Error propagation

l1 is A+, l2 is A-

```
(%i1) l1 : -(e3*N3 - (e1*N1 + N2))/(e3*N3 + (e1*N1 + N2));
```

$$\frac{-N3\,e3+N1\,e1+N2}{N3\,e3+N1\,e1+N2} \tag{l1}$$

```
(%i2) l2 : -((e3*N3 + N2) - e1*N1)/((e3*N3 + N2) + e1*N1);
```

$$\frac{-N3\,e3+N1\,e1-N2}{N3\,e3+N1\,e1+N2} \tag{l2}$$

1 Error propagation from N1, N2, N3 and standard model error to e1, e3

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(%i3) e_sol : solve([Ap_sm = l1, Am_sm = l2], [e1, e3])[1];
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$$\left[e1=\frac{(Am\_sm+1)\,N2}{(Ap\_sm-Am\_sm)\,N1}\,,e3=-\frac{(Ap\_sm-1)\,N2}{(Ap\_sm-Am\_sm)\,N3}\right] \tag{e\_ sol}$$

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(%i4) e1_expr : rhs(e_sol[1]); e3_expr : rhs(e_sol[2]);
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$$\frac{(Am\_sm+1)\,N2}{(Ap\_sm-Am\_sm)\,N1} \tag{e1\_ expr}$$

$$-\frac{(Ap\_sm-1)\,N2}{(Ap\_sm-Am\_sm)\,N3} \tag{e3\_ expr}$$

Both e1 and e3 do not use all the N1,N2,N3 but it is alright to use them in the general expression because those terms are multiplied by zero once you do the derivative.

```
(%i6) σ_e(f) := sqrt(diff(f, Ap_sm)^ 2 *~ σ_Ap_sm^ 2 + diff(f, Am_sm)^ 2 *
σ_Am_sm^ 2 + diff(f,N1)^ 2 * σ_N1^ 2 + diff(f,N2)^ 2 *σ_N2^ 2 + diff(f,N3)^
2 * σ_N3^ 2~ );
```

$$\sigma\_e(f):=\sqrt{\left(\frac{\partial}{\partial Ap\_sm}f\right)^2\sigma\_Ap\_sm^2+\left(\frac{\partial}{\partial Am\_sm}f\right)^2\sigma\_Am\_sm^2+\left(\frac{\partial}{\partial N1}f\right)^2{\sigma_{N1}}^2+\left(\frac{\partial}{\partial N2}f\right)^2{\sigma_{N2}}^2+} \tag{\% o6}$$

-> $\sigma\_e1\_expr :\sim \sigma\_e(e1\_expr);$

$$\sqrt{\frac{(\mathrm{Am\_sm}+1)^2\sigma_{\mathrm{N2}}{}^2}{(\mathrm{Ap\_sm}-\mathrm{Am\_sm})^2\mathrm{N1}^2}+\frac{(\mathrm{Am\_sm}+1)^2\mathrm{N2}^2\sigma_{\mathrm{N1}}{}^2}{(\mathrm{Ap\_sm}-\mathrm{Am\_sm})^2\mathrm{N1}^4}+\frac{(\mathrm{Am\_sm}+1)^2\mathrm{N2}^2\sigma\_\mathrm{Ap\_sm}^2}{(\mathrm{Ap\_sm}-\mathrm{Am\_sm})^4\mathrm{N1}^2}+\left(\frac{\mathrm{N2}}{(\mathrm{Ap\_sm}-\mathrm{Am}}\right.}$$

$$(\sigma\_\ \mathrm{e1}\_\ \mathrm{expr})$$

-> $\sigma\_e3\_expr : \sigma\_e(e3\_expr);$

$$\sqrt{\frac{(\mathrm{Ap\_sm}-1)^2\mathrm{N2}^2\sigma_{\mathrm{N3}}{}^2}{(\mathrm{Ap\_sm}-\mathrm{Am\_sm})^2\mathrm{N3}^4}+\frac{(\mathrm{Ap\_sm}-1)^2\sigma_{\mathrm{N2}}{}^2}{(\mathrm{Ap\_sm}-\mathrm{Am\_sm})^2\mathrm{N3}^2}+\left(\frac{(\mathrm{Ap\_sm}-1)\,\mathrm{N2}}{(\mathrm{Ap\_sm}-\mathrm{Am\_sm})^2\mathrm{N3}}-\frac{\mathrm{N2}}{(\mathrm{Ap\_sm}-\mathrm{Am\_sm})}\right.}$$

