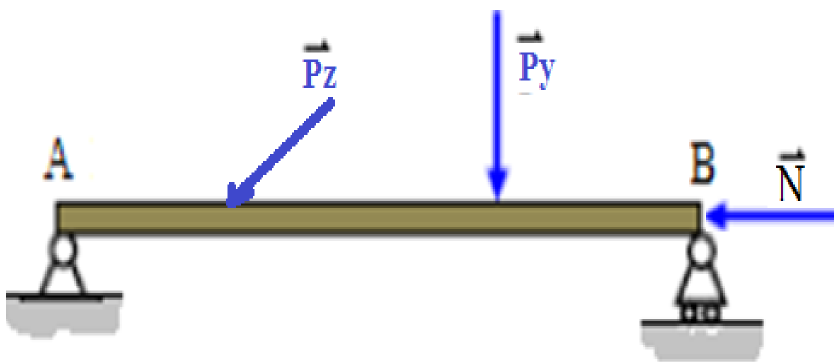
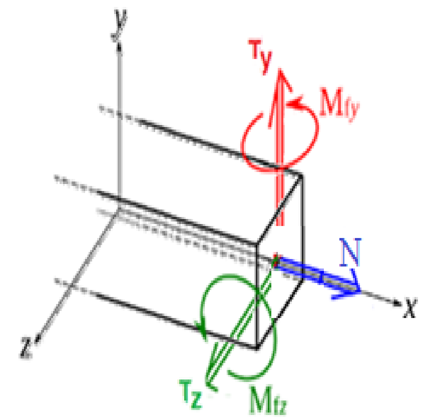
FLEXION COMPOSEE

1- Flexion composée avec traction ou compression

C’est le cas général d’une poutre soumise à des chargements transversaux et longitudinaux, les efforts Mfy, Mfz ainsi que N sont présents.





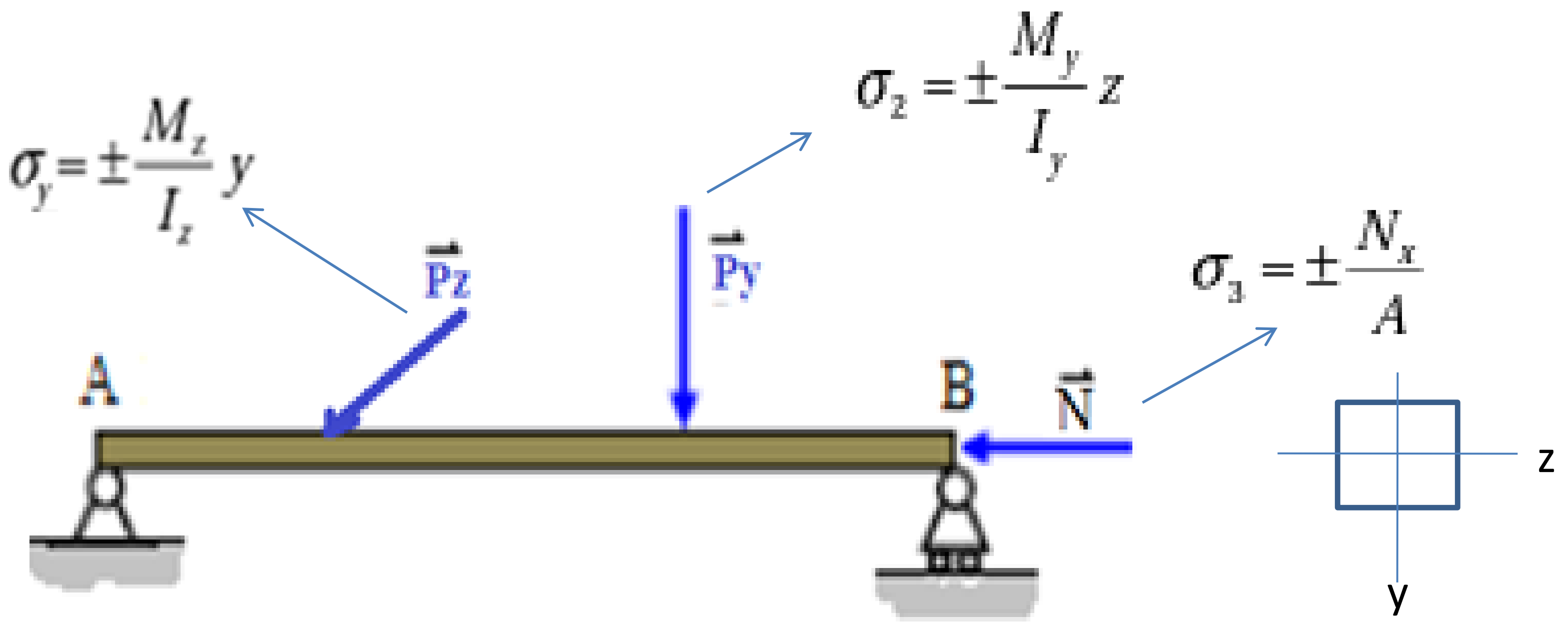
Contraintes

l’équation générale donnant la valeur de la contrainte globale est donné par :

