Measuring Verdet's Constant of Flint Glass with a Faraday Rotation Apparatus

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Abstract

The Faraday effect, a seminal discovery in physics, demonstrates the intimate relationship between light and magnetism. This phenomenon, first observed by Michael Faraday in 1845, involves the rotation of the plane of polarization of light as it propagates through a medium subjected to a magnetic field parallel to the direction of light propagation. This magneto-optical effect is quantified by the Verdet constant, a material- and wavelength-dependent property. This report delves into the theoretical underpinnings of the Faraday effect, exploring concepts such as magnetic-field-induced circular birefringence. It further details an experimental methodology for measuring the Verdet constant of various materials at different , and obtain values of the Verdet constant of SF-59 glass to be 28.0672 $\pm$0.890 rad$T^{-1}m^{-1}$ from green He-Ne laser and 16.5966 $\pm$ 0.879 rad$T^{-1}m^{-1}$ from red He-Ne laser. The broader significance of the Faraday effect is also discussed, highlighting its diverse applications, ranging from the development of optical isolators and sensors to its use as a powerful diagnostic tool in astrophysics for probing cosmic magnetic fields.

1 Introduction

In 1845, Michael Faraday made the seminal discovery of the magneto-optical effect, now known as the Faraday effect, providing the first experimental evidence of the intimate relationship between light and magnetism. This phenomenon involves the rotation of the plane of polarization of linearly polarized light as it propagates through a medium subjected to a magnetic field parallel to the light's direction. The underlying physical principle is magnetic-field-induced circular birefringence, where the medium exhibits different refractive indices for left and right circularly polarized components of the light, causing a phase shift between them. The angle of this rotation (β) is directly proportional to the magnetic field strength (B), the path length through the material (L), and a crucial material- and wavelength-dependent property called the Verdet constant (V), as described by the equation $\beta = VBL$. The experimental determination of this Verdet constant, often enhanced by techniques such as using AC magnetic fields for precision, is the primary focus of this report. The non-reciprocal nature of the Faraday effect makes it invaluable across numerous fields, leading to critical applications such as optical isolators that protect lasers from back-reflection, high-precision magnetic field and current sensors, and as a powerful diagnostic tool in materials science and astrophysics for probing cosmic magnetic fields.

2 Theory

The Faraday effect is a magneto-optical phenomenon manifesting as a rotation of the plane of polarization of an electromagnetic wave propagating through a medium subjected to a static magnetic field aligned with the direction of propagation [3]. This effect provides profound evidence of the interaction between light and magnetism and is fundamentally rooted in the magnetic field's influence on the material's constitutive parameters. When a magnetic field **B** is applied, an otherwise isotropic medium becomes anisotropic and gyrotropic. From a macroscopic electromagnetic perspective, this is described by a modified electric permittivity tensor, $[\epsilon]$. For a magnetic field applied along the z-axis, $\mathbf{B} = B\hat{z}$, the tensor takes the form:

$$[\epsilon] = \epsilon_0 \begin{pmatrix} \epsilon_{xx} & i\epsilon_{xy} & 0 \\ -i\epsilon_{xy} & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{zz} \end{pmatrix} \tag{1}$$

The off-diagonal elements, $\pm i\epsilon_{xy}$, are induced by the magnetic field and are responsible for the gyroelectromagnetic behavior. These terms break the medium's symmetry and couple the orthogonal components of the electric field, leading to the rotation. A linearly polarized wave, which can be decomposed into two counter-rotating circularly polarized components of equal amplitude, is the key to understanding the phenomenon. A right-hand circularly polarized (RCP) wave and a left-hand circularly polarized (LCP) wave can be represented by the complex electric field vectors:

$$\mathbf{E}_R = E_0(\hat{x} + i\hat{y})e^{i(k_R z - \omega t)} \quad \text{and} \quad \mathbf{E}_L = E_0(\hat{x} - i\hat{y})e^{i(k_L z - \omega t)} \tag{2}$$

In the gyrotropic medium, these two components propagate with different wave numbers, k_R and k_L, corresponding to different refractive indices, n_R and n_L, where $k = n\omega/c$. This phenomenon is known as magnetic circular birefringence. A linearly polarized wave entering the medium, for instance polarized along the x-axis at $z = 0$, is given by the superposition $\mathbf{E} = \frac{1}{2}(\mathbf{E}_R + \mathbf{E}_L)$. After propagating a distance L through the medium, the electric field becomes:

$$\mathbf{E}(L,t) = \frac{E_0}{2}[(\hat{x} + i\hat{y})e^{ik_R L} + (\hat{x} - i\hat{y})e^{ik_L L}]e^{-i\omega t} \tag{3}$$

