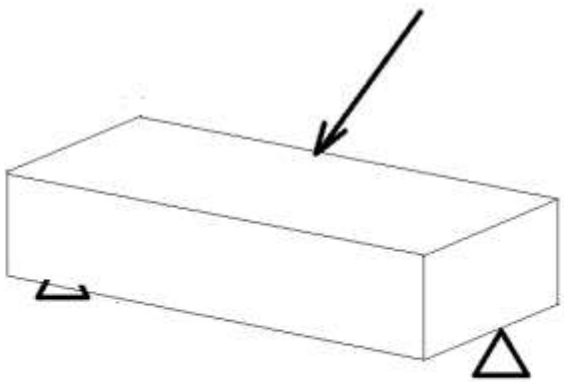
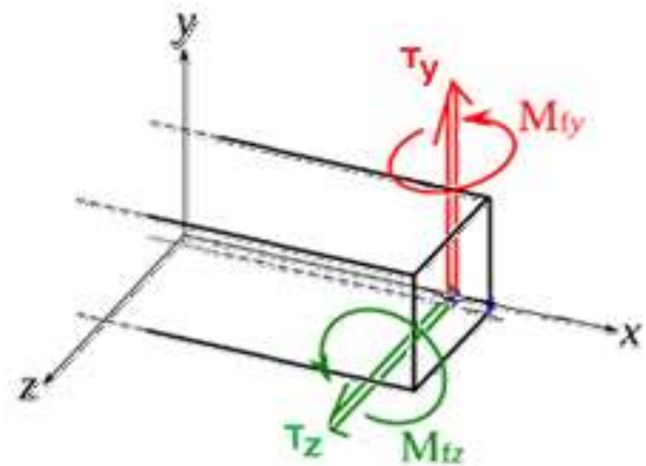
FLEXION DEVIEE

FLEXION DEVIEE

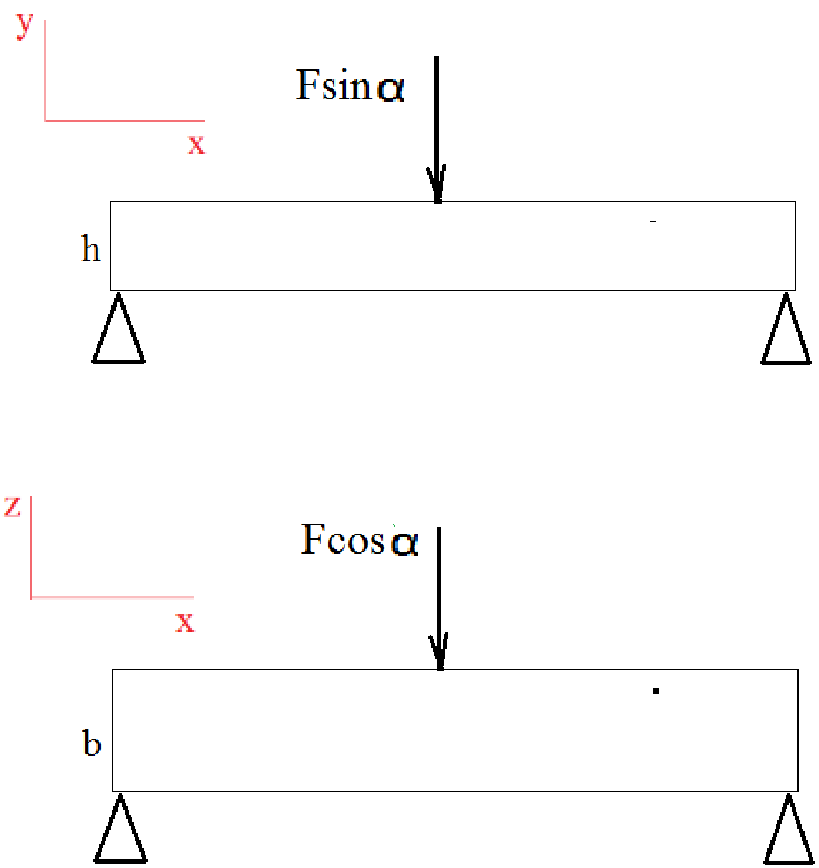
La flexion déviée est définie comme une combinaison de deux flexions planes, si les charges sont appliquées aux axes principaux (deux composantes My et Mz avec deux efforts tranchants Ty et Tz). L'étude de la flexion déviée revient à décomposer les sollicitations en deux flexions planes suivant les plans principaux.

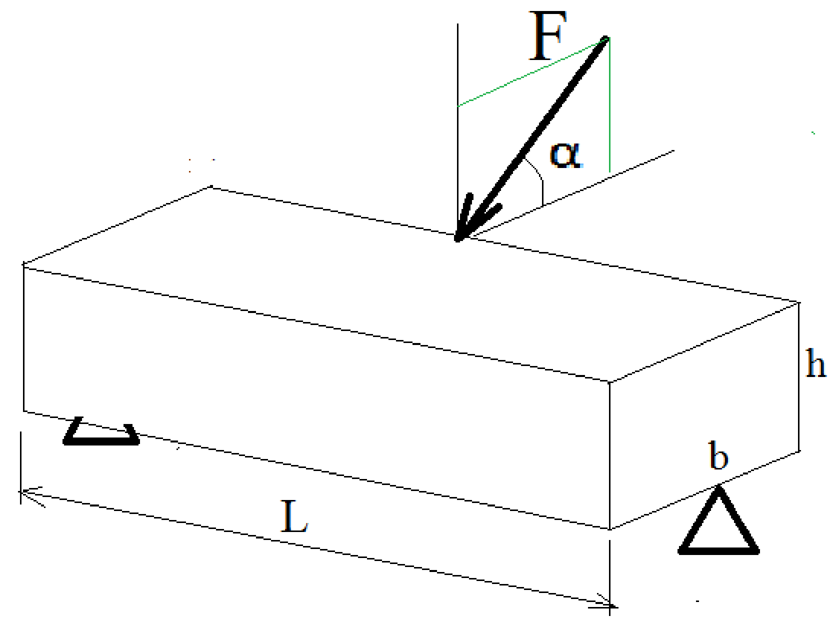




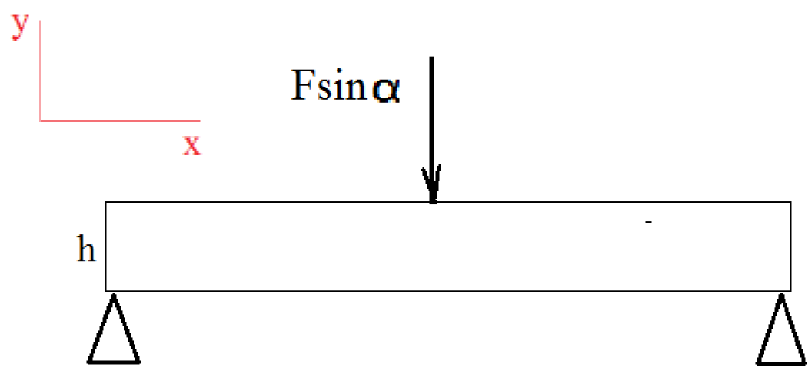
Flexion déviée = flexion plane/xoy + flexion plane/xoz.

