DIFFERENTIAL EQUATIONS PROJECT REPORT

AALYAN RAZA	22I-0833
ANAS BIN RASHID	22I-0907
NAJAMUDDIN HASSAN	22I-1332

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OBJECTIVES AND INTRODUCTION

The problem that was given to us dealt with the application of a suspension system where the task was to find out the best and most cost effective shock out of the three available options which can then be used for the suspension system of the STT. The solution methodology which was adopted was based on pure implementation of the studied topics of differential equations of higher orders and their applications in real world scenarios.

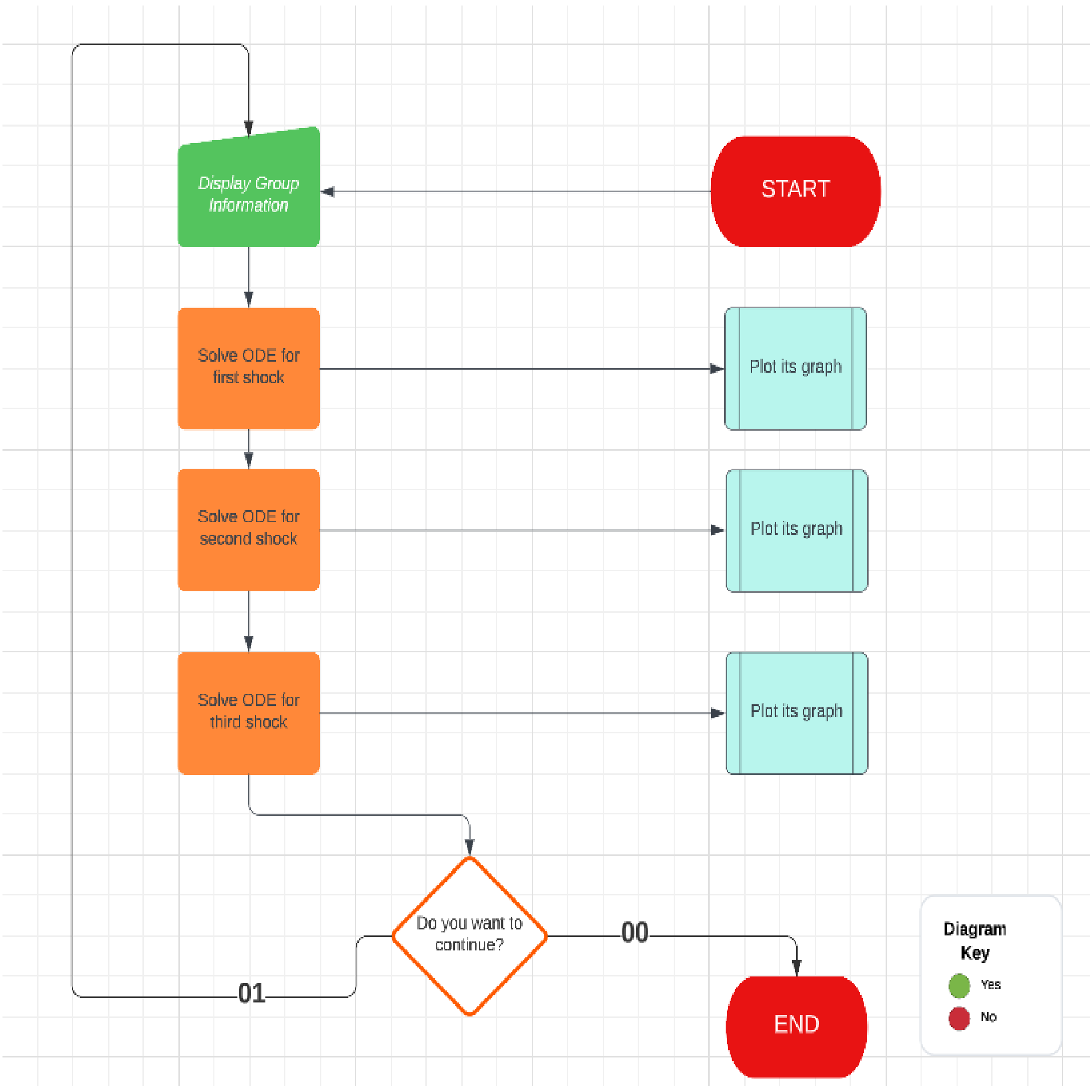

Figure 1 ***Flowchart***

ANALYTICAL SOLUTION

Part 1, Part 2

have been done By-Hand and attached.