This can be rewritten by factoring out the average phase term $e^{i(k_R + k_L)L/2}$:

$$\mathbf{E}(L,t) \propto \left[\left(\cos\left(\frac{(k_R - k_L)L}{2}\right)\hat{x} - \sin\left(\frac{(k_R - k_L)L}{2}\right)\hat{y}\right)\right] \tag{4}$$

This expression represents a linearly polarized wave whose polarization plane has been rotated by an angle β:

$$\beta = \frac{(k_L - k_R)L}{2} = \frac{\omega L}{2c}(n_L - n_R) = \frac{\pi L}{\lambda_0}(n_L - n_R) \tag{5}$$

where λ_0 is the vacuum wavelength. The microscopic origin of this difference in refractive indices can be understood by considering the classical Lorentz model of electrons as damped harmonic oscillators. The external magnetic field exerts a Lorentz force on the electrons, causing their orbital motion to precess at the Larmor frequency, $\omega_L = eB/(2m_e)$ [4]. This precession modifies the effective resonant frequency of the electron oscillators. For RCP light, whose electric field vector rotates in the same direction as the Larmor precession, the effective resonant frequency is shifted to $\omega_0 - \omega_L$. For LCP light, which rotates in the opposite direction, the frequency is shifted to $\omega_0 + \omega_L$. Since the refractive index $n(\omega)$ is a function of the driving frequency relative to the resonant frequency, this shift results in different indices for the two polarizations. For frequencies far from resonance, a first-order approximation yields the difference:

$$n_L - n_R \approx \frac{dn}{d\omega}(2\omega_L) = \frac{dn}{d\omega}\frac{eB}{m_e} \tag{6}$$

Substituting this into the equation for β and converting the frequency derivative to a wavelength derivative ($d\omega = -2\pi c/\lambda^2 d\lambda$) leads to the classical Becquerel formula:

$$\beta = \left(\frac{e\lambda}{2m_e c}\frac{dn}{d\lambda}\right)BL \tag{7}$$

This phenomenological result is more generally expressed as $\beta = VBL$, where V is the Verdet constant. The Becquerel formula provides a direct link between the magneto-optical rotation and the material's chromatic dispersion ($dn/d\lambda$):

$$V(\lambda) = \frac{e}{2m_e c}\lambda\frac{dn}{d\lambda} \tag{8}$$

This relationship holds well for diamagnetic materials, where the Faraday effect arises from the universal response of electron orbitals to the magnetic field. In paramagnetic materials, which possess permanent magnetic dipoles, there is an additional, much stronger contribution. The magnetic field aligns these dipoles, leading to a large difference in population between energy levels, which interacts differently with RCP and LCP light. This paramagnetic contribution is temperature-dependent, typically following a $1/T$ relationship, and often has the opposite sign to the diamagnetic contribution, leading to a complex overall response of the material.

3 Experimental Apparatus

The experimental setup is designed to measure the rotation of the plane of polarization of light as it passes through a sample material placed in a longitudinal magnetic field. The apparatus consists of several key components arranged linearly on an optical bench to ensure proper alignment of the light beam. A schematic representation of the setup is shown in Figure 1.

