Cosine-Weighted Discrete Ordinates Selection For RPV Inversion

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abstract

Cosine-Weighted hemisphere sampling is a well-known technique for radiation tracking, in order to eliminate the cosine factor of irradiance over radiance to have a uniformly distributed irradiance during tracking. This technique is specifically designed for Monte-Carlo sampling to reduce the noise. However, for the equally widely-adapted discrete-ordinate flux tracking technique, a uniformly distribution of the sampling is classically prefered. The objective of this report is to find a way to adapt the cosine weighted distribution to discrete ordinates method for the unity that combine these two techniques.

DART directions

DART combine exact kernel and discrete ordinate techniques for solving the transport equation. They discretize the 4π space into a finite number of angular sectors, with directions along which radiation propagates. The latest direction discretization technique generates a set of uniformly distributed directions with equal solid angles and well-defined shapes. Zenith (θ) and azimuth (ϕ) boundaries are quantified, and they are adjoint to each other. Table 1 gives the attributes of any direction (Ω, $\Delta\Omega$).

The center of the solid angle the center of mass: it devides the solid angle (Ω, $\Delta\Omega$) into 2 equal solid angles ($\Delta\Omega/2$). The center of (Ω, $\Delta\Omega$) is defined by:

$$\theta_c = \arccos(\cos\theta_0 - \Delta\cos\theta/2) = \arccos(\frac{\cos\theta_0 + \cos\theta_1}{2}) \tag{1}$$

$$\phi_c = \frac{\phi_0 + \phi_1}{2} \tag{2}$$

and the solid angle is calculated as:

$$\Delta\Omega = \Delta\cos\theta \times \Delta\phi \tag{3}$$

Table 1: Attributes of direction unit

symbol	expression	representation
θ_0		The upper θ boundary of Ω
θ_1		The lower θ boundary of Ω
ϕ_0		The left ϕ boundary of Ω
ϕ_1		The right ϕ boundary of Ω
θ_c	Eqa.(1)	The θ of the center of Ω
ϕ_c	Eqa.(2)	The ϕ of the center of Ω
$\Delta\theta$	$\theta_1 - \theta_0$	The difference between the θ boundary values
$\Delta\phi$	$\phi_1 - \phi_0$	The difference between the ϕ boundary values
$\Delta\cos\theta$	$\cos\theta_0 - \cos\theta_1$	The difference between the $\cos\theta$ boundary values
$\Delta\Omega$	Eqa.(3)	The solid angle of Ω

The zenith angles of the directions are equally spaced. It leads to several layers, with different number of directions within each layer. Another condition is used in order to ensure the shape of solid angles is more or less a square: the within solid angle vertical and horizontal arc that pass through the direction center (θ_c, ϕ_c) have equal lengths. It is:

$$\Delta\theta = \sin\theta_c \Delta\phi \tag{4}$$

Substitute Eqa.(4) into Eqa.(3), we get:

$$\Delta\Omega = (\cos\theta_0 - \cos\theta_1)\frac{\Delta\theta}{\sin\theta_c}$$

If $\Delta\theta$ is determined, the corresponding layer can be generated. Assume the superscript l to be the layer index, the total solid angle of a layer is:

$$\Delta\Omega^l = (\cos(\theta_0^l) - \cos(\theta_1^l)) \times 2\pi$$

Therefore, the number of directions n in layer l is:

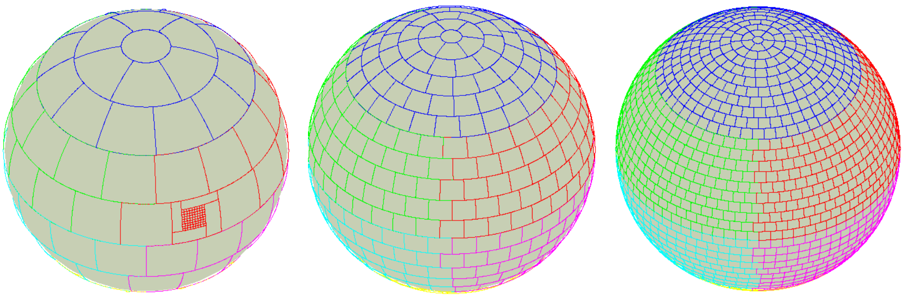
$$n^l = \frac{2\pi \times \sin\theta_c}{\Delta\theta^l}$$