$$\sigma = \pm \frac{M_z}{I_z} y \pm \frac{M_y}{I_y} z \pm \frac{N_x}{A}$$

FLEXION COMPOSEE

Contraintes

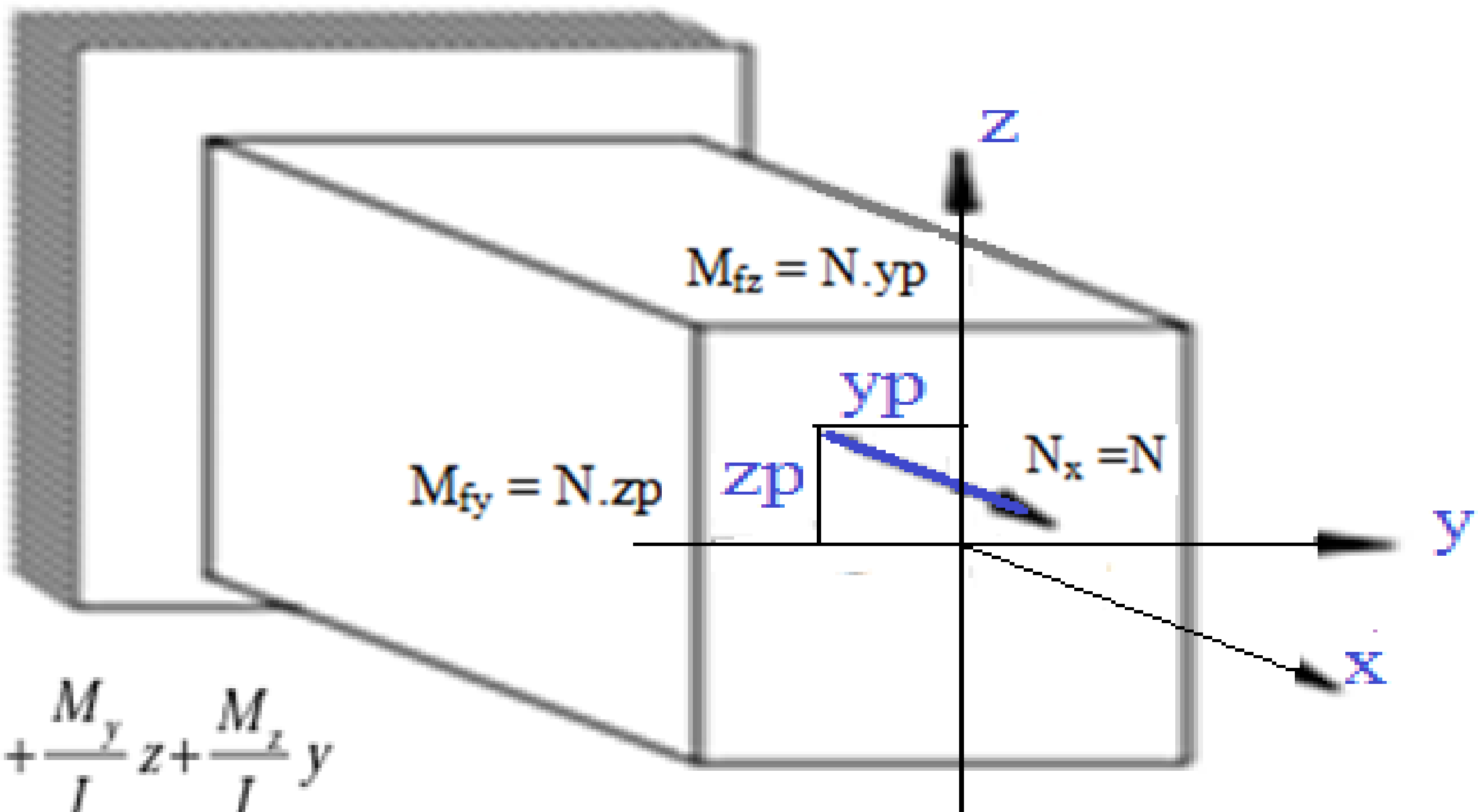


$$\sigma = \pm \frac{M_z}{I_z} y \pm \frac{M_y}{I_y} z \pm \frac{N_x}{A}$$

FLEXION COMPOSEE

2- Traction ou compression excentrée

Si une force excentrée dont les coordonnées du point d'application sont yp, zp, on peut le remplacer par un effort de compression équivalent N passant par le centre de gravité de la section, plus deux moments fléchissant Mfy et Mfz.