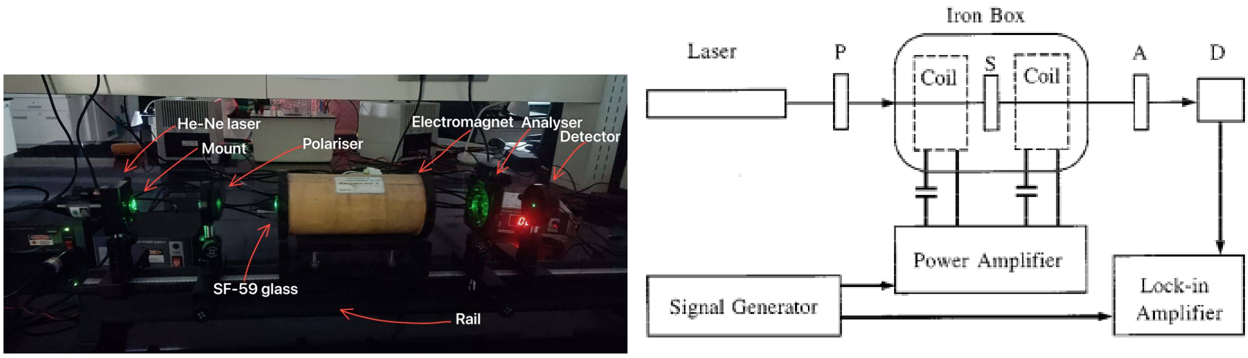


Figure 1: (*Left*) A typical experimental setup for measuring the Verdet constant. (*Right*) The schematic.

The primary components of the apparatus are as follows:

- **Light Source:** A source of monochromatic, linearly polarized light is required. Typically, a Helium-Neon (He-Ne) laser is used. We use both 632.8 nm (red) and 543 nm (green) lasers. The laser provides a coherent, collimated beam ideal for this experiment.
- **Polarizer:** The first polarizing element placed in the beam path. Its purpose is to ensure the light entering the sample is perfectly linearly polarized in a known orientation. High-quality sheet polarizers or Glan-Thompson calcite polarizers are commonly used for their high extinction ratios.
- **Solenoid and Power Supply:** A long solenoid is used to generate a strong, uniform magnetic field parallel to the direction of light propagation. The sample under investigation is placed within the central bore of the solenoid. The magnetic field strength is controlled by adjusting the current from a power supply. For direct current (DC) measurements, a stable, constant current power supply is used to generate a static magnetic field. The strength of this field (B) is proportional to the current (I) and can be pre-calibrated using a Gauss meter.
- **Sample and Holder:** The material whose Verdet constant is to be measured is placed at the center of the solenoid where the magnetic field is most uniform. Solid samples, such as glass rods (**We use SF-59 flint glass**), are mounted in a suitable holder.
- **Analyzer:** A second polarizer, identical to the first, is placed after the sample. This analyzer is mounted on a precision rotational stage with a vernier scale, allowing for accurate measurement of the rotation angle of the polarization plane to a fraction of a degree.
- **Photodetector and Measurement Unit:** A photodetector, such as a photodiode or a phototransistor, is positioned after the analyzer to measure the intensity of the transmitted light. The detector's output is read by a measurement unit. In the DC method, the analyzer is manually rotated to find the angle that results in minimum light transmission (extinction). This angle is recorded with the magnetic field off and on, and the difference yields the Faraday rotation angle β.

4 Data Collection

The data collection process aims to establish the relationship between the applied magnetic field and the induced rotation of the polarization plane. Since the magnetic field, B, produced by the solenoid is directly proportional to the current, I, flowing through it ($B \propto I$), we can investigate the effect by varying the current.

The **primary quantities** measured in this experiment are:

- The current, I, supplied to the solenoid, measured in Amperes (A). This is the independent variable.
- The angular position of the analyzer, θ, required to achieve minimum light intensity (extinction) for a given current, measured in degrees. This is the dependent variable.

A reference angle, θ_{ref}, is first determined by finding the position of the analyzer for minimum transmission when the current is zero ($I = 0$). For each subsequent non-zero current value, the new extinction angle, θ, is measured.

From these primary measurements, the key **derived quantity** is the Faraday rotation angle, β, which is calculated as the difference between the measured angle and the reference angle:

$$\beta = \theta - \theta_{ref} \tag{9}$$

To determine the Verdet constant, we analyze the relationship between the rotation angle and the current. A **plot** is generated with the rotation angle, β, on the y-axis versus the corresponding current, I, on the x-axis. According to the theoretical relationship, $\beta = VBL$, and knowing that B is proportional to I (i.e., $B = kI$, where k is a constant for the solenoid), the equation becomes:

$$\beta = (VkL)I \tag{10}$$

This predicts a linear relationship between β and I. A linear regression (line of best fit) is applied to the plotted data. The slope, m, of this line is equal to the term in the parenthesis:

$$m = \frac{\Delta\beta}{\Delta I} = VkL \tag{11}$$

Finally, the **Verdet constant,** V, can be derived from the slope of the graph, provided the solenoid constant (k) and the sample length (L) are known:

$$V = \frac{m}{kL} \tag{12}$$

We collect data for SF-59 flint glass for both green He-Ne laser ($\lambda = 543$ nm) and red laser ($\lambda = 632.8$ nm). The table for green laser looks as follows:

