

# Exam 18.10.2024

Mathematics for Programmers, ID00EK08-3001

## Basic math

1. (a) Simplify the expression

$$\left(\frac{3}{2} + \frac{2}{5}\right) \bigg/ \left(3 - \frac{1}{6}\right)$$

to the form  $\frac{p}{q}$ , where  $p$  and  $q$  are integers.

- (b) Simplify by opening the parentheses

$$(4x - 3)^2 - 3(5 - 2x).$$

2. (a) Solve the quadratic equation

$$x^2 - 7x + 10 = 0.$$

- (b) Solve the unknowns  $x$  and  $y$  from the pair of equations

$$\begin{cases} 5x + 2y &= 6 \\ 2x - y &= 3. \end{cases}$$

3. (a) Solve  $x$  when  $2^{7x} = 3^{x+2}$ .

- (b) Solve  $x$  when  $2 \ln(5x^3) - 5 \ln(x) = \ln(7x + 3)$ .

## Calculus

4. Find  $f'(x)$ , when

(a)  $f(x) = 3x^4 - \sqrt{5x} + \frac{1}{x^7}$

(b)  $f(x) = 3 \sin(2x) - 3e^{x^3}$

(c)  $f(x) = \cos(x) \ln(x)$

5. Find  $x > 0$  which is the maximum of  $f(x) = x^5 e^{-7x}$ .

6. Calculate

(a)  $\int 3x + \sqrt{x} dx$

(b)  $\int \cos(3x) dx$

(c)  $\int_0^1 e^{-x} + 1 dx$

# Calculus formulas

$$y - y_1 = k(x - x_1), \quad y = kx + b, \quad k = \frac{y_2 - y_1}{x_2 - x_1} = f'(x_1)$$

$$\int_a^b f(x)dx = \left|_a^b F(x) = F(b) - F(a), \quad F'(x) = f(x)\right.$$

## Differentiation

$$Dx^n = nx^{n-1}$$

$$De^x = e^x$$

$$Db^x = b^x \ln(b)$$

$$D \ln(x) = \frac{1}{x}$$

$$D \ln|x| = \frac{1}{x}$$

$$D \log_a(x) = \frac{1}{x \ln(a)}$$

$$D \log_a|x| = \frac{1}{x \ln(a)}$$

$$D \sin(x) = \cos(x)$$

$$D \cos(x) = -\sin(x)$$

$$D \tan(x) = 1 + \tan^2(x)$$

$$Dx \ln(x) - x = \ln(x)$$

$$D \arcsin(x) = \frac{1}{\sqrt{1-x^2}}$$

$$D \arccos(x) = \frac{1}{-\sqrt{1-x^2}}$$

$$D \arctan(x) = \frac{1}{1+x^2}$$

$$D \sinh(x) = \cosh(x)$$

$$D \cosh(x) = \sinh(x)$$

$$D \tanh(x) = \frac{1}{\cosh^2(x)}$$

## Integration

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int e^x dx = e^x + C$$

$$\int b^x dx = \frac{b^x}{\ln(b)}$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int \cos(x) dx = \sin(x) + C$$

$$\int \sin(x) dx = -\cos(x) + C$$

$$\int 1 + \tan^2(x) dx = \tan(x) + C$$

$$\int \ln(x) dx = x \ln(x) - x + C$$

$$\int \frac{1}{\sqrt{1-x^2}} = \arcsin(x) + C$$

$$\int \frac{1}{-\sqrt{1-x^2}} = \arccos(x) + C$$

$$\int \frac{1}{1+x^2} = \arctan(x) + C$$

## Differentiation

$$Df(g(x)) = f'(g(x))g'(x)$$

Special cases

$$D \ln(f(x)) = \frac{f'(x)}{f(x)}$$

$$De^{f(x)} = e^{f(x)} f'(x)$$

$$Dfg = f'g + fg'$$

$$D(f/g) = (gf' - fg')/g^2$$

## Integration

$$\int f'(g(x))g'(x) dx = f(g(x)) + C$$

$$\int \frac{f'(x)}{f(x)} dx = \ln(f(x)) + C$$

$$\int f'(x)e^{f(x)} dx = e^{f(x)} + C$$

$$\int f'g dx = fg - \int fg' dx$$

# Basic formulas

Fractions

$$\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad + bc}{bd}, \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}, \quad \frac{a}{b} \bigg/ \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

Powers

$$a^b a^c = a^{b+c}, \quad \frac{a^b}{a^c} = a^{b-c}, \quad (a^b)^c = a^{bc}, \quad (ab)^c = a^b a^c, \quad \left(\frac{a}{b}\right)^c = \frac{a^b}{b^c}$$