$$(\sigma\_\ \mathrm{e3}\_\ \mathrm{expr})$$

2 Error propagation of N1, N2, N3, e1, e3 to A±

-> $\sigma\_A(f) := \mathrm{sqrt}(\mathrm{diff}(f,N1)\hat{}\ 2 * \sigma\_N1\hat{}\ 2 + \mathrm{diff}(f,N2)\hat{}\ 2 * \sigma\_N2\hat{}\ 2 + \mathrm{diff}(f,N3)\hat{}\ 2 * \sigma\_N3\hat{}\ 2 + \mathrm{diff}(f,e1)\hat{}\ 2 * \sigma\_e1\hat{}\ 2 +\sim \mathrm{diff}(f,e3)\hat{}\ 2 * \sigma\_e3\hat{}\ 2);$

$$\sigma\_\mathrm{A}(f) := \sqrt{\left(\frac{\partial}{\partial \mathrm{N1}}f\right)^2\sigma_{\mathrm{N1}}{}^2+\left(\frac{\partial}{\partial \mathrm{N2}}f\right)^2\sigma_{\mathrm{N2}}{}^2+\left(\frac{\partial}{\partial \mathrm{N3}}f\right)^2\sigma_{\mathrm{N3}}{}^2+\left(\frac{\partial}{\partial \mathrm{e1}}f\right)^2\sigma_{\mathrm{e1}}{}^2+\left(\frac{\partial}{\partial \mathrm{e3}}f\right)^2\sigma_{\mathrm{e3}}{}^2}$$

(% o9)

-> $\sigma\_Ap\_expr :\sim \sigma\_A(l1), \mathrm{factor};$

$$\frac{2\sqrt{\left(\mathrm{N1}^2\mathrm{N3}^2\mathrm{e1}^2+2\mathrm{N1}\,\mathrm{N2N3}^2\mathrm{e1}+\mathrm{N2}^2\mathrm{N3}^2\right)\sigma_{\mathrm{e3}}{}^2+\mathrm{N1}^2\mathrm{N3}^2\mathrm{e3}^2\sigma_{\mathrm{e1}}{}^2+\left(\mathrm{N1}^2\mathrm{e1}^2+2\mathrm{N1}\,\mathrm{N2}\,\mathrm{e1}+\mathrm{N2}^2\right)\mathrm{e3}^2\sigma_{\mathrm{N3}}{}^2+\mathrm{N}}}{\left(\mathrm{N3}\,\mathrm{e3}+\mathrm{N1}\,\mathrm{e1}+\mathrm{N2}\right)^2}$$

$$(\sigma\_\ \mathrm{Ap}\_\ \mathrm{expr})$$

-> $\sigma\_Am\_expr : \sigma\_A(l2), \mathrm{factor};$

$$\frac{2\sqrt{\mathrm{N1}^2\mathrm{N3}^2\mathrm{e1}^2\sigma_{\mathrm{e3}}{}^2+\left(\mathrm{N1}^2\mathrm{N3}^2\mathrm{e3}^2+2\mathrm{N1}^2\mathrm{N2}\,\mathrm{N3}\,\mathrm{e3}+\mathrm{N1}^2\mathrm{N2}^2\right)\sigma_{\mathrm{e1}}{}^2+\mathrm{N1}^2\mathrm{e1}^2\mathrm{e3}^2\sigma_{\mathrm{N3}}{}^2+\mathrm{N1}^2\mathrm{e1}^2\sigma_{\mathrm{N2}}{}^2+\left(\mathrm{N3}^2\mathrm{e1}^2\right.}}{\left(\mathrm{N3}\,\mathrm{e3}+\mathrm{N1}\,\mathrm{e1}+\mathrm{N2}\right)^2}$$

$$(\sigma\_\ \mathrm{Am}\_\ \mathrm{expr})$$

3 Error propagation from A± to helicity fractions

```
->  fr : 1/(1-β) + (am - β*ap)/(3*β*(1-β^ 2));
```

$$\frac{\mathrm{am} - \mathrm{ap}\beta}{3\beta\,(1-\beta^2)} + \frac{1}{1-\beta} \tag{fr}$$

```
->  fl : (1/(1-β)) - (ap - β*am)/(3*β*(1-β^ 2));
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$$\frac{1}{1-\beta} - \frac{\mathrm{ap} - \mathrm{am}\beta}{3\beta\,(1-\beta^2)} \tag{fl}$$

```
->  f0 : -(1+β)/(1-β) + (ap - am)/(3*β*(1-β));
```

$$\frac{\mathrm{ap} - \mathrm{am}}{3\,(1-\beta)\,\beta} + \frac{-\beta - 1}{1-\beta} \tag{f0}$$