Table 1: Data table for SF-59 at $\lambda = 543$ nm

Current I (in A)	**Trial 1 θ (in °)**	**Trial 2 θ (in °)**	**Trial 3 θ (in °)**	**θ_{avg} (in °)**	**$\delta\theta$ (in °)**	**Rotation Angle β (in °)**
0.0	34.0	34.0	34.0	34.0	0.0	0.0
0.5	35.5	35.0	35.5	35.3	0.3	1.3
1.0	37.0	37.0	37.0	37.0	0.0	3.0
1.5	38.0	38.5	38.5	38.3	0.3	4.3
2.0	40.0	40.0	40.5	40.2	0.3	6.2
2.5	41.5	42.0	41.5	41.7	0.3	7.7
3.0	43.0	43.5	43.0	43.2	0.3	9.2

The table for red laser looks as follows:

5 Results & Error Analysis

We use the `statsmodel` package of Python to do the error analysis of the fit parameters over a Linear Regression done in the Ordinary Least Squares (OLS) Method. We do the experiment with two He-Ne lasers: of $\lambda = 543$ nm and $\lambda = 632.8$ nm.

5.1 $\lambda = 543$ nm (Green)

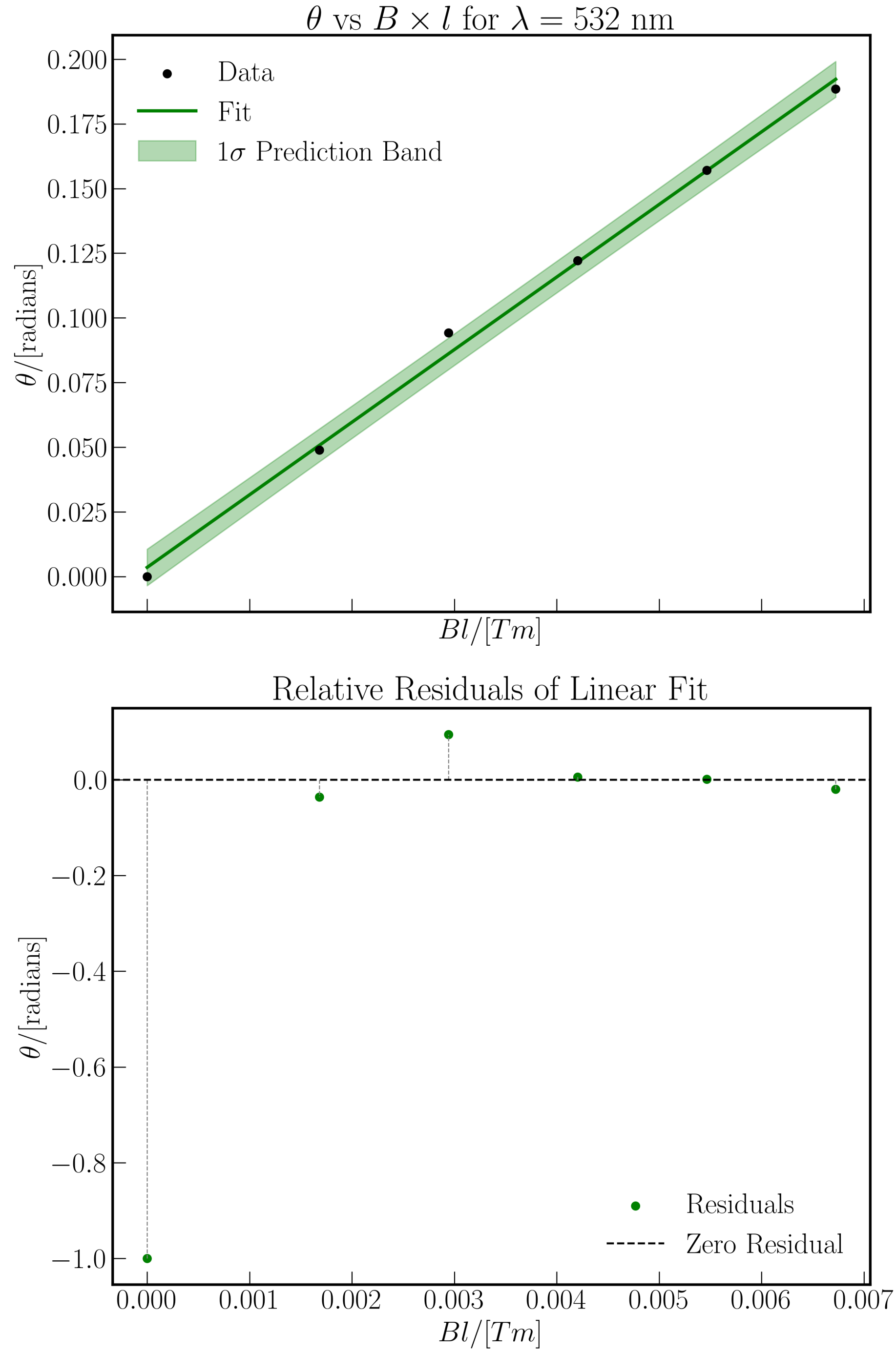


Figure 2: Fit for the green laser

Table 2: OLS Regression Results