Let N be the total number of directions, the probability of a direction located in layer l is:

$$P^l = \frac{2\pi \times \sin\theta_c}{\Delta\theta^l \times N}$$

This expression can be converted to an average probability density $(p(\theta_c))$ of layer l in terms of θ, it gives:

$$p(\theta_c)^l = \frac{2\pi \times \sin\theta_c}{\Delta\theta^l \times \Delta\theta^l \times N} \tag{5}$$



(a) 100 directions with region oversampling

(b) 500 directions

(c) 2000 directions

Figure 1: 3D spherical distribution of directions for 100, 500 and 2000 directions

We have:

$$p(\theta) \propto \sin\theta \tag{6}$$

Within each layer, the solid angle of each direction is equal, so the ϕ is uniformly distributed. The resultant DART direction is shown in Figure 1:

As $N \to 10^6$, the histogram of direction distribution of upper hemisphere over θ is shown in Figure 2, which is corresponding to Eqa.(6).

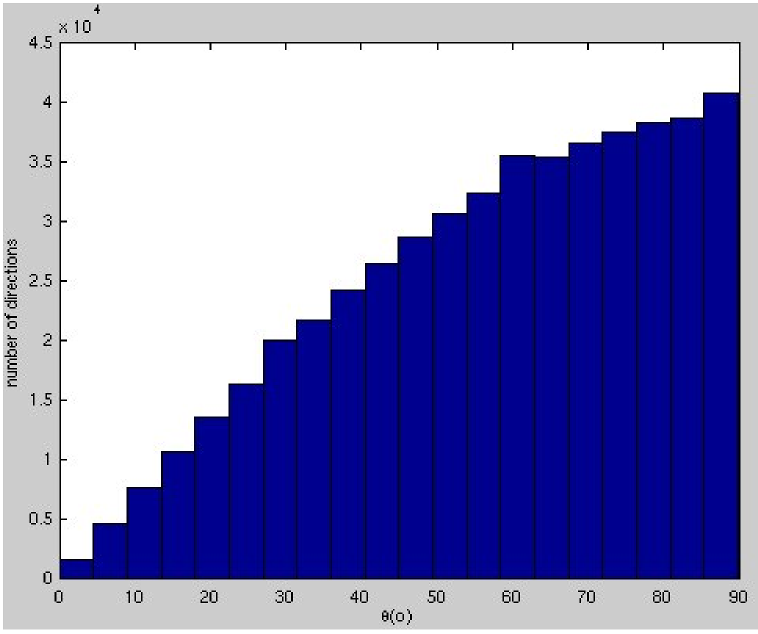


Figure 2: The histogram of 5×10^5 DART directions over upper hemisphere

Cosine-Weighted Sampling

The cosine-weighted hemisphere sampling method provides a 3D distribution of rays. The angle density of which on the sphere is proportional to that of a Lambertian surface. The energy that a Lambertian surface has in a solid angle $d\Omega = sin\theta d\theta d\phi$ is $\frac{1}{2}Lsin(2\theta)d\theta d\phi$, where L is the surface radiance. The classical random method for obtaining the rays (i.e., directions) is shown below:

1. Choose 2 random number $r1$, $r2$ in the range (0, 1).
2. $\phi(r1) = 2\pi \times r1$
3. $\theta(r2) = arccos(\sqrt{r2})$

It implies that ϕ is uniformly distributed in the 2π space. θ is not uniformly distributed. The probability density function (pdf) of $r2$ is a constant equals 1 ($f_X(r2) = 1$). According to the "change of variable" rule of pdf:

$$\begin{aligned} f_Y(\theta) &= |\frac{d}{d\theta}r2(\theta)| \times f_X(r2(\theta)) \\ &= |\frac{d}{d\theta}(cos^2(\theta))| \\ &= |-2cos(\theta)sin(\theta)| \\ &= sin(2\theta) \end{aligned} \tag{7}$$

Figure 3 shows a histogram of 5×10^5 cosine-weighted sampling, which corresponds to Eqa. (7).