```
->  σ_f(f) := sqrt(diff(f,ap)^ 2 * σ_ap^ 2 +~ diff(f,am)^ 2 * σ_am^ 2);
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$$\sigma\_\mathrm{f}(f) := \sqrt{\left(\frac{\partial}{\partial \mathrm{ap}} f\right)^2 {\sigma_{\mathrm{ap}}}^2 + \left(\frac{\partial}{\partial \mathrm{am}} f\right)^2 {\sigma_{\mathrm{am}}}^2} \tag{\% o15}$$

```
->  σ_fr_expr :~ σ_f(fr);
```

$$\sqrt{\frac{{\sigma_{\mathrm{ap}}}^2}{9(1-\beta^2)^2} + \frac{{\sigma_{\mathrm{am}}}^2}{9\beta^2(1-\beta^2)^2}} \tag{σ\_ fr\_ expr}$$

```
->  σ_fl_expr : σ_f(fl);
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$$\sqrt{\frac{{\sigma_{\mathrm{ap}}}^2}{9\beta^2(1-\beta^2)^2} + \frac{{\sigma_{\mathrm{am}}}^2}{9(1-\beta^2)^2}} \tag{σ\_ fl\_ expr}$$

```
->  σ_f0_expr : σ_f(f0);
```

$$\sqrt{\frac{{\sigma_{\mathrm{ap}}}^2}{9(1-\beta)^2\beta^2} + \frac{{\sigma_{\mathrm{am}}}^2}{9(1-\beta)^2\beta^2}} \tag{σ\_ f0\_ expr}$$

4 Inserting values

Data A∼:∼ N1 = 5829, N2 = 840, N3 = 4989Data B :∼ N1 = 21888, N2 = 3186, N3 = 18702Data C :∼ N1 = 31776, N2 = 4208, N3 = 27568Data D :∼ N1 = 51015, N2 = 7042, N3 = 43973 Using sum of N's for N1,N2,N3

-> vals :∼ [N1= 116337, N2=15276, N3=95232, β = 2^ (1/3) -1, Ap_sm = 0.537, Am_sm = -0.841];

$$\left[\text{N1} = 116337\,,\text{N2} = 15276\,,\text{N3} = 95232\,,\beta = 2^{\frac{1}{3}} - 1\,,\text{Ap\_sm} = 0.537\,,\text{Am\_sm} = -0.841\right] \quad \text{(vals)}$$

-> sigmas :∼ [σ_N1 = sqrt(N1), σ_N2 = sqrt(N2), σ_N3 = sqrt(N3), σ_Ap_sm = 0.004, σ_Am_sm = 0.006],vals;

$$\left[\sigma_{\text{N1}} = \sqrt{116337}\,,\sigma_{\text{N2}} = 2\sqrt{3819}\,,\sigma_{\text{N3}} = 32\sqrt{93}\,,\sigma\_\text{Ap\_sm} = 0.004\,,\sigma\_\text{Am\_sm} = 0.006\right] \quad \text{(sigmas)}$$

The following expressions are extremely long, so they are hidden.

-> σ_f0_expr2 :∼ σ_f0_expr, σ_ap = σ_Ap_expr, σ_am = σ_Am_expr$

-> σ_f0_expr3 :∼ σ_f0_expr2, e1 = e1_expr, e3 = e3_expr, σ_e1 = σ_e1_expr, σ_e3 = σ_e3_expr$

-> σ_f0 :∼ σ_f0_expr3, vals, sigmas, numer, eval;

0.01553843758527783 (σ_ f0)

-> σ_fl_expr2 : σ_fl_expr, σ_ap = σ_Ap_expr, σ_am = σ_Am_expr$

-> σ_fl_expr3 : σ_fl_expr2, e1 = e1_expr, e3 = e3_expr, σ_e1 = σ_e1_expr, σ_e3 = σ_e3_expr$

-> σ_fl : σ_fl_expr3, vals, sigmas, numer, eval;

0.008931261372069557 (σ_ fl)

-> σ_fr_expr2 : σ_fr_expr, σ_ap = σ_Ap_expr, σ_am = σ_Am_expr$

-> σ_fr_expr3 : σ_fr_expr2, e1 = e1_expr, e3 = e3_expr, σ_e1 = σ_e1_expr, σ_e3 = σ_e3_expr$

-> σ_fr : σ_fr_expr3, vals, sigmas, numer, eval;

0.00908888578467536 (σ_ fr)

->