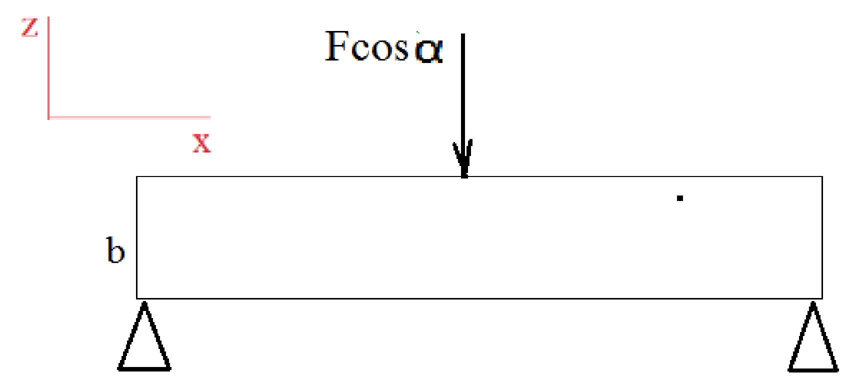
FLEXION DEVIEE

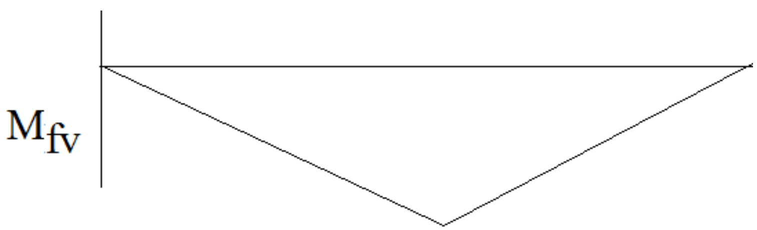


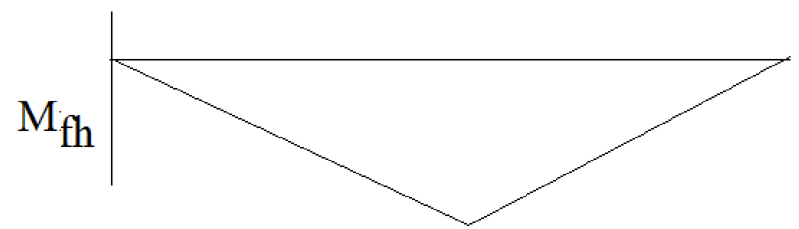


FLEXION DEVIEE

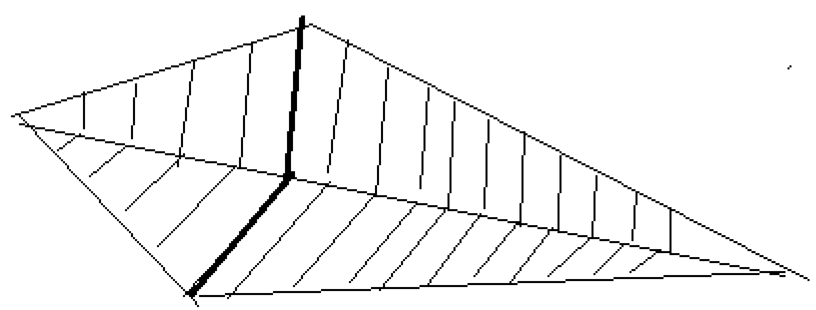




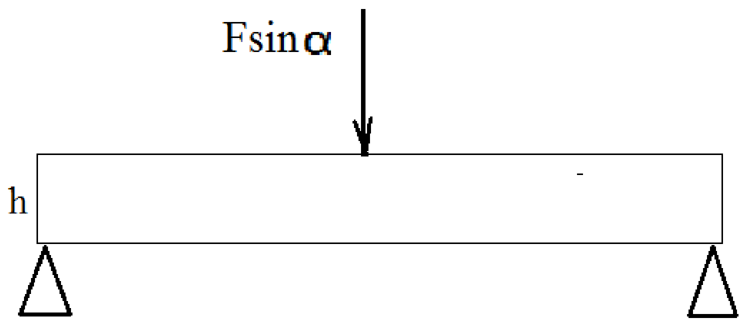


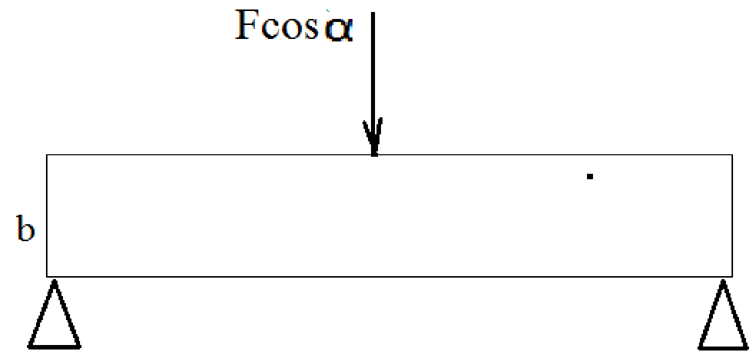


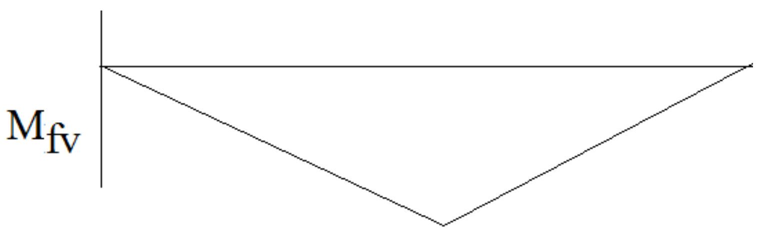
$$M_f = \sqrt{M_h^2 + M_v^2}$$

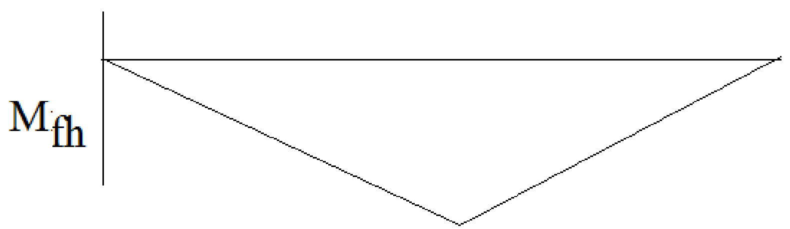


FLEXION DEVIEE

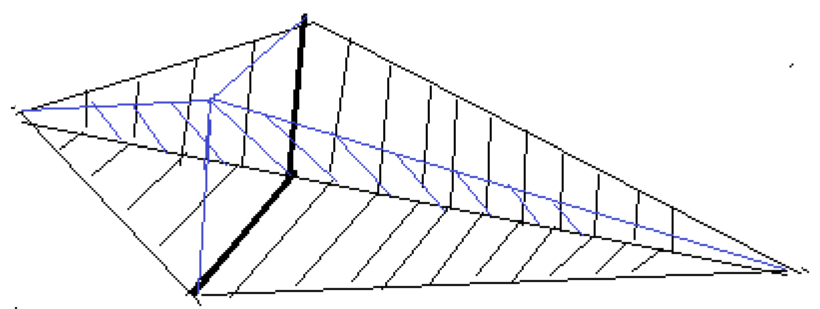




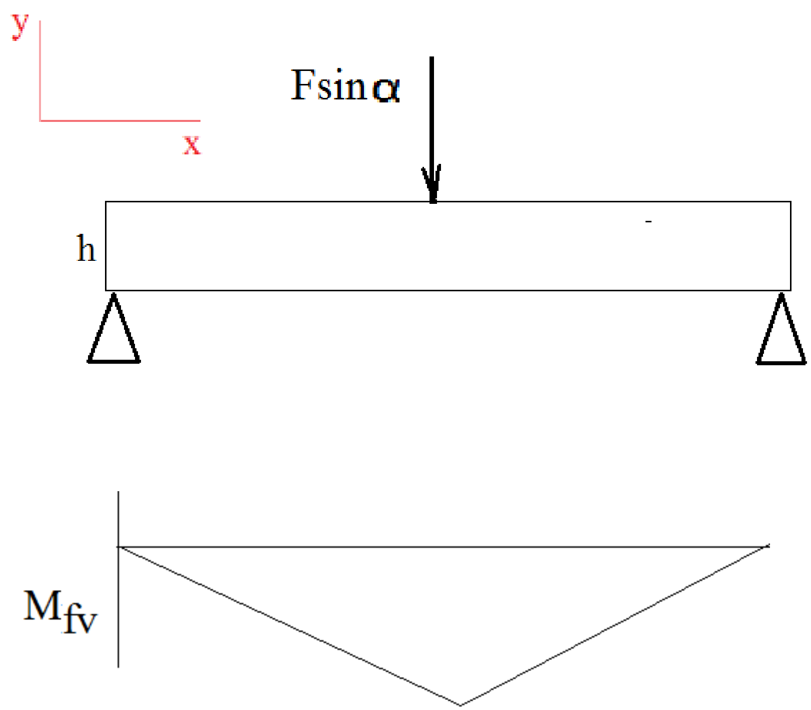


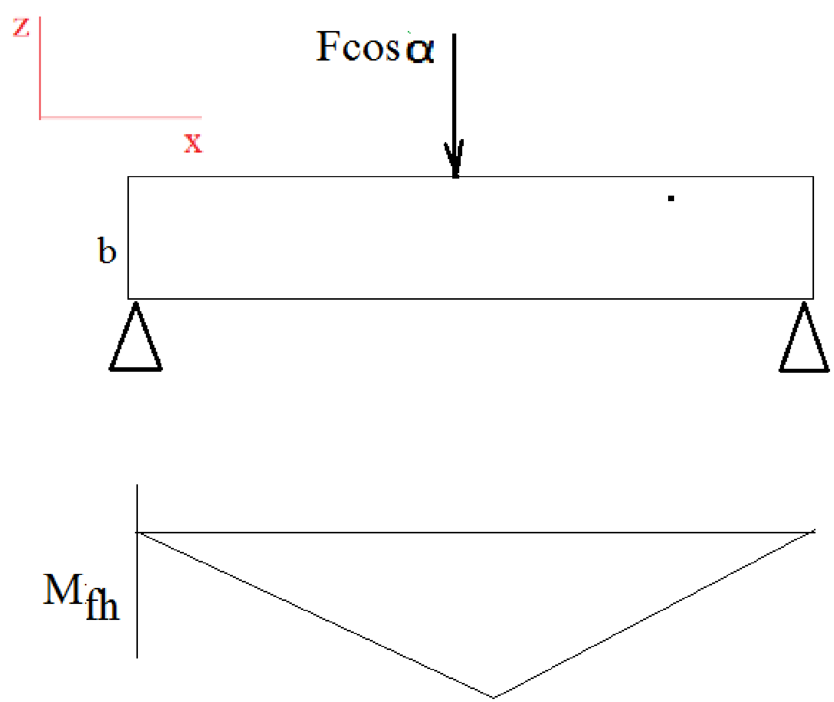


$$M_f=\sqrt{M_h^2+M_v^2}$$



FLEXION DEVIEE

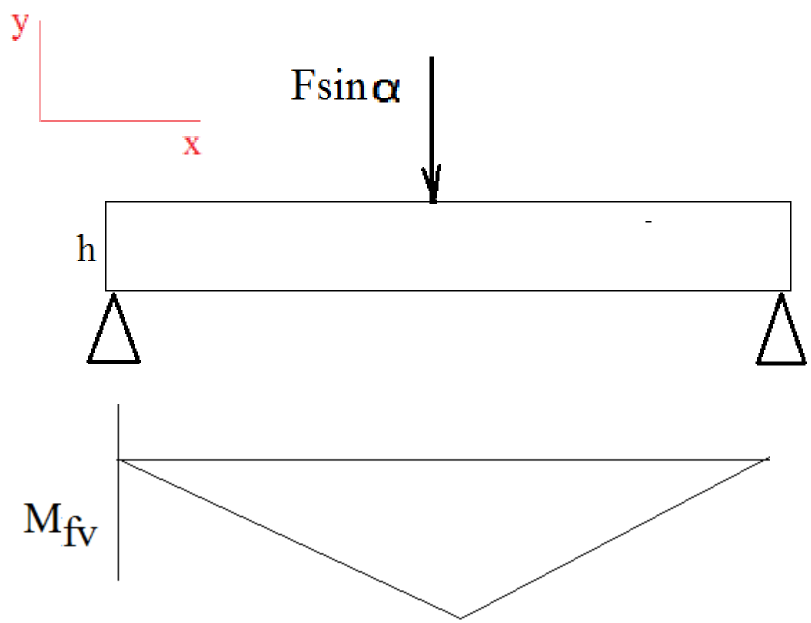


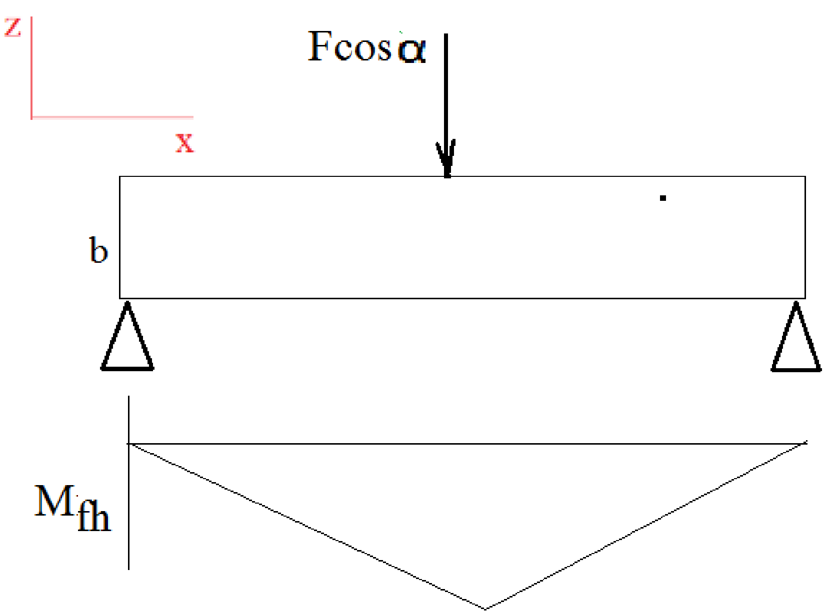


$$\sigma_v = \frac{M_{fv}}{I_z} h/2$$

$$\sigma_h = \frac{M_{fh}}{I_y} b/2$$

FLEXION DEVIEE





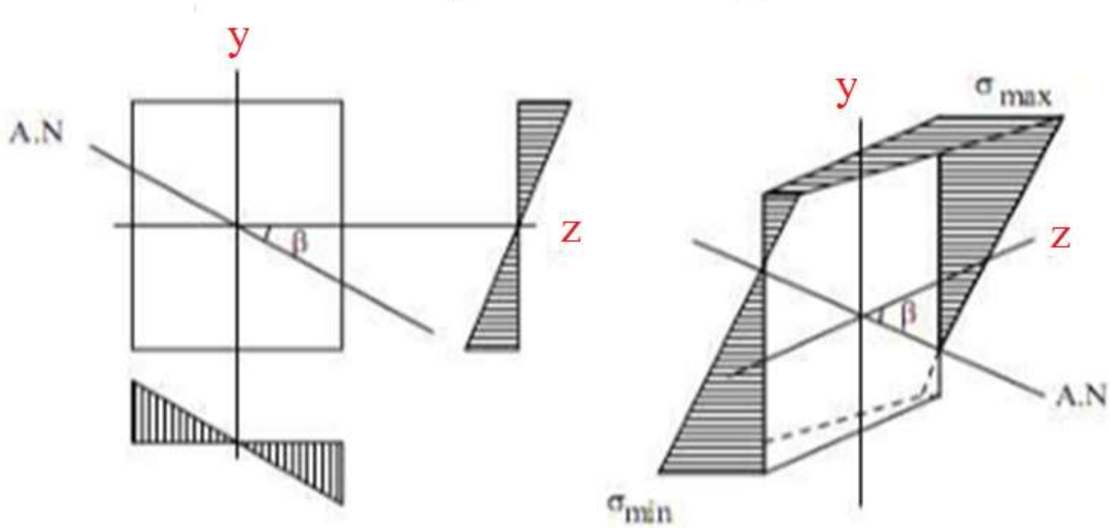
$$\sigma_v = \frac{M_{fv}}{I_z} h/2$$

$$\sigma_h = \frac{M_{fh}}{I_y} b/2$$

$$\sigma = \frac{M_{fh}}{I_y} b/2 + \frac{M_{fv}}{I_z} h/2$$

FLEXION DEVIEE

$$\sigma = \frac{M_{fh}}{I_y} b/2 + \frac{M_{fv}}{I_z} h/2$$



peut alors déduire le lieu des contraintes σ est nulle est l'axe neutre, qui a pour équation :