$$\sigma = \frac{N}{A} + \frac{M_y}{I_y} z + \frac{M_z}{I_z} y$$

FLEXION COMPOSEE

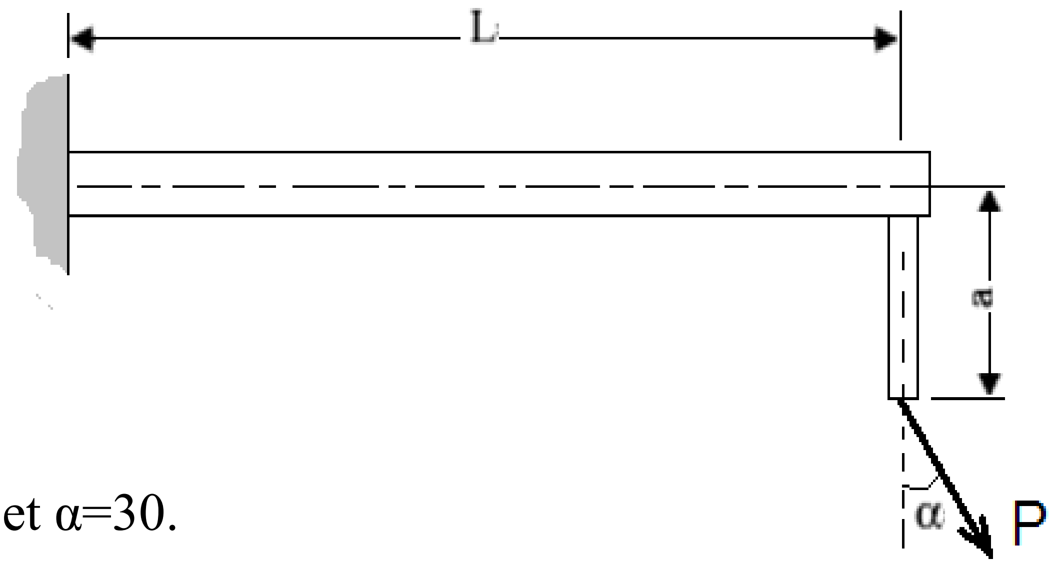
CONDITION DE RESISTANCE

Pour une section symétrique, la condition de résistance s’écrit :

$$\pm\frac{M_z}{I_z}y\pm\frac{M_y}{I_y}z\pm\frac{N_x}{A}\leq[\sigma]$$

FLEXION COMPOSEE

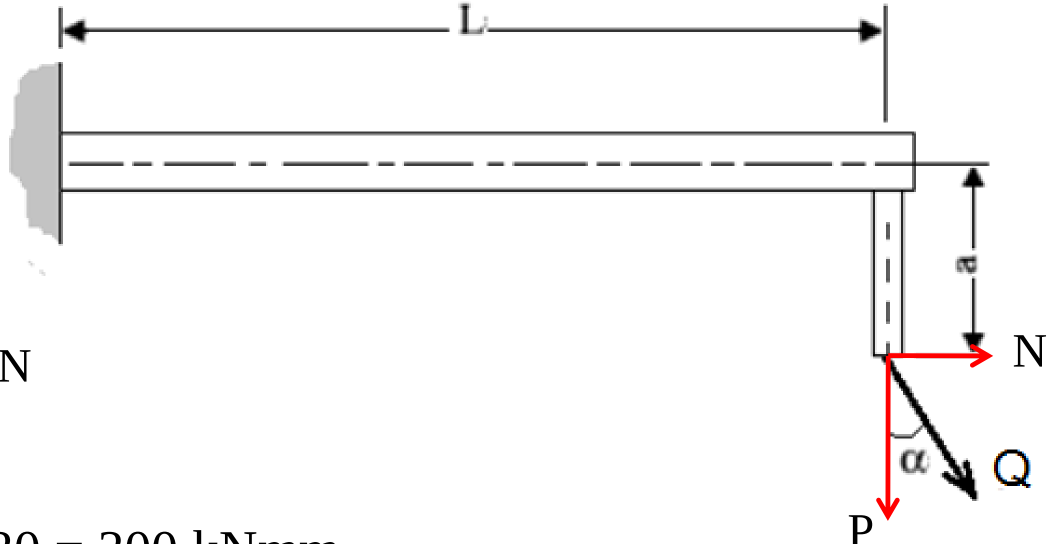
Exo.1 : Soit une poutre cylindrique encastrée de diamètre 30mm. A l’extrémité libre est appliquée une charge P de 3kN comme il est indiqué sur la figure. Déterminer la contrainte maximale de la poutre suivante, sachant que : L =1m; a=0,2m et α=30.



d = 30mm,
P = 3kN,
L =1m; a=0,2m et α=30.

