$$b = (r\rho + \rho r^\dagger) - Tr(r\rho + \rho r^\dagger)\rho$$
$$b' = r + r^\dagger - (r + r^\dagger)\rho - Tr(r\rho + \rho r^\dagger)$$
$$\underbrace{\left(Tr((\rho) + Tr(\rho r^\dagger)\right)}_{r + r^\dagger}$$

$$\frac{\partial\, Tr(A)}{\partial\, A}$$

$$\frac{\partial\, Tr(r\rho + \rho r^\dagger)}{\partial\,(r\rho + \rho r^\dagger)} \quad \frac{\partial\, r\rho + \rho r^\dagger}{\partial\, \rho}$$

$$u = (r\rho + \rho r^\dagger)$$
$$V = Tr(u)$$
$$w = r + r^\dagger$$
$$b = u - V\rho$$
$$b' = w - w\rho - V$$

$$b = (r\rho + \rho r^\dagger) - Tr(r\rho + \rho r^\dagger)\rho$$
$$b' = r + r^\dagger - (r + r^\dagger)\rho - Tr(r\rho + \rho r^\dagger)$$
$$r = dZ^* C_{out}$$

$$r = dZ^* C_{out}$$

$$dZ^* C_{out} \qquad \xrightarrow{dag} \qquad C_{out}^\dagger dZ$$

Factorization

stochastic Part:

$$S = \left(dZ^* C_{out}\rho + \rho\, C_{out}^\dagger dZ\right) - Tr\left(dZ^* C_{out}\,\rho + \rho\, C_{out}^\dagger dZ\right)\rho$$

$$S = \left(dZ^* C_{out}\rho + \rho\, C_{out}^\dagger dZ\right) - \left(Tr\left(dZ^* C_{out}\,\rho\right) + Tr\left(\rho\, C_{out}^\dagger dZ\right)\right)\rho$$

$$dZ = a + bi = dW_x + dW_y\, i$$

$$S = \left((a - bi)\, C_{out}\rho + \rho\, C_{out}^\dagger (a + bi)\right) - \left((a - bi)\, Tr\left(C_{out}\,\rho\right) + (a + bi)\, Tr\left(\rho\, C_{out}^\dagger\right)\right)\rho$$

$$S = -i\left(b\, Tr\left(\rho\, C_{out}^\dagger\right)\rho\right) - a\, Tr\left(\rho c_{out}^\dagger\right)\rho + i\, b\left(\rho c_{out}^\dagger\right) + a\left(\rho c_{out}^\dagger\right) + i\left(b\, Tr\left(C_{out}\,\rho\right)\rho\right)$$
$$\quad - a\, Tr\left(C_{out}\,\rho\right)\rho - i b\left(C_{out}\,\rho\right) + a\left(C_{out}\,\rho\right)$$

$$S = S_a + S_b$$

$$S_a = -a\, Tr\left(\rho c_{out}^\dagger\right)\rho + a\left(\rho c_{out}^\dagger\right) - a\, Tr\left(C_{out}\,\rho\right)\rho + a\left(C_{out}\,\rho\right)$$
$$\quad = a\left(-Tr\left(\rho c_{out}^\dagger\right)\rho + \rho c_{out}^\dagger - Tr\left(C_{out}\,\rho\right)\rho + C_{out}\,\rho\right)$$

$$S_b = -i\left(b\, Tr\left(\rho\, C_{out}^\dagger\right)\rho\right) + i\, b\left(\rho c_{out}^\dagger\right) + i\left(b\, Tr\left(C_{out}\,\rho\right)\rho\right) - i b\left(C_{out}\,\rho\right)$$
$$\quad = i b\left(\underbrace{-Tr\left(\rho\, C_{out}^\dagger\right)\rho + \rho c_{out}^\dagger}_{t_1} + \underbrace{Tr\left(C_{out}\,\rho\right)\rho - C_{out}\,\rho}_{t_2}\right)$$

$$S = a\left(t_1 - t_2\right) + i b\left(t_1 + t_2\right) = (t_1 - t_2)dW_x + i\,(t_1 + t_2)dW_y$$

Optimization

$$t_1 = -Tr\left(\rho\, c_{out}^+\right)\rho + \rho c_{out}^+ \qquad t_2 = Tr\left(c_{out}\,\rho\right)\rho - c_{out}\rho$$

$$u_1 = \rho c_{out}^+ \qquad u_2 = c_{out}\rho$$

$$t_1 = -Tr\left(u_1\right)\rho + u_1 \qquad t_2 = Tr\left(u_2\right)\rho + u_2$$

The Milstein b'

todo

$$b_1' = (t_1 - t_2) \qquad\qquad b_2 \;\; i(t_1 + t_2)$$

todo: $\qquad b' = r + r^+ - (r + r^+)\rho - Tr\left(r\rho + \rho r^+\right)$