(Scanned Images on Following Pages)

Part 3, Part 4, Part 6

have been done on MATLAB.

Part 5, Part 7

included after MATLAB Solution.

Analytical Solution

day / date:

The mass spring system equation is implemented here as:

$$m\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + kx = F(x)$$

Here $m = 1800$, $\beta = 100$, $k = 220$, $F(x) = 0$

$$\therefore \quad 1800\frac{d^2x}{dt^2} + 100\frac{dx}{dt} + 220x = 0$$

is our required DE to solve for first shock.

$$1800m^2 + 100m + 220 = 0$$

Solving for m, we get:

$$m = -0.03 \pm 0.34i$$

The equation for solution of $x(t)$ becomes:

$$x(t) = e^{-0.03t}\left[c_1\cos 0.34t + C_2\sin 0.34t\right]$$

The initial conditions are

$$x(0) = 3\text{ cm} = 0.03\text{ m} \rightarrow (i)$$

$$x'(0) = 0 \rightarrow (ii)$$

At $t = 0$, $x = 0.03$

$$C_1 = 0.03$$

Now for second initial condition:

$$x'(t) = C_1\left[-0.03e^{-0.03t}\cos 0.34t - 0.34\sin 0.34t\, e^{-0.03t}\right] + C_2\left[-0.03e^{-0.03t}\sin 0.34t + e^{-0.03t}\, 0.34\cos 0.34t\right]$$

At $t = 0$, $x'(t) = 0$

$$0 = -C_1[0.03] + C_2[0.34]$$

$$C_2 = 0.00264$$

$$\therefore x(t) = e^{-0.03t}\left[0.03\cos(0.34t) + 0.00264\sin(0.34t)\right]$$

Figure 2 ***Analytical Solution***

MATLAB CODE

```
t=1;                    %% Condition for first iteration of loop
while t==1              %% While loop so program continues till user wants

%% Displaying Group Information

fprintf('\n\nSoftware House:\t\tThe Nebula Webb\n\nAalyan Raza\t\t22I-0833\nAnas Bin
Rashid\t\t22I-0907\nNajamuddin Hassan\t22I-1332\n\n');

%% Press Any Key to Continue Prompt

fprintf("\nPress any key to continue: ");
pause;

%% Define the ODE for first shock as well as define Initial Conditions

syms x(t)
ode = 1800*diff(x, t, t) + 100*diff(x, t) + 220*x == 0;
xprime = diff(x);
ic = [x(0)==0.030, xprime(0)==0];

%% Solve the ODE for first shock using dsolve

sol = dsolve(ode, ic);

%% Displaying Solution for first shock

fprintf("\n\nFirst Shock:\t")
disp(sol)

%% Plotting Graph for First Shock

fplot(sol, [0, 200]);
hold on;

%% Define the ODE for second shock as well as define Initial Conditions

syms x(t)
ode = 1800*diff(x, t, t) + 1050*diff(x, t) + 59*x == 0;
xprime = diff(x);
ic = [x(0)==0.030, xprime(0)==0];

%% Solve the ODE for second shock using dsolve

sol = dsolve(ode, ic);

%% Displaying Solution for second shock
```

```
fprintf("\nSecond Shock:\t")
disp(sol)

%% Plotting Graph for Second Shock

fplot(sol, [0, 200]);
hold on;

%% Define the ODE for third shock as well as define Initial Conditions

syms x(t)
ode = 1800*diff(x, t, t) + 1110*diff(x, t) + 170*x == 0;
xprime = diff(x);
ic = [x(0)==0.030, xprime(0)==0];

%% Solve the ODE for third shock using dsolve

sol = dsolve(ode, ic);

%% Displaying Solution for third shock

fprintf("\nThird Shock:\t")
disp(sol)

%% Plotting Graph for Third Shock

fplot(sol, [0, 200]);
hold on;

%% Input for next iteration of loop

t=input('\nPress 1 if you wish to continue: ');
end
```

