Gravitational Lensing in Astrophysics

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Abstract

Gravitational lensing is a powerful astrophysical phenomenon, predicted by Einstein's theory of General Relativity. This report presents a broad overview of this subject, based on a more comprehensive presentation given in class. An intriguing problem and approaches to solve it are also discussed. The report concludes with future directions for gravitational lensing.

1 Introduction

Although this discovery of the phenomenon was made only in the last century, the possibility that there could be such a deflection had been suspected much earlier, by Newton and Laplace among others. Luminaries in the field, including both theoretical physicists and astrophysicists, have made important contributions to the rich mathematical framework that we know today. The power of the framework is further enhanced by the fact that we have many observational followups to refine and test our theoretical predictions.

2 Framework

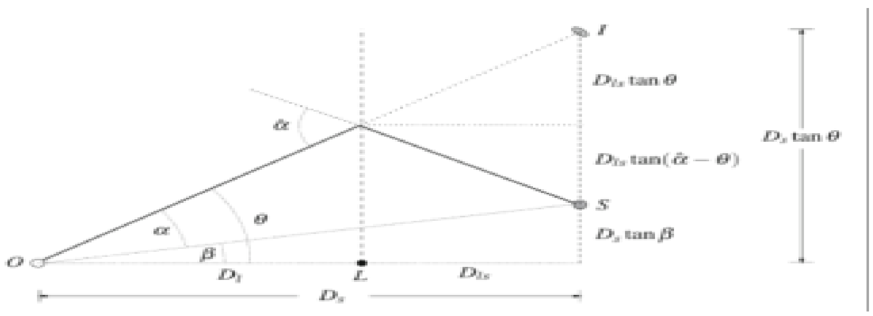

Figure 1: Geometric angle-chasing leads to the lens equation

Assumptions:

- Locally flat spacteime
- $|\Phi| << c^2$
- All the machinery of geometric optics is carried over

The fundamental calculation is the bending angle, $\hat{\alpha}$, given by the formula:

$$\hat{\alpha} = \frac{4GM}{c^2\xi}$$

where G is the gravitational constant, M is the mass of the lens, c is the speed of light, and ξ is the impact parameter (the closest distance the light ray passes to the lens).

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This leads to the "Lens Equation," which relates the true position of the source ($\vec{\beta}$) to the observed position of the image(s) ($\vec{\theta}$). This is given in its vector form as:

$$\boxed{\vec{\theta} - \vec{\beta} = \vec{\hat{\alpha}}(\vec{\theta})}$$

The important observables that furnish a complete description of the system are the magnification (μ) and the time delay function. The goal is to predict the image properties for various lens models. The more physically relevant ones that we have discussed in detail are the point mass lens and the singular isothermal sphere (and some of its modifications). These models can be described completely in terms of the **Einstein radius** θ_E and the **projected mass density** Σ_{crit}, related as follows:

$$M(< \theta_E) = \pi \theta_E^2 D_L^2 \Sigma_{\text{crit}}$$

3 Applications

Strong lensing by galaxies occurs when the projected surface mass density of the lens exceeds the critical density, enabling the formation of multiple images, arcs, and Einstein rings of background sources [10]. Galaxies are well described by the Singular Isothermal Sphere (SIS) model, whose constant velocity dispersion produces a constant deflection angle and an Einstein radius of order an arcsecond for massive ($\sim 10^{11} M_\odot$) galaxies at cosmological distances. Sources located within approximately θ_E of the optical axis experience large magnifications and strong distortions, often producing two images with opposite parities or a full Einstein ring in the case of perfect alignment. Strong lens systems allow accurate mass measurements of galaxy interiors, independent of light, and act as "cosmic telescopes" that magnify faint distant galaxies. The image geometry and measured time delays between lensed images of variable sources additionally provide constraints on the Hubble constant and the distribution of dark matter.

Microlensing occurs when a compact object—typically a star, stellar remnant, or planetary system—acts as a point-mass gravitational lens, producing no resolvable multiple images but instead a time-varying magnification of a background source. The characteristic angular scale is the Einstein radius, which for stellar lenses is of order milliarcseconds, making image splitting unobservable; the observable signature is thus a symmetric, achromatic brightening with a timescale $t_0 \sim D_d \theta_E / v$, ranging from hours to years depending on the lens mass and geometry [3]. Because the magnification depends sensitively on the lens–source alignment, deviations from the standard single-lens light curve can reveal planetary companions or binary lenses via caustic crossings. Microlensing therefore provides mass-dependent variability without relying on the lens's luminosity, enabling the detection of otherwise invisible objects such as isolated black holes, faint M-dwarfs, and cold exoplanets.