$$\frac{M_{fh}}{I_y} b/2 + \frac{M_{fv}}{I_z} h/2 = 0$$

FLEXION DEVIEE

VERIFICATION A LA RESISTANCE

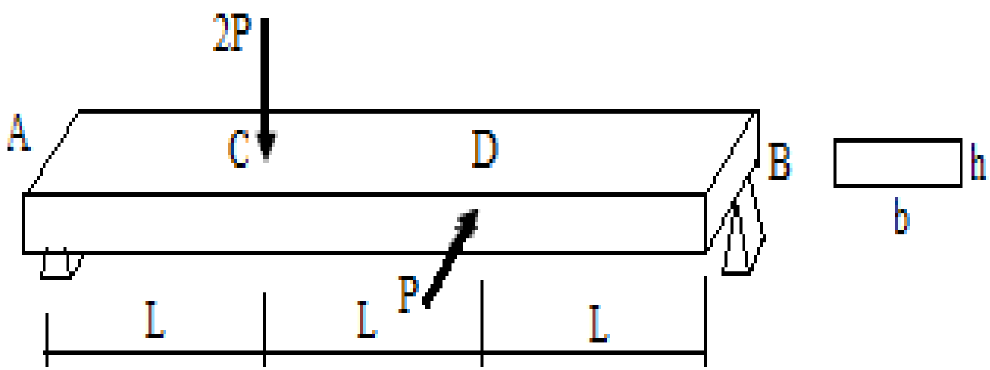
Le calcule de vérification de la résistance s'effectue à la base des données sur la contrainte totale maximale.

$$\sigma \leq \sigma_{adm}$$

$$\sigma = \frac{M_{fh}}{I_y} b/2 + \frac{M_{fv}}{I_z} h/2 \leq \sigma_{adm}$$

FLEXION DEVIEE

Exemple : Déterminer la contrainte en C et D . On suppose que; P= 10N, L=200mm, h=4mm et b=2h.

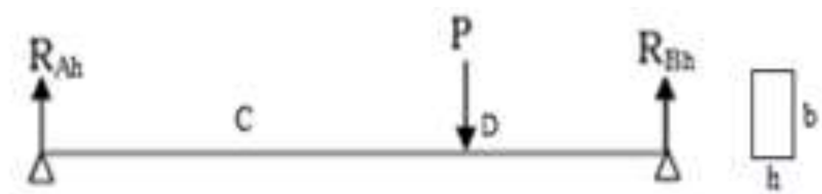


Plan vertical (xoy)

$$\begin{cases} R_{Av} + R_{Bv} = 2P \\ -R_{Av}.3L+2P.2L= 0 \end{cases}$$

R_{Av}= 4.P/3 R_{Bv} = 2P/3

Plan horizontal (xoz)



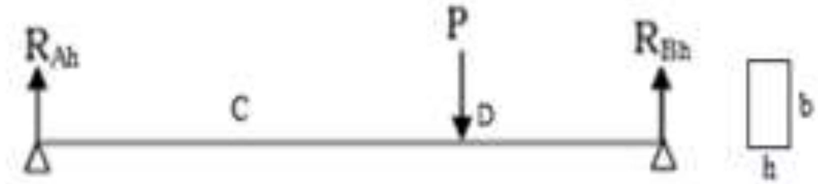
$$\begin{cases} R_{Ah} + R_{Bh} = P \\ -R_{Ah}.3L+P.\ L= 0 \end{cases}$$

R_{Ah}= P/3 R_{Bv} = 2P/3

FLEXION DEVIEE

Plan vertical (xoy)

Plan horizontal (xoz)



$R_{Av}= 4.P/3 \qquad R_{Bv} = 2P/3$

$R_{Ah}= P/3 \qquad R_{Bv} = 2P/3$

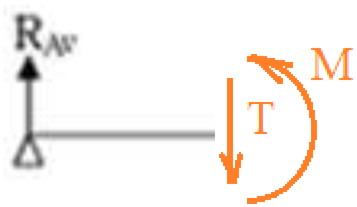
$0 < x < L$

$T = R_{Av}= 4.P/3$

$M_v = R_{Av}.x =, 4.P.x/3$

$x =0; \;\; M_v = 0$

$x =L; \;\; M_v = 4.P.L/3$



$L < x < 3L$

$T = R_{Av} – 2P = -2.P/3$

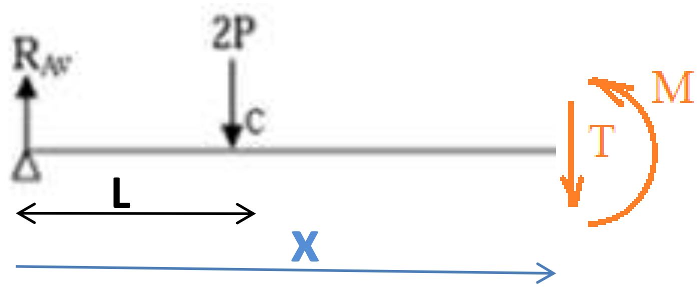
$M_v = R_{Av}.x - 2P.(x-L) = 4.P.x/3 -2P.(x-L)$

$M_v = R_{Av}.x - 2P.(x-L) = 2P.L-2.P.x/3$

$x =L; \;\; M_v = 4.P.L/3$

$x =2L; \;\; M_v = 2.P.L/3$

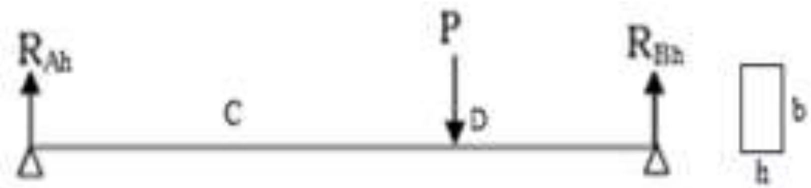
$x =3L; \;\; M_v = 0$



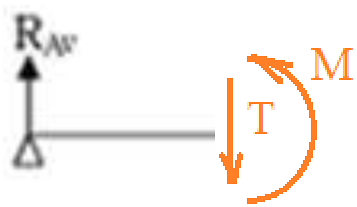
FLEXION DEVIEE

Plan vertical (xoy)

Plan horizontal (xoz)



$R_{Ah} = P/3$ $R_{Bv} = 2P/3$

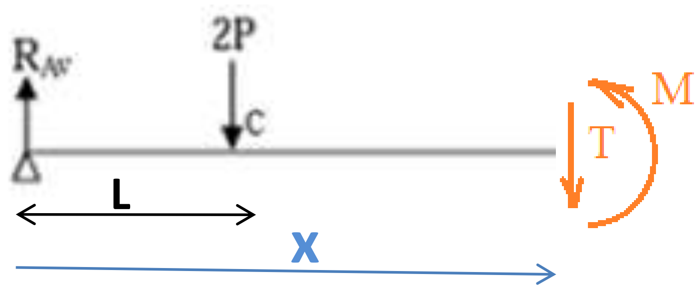


$0 < x < L$

$T = R_{Av} = 4.P/3$

$x = 0;\ M_v = 0$

$x = L;\ M_v = 4.P.L/3$



$L < x < 3L$

$T = R_{Av} - 2P = -2.P/3$

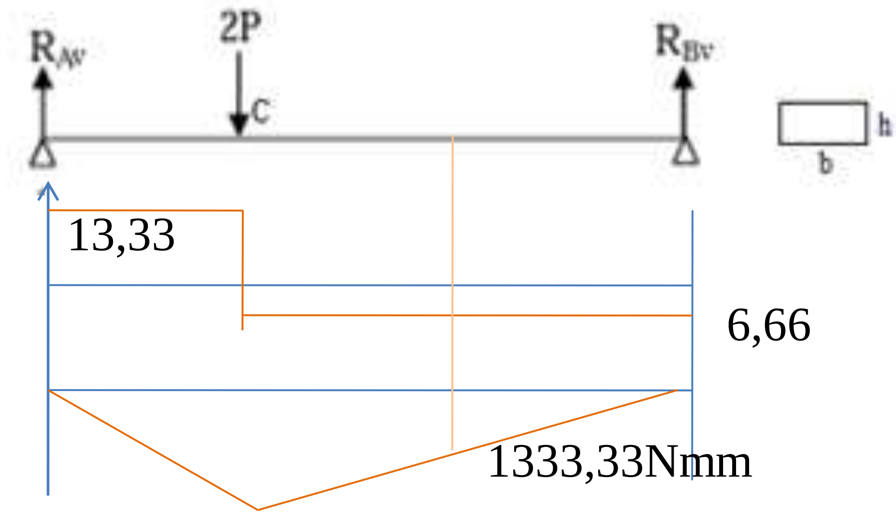
$x = 2L;\ M_v = 2.P.L/3$

$x = 3L;\ M_v = 0$

AN; P= 10N, L=200mm, h=4mm

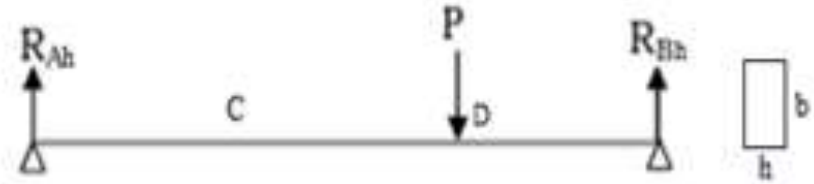
FLEXION DEVIEE

Plan vertical (xoy)



2666,66Nmm

Plan horizontal (xoz)



$R_{Ah} = P/3 \quad R_{Bv} = 2P/3$

$$\boxed{\sigma = \frac{M_{fh}}{I_y} b/2 + \frac{M_{fv}}{I_z} h/2 \leq \sigma adm}$$