FLEXION COMPOSEE

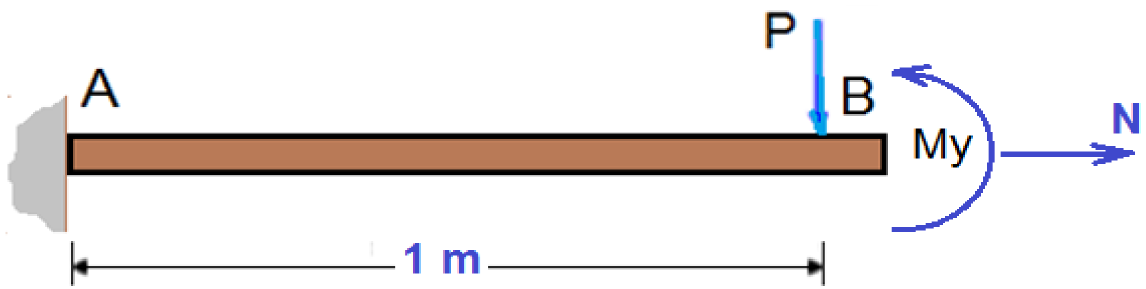
d = 30mm,
P = 3kN,
L =1m; a=0,2m et α=30.



P = Q.cosα = 3000.cos30 = 2,598 kN

N = Q.sinα = 3. .sin30 = 1,500 kN

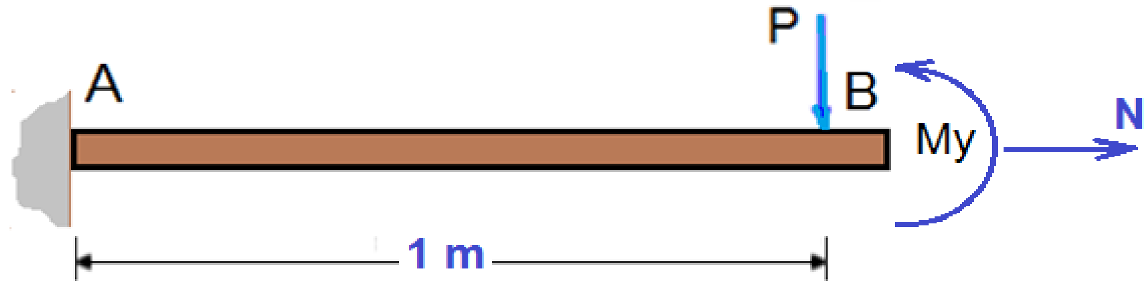
My =N.a = Q.a.sinα = 0,2.3.sin30 = 300 kNmm



Déterminons les réactions aux appuis:

FLEXION COMPOSEE

d = 30mm,
P = 3kN,
L =1m; a=0,2m et α=30.

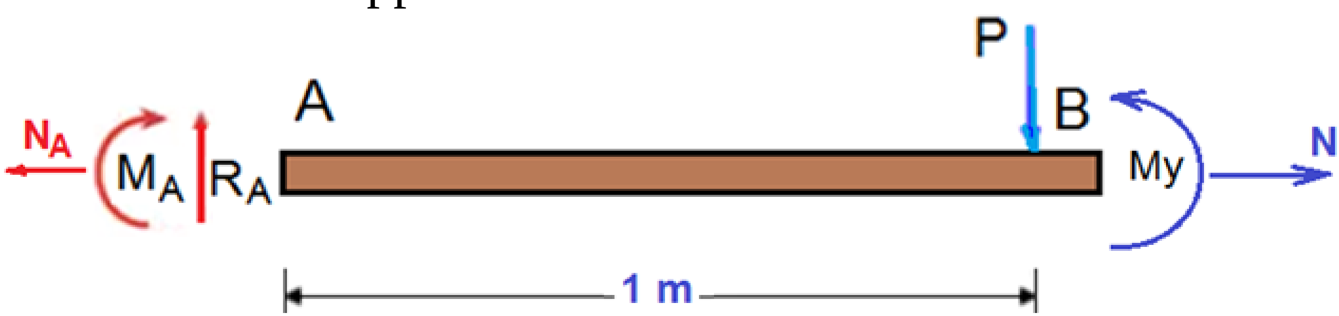


P = 2,598 kN

N = 1,500 kN

My =300 kNmm

Déterminons les réactions aux appuis:



D'apes le principe fondamental de la statique

ΣFx =0

ΣFy =0

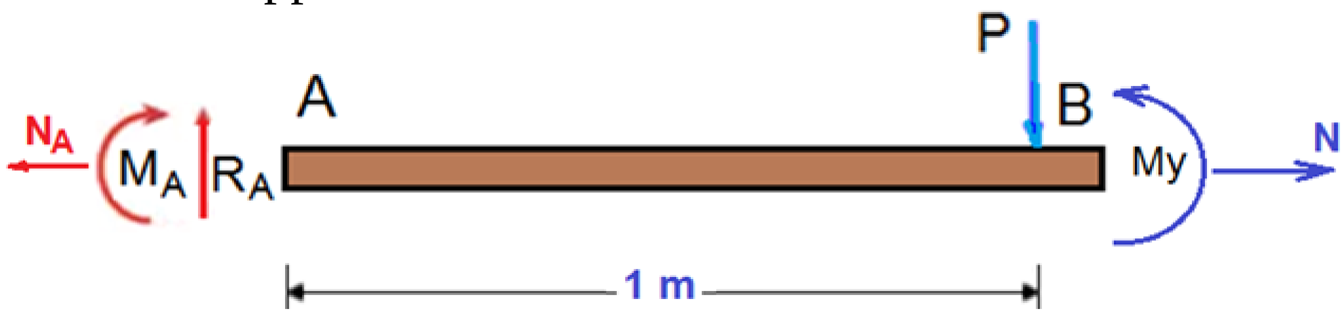
$\Sigma M_{/A}=0$

FLEXION COMPOSEE

d = 30mm,
P = 3kN,
L =1m; a=0,2m et α=30.