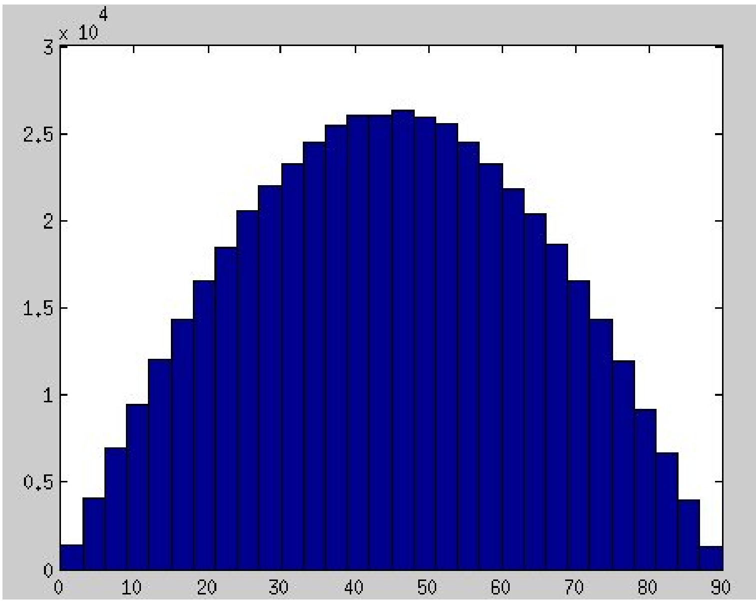


Figure 3: The histogram of 5×10^5 cosine-weighted sampling of upper hemisphere

Generation of cosine-weighte discrete ordinates

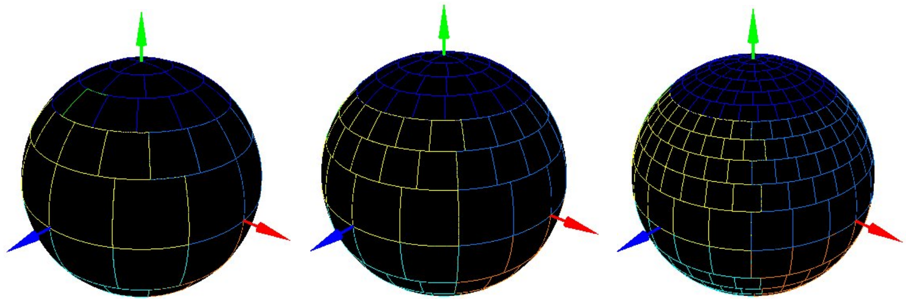
Cosine-weighted discrete ordinates can be difined through Eqa.(5) and Eqa.(7). The only parameter that is flexible is $\Delta\theta$. It means that $\Delta\theta$ must be such that the density of DART directions equals the probability density of cosine-weighted sampling. We must have:

$$(\Delta\theta^l)^2 \propto \frac{1}{cos(\theta_c^l)} \tag{8}$$

which means for every layer of the sphere of directions, the term $(\theta_1-\theta_0)^2 \times \frac{cos(\theta_0)+cos(\theta_1)}{2}$ must be considered. We can reduce the computation complexity with the approximation $(\theta_1-\theta_0)\times\frac{cos(\theta_0)+cos(\theta_1)}{2} = sin(\theta_1)-sin(\theta_0)$. If the total number of layers is L, we have $L-1$ equations with $L-1$ unknowns (subscript as the index):