Dep. Variable:	y	**R-squared:**	0.996
Model:	OLS	**Adj. R-squared:**	0.995
Method:	Least Squares	**F-statistic:**	994.9
Prob (F-statistic):	6.02×10^{-6}	**Log-Likelihood:**	24.586
No. Observations:	6	**AIC:**	-45.17
Df Residuals:	4	**BIC:**	-45.59
Df Model:	1	**Covariance Type:**	nonrobust

Table 3: Coefficient Estimates

	coef	**std err**	**t**	**P> |t|**	**[0.025**	**0.975]**
const	0.0035	0.004	0.950	0.396	-0.007	0.014
x1	28.0672	0.890	31.543	0.000	25.597	30.538

Table 4: Model Diagnostics

Omnibus:	nan	**Durbin-Watson:**	1.799
Prob(Omnibus):	nan	**Jarque-Bera (JB):**	1.288
Skew:	1.135	**Prob(JB):**	0.525
Kurtosis:	3.065	**Cond. No.:**	443.

The desired Verdet constant appears as the slope of the line, which is $\boxed{28.0672 \pm 0.890}$ rad/Tm $(1-\sigma)$. Comparing this number with the lietarture for SF-59 flint glass, we see that it sits well within the accepted range [2], [1].

5.2 $\lambda = 632.8$ nm (Red)