P = 2,598 kN
N = 1,500 kN
My =300 kNmm

Déterminons les réactions aux appuis:

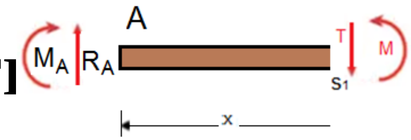


$\Sigma Fx = 0$; $N - N_A = 0 \Rightarrow N_A = N = 1{,}5$ kN

$\Sigma Fy = 0$; $P - R_A = 0 \Rightarrow R_A = P = 2{,}598$ kN

$\Sigma M_{/A} = 0$; $R_A.1000 + M_A - My = 0 \Rightarrow M_A = -P.1000 + My$

$M_A = -2{,}598.1000 + 300 = -2298$ kNmm

FI )SEE

d = 30mm,
P = 3kN,
L =1m; a=0,2m et α=30.

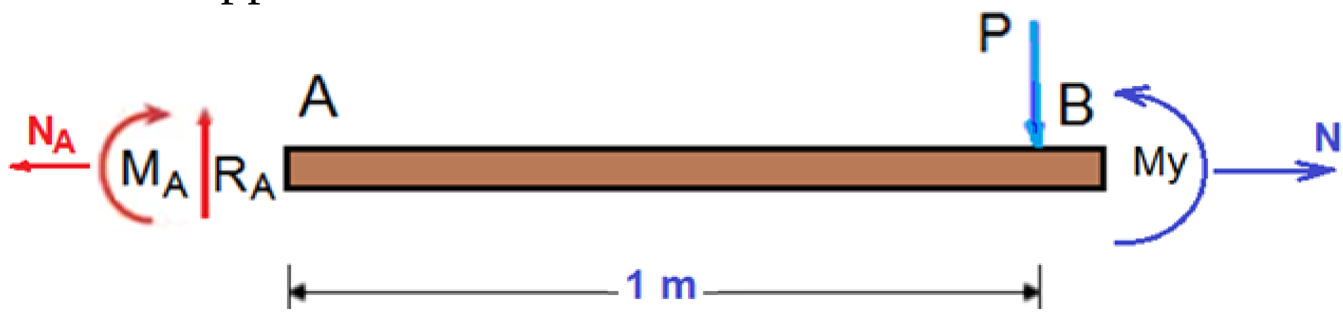
P = 2,598 kN
N = 1,500 kN
M_y =300 kNmm

Déterminons les réactions aux appuis:

N_A = 1,5 kN

R_A = 2,598 kN

M_A = -2298 kNmm

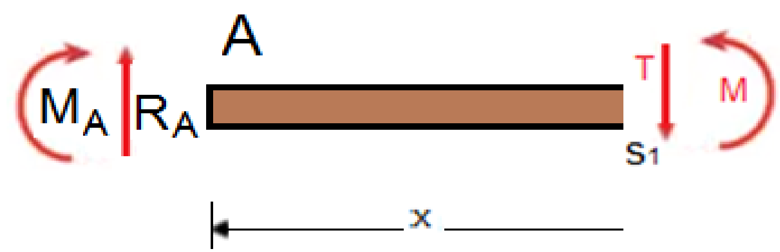


On a donc dans le premier tronçon jusqu'à P (0< x <1000):

$\Sigma F_y = 0$; $-R_A + T = 0$; $T = 2{,}598$ kN

$\Sigma M_{/s1} = 0$; $-R_A . x - M_A + M = 0$;

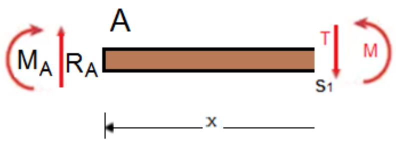
$M = 2598x - 2298000$



Pour x =0; M = -2298 kNmm

Pour x =L; M = 300 kNmm ;

FLEXION COMPOSEE



d = 30mm,
P = 3kN,
L =1m; a=0,2m et α=30.