0 < x < L

$T = R_{Av} = 13,33N$

$x = 0; \; M_v = 0$

$x = L; \; M_v = 2666,66Nmm$

L < x < 3L

$T = R_{Av} - 2P = -6,66N$

$x = 2L; \; M_v = 1333,33Nmm$

$x = 3L; \; M_v = 0$

AN; P= 10N, L=200mm, h=4mm

$\sigma_v = (M_v.\ h/2)/Iz$

$Iz = b.h^3/12$

$Iz = 8.4^3/12 = 42,6mm^4$

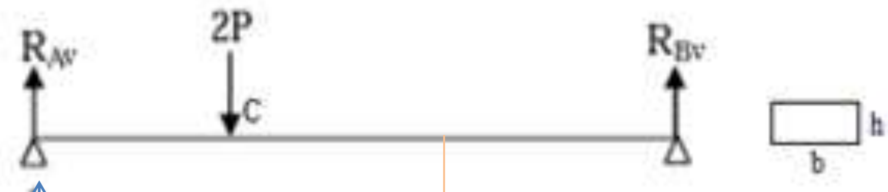
$h/2 = 2mm$

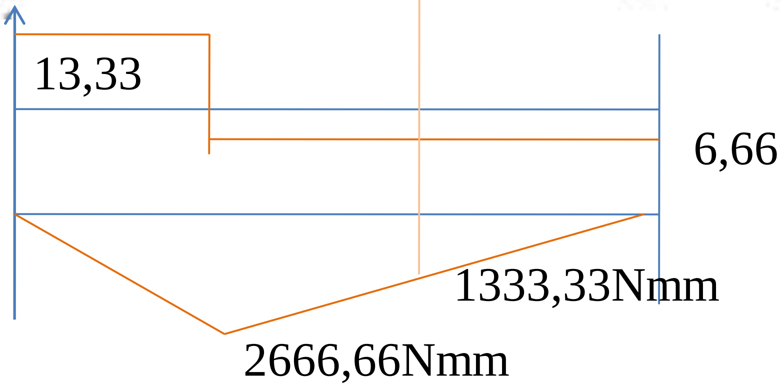
$\sigma_{cv} = 2666,66.2/42,6 = 125,2N/mm^2$

$\sigma_{Dv} = 1333,33.2/42,6 = 62,6N/mm^2$

FLEXION DEVIEE

Plan vertical (xoy)



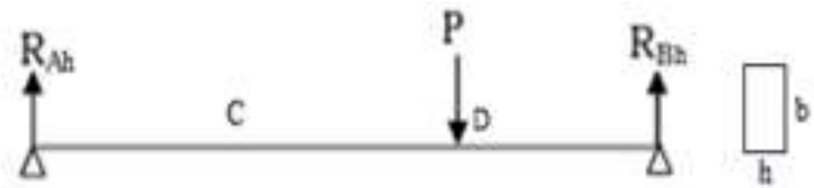


σ_{cv} =125,2N/mm²

σ_{Dv} =62,6N/mm²

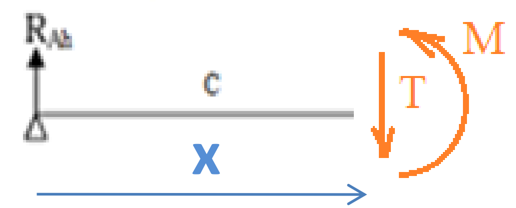
AN; P= 10N, L=200mm, h=4mm

Plan horizontal (xoz)



R_{Ah}= P/3 $\quad$ R_{Bv} = 2P/3

0 < x < 2L



$T = R_{Ah} = P/3$

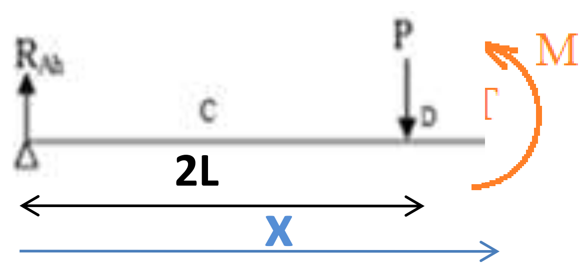
$M_h = R_{Ah}.x = P.x/3$

$x = 0;\ M_h = 0$

$x = 2L;\ M_h = 2.P.L/3$

$x = L;\ M_h = P.L/3$

2L < x < 3L



$T = R_{Ah} – P = -2.P/3$

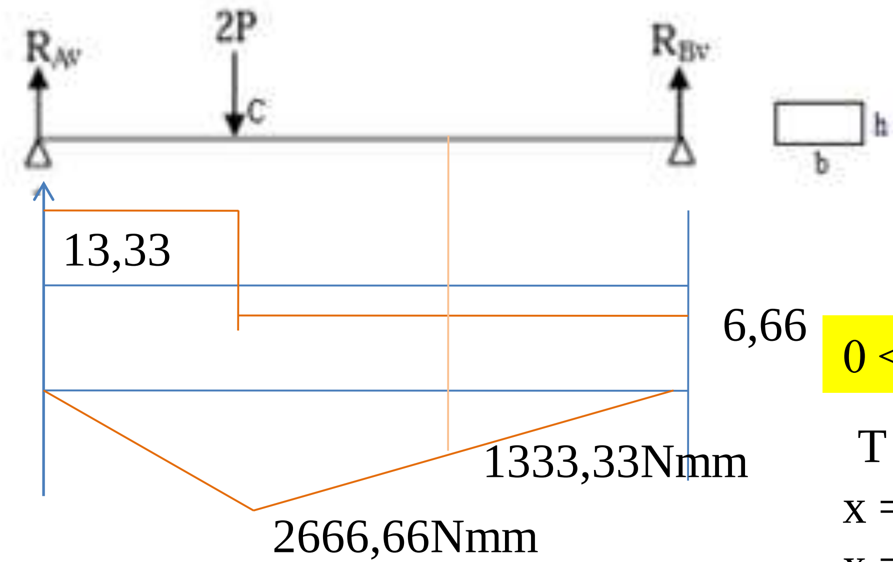
$M_h = R_{Ah}.x - P.(x-2L) = 2.P.L- 2P.x/3$

$x = 2L;\ M_h = 2.P.L/3$

$x = 3L;\ M_h = 0$

FLEXION DEVIEE

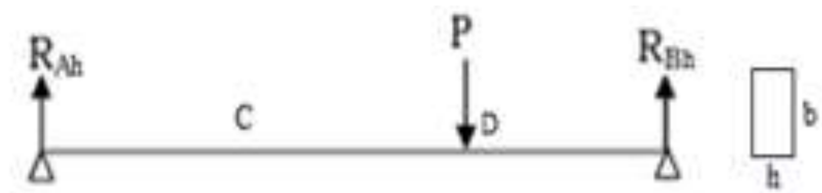
Plan vertical (xoy)



σ_{cv} =125,2N/mm^2

σ_{Dv} =62,6N/mm^2

Plan horizontal (xoz)



R_{Ah}= P/3 $\quad$ R_{Bv} = 2P/3

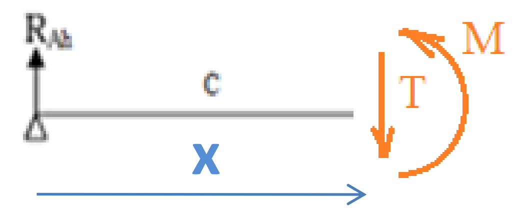
$0 < x < 2L$

T = P/3

x =0; M_h = 0

x =2L; M_h = 2.P.L/3

x =L; M_h = P.L/3

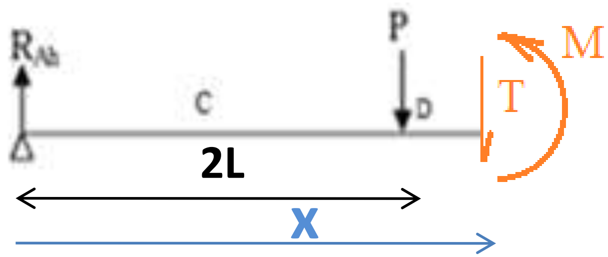


$2L < x < 3L$

T = -2.P/3

x =2L; M_h = 2.P.L/3

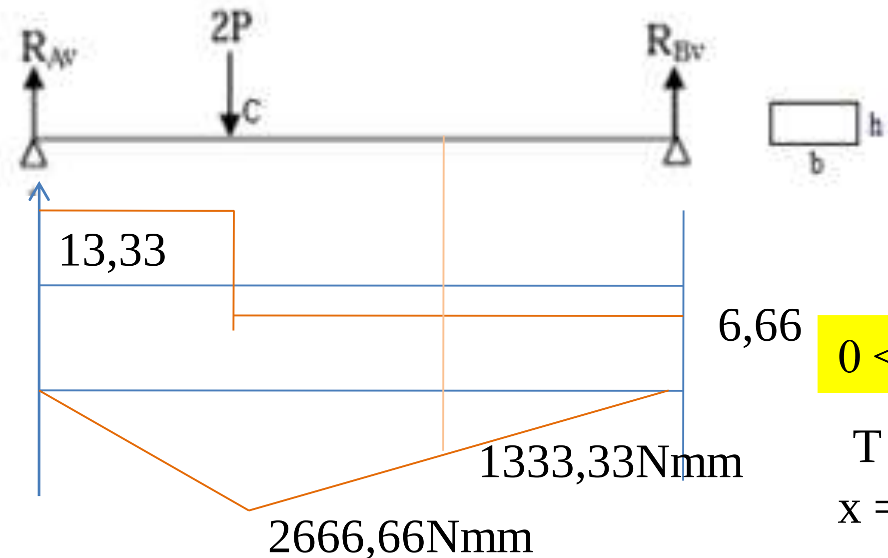
x =3L; M_h = 0



AN; P= 10N, L=200mm, h=4mm

FLEXION DEVIEE

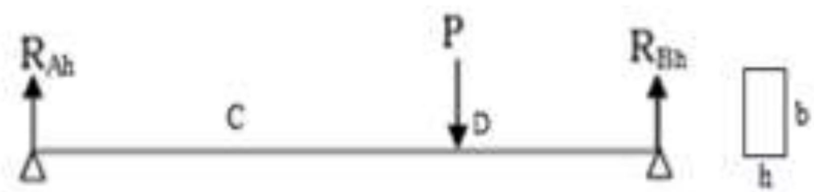
Plan vertical (xoy)



σ_{cv} =125,2N/mm^2

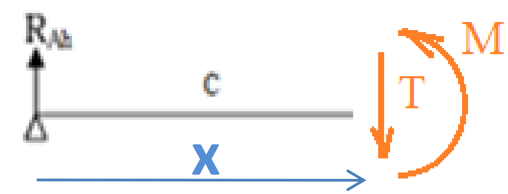
σ_{Dv} =62,6N/mm^2

Plan horizontal (xoz)



