Early exam, sample 2.9.2024

Mathematics for Programmers, ID00EK08-3001

Basic math

1. (a) Simplify the expression

$$\left(\frac{1}{7} - \frac{1}{10}\right) \Big/ \left(\frac{1}{7} + \frac{1}{2}\right)$$

to the form $\frac{p}{q}$, where p and q are integers.

(b) Simplify by opening the parentheses

$$(3x + 2)^2 - 2(6x - 2) + 5.$$

2. (a) Solve the quadratic equation $2x^2 - 8x + 6 = 0$.

(b) Solve the unknowns x and y from the pair of equations

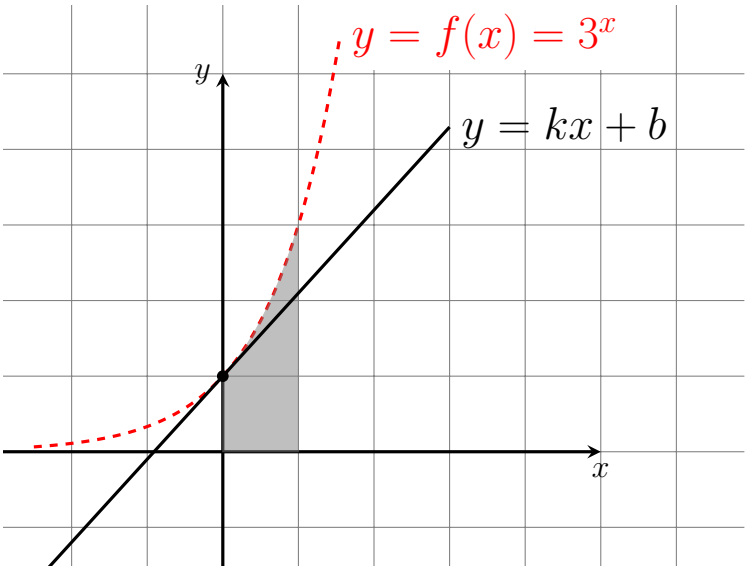
$$\begin{cases} 6x + 7y &= 4 \\ x + 2y &= 2. \end{cases}$$

3. (a) Solve x when $5^{4x} = 3^{x+2}$.

(b) Solve x when $2\ln(3x) - \ln(x) = \ln(x + 2)$.

Calculus

1. Find $f'(x)$, when
   (a) $f(x) = 4x^5 - \sqrt{2x} + \frac{1}{x^3}$
   (b) $f(x) = 2\sin(3x) - 7e^{x^4}$
   (c) $f(x) = e^x \ln(x)$



Kuva 1: A graph of a function, a tangent line and a shaded area. Gridlines are one unit apart.

2. Find the equation $y = kx + b$ of the line in Figure 1.
3. Calculate the shaded area in Figure 1.
4. Find x > 0 which is the maximum of $f(x) = x^8 e^{-3x}$.
5. Calculate
   (a) $\int 4x + \sqrt{x} dx$
   (b) $\int \cos(2x) dx$
   (c) $\int_2^3 e^{-x} + 1 dx$

Calculus formulas

$$y - y_1 = k(x - x_1), \quad y = kx + b, \quad k = \frac{y_2 - y_1}{x_2 - x_1} = f'(x_1)$$

$$\int_a^b f(x)dx = \Big|_a^b F(x) = F(b) - F(a), \quad F'(x) = f(x)$$

Differentiation		**Integration**	
Dx^n	$= nx^{n-1}$	$\int x^n dx$	$= \frac{x^{n+1}}{n+1} + C$
De^x	$= e^x$	$\int e^x dx$	$= e^x + C$
Db^x	$= b^x \ln(b)$	$\int b^x dx$	$= \frac{b^x}{\ln(b)}$
$D\ln(x)$	$= \frac{1}{x}$		
$D\ln\lvert x\rvert$	$= \frac{1}{x}$	$\int \frac{1}{x} dx$	$= \ln\lvert x\rvert + C$
$D\log_a(x)$	$= \frac{1}{x\ln(a)}$		
$D\log_a\lvert x\rvert$	$= \frac{1}{x\ln(a)}$		
$D\sin(x)$	$= \cos(x)$	$\int \cos(x)dx$	$= \sin(x) + C$
$D\cos(x)$	$= -\sin(x)$	$\int \sin(x)dx$	$= -\cos(x) + C$
$D\tan(x)$	$= 1 + \tan^2(x)$	$\int 1 + \tan^2(x)dx$	$= \tan(x) + C$
$Dx\ln(x) - x$	$= \ln(x)$	$\int \ln(x)dx$	$= x\ln(x) - x + C$
$D\arcsin(x)$	$= \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}}$	$= \arcsin(x) + C$
$D\arccos(x)$	$= \frac{1}{-\sqrt{1-x^2}}$	$\int \frac{1}{-\sqrt{1-x^2}}$	$= \arccos(x) + C$
$D\arctan(x)$	$= \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2}$	$= \arctan(x) + C$
$D\sinh(x)$	$= \cosh(x)$		
$D\cosh(x)$	$= \sinh(x)$		
$D\tanh(x)$	$= \frac{1}{\cosh^2(x)}$		