P = 2,598 kN

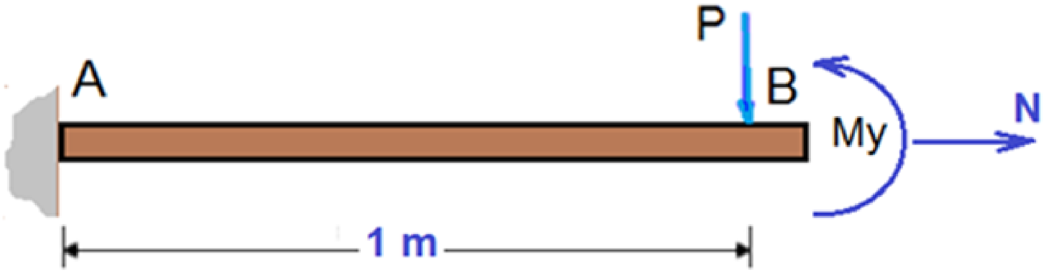
My =300 kNmm

N = 1,500 kN

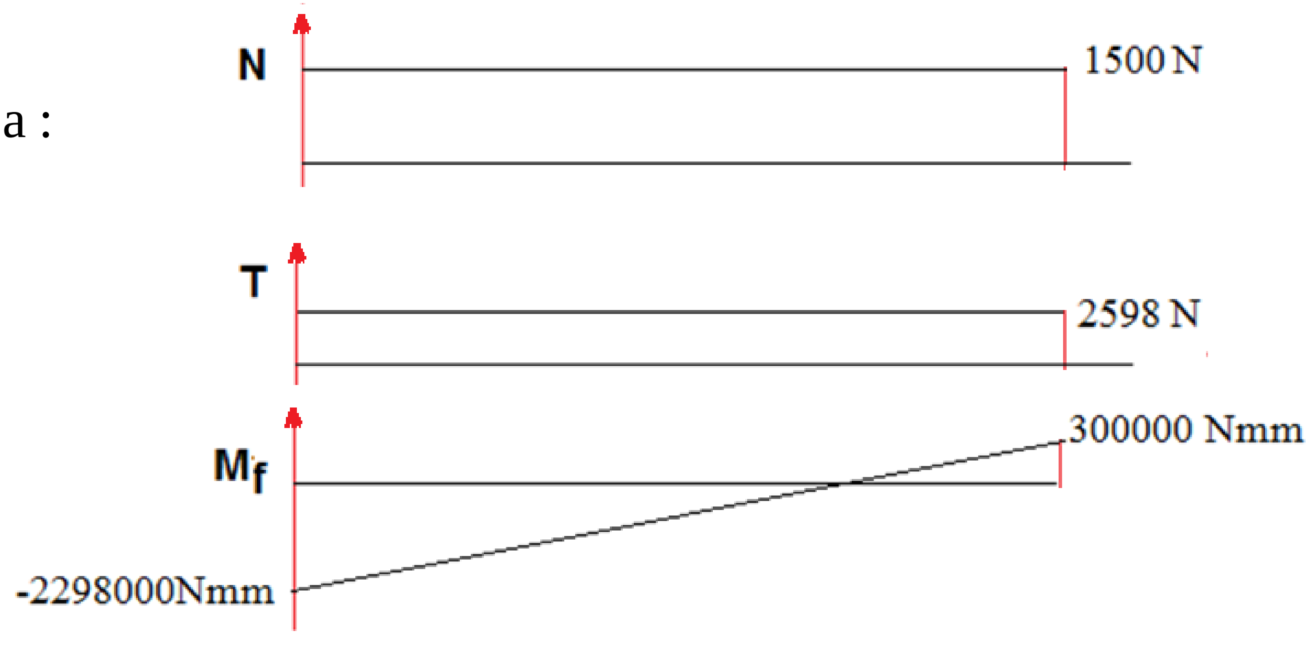
T = 2,598 kN

Pour x =0; M = -2298 kNmm

Pour x =L; M = 300 kNmm ;



D'après les graphes, on a :
M_{fmax} = 2,298 kNm
N_{max} = 1,5 kN



FLEXION COMPOSEE

d = 30mm,
P = 3kN,
L =1m; a=0,2m et α=30.

P = 2,598 kN

My =300 kNmm

N = 1,500 kN

T = 2,598 kN

D'après les graphes, on a :

