ARCH MODEL

November 19, 2023

Problem Statement:
Consider the data set "byd" from R. Plot the data and from the histogram check the ARCH effect from the residual analysis.

Solution

The ARCH model assumes that the conditional mean of the error term in a time series model is constant (zero), unlike the non stationary series we have discussed so far), but its conditional variance is not. Such a model can be described as in Equations 1, 2 and 3.

$$y_t = \phi + e_t \tag{1}$$

$$e_t | I_{t-1} \sim \mathcal{N}(0, h_t) \tag{2}$$

$$h_t = \alpha_0 + \alpha_1 e_{t-1}^2, \quad \alpha_0 > 0, \quad 0 \le \alpha_1 < 1 \tag{3}$$

Equations 4 and 5 give both the test model and the hypotheses to test for ARCH effects in a time series, where the residuals $\hat{e}_t$ come from regressing the variable y_t on a constant, such as 1 , or on a constant plus other regressors; the test shown in Equation 4 may include several lag terms, in which case the null hypothesis (Equation 5) would be that all of them are jointly insignificant.

$$e_t^2 = \gamma_0 + \gamma_1 e_{t-1}^2 + \ldots + \gamma_q e_{t-q}^2 + \nu_t \tag{4}$$

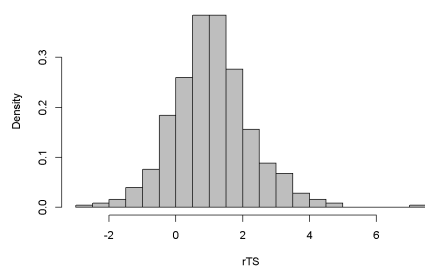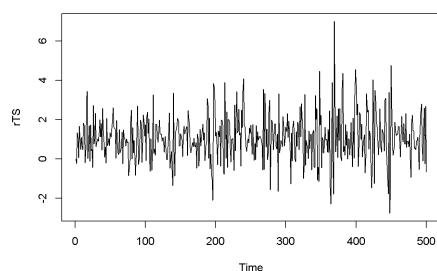$$H_0 : \gamma_1 = \ldots = \gamma_q = 0$$

$$H_A : \gamma_1 \neq 0 \ldots \gamma_q \neq 0$$

The null hypothesis is that there are no ARCH effects. The test statistic is

$$(T - q)R^2 \sim \chi^2(1 - \alpha, q) \tag{5}$$

The following example uses the dataset byd , which contains 500 generated observations on the returns to shares in BrightenYourDay Lighting.

The plots are as follows-

Let us first perform, step by step, the ARCH test described in Equations 4 and 5 , on the variable r from dataset byd

```
##
## Time series regression with "ts" data:
## Start = 1, End = 500
##
## Call:
## dynlm(formula = rTS ~ 1)
##
## Residuals:
##    Min    1Q Median    3Q    Max
## -3.847 -0.723 -0.049  0.669  5.931
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)    1.078      0.053     20.4   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.19 on 499 degrees of freedom
```

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```
ehatsq <- ts(resid(byd.mean)^2)
byd.ARCH <- dynlm(ehatsq~L(ehatsq))
summary(byd.ARCH)
```

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The summary is described as follows-

```
## Time series regression with "ts" data:
## Start = 2, End = 500
##
## Call:
## dynlm(formula = ehatsq ~ L(ehatsq))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -10.802  -0.950  -0.705   0.320  31.347
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)     0.908      0.124    7.30  1.1e-12 ***
## L(ehatsq)       0.353      0.042    8.41  4.4e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.45 on 497 degrees of freedom
## Multiple R-squared:  0.125,  Adjusted R-squared:  0.123
## F-statistic: 70.7 on 1 and 497 DF,  p-value: 4.39e-16
```

The result is the LM statistic, equal to 62.16, which is to be compared
to the critical chi-squared value with $\alpha = 0.05$ and $q = 1$ degrees of
freedom; this value is $\chi^2(0.95, 1) = 3.84$. This indicates that the null
hypothesis is rejected, concluding that the series has ARCH effects.