Roots

$$(a^b)^{\frac{1}{b}} = a^{b \cdot \frac{1}{b}} = a^1 = a, \quad \text{if } a > 0, \quad \sqrt{a} = a^{\frac{1}{2}}, \quad \sqrt[3]{a} = a^{\frac{1}{3}}$$

First degree equation

$$ax = b \quad \Leftrightarrow \quad x = \frac{b}{a}$$

Quadratic equation

$$ax^2 + bx + c = 0 \quad \Leftrightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

System of linear equations

$$\begin{cases} ax + by = U \\ cx + dy = V \end{cases} \Rightarrow \begin{cases} adx + bdy = Ud \\ -bcx - bdy = -bV \end{cases} \Rightarrow \begin{cases} (ad - bc)x = Ud - bV \end{cases} \Rightarrow x = \frac{Ud - bV}{ad - bc}$$

and

$$\begin{cases} ax + by = U \\ cx + dy = V \end{cases} \Rightarrow \begin{cases} -acx - bcy = -Uc \\ acx + ady = aV \end{cases} \Rightarrow \begin{cases} (ad - bc)y = aV - Uc \end{cases} \Rightarrow y = \frac{aV - Uc}{ad - bc}$$

Function  $f(x)$  and inverse function  $g(x) = f^{-1}(x)$

$$f(g(x)) = x, \quad g(f(x)) = x$$

Powers	Logarithms
$a^1 = a$	$\log_a(a) = 1$
$a^0 = 1$	$\log_a(1) = 0$
$a^b = c \quad \Leftrightarrow \quad a = c^{\frac{1}{b}}$	$\log_a(c) = \frac{1}{\log_c(a)}$
$a^b a^c = a^{b+c}$	$\log_a(bc) = \log_a(b) + \log_a(c)$
$\frac{a^b}{a^c} = a^{b-c}$	$\log_a \frac{b}{c} = \log_a(b) - \log_a(c)$
$(a^b)^c = a^{bc}$	$\log_a(b) = \frac{\log_c(b)}{\log_c(a)}$
	$\log_a(b^c) = c \log_a(b)$

$$\log_{10}(x) \approx \frac{x-1}{x+1}, \quad \frac{1}{5} \leq x \leq 5,$$

$$\log_e(x) \approx 2 \frac{x-1}{x+1}, \quad \frac{1}{3} \leq x \leq 3,$$

$$\log_2(x) \approx 3 \frac{x-1}{x+1}, \quad \frac{1}{2} \leq x \leq 2,$$

$\log_2(2) = 1$	$\log_2(e) \approx 1.4427$	$\log_2(10) \approx 3.3219$
$\log_e(2) \approx 0.6931$	$\log_e(e) = 1$	$\log_e(10) \approx 2.3026$
$\log_{10}(2) \approx 0.30103$	$\log_{10}(e) \approx 0.4343$	$\log_{10}(10) = 1$

Powers

$$a^b a^c = a^{b+c}, \quad \frac{a^b}{a^c} = a^{b-c}, \quad (a^b)^c = a^{bc}, \quad (ab)^c = a^b a^c, \quad \left(\frac{a}{b}\right)^c = \frac{a^b}{b^c}$$

Logarithms

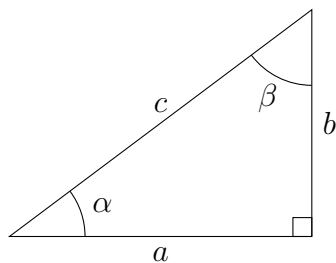
$$\ln(ab) = \ln(a) + \ln(b), \quad \ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b), \quad \ln(a^b) = b \ln(a)$$

$$\log_a(x) = y \quad \Leftrightarrow \quad a^y = x$$

$$\log_a(1) = 0, \quad \log_a(a) = 1, \quad \log_a(a^x) = x, \quad a^{\log_a(x)} = x$$

$$\begin{aligned} \log_a(b^c) &= c \log_a(b) \\ \log_a(xy) &= \log_a(x) + \log_a(y) \\ \log_a\left(\frac{x}{y}\right) &= \log_a(x) - \log_a(y) \\ \log_a(x) &= \frac{\log_b(x)}{\log_b(a)} \end{aligned}$$

$$\text{lb}(x) = \log_2(x), \quad \text{lg}(x) = \log_{10}(x), \quad \ln(x) = \log_e(x), \quad e \approx 2,72$$



Trigonometry

$$\begin{aligned} c^2 &= a^2 + b^2 \\ \sin(\alpha) &= \frac{b}{c}, \quad \cos(\alpha) = \frac{a}{c}, \quad \tan(\alpha) = \frac{b}{a}, \\ \alpha &= \arcsin \frac{b}{c}, \quad = \arccos \frac{a}{c}, \quad = \arctan \frac{b}{a}, \\ \alpha &= \text{imag}(\ln(a + bi)) \cdot \frac{180^\circ}{\pi} \end{aligned}$$