M_{fmax} = 2,298 kNm
N_{max} = 1,5 kN

$$\sigma_{max} = \sigma_{tra} + \sigma_f$$

$\sigma_{tra} = N_{max} / S$

$S = \pi.d^2/4 = \pi.30^2/4 = 706,5 \text{ mm}^2$

σ_{tra} = 1500/706,5 = 2,12MPa

$\sigma_f = (M_f.d/2)/ I_y$

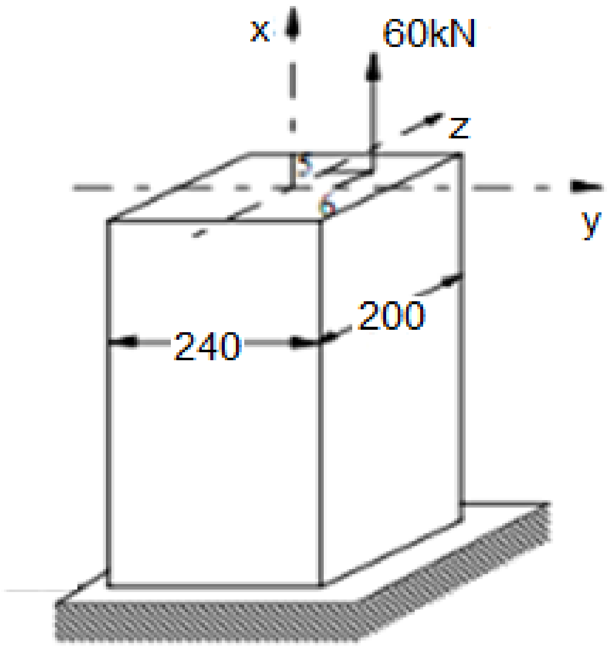
$I_y = (\pi.d^4)/64 = (\pi.30^4)/64 = 39740,6 \text{mm}^4$

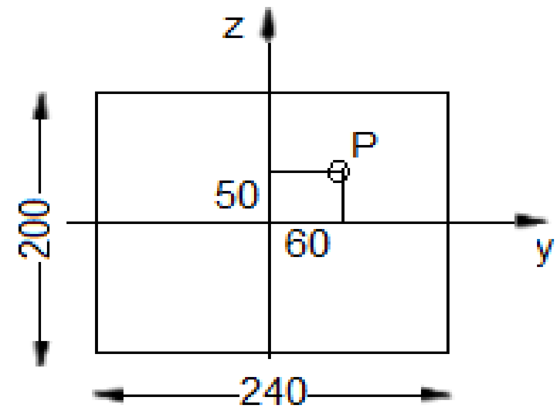
σ_f = 2298000.15/39740,6= 867,4 MPa

$$\sigma_{max} = \sigma_{tra} + \sigma_f = 869,5 \text{ MPa}$$

FLEXION COMPOSEE

Exo. 2 : Déterminer la contrainte normale σ_{max} dans la section dangereuse de la poutre ci-dessous avec P = 60 kN, zp = 60mm, yp = 50mm.





FLEXION COMPOSEE

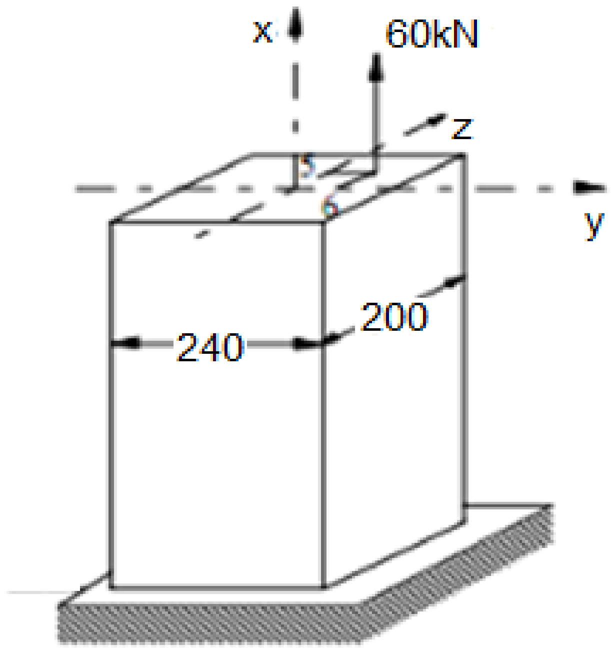
Traction;

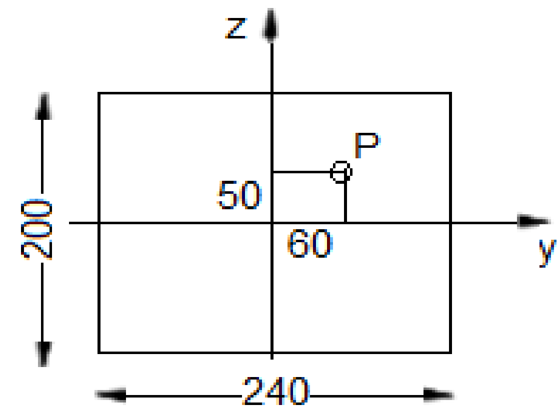
σ_{tra} = N/S

N = P = 60000N

S = 240.200 = 48000 mm^2

σ_{tra} = 60000/48000 = 1,25MPa





FLEXION COMPOSEE

Traction;