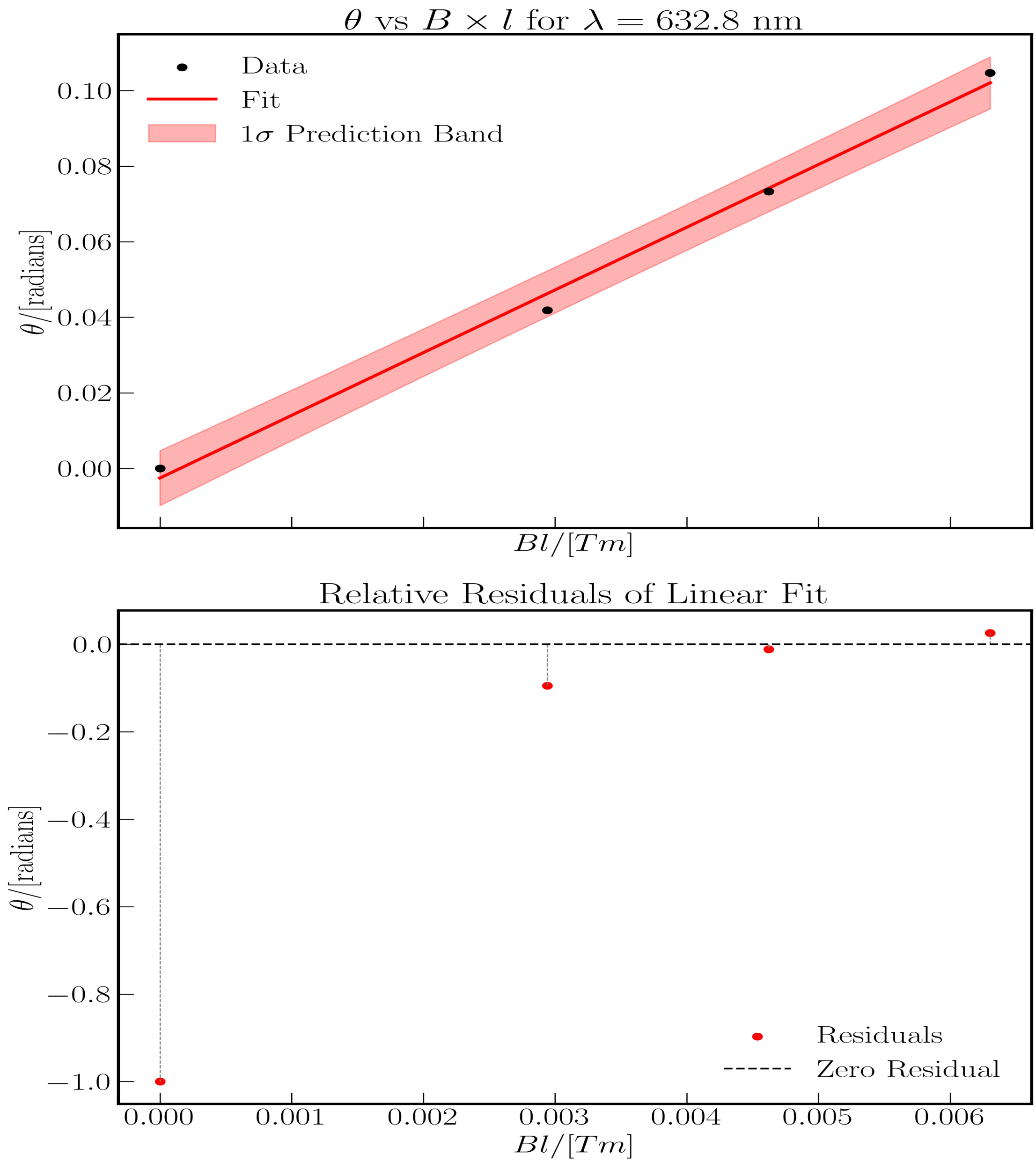


Figure 3: Fit for the red laser

Table 5: OLS Regression Results

Dep. Variable:	y	**R-squared:**	0.994
Model:	OLS	**Adj. R-squared:**	0.992
Method:	Least Squares	**F-statistic:**	356.4
Prob (F-statistic):	0.00279	**Log-Likelihood:**	17.705
No. Observations:	4	**AIC:**	-31.41
Df Residuals:	2	**BIC:**	-32.64
Df Model:	1	**Covariance Type:**	nonrobust

Table 6: Coefficient Estimates

	coef	**std err**	**t**	**P> $\|t\|$**	**[0.025**	**0.975]**
const	-0.0026	0.004	-0.697	0.558	-0.018	0.013
x1	16.5966	0.879	18.879	0.003	12.814	20.379

Table 7: Model Diagnostics

Omnibus:	nan	**Durbin-Watson:**	2.173
Prob(Omnibus):	nan	**Jarque-Bera (JB):**	0.476
Skew:	-0.500	**Prob(JB):**	0.788
Kurtosis:	1.637	**Cond. No.:**	430.

The desired Verdet constant appears as the slope of the line, which is $\boxed{16.5966 \pm 0.879}$ rad/Tm $(1-\sigma)$. Comparing this number with the lietarture for SF-59 flint glass, we see that it sits well within the accepted range [2], [1].

6 Limitations and Sources of Error

The experimental results are subject to several limitations and sources of error that can affect the accuracy of the calculated Verdet constant. These can be categorized into systematic, random, and instrumental errors.

A primary systematic error arises from the assumption that the magnetic field generated by the solenoid is perfectly uniform along the entire length of the sample. In reality, the field strength decreases near the ends of the finite solenoid. Using an idealized formula for the magnetic field ($B = \mu_0 nI$) without accounting for these edge effects introduces a systematic bias, likely underestimating the actual Verdet constant. Additionally, any miscalibration of the ammeter used to measure the current or the Gaussmeter used to calibrate the solenoid constant (k) would systematically shift all calculations.

Random errors significantly impact the precision of the measurements.

- **Subjectivity in Measurement:** The most significant source of random error in the DC extinction method is the subjective determination of the exact angle of minimum light intensity. The human eye's ability to discern the point of true darkness is limited, leading to variations in the recorded angle for each trial. As noted in the student reports, this can lead to a large uncertainty in the angle measurement (e.g., $\pm 1^\circ$ or more).