MATLAB SOLUTION AND RESULTS

An overall display of the MATLAB code output is attached below which is then followed by a complete breakdown of the MATLAB solutions and results:

```
Software House:        The Nebula Webb

Aalyan Raza            22I-0833
Anas Bin Rashid        22I-0907
Najamuddin Hassan      22I-1332


Press any key to continue:

First Shock:    (3*exp(-t/36)*(787*cos((3935^(1/2)*t)/180) + 3935^(1/2)*sin((3935^(1/2)*t)/180)))/78700


Second Shock:   (753^(1/2)*exp(t*(753^(1/2)/120 - 7/24))*(753^(1/2) + 35))/50200 - exp(-t*(753^(1/2)/120 + 7/24))*((7*753^(1/2))/10040 - 3/200)


Third Shock:    (exp(-t/3)*(20*exp(t/20) - 17))/100


Press 1 if you wish to continue:
1


Software House:        The Nebula Webb

Aalyan Raza            22I-0833
Anas Bin Rashid        22I-0907
Najamuddin Hassan      22I-1332


Press any key to continue:
```

Figure 3 ***Overall MATLAB Output***

The program begins by displaying the name of the Software House along with the Names and the Roll Numbers of the Group Members:

```
Software House:        The Nebula Webb

Aalyan Raza            22I-0833
Anas Bin Rashid        22I-0907
Najamuddin Hassan      22I-1332
```

Figure 4 ***Group Description***

Then there is a message asking the user to press any key to continue:

```
Press any key to continue:
```

Figure 5 ***Press any Key***

After the user presses any key, the solution for the First Shock is displayed:

```
First Shock:    (3*exp(-t/36)*(787*cos((3935^(1/2)*t)/180) + 3935^(1/2)*sin((3935^(1/2)*t)/180)))/78700
```

Figure 6 ***First Shock***

First Shock:

```
(3*exp(-t/36)*(787*cos((3935^(1/2)*t)/180)+3935^(1/2)*sin((3935^(1/2)*t)/180)))/78700
```

Followed by the display of the solution for the Second Shock:

```
Second Shock:   (753^(1/2)*exp(t*(753^(1/2)/120 - 7/24))*(753^(1/2) + 35))/50200 - exp(-t*(753^(1/2)/120 + 7/24))*((7*753^(1/2))/10040 - 3/200)
```

Figure 7 ***Second Shock***

Second Shock:

```
(753^(1/2)*exp(t*(753^(1/2)/120 - 7/24))*(753^(1/2) + 35))/50200 - exp(-
t*(753^(1/2)/120 + 7/24))*((7*753^(1/2))/10040 - 3/200)
```

And finally, the solution for the Third Shock is displayed:

```
Third Shock:    (exp(-t/3)*(20*exp(t/20) - 17))/100
```

Figure 8 ***Third Shock***

Third Shock:

```
(exp(-t/3)*(20*exp(t/20) - 17))/100
```

These differential equations have been solved using the **dsolve** command, which basically gives the solution of an ordinary differential equation.

The graph is then displayed which has been made for all the three shocks on a single plane for better representation and comparison of the three shocks using **fplot** command. The blue line is for Shock 1, Yellow is for Shock 2 and Orange is for Shock 3:

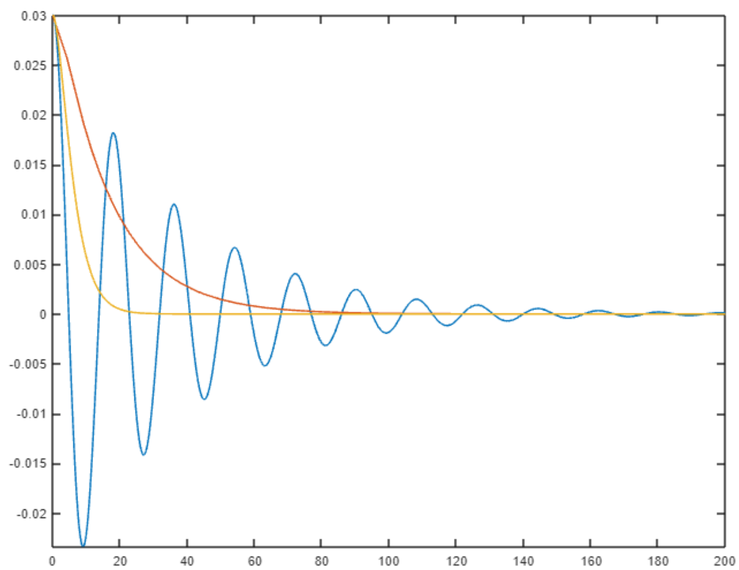

Figure 9 ***Graphs for the Three Shocks***

At the end of the program, there is a prompt asking the user if he/she wants to continue the program or terminate it. 1 should be enter for continuation, any other input will end the program.