Differentiation		**Integration**	
$Df(g(x))$	$= f'(g(x))g'(x)$	$\int f(g(x))g'(x)dx$	$= f(g(x)) + C$
Special cases			
$D\ln(f(x))$	$= \frac{f'(x)}{f(x)}$	$\int \frac{f'(x)}{f(x)}dx$	$= \ln(f(x)) + C$
$De^{f(x)}$	$= e^{f(x)}f'(x)$	$\int f'(x)e^{f(x)}dx$	$= e^{f(x)} + C$
Dfg	$= f'g + fg'$	$\int f'gdx$	$= fg - \int fg'dx$
$D(f/g)$	$= (gf' - fg')/g^2$		

Basic formulas

Fractions

$$\frac{a}{b}+\frac{c}{d}=\frac{ad}{bd}+\frac{bc}{bd}=\frac{ad+bc}{bd},\quad \frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd},\quad \frac{a}{b}\Big/\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{ad}{bc}$$

Powers

$$a^b a^c=a^{b+c},\quad \frac{a^b}{a^c}=a^{b-c},\quad (a^b)^c=a^{bc},\quad (ab)^c=a^b a^c,\quad \left(\frac{a}{b}\right)^c=\frac{a^b}{b^c}$$

Roots

$$(a^b)^{\frac{1}{b}}=a^{b\cdot\frac{1}{b}}=a^1=a,\quad \text{jos}\quad a>0,\quad \sqrt{a}=a^{\frac{1}{2}},\quad \sqrt[3]{a}=a^{\frac{1}{3}}$$

First degree equation

$$ax=b\quad\Leftrightarrow\quad x=\frac{b}{a}$$

Quadratic equation

$$ax^2+bx+c=0\quad\Leftrightarrow\quad x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

System of linear equations

$$\begin{cases} ax+by &= U \\ cx+dy &= V \end{cases} \Rightarrow \begin{cases} acx+bcy &= cU \\ -acx+-ady &= -aV \end{cases} \Rightarrow \dots$$

$$\begin{cases} ax+by &= U \\ cx+dy &= V \end{cases} \Rightarrow y=\frac{U-ax}{b} \Rightarrow cx+d\frac{U-ax}{b}=V \Rightarrow \dots$$

$$\begin{cases} ax+by &= U \\ cx+dy &= V \end{cases} \Rightarrow \begin{cases} x &= \frac{Ud-bV}{ad-bc} \\ y &= \frac{aV-Uc}{ad-bc} \end{cases}$$

Function $f(x)$ and inverse function $g(x)=f^{-1}(x)$

$$f(g(x))=x,\quad g(f(x))=x$$

Logarithms

$$\ln(ab)=\ln(a)+\ln(b),\quad \ln(\frac{a}{b})=\ln(a)-\ln(b),\quad \ln(a^b)=b\ln(a)$$

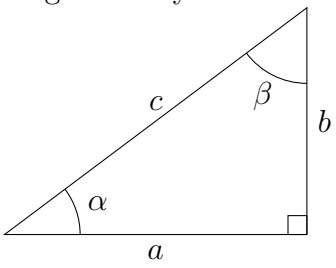
$$\log_a(x)=y\quad\Leftrightarrow\quad a^y=x$$

$$\log_a(1)=0,\quad \log_a(a)=1,\quad \log_a(a^x)=x,\quad a^{\log_a(x)}=x$$

$$\begin{aligned} \log_a(b^c)&=c\log_a(b)\\ \log_a(xy)&=\log_a(x)+\log_a(y)\\ \log_a\left(\frac{x}{y}\right)&=\log_a(x)-\log_a(y)\\ \log_a(x)&=\frac{\log_b(x)}{\log_b(a)} \end{aligned}$$

$$\mathrm{lb}(x) = \log_2(x), \quad \lg(x) = \log_{10}(x), \quad \ln(x) = \log_e(x), \quad e \approx 2,72$$

Trigonometry



$$c^2 = a^2 + b^2$$

$$\sin(\alpha) = \frac{b}{c}, \quad \cos(\alpha) = \frac{a}{c}, \quad \tan(\alpha) = \frac{b}{a},$$

$$\alpha = \arcsin\frac{b}{c}, \quad = \arccos\frac{b}{c}, \quad = \arctan\frac{b}{c},$$

$$\alpha = \mathrm{imag}(\ln(a + bi)) \cdot \frac{180^\circ}{\pi}$$