R_{Ah}= P/3 R_{Bv} = 2P/3

$0 < x < 2L$



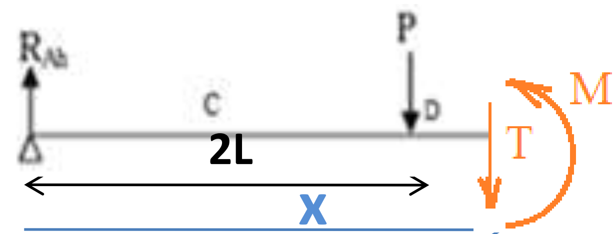
T = 3,33N

x =0; M_h = 0

x =2L; M_h = 1333,33Nmm

x =L; M_h = 666,66Nmm

$2L < x < 3L$



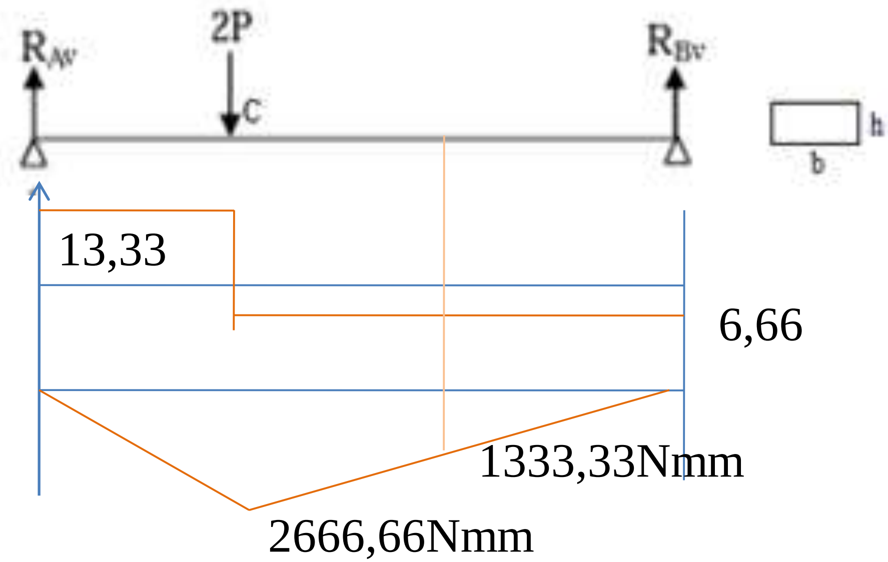
T = -6,66N

x =3L; M_h = 0

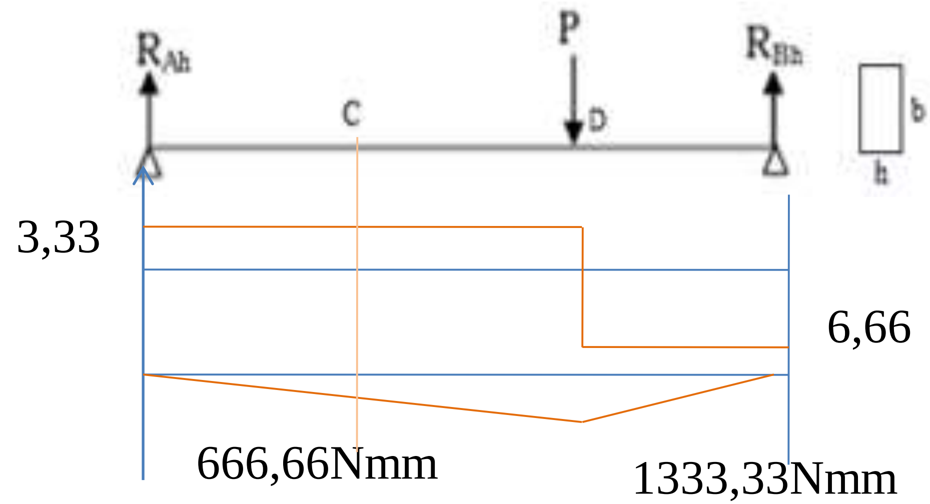
AN; P= 10N, L=200mm, h=4mm

FLEXION DEVIEE

Plan vertical (xoy)



Plan horizontal (xoz)



σ_{cv} =125,2N/mm^2

σ_{Dv} =62,6N/mm^2

$0 < x < 2L$

T = 3,33N

x =0; M_h = 0

x =2L; M_h = 1333,33Nmm

x =L; M_h = 666,66Nmm

$2L < x < 3L$

T = -6,66N

x =3L; M_h = 0

$\sigma_h = (M_v . b/2)/Iy$

$Iy = h.b^3/12$

$Iy = 4.8^3/12 = 170{,}6mm^4$

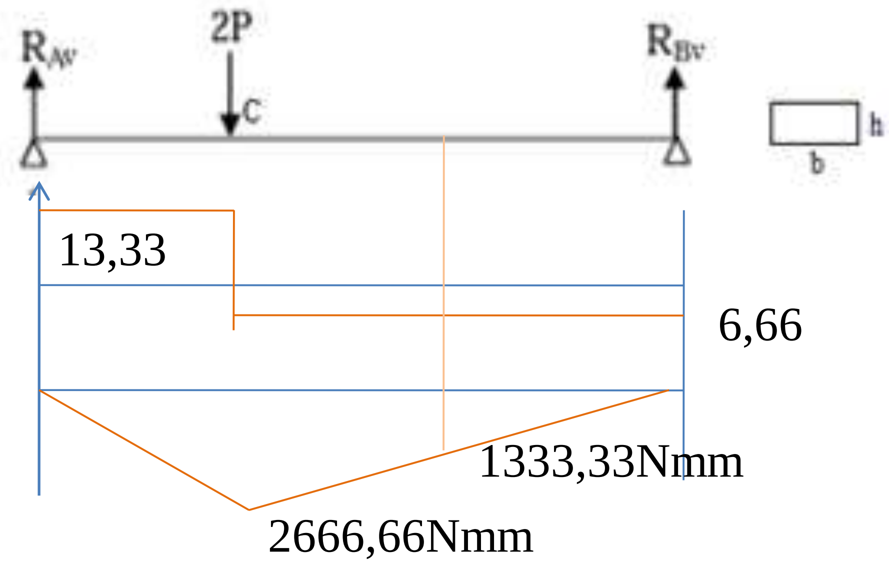
b/2 = 4mm

σ_{ch} =666,7.4/170,6 = 15,6N/mm^2

σ_{Dh} =1333,3.4/170,6 = 31,3N/mm^2

FLEXION DEVIEE

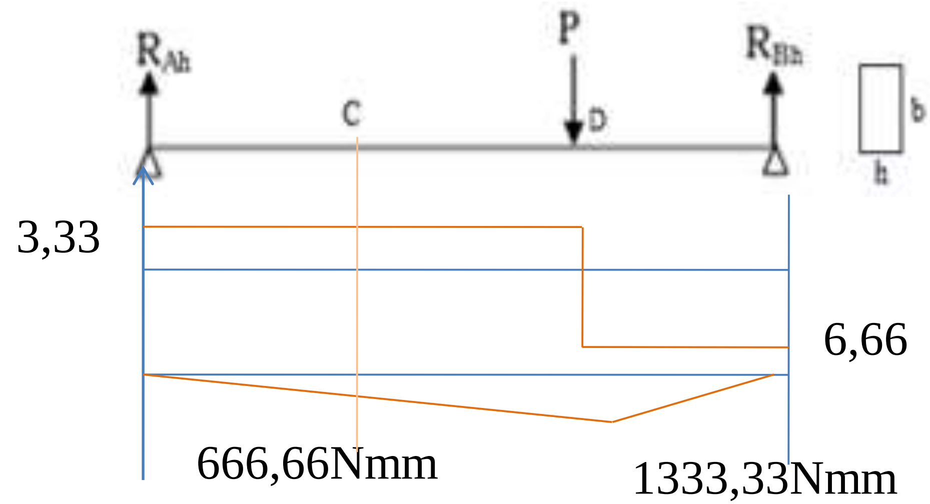
Plan vertical (xoy)



σ_{cv} =125,2N/mm^2

σ_{Dv} =62,6N/mm^2

Plan horizontal (xoz)



σ_{ch} = 15,6N/mm^2

σ_{Dh} = 31,3N/mm^2

$$\boxed{\sigma = \frac{M_{fh}}{I_y} b/2 + \frac{M_{fv}}{I_z} h/2 \ \leq \sigma adm}$$

Point C: σ_c = 125,2 +15,6 = 140,8 N/mm^2

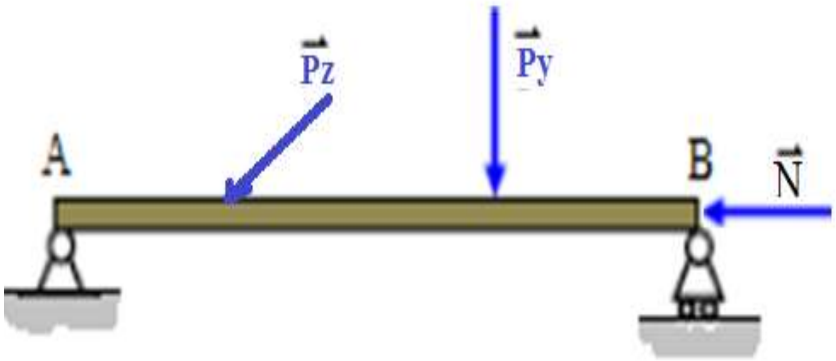
Point D: σ_D = 62,6 + 31,3 = 93,9 N/mm^2

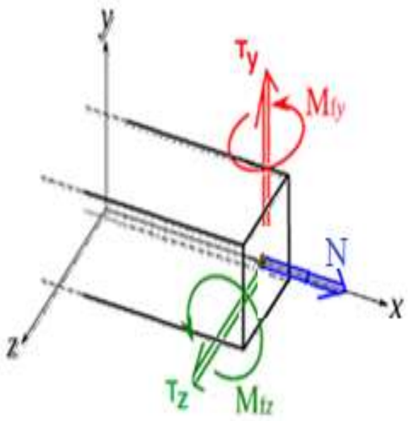
$\boldsymbol{\sigma_{max}}$ **= 140,8 N/mm^2**

FLEXION COMPOSEE

1- Flexion composée avec traction ou compression

C'est le cas général d'une poutre soumise à des chargements transversaux et longitudinaux, les efforts Mfy, Mfz ainsi que N sont présents.





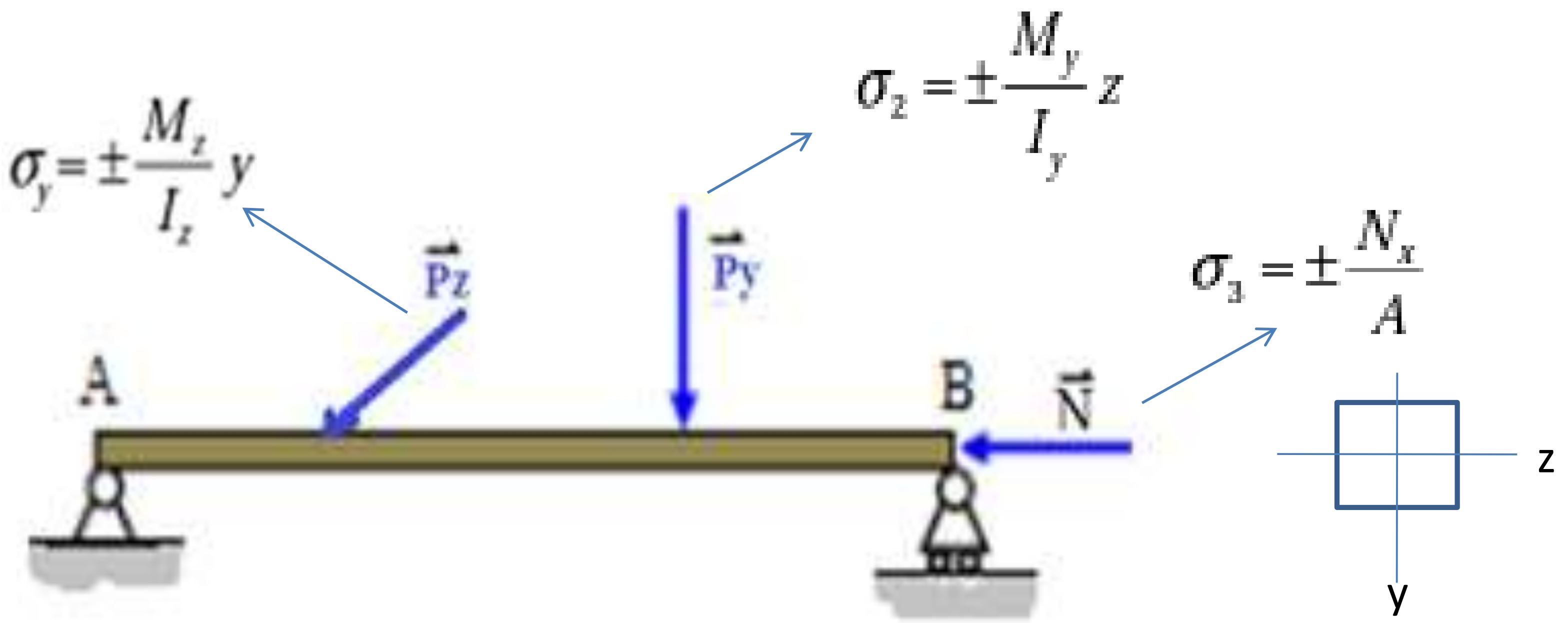
Contraintes

l'équation générale donnant la valeur de la contrainte globale est donné par :