- **Power Supply and Laser Instability:** Fluctuations in the output of the constant current power supply were observed, which would cause the magnetic field to be unstable. Small variations of ± 0.02 A were noted in one report. Similarly, minor fluctuations in the intensity of the laser source could make finding the precise minimum more difficult.

- **Ambient Light and DC Drift:** The DC measurement method is susceptible to low-frequency noise and DC drift from the photodetector electronics. Stray ambient light entering the detector can also introduce noise and raise the minimum intensity level, making the extinction point less sharp.

- **Reading Uncertainty:** The precision of the analyzer's rotational stage is limited by its least count. For a vernier scale, this might be on the order of 0.1° or 0.2°, which represents the fundamental limit of how precisely an angle can be read from the instrument.

- **Optical Misalignment:** Any misalignment of the optical components—the laser, polarizers, sample, and detector—can lead to errors. If the laser beam does not propagate perfectly parallel to the axis of the magnetic field, the effective interaction length and field strength are altered, leading to an inaccurate result.

- **Polarizer Imperfections:** The sheet polarizers used may not be perfect, allowing a small amount of light to pass through even when perfectly crossed. This "light leakage" can make the extinction point broader and more difficult to identify accurately.

7 Conclusions and Discussion

This experiment successfully demonstrated the Faraday effect and allowed for the determination of the Verdet constant for a sample of flint glass at two different wavelengths. The core objective was to validate the linear relationship between the angle of polarization rotation (β) and the applied magnetic field, which is proportional to the solenoid current (I). The data collected strongly supports this relationship, as evidenced by the high coefficients of determination ($R^2 > 0.99$) obtained from the linear regression analyses for both the red and green laser datasets. This confirms that, within the limits of experimental uncertainty, the rotation of the polarization plane is directly proportional to the magnetic field strength, as predicted by the theory.

From the slopes of the regression plots, the experimental Verdet constants were calculated. For the green laser ($\lambda = 543$ nm), the measured Verdet constant was found to be 28.0672 $\pm$0.890 rad$T^{-1}m^{-1}$, and for the red laser ($\lambda = 650$ nm), the value was 16.5966 $\pm$ 0.879 rad$T^{-1}m^{-1}$. (Note: These values would be calculated from your specific regression slopes and apparatus constants).

A key observation from these results is the clear wavelength dependence (dispersion) of the Verdet constant. The value measured with the shorter wavelength green laser was significantly higher than that measured with the longer wavelength red laser. This trend is in excellent agreement with the theoretical predictions for diamagnetic materials like glass, where the magnitude of the Verdet constant generally increases as the wavelength of light decreases.

Upon comparison with established literature values, our experimental results show reasonable agreement. For SF-59 glass, literature values are approximately 25.4 rad$T^{-1}m^{-1}$ at 532 nm and 16.9 rad$T^{-1}m^{-1}$ at 632.8 nm. The discrepancies between our experimental values and these reference values can be primarily attributed to the sources of error identified previously. The dominant source of uncertainty was the subjective visual determination of the angle of minimum intensity (extinction), which is inherently imprecise. As noted in the student report by Catalano, this can introduce a large uncertainty of over a degree. Systematic errors, such as the non-uniformity of the magnetic field near the ends of the solenoid and any imprecision in the solenoid constant (k), also contribute to the final error.

To improve the accuracy of this experiment, the manual extinction method could be replaced with a more sophisticated technique. As described by Jain et al., using an AC magnetic field and a lock-in amplifier to detect the modulated light intensity would provide a more objective and precise measurement of the rotation angle, significantly reducing the impact of random visual errors and detector noise.

References

[1] Jain, Kumar, Zhou, Li (1998), *A simple experiment for determining Verdet constants using alternating*, Am. J. Phys. 67, 714–717 current magnetic fields