$\sigma_{tra} = 1,25\text{MPa}$

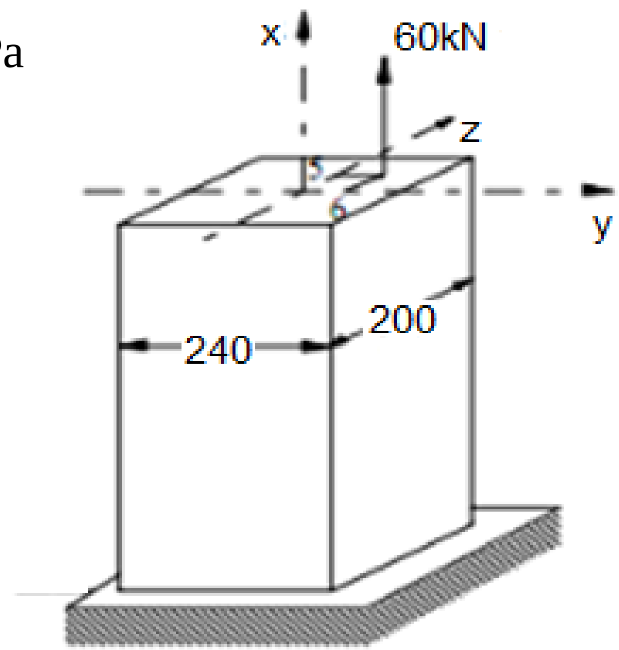
Flexion;

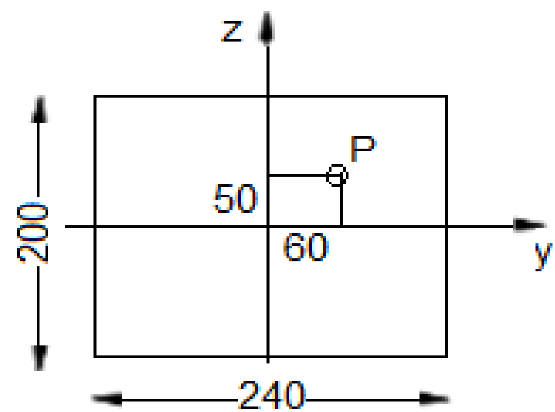
$\sigma_{fz} = M_{fx}\, z / I_y$

$M_{fx} = P.zp = 60000.50 = 3000000\text{Nmm}$

$I_y = 240.200^3/12 = 160000000\ \text{mm}^4$

$\sigma_{fz} = 3000000.100 / I_z = 1,88\text{MPa}$





FLEXION COMPOSEE

Traction;

$\sigma_{tra} = 1{,}25MPa$

Flexion;

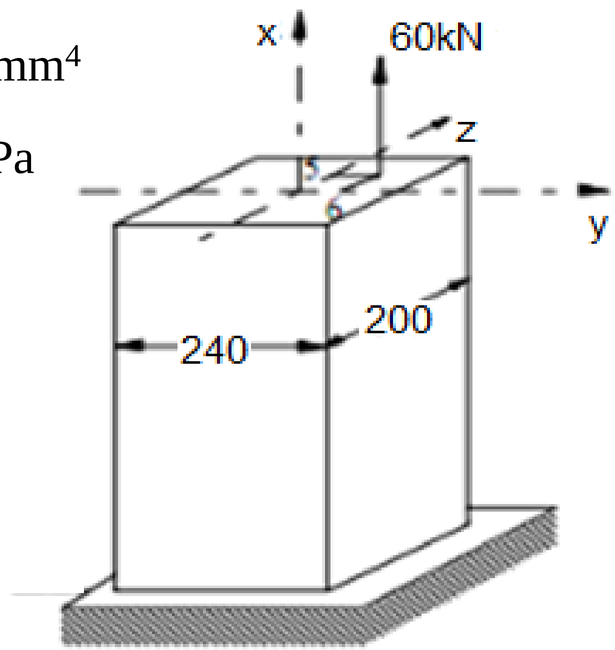
$\sigma_{fz} = 1{,}88MPa$

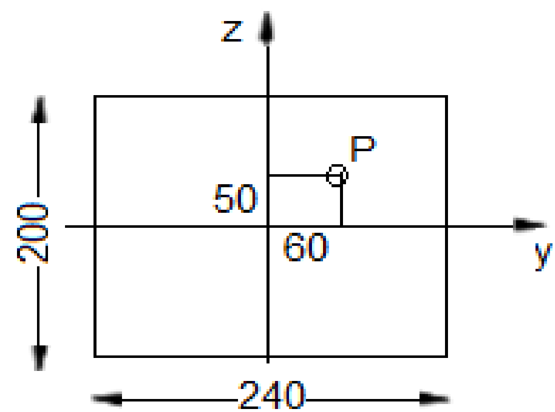
$\sigma_y = M_{fx}\, y / I_z$

$M_{fx} = P.yp = 60000.60 = 3600000Nmm$

$I_y = 200.240^3/12 = 230400000mm^4$

$\sigma_{fy} = 3600000.120 / I_y = 1{,}88\ MPa$





FLEXION COMPOSEE

Traction;

$\sigma_{tra} = 1{,}25\text{MPa}$

Flexion;

$\sigma_{fz} = 1{,}88\text{MPa}$

$\sigma_{fy} = 1{,}88\ \text{MPa}$

la contrainte normale σ_{max}

$$\sigma_{max} = \sigma_{tra} + \sigma_{fz} + \sigma_{fy}$$

$$\sigma_{max} = 1{,}25+1{,}88+1{,}88 = 5{,}01\ \text{MPa}$$