$$\sigma = \pm \frac{M_z}{I_z} y \pm \frac{M_y}{I_y} z \pm \frac{N_x}{A}$$

FLEXION COMPOSEE

Contraintes

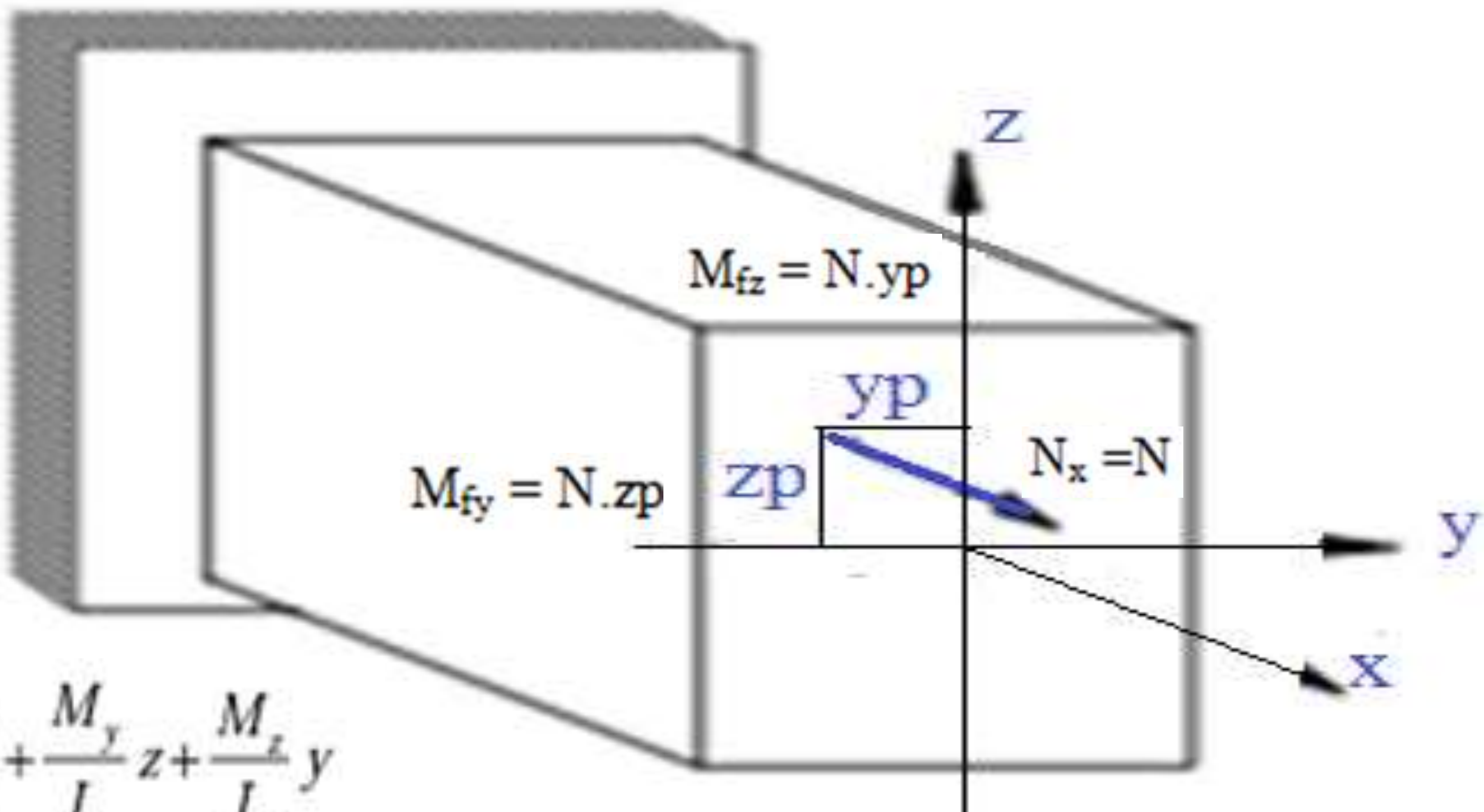


$$\sigma = \pm \frac{M_z}{I_z} y \pm \frac{M_y}{I_y} z \pm \frac{N_x}{A}$$

FLEXION COMPOSEE

2- Traction ou compression excentrée

Si une force excentrée dont les coordonnées du point d'application sont yp, zp, on peut le remplacer par un effort de compression équivalent N passant par le centre de gravité de la section, plus deux moments fléchissant Mfy et Mfz.



$$\sigma = \frac{N}{A} + \frac{M_y}{I_y} z + \frac{M_z}{I_z} y$$

FLEXION COMPOSEE

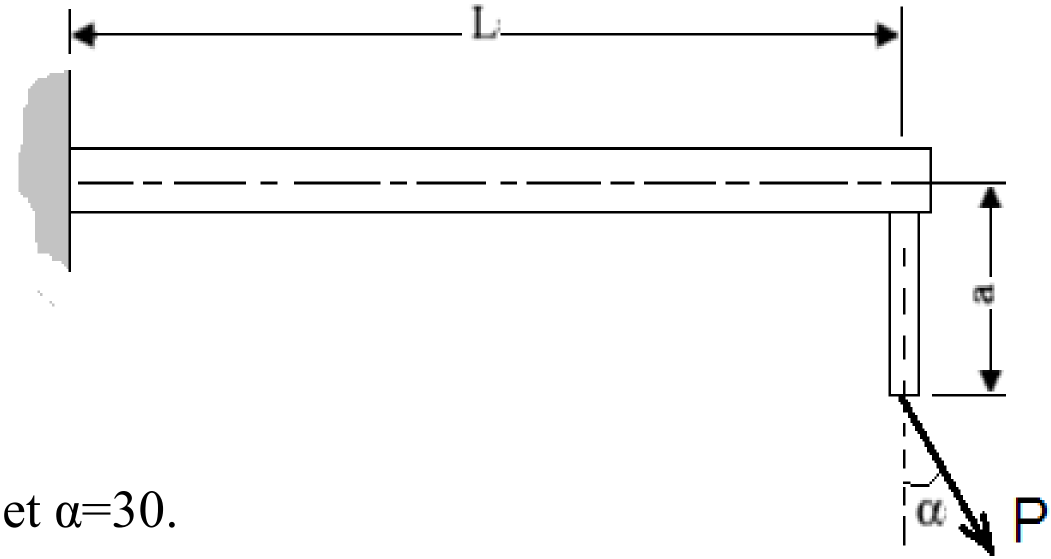
CONDITION DE RESISTANCE

Pour une section symétrique, la condition de résistance s’écrit :

$$\pm \frac{M_z}{I_z} y \pm \frac{M_y}{I_y} z \pm \frac{N_x}{A} \leq [\sigma]$$

FLEXION COMPOSEE

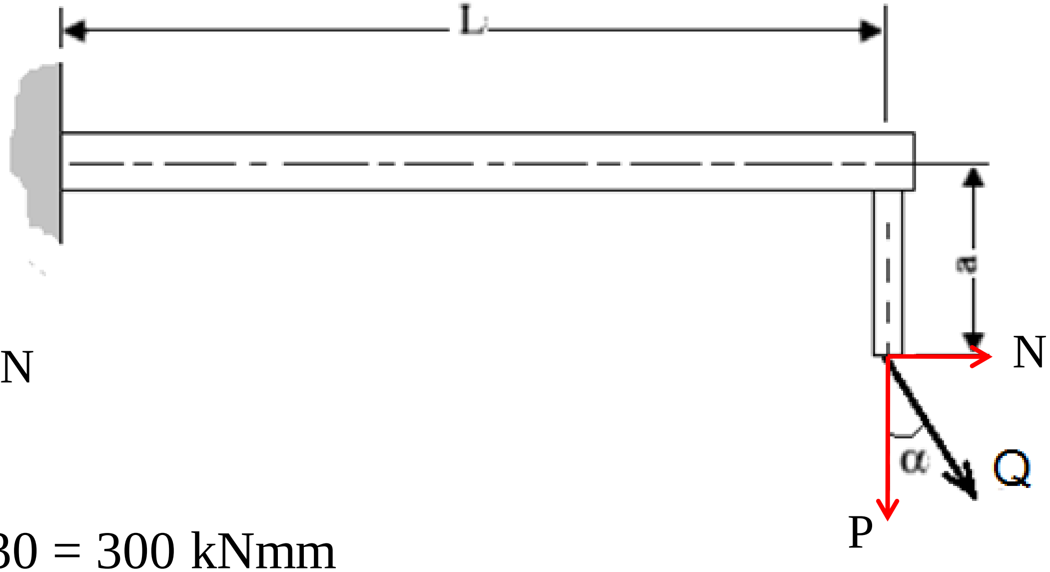
Exo.1 : Soit une poutre cylindrique encastrée de diamètre 30mm. A l’extrémité libre est appliquée une charge P de 3kN comme il est indiqué sur la figure. Déterminer la contrainte maximale de la poutre suivante, sachant que : L =1m; a=0,2m et α=30.



d = 30mm,
P = 3kN,
L =1m; a=0,2m et α=30.

FLEXION COMPOSEE

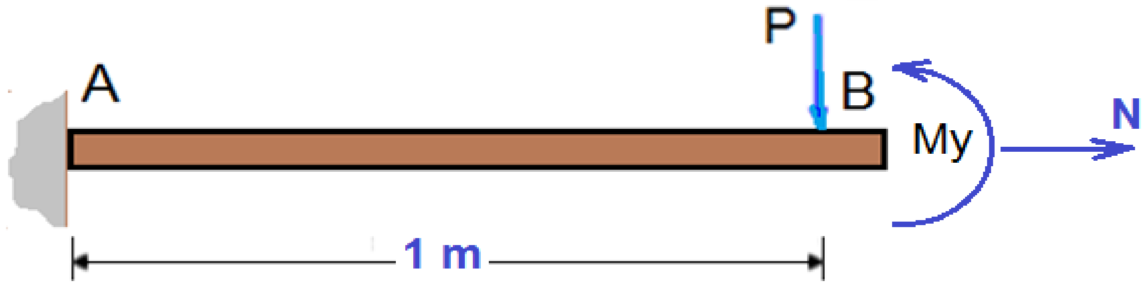
d = 30mm,
P = 3kN,
L =1m; a=0,2m et α=30.