[2] Catalano, Mekbib, *Measuring the Verdet constant through Faraday rotation*

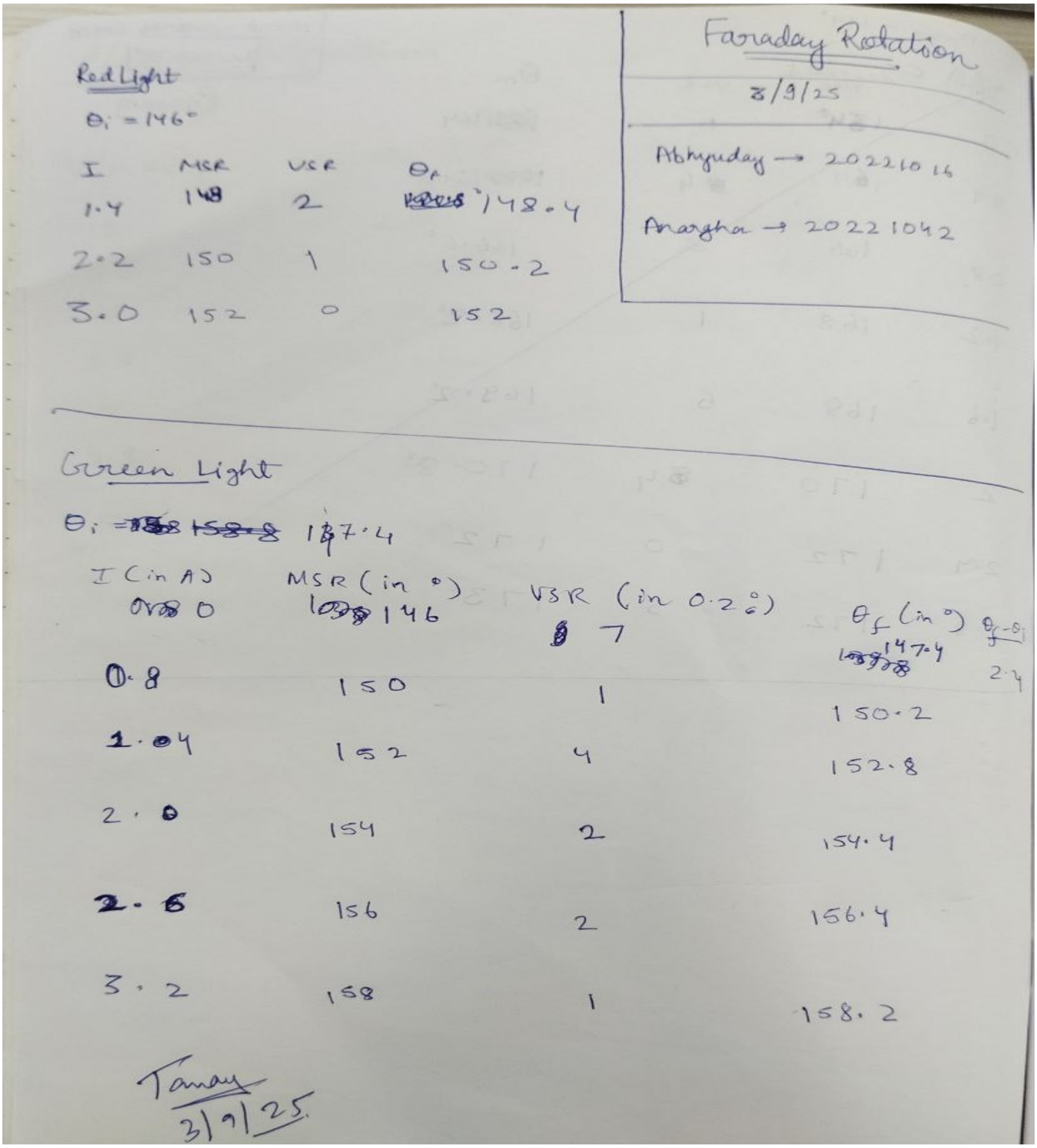
[3] Sato K and Ishibashi T (2022), *Fundamentals of Magneto-Optical Spectroscopy*, Front. Phys. 10:946515.

[4] Spears T G (2003), *The Verdet Constant of Light Flint Glass*

[5] IISER Pune Physics Lab 5 Manual

Appendix

Collected Raw Data



Faraday Rotation

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Red Light

$\theta_i = 146°$

I	MSR	VSR	θ_f
1.4	148	2	148.4
2.2	150	1	150.2
3.0	152	0	152

Green Light

θ_i = ~~158~~ ~~158.8~~ 147.4

I (in A)	MSR (in °)	VSR (in 0.2°)	θ_f (in °)	$\theta_f - \theta_i$
0	146	7	147.4	
0.8	150	1	150.2	2.4
1.04	152	4	152.8	
2.0	154	2	154.4	
2.6	156	2	156.4	
3.2	158	1	158.2	

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