$$\begin{cases} sin(\theta_1)\times\theta_1 = (sin(\theta_2)-sin(\theta_1))*(\theta_2-\theta_1) \\ (sin(\theta_2)-sin(\theta_1))*(\theta_2-\theta_1) = (sin(\theta_3)-sin(\theta_2))*(\theta_3-\theta_2) \\ \quad . \\ \quad . \\ (sin(\theta_{L-2})-sin(\theta_{L-3}))*(\theta_{L-2}-\theta_{L-3}) = (sin(\theta_{L-1})-sin(\theta_{L-2}))*(\theta_{L-1}-\theta_{L-2}) \\ (sin(\theta_{L-1})-sin(\theta_{L-2}))*(\theta_{L-1}-\theta_{L-2}) = (1-sin(\theta_{L-1}))*(\pi/2-\theta_{L-1}) \end{cases} \tag{9}$$

There is no simple analytical solution to the above equation set. However, the Newton-Raphson Method for Nonlinear Systems of Equations gives fairly good and fast estimation of the roots. Details can be found from Numerical Recipes (C++) 3rd Edition (2007). Figure 4 shows the resultant cosine-weighted discrete ordinate directions. . Figure 5 is the histogram of 431464 cosine-weighted discrete ordinates in the upper sphere. It samples very well to the histogram of the consine-weighted directions that are obtained by the random pulling method (Figure 5).



(a) 110 directions with region oversampling

(b) 216 directions

(c) 528 directions

Figure 4: The 3D spherical distribution of directions for cosine-weighted 110, 216 and 528 ordinates

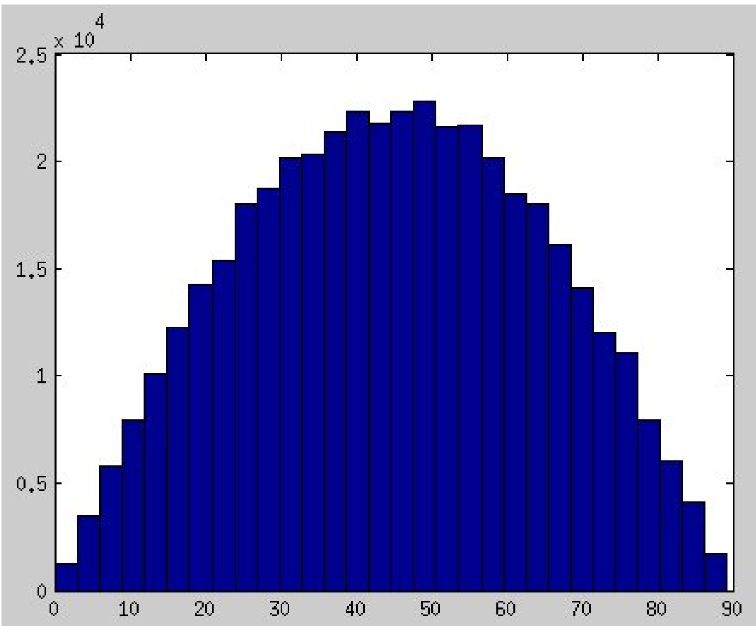


Figure 5: The histogram of 431464 cosine-weighted directions in the upper sphere

Approach RPV parameter fitting

The cosine-weighted discrete ordinates have some advantages.

1. It generate cosine-weighted angle adapted to flux-tracking and sampling mode.

2. User can define the approximate number of directions.

3. With limited number of samplings, it provides a better distribution than a random sampling, which is more suitable for the parameter fitting purpose.

Since a single DART simulation provides N radiance (reflectance) values for N viewing directions for a single sun direction. We suggest to generate 30 cosine-weighted discrete ordinate within upper hemisphere to be used as sun directions. For each sun direction, a DART simulation gives the radiance (reflectance) for 100 discrete ordinates. Then, an RPV fitting technique cam be applied to the 30×100 reflectance values.

References

A new approach of direction discretization and oversampling for 3D anisotropic radiative transfer modeling. Tiangang Yin, Jean-Philippe Gastellu-Etchegorry, Nicolas Lauret, Eloi Grau, Jeremy Rubio. Remote Sensing of Environment. Volume 135, August 2013, Pages 213–223