P = Q.cosα = 3000.cos30 = 2,598 kN

N = Q.sinα = 3. .sin30 = 1,500 kN

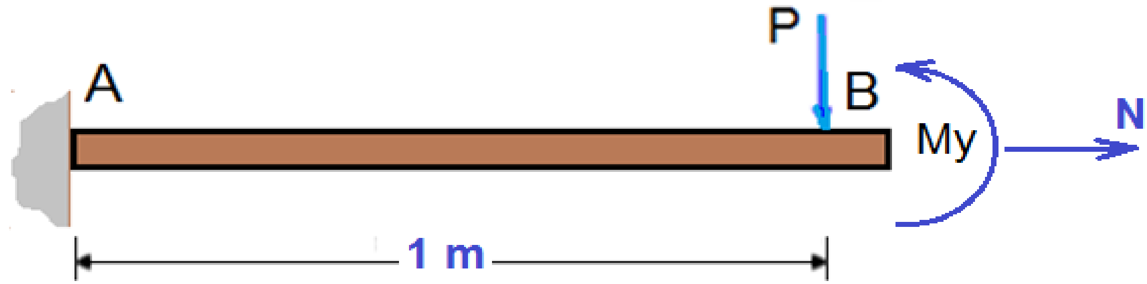
My =N.a = Q.a.sinα = 0,2.3.sin30 = 300 kNmm



Déterminons les réactions aux appuis:

FLEXION COMPOSEE

d = 30mm,
P = 3kN,
L =1m; a=0,2m et α=30.

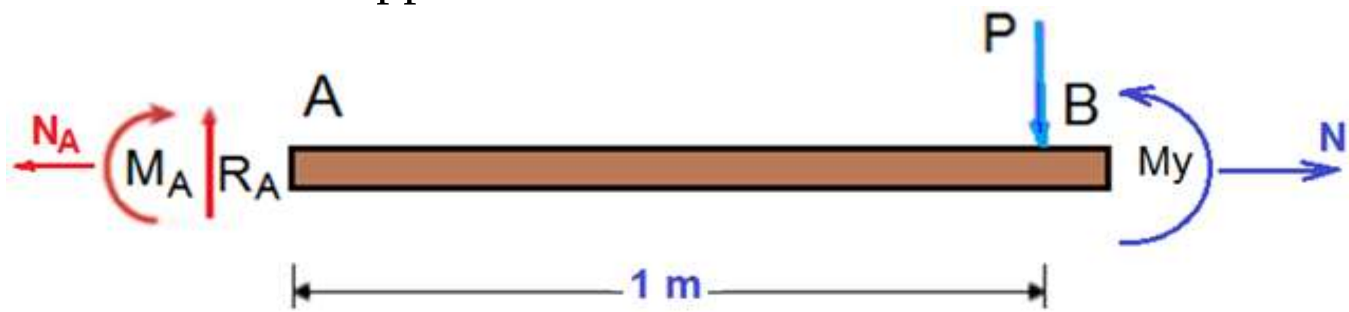


P = 2,598 kN

N = 1,500 kN

My =300 kNmm

Déterminons les réactions aux appuis:



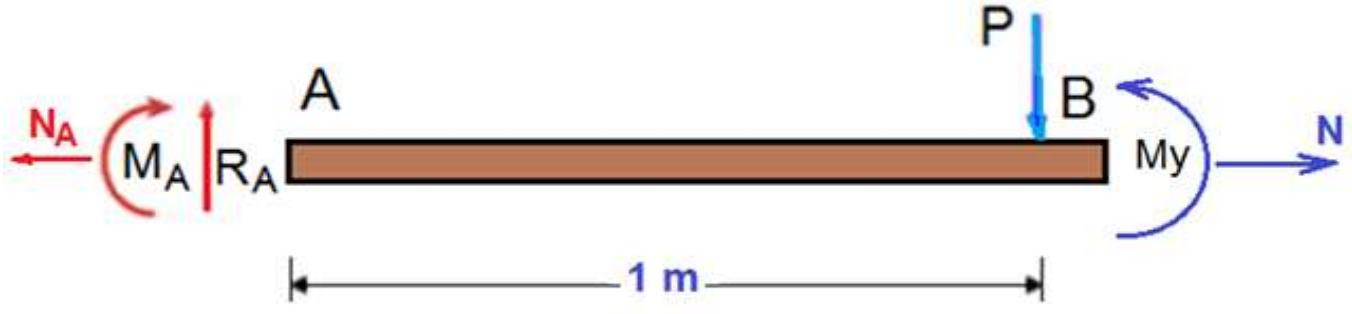
D’apes le principe fondamental de la statique
$\Sigma Fx = 0$
$\Sigma Fy = 0$
$\Sigma M_{/A} = 0$

FLEXION COMPOSEE

d = 30mm,
P = 3kN,
L =1m; a=0,2m et α=30.

P = 2,598 kN
N = 1,500 kN
My =300 kNmm

Déterminons les réactions aux appuis:

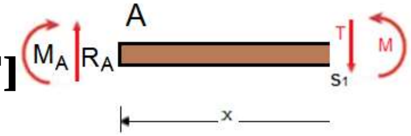


$\Sigma Fx = 0$; $N - N_A = 0 \Rightarrow N_A = N = 1,5$ kN

$\Sigma Fy = 0$; $P - R_A = 0 \Rightarrow R_A = P = 2,598$ kN

$\Sigma M_{/A} = 0$; $R_A.1000 + M_A - My = 0 \Rightarrow M_A = -P.1000 + My$

$M_A = -2,598.1000 + 300 = -2298$ kNmm

FI  OSEE

d = 30mm,
P = 3kN,
L =1m; a=0,2m et α=30.

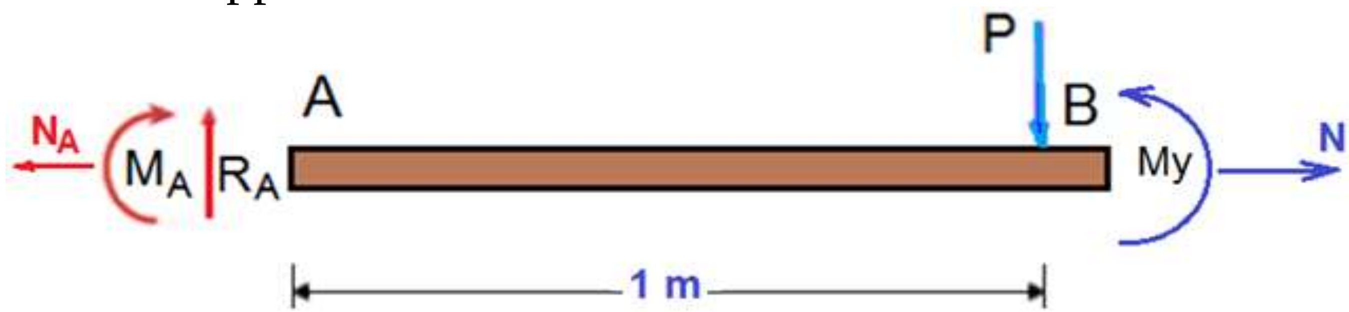
P = 2,598 kN
N = 1,500 kN
M_y =300 kNmm

Déterminons les réactions aux appuis:

N_A = 1,5 kN

R_A = 2,598 kN

M_A= -2298 kNmm

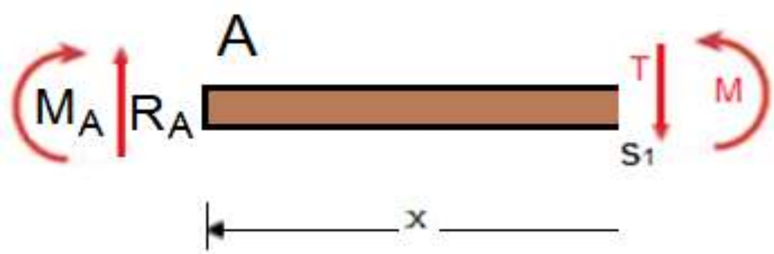


On a donc dans le premier tronçon jusqu'à P (0< x <1000):

$\Sigma F_y = 0$; $-R_A + T = 0$; $T = 2,598$ kN

$\Sigma M_{/s1} = 0$; $-R_A.x - M_A + M = 0$;

$M = 2598x - 2298000$



Pour x =0; M = -2298 kNmm

Pour x =L; M = 300 kNmm ;

FI  OSEE

d = 30mm,
P = 3kN,
L =1m; a=0,2m et α=30.

N = 1,500 kN

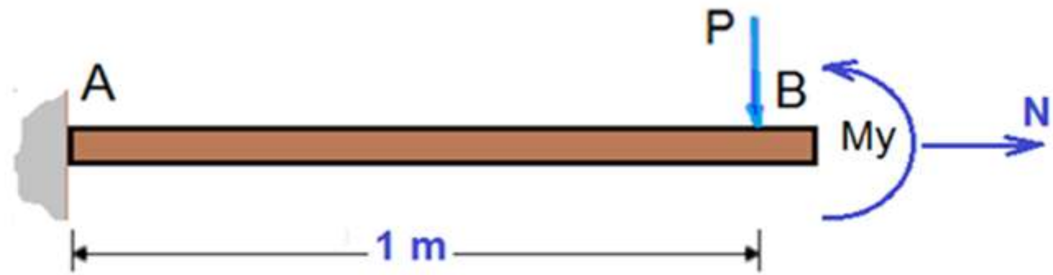
T = 2,598 kN

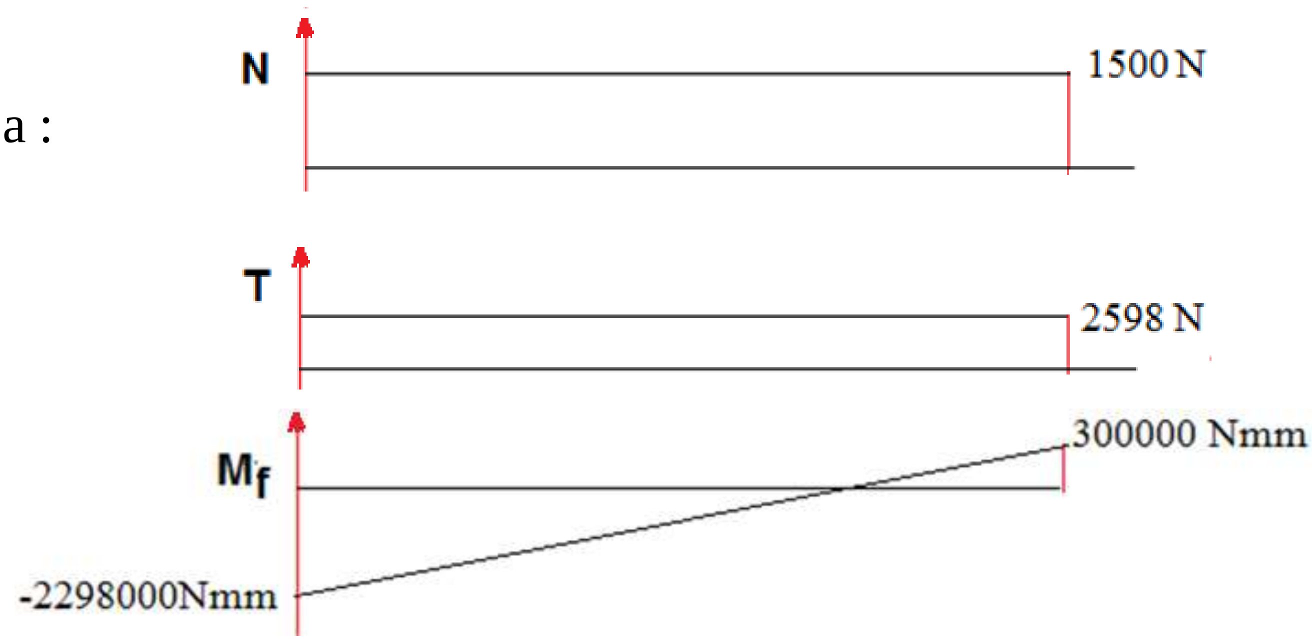
Pour x =0; M = -2298 kNmm

Pour x =L; M = 300 kNmm ;

P = 2,598 kN

My =300 kNmm





D'après les graphes, on a :
M_{fmax} = 2,298 kNm
N_{max} = 1,5 kN

FLEXION COMPOSEE

d = 30mm,

P = 3kN,

L =1m; a=0,2m et α=30.