4 Science with Lensing

Gravitational lensing has developed into a precision tool for astrophysics, enabling a wide range of scientific discoveries that are independent of the luminosity or composition of the lens. In the context of **microlensing**, the point-mass lens regime has led to direct detections of compact, otherwise invisible objects in the Milky Way. Photometric microlensing surveys such as MACHO [4], OGLE [5], EROS [6], and DUO [7] demonstrated that stellar-mass lenses can be identified through their symmetric, achromatic light curves, leading to measurements of the Galactic microlensing optical depth and constraints on the contribution of MACHOs to the dark matter halo. Microlensing has also revolutionized exoplanet studies: planetary companions are revealed as short-lived anomalies in the light curve, enabling the discovery of cold, low-mass planets beyond the snow line—including sub-Neptune planets and Jupiter/Saturn analogs—in regimes inaccessible to the transit or radial-velocity methods [8]. The next generation of surveys, particularly the *Roman Space Telescope*, will detect thousands of microlensing planets and provide astrometric microlensing measurements to determine individual lens masses, including isolated black holes [9].

Strong lensing by galaxies enables complementary science on much larger mass scales. Observations of strong lenses—from early systems like QSO 0957+561 to modern samples from HST surveys such as SLACS—have yielded accurate measurements of galaxy mass profiles, evidence for dark matter on kiloparsec scales, and magnified views of distant galaxies acting as "cosmic telescopes"[10]. Time delays between multiply imaged quasars provide an independent method to infer the Hubble constant, while detailed modeling of arcs and rings constrains the substructure content of dark matter halos [11]. Future wide-field optical surveys, especially LSST and *Euclid*, are expected to increase the known sample of strong lenses by orders of magnitude, enabling high-precision cosmology and detailed studies of dark matter through lens statistics, time delays, and resolved Einstein rings.

5 Interesting Problem

The objective of the lens equation (along with the magnification) is to describe the parity, nature and position of the images. **What if we pose the inverse problem, i.e., we want to study the source properties by delensing the lensed signal.** This problem is in fact, more sophisticated and harder to tackle than the 'forward' problem, since unlike passive measurements of lensing effects, delensing is an active reconstruction technique that requires precise knowledge or estimation of the mass distribution along the line of sight [13]. However, it is a well-posed problem, and could be of much importance. Several ways to tackle this problem has already been proposed.

A direction that has been pursued with interest is CMB delensing [12] [13] [15]. although this is more a focus of cosmology than astrophysics, but it has yielded fruitful results for suture applications in astrophysics such as probing the inner structure of the accretion discs of black holes [14].

Delensed CMB spectra have sharper acoustic peaks and more prominent damping tails, allowing for improved inferences of cosmological parameters that impact those features. Delensing reduces B-mode power, aiding the search for primordial gravitational waves and allowing for lower variance reconstruction of lensing and other sources of secondary CMB anisotropies. Lensing-induced power spectrum covariances are reduced by delensing, simplifying analyses and improving constraints on primordial non-Gaussianities. Biases that result from incorrectly modeling nonlinear and baryonic feedback effects on the lensing power spectrum are mitigated by delensing. All of these benefits are possible without any changes to experimental or survey design [12].

This problem can be pursued in a myriad ways. Neural network delensing methods have proven more fruitful than the traditional quadratic estimator analysis [15]. Another approach is using denoising diffusion models in performing Bayesian lensing reconstruction [16]. Yet other methods invoke non-parametric lens inversion techniques [17]. As is evident, the problem almost singularly deals with weak (de-)lensing of the CMB, yet this technique in general can have astrophysical applications, as mentioned above [14].

6 Future Directions

The coming decade will dramatically expand the scientific impact of both strong lensing and microlensing. Wide-field surveys such as LSST and *Euclid* will increase the known strong-lens population by orders of magnitude, enabling precise measurements of galaxy mass profiles, dark-matter substructure, and cosmological distances through time-delay lensing. High-resolution follow-up with JWST and future space telescopes will convert many of these systems into detailed probes of baryonic and dark matter physics across cosmic time.

Microlensing will see an equally significant leap. While current surveys like OGLE and MOA have revealed exoplanets, stellar remnants, and free-floating planets, the *Roman Space Telescope* will detect thousands more and provide astrometric mass measurements of isolated compact objects.

Across both domains, AI-assisted discovery and automated de-lensing will be essential. Machine-learning techniques will process billions of sources, identify strong-lens morphologies, and track microlensing variability in real time. Combined with next-generation observatories, these advances position gravitational lensing to deliver major breakthroughs in dark-matter physics, galaxy evolution, and exoplanet science.

Due to brevity constraints, we have not digressed into a discussion on lensing of *gravitational waves*, although they are a very strong frontier for research in the future. We have also refrained from discussing *weak lensing*, an already well-established paradigm for cosmological studies. Important work is being done in these fields to constrain cosmological parameters, distance measurements and models of large-scale structure formation.

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