_{1,2}$	$P_{0,1,2}$	
$x_3 = 0.75$	$P_3 = 8$	$P_{2,3} = 2.4$	$P_{1,2,3} = 2.96$	$P_{0,1,2,3} = 3.016$

   Determine $P_2 = f(0.5)$.

4. [10 points] Show that $H_{2n+1}(x)$ is the unique polynomial of least degree agreeing with f and f' at $x_0, \ldots, x_n$. Assume that $P(x)$ is another such polynomial and consider $D(x) = H_{2n+1}(x) - P(x)$ and $D'(x)$ at $x = x_0, x_1, \ldots, x_n$.

5. [10+5 points] Let $(x_0, y_0) = (0, 0)$ and $(x_1, y_1) = (5, 2)$ be the endpoints of a curve, and let $(1, 1)$ and $(6, 3)$ be the given guidepoints, respectively.

   (a) Construct a parametric cubic Hermite approximations $(x(t), y(t))$ to the curve.

   (b) Draw a graph of the approximation (possibly by MATLAB).

6. [10+10+10 points]

   (a) Implement the Newton divided difference formula via MATLAB grader.

   (b) Implement the divided difference formula for Hermite Polynomials via MATLAB grader.

   (c) Implement the cubic spline interpolations via MATLAB grader.