P = 2,598 kN

My =300 kNmm

N = 1,500 kN

T = 2,598 kN

D'après les graphes, on a :

M_{fmax} = 2,298 kNm

N_{max} = 1,5 kN

$$\sigma_{max} = \sigma_{tra} + \sigma_f$$

$\sigma_{tra} = N_{max} / S$

$S = \pi.d^2/4 = \pi.30^2/4 = 706,5\ mm^2$

σ_{tra} = 1500/706,5 = 2,12MPa

$\sigma_f = (M_f.d/2)/ I_y$

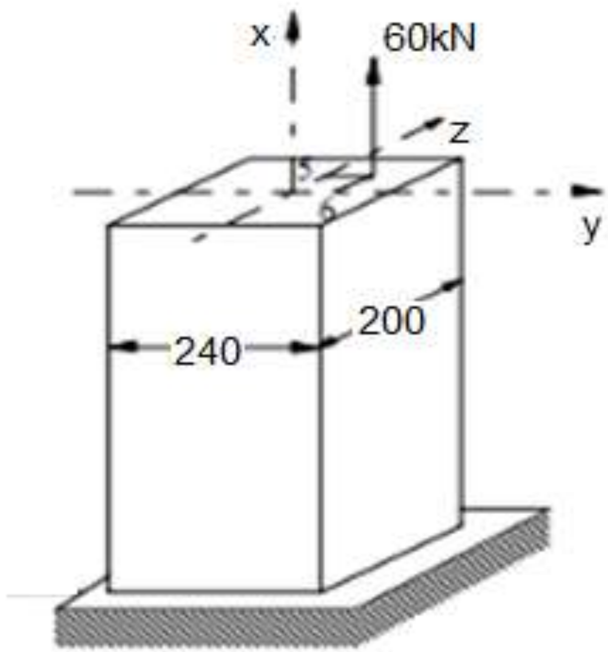
$I_y = (\pi.d^4)/64 = (\pi.30^4)/64 = 39740,6 mm^4$

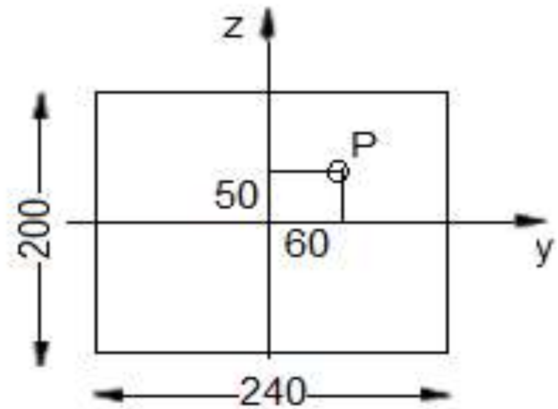
σ_f = 2298000.15/39740,6= 867,4 MPa

$$\sigma_{max} = \sigma_{tra} + \sigma_f = 869,5\ MPa$$

FLEXION COMPOSEE

Exo. 2 : Déterminer la contrainte normale σ_{max} dans la section dangereuse de la poutre ci-dessous avec P = 60 kN, zp = 60mm, yp = 50mm.





FLEXION COMPOSEE

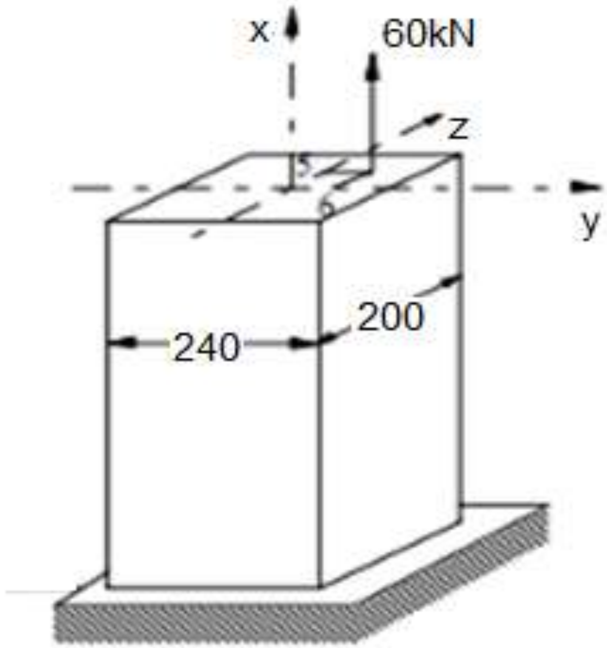
Traction;

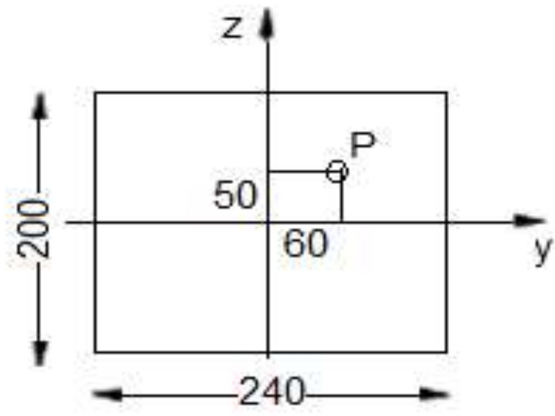
σ_{tra} = N/S

N = P = 60000N

S = 240.200 = 48000 mm^2

σ_{tra} = 60000/48000 = 1,25MPa





FLEXION COMPOSEE

Traction;

σ_{tra} = 1,25MPa

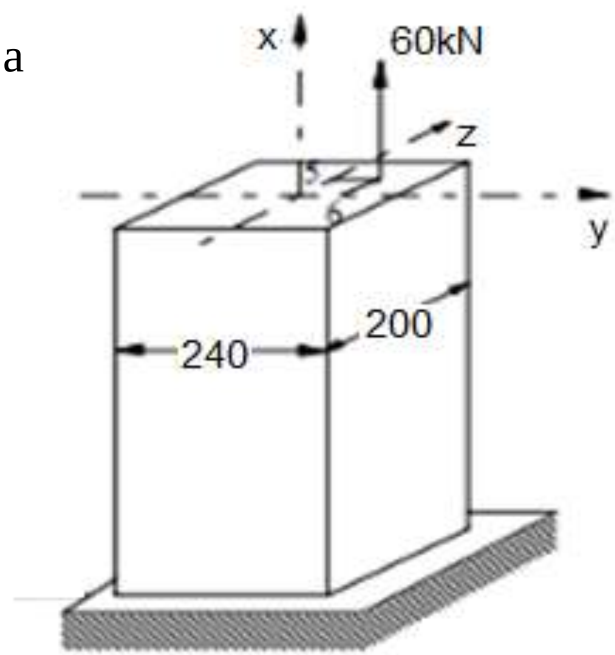
Flexion;

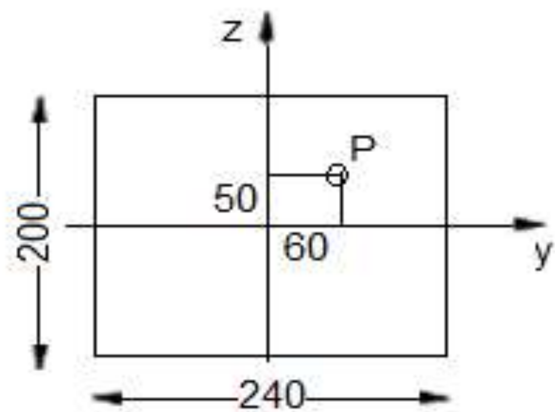
$\sigma_{fz} = M_{fx}\ z/\ I_y$

M_{fx} = P.zp = 60000.50 =3000000Nmm

$I_y = 240.200^3/12 = 160000000\ mm^4$

σ_{fz} =3000000.100/ I_z = 1,88MPa





FLEXION COMPOSEE

Traction;

σ_{tra} = 1,25MPa

Flexion;

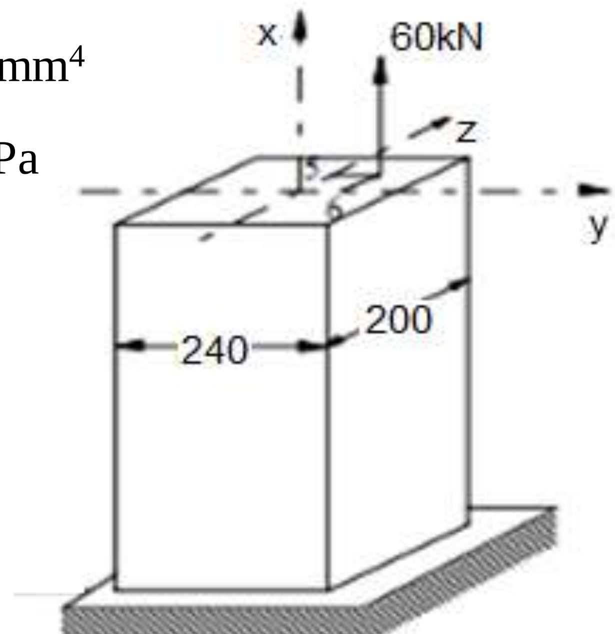
σ_{fz} = 1,88MPa

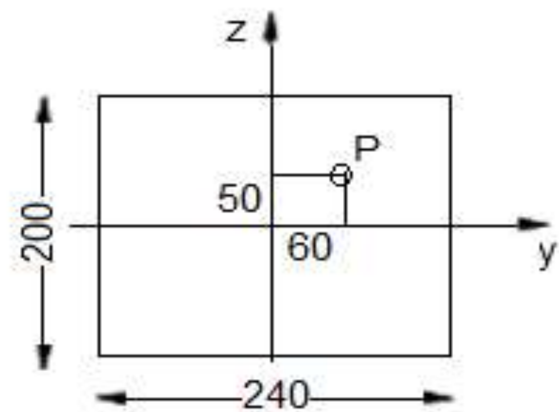
$\sigma_y = M_{fx}\, y / I_z$

M_{fx} = P.yp = 60000.60 = 3600000Nmm

$I_y = 200.240^3/12 = 230400000 mm^4$

$\sigma_{fy} = 3600000.120 / I_y$ = 1,88 MPa





FLEXION COMPOSEE

Traction;

$\sigma_{tra} = 1{,}25\text{MPa}$

Flexion;

$\sigma_{fz} = 1{,}88\text{MPa}$

$\sigma_{fy} = 1{,}88\ \text{MPa}$

la contrainte normale σ_{max}

$$\sigma_{max} = \sigma_{tra} + \sigma_{fz} + \sigma_{fy}$$

$$\sigma_{max} = 1{,}25+1{,}88+1{,}88 = 5{,}01\ \text{MPa}$$