For example, if 1 is entered:

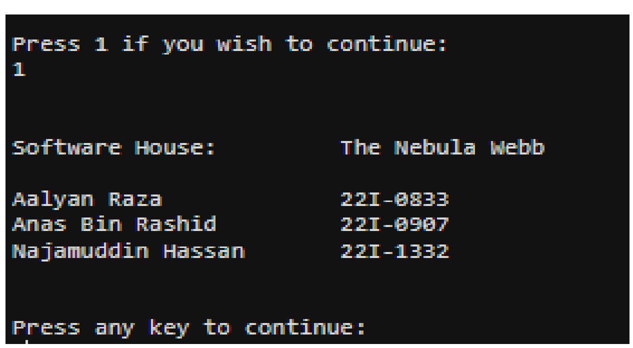

Figure 10 ***If 1 is entered***

On the other hand:

```
Press 1 if you wish to continue:
0
>> |
```

Figure 11 ***If 0 is entered***

Difference between By-Hand Solutions and MATLAB Results

The slight difference between the outputs of Analytical Solution and MATLAB outputs as the MATLAB calculation is extremely precise and the ones used in calculators are rounded-off to an easy to calculate decimal place. Therefore, there is a difference between the two results to some extent even when the methodology was same.

For example, the MATLAB solution for the First Shock is:

```
(3*exp(-t/36)*(787*cos((3935^(1/2)*t)/180)+3935^(1/2)*sin((3935^(1/2)*t)/180)))/78700
```

Which, on simplification, is the same as:

```
e^(-t/36)*[0.03cos(0.348t)+0.00239sin(0.348t)]
```

And is equal to the Analytical Solution found earlier.

Part 5

Comparison between Analytical Solution & MATLAB Result for First Shock:

After analyzing the Analytical Solutions for the First Shock and the MATLAB Solution for the same shock, the conclusion achieved is that both the solutions point to the same very result. As demonstrated in the previous section as well, the MATLAB solution for the First Shock is:

```
(3*exp(-t/36)*(787*cos((3935^(1/2)*t)/180)+3935^(1/2)*sin((3935^(1/2)*t)/180)))/78700
```

Which, on simplification, is the same as:

```
e^(-t/36)*[0.03cos(0.348t)+0.00239sin(0.348t)]
```

And is equal to the Analytical Solution found earlier.

Henceforth, there is no major or considerable difference between the two.

Part 7

Comparison between the MATLAB Results of the Three Shocks:

On comparison between the MATLAB solutions of the three shocks and analyzing the graphs obtained for damping over the period of time, it is quite easy to say that the **First Shock** is the best to be opted for in the suspension system of the STT, due to the perfect damping curve and the overall result being the one that is similar for both the MATLAB and Analytical Solutions as illustrated before too.

CONCLUSIONS

The project dealt with choosing the best and the most cost-effective shocks for the suspension system used in the construction of an STT. As per the provided details, the three ordinary differential equations for the three shocks were obtained and solved, using two different solution techniques. However, both the methods for the solution of the problem were leading to the conclusion that the First Shock is the best choice and should be opted for in the construction process.

CONTRIBUTION

The group had three members. The project was divided between the three members based on the command each member has on their part. The first member Aalyan Raza **(22I-0833)** was mainly involved in doing the programming portion of the project that had to be done on MATLAB. The second member was Anas Bin Rashid **(22I-0907)**, mainly doing report writing, research work and contributing to the programming part as well. The third member was Najamuddin Hassan **(22I-1332)** who did the theoretical work of the project and assisted the others as well.

The main problems that were faced during this project were the use of new functions in MATLAB such as dsolve, but they were resolved through research work and understanding their syntax and correct method of use by getting some online help.