Talha Rana 19K-0274 Section: C

DISCRETE ASSIGNMENT-1

5/5

Q1. a) True.

b) True.

c) False.

d) False.

e) False.

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Q2.**p: I bought a lottery ticket this week.**

q: I won the million-dollar jackpot.

bought

a). ¬p = I did not a lottery ticket this week.

b). p V q = I bought a lottery ticket this week or I won the million-dollar jackpot.

c). p →q = If I buy a lottery ticket this week, then I will win the million-dollar jackpot.

d). p Λq = I bought a lottery ticket this week and I won the million-dollar jackpot.

e). p ↔ q = I will buy the lottery ticket this week if and only if I will win the million-dollar jackpot.

f). ¬p → ¬q = If I will not buy the lottery ticket this week, then I will not win the million-dollar jackpot.

g). ¬p Λ ¬q = I did not buy the lottery ticket this week and I did not win the million-dollar jackpot.

h). ¬p V (p Λ q) = I did not buy the lottery ticket this week or I bought the lottery ticket this week and I won the million-dollar jackpot.

Q3. Let p, q, and r be the propositions

p: You have the flu.

q: You miss the final examination.

r: You pass the course.

5.5/6

a) $p \rightarrow q$ = If you have the flu, then you will miss the final examination.

b) $\neg q \leftrightarrow r$ = You will not miss the final examination if and only if you pass the course. .5

c) $q \rightarrow \neg r$ = If you will miss the final examination, then you will not pass the course.

d) $p \vee q \vee r$ = You have the flu or you miss the final examination or you pass the course.

e) $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$ = If you have the flu, then you will not pass the course or if you will miss the final examination, then you will not pass the course.

f) $(p \wedge q) \vee (\neg q \wedge r)$ = You have the flu and you will miss the final examination or you will not miss the final examination and you will pass the course.

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Q4. a). Inclusive OR: Because you can either have c++ or Java or both.

b). Exclusive OR: You can only one from choose soup or salad.

c). Inclusive OR: You need a passport or voter registration card or both.

d). Exclusive OR: You can publish or you can perish and you cannot do both at same time.

Q5. State the converse, contrapositive, and inverse of each of these conditional statements.

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a) If it snows tonight, then I will stay at home.

Converse: If I will stay at home, then it will snow tonight.

Statement of Queen
only one of these inscription is true.

trunk 1 and 2 inscribed with message the trunk is empty.

trunk 3 inscribed with treasure is in trunk 2

P_1 treasure in trunk 1
P_2: treasure in trunk 2
P_3: treasure in trunk 3

$\neg P1, \neg P2, P2$

$(\neg P1 \wedge \neg(\neg P2), \neg P2) \vee (\neg(\neg P1) \wedge \neg P2, \neg P2)$

$\vee(\neg(\neg P1) \wedge \neg(\neg P2), P2)$

$(\neg P1 \wedge \overset{T}{P2} \wedge \overset{F}{\neg P2}) \vee (P1 \wedge \neg P2 \wedge \neg P2)$

$\vee(P1 \wedge P2 \wedge P2)$

$(\neg P1 \wedge \neg P2) \vee (P1 \wedge \neg P2) \vee (P1 \wedge P2)$

contradiction $\cancel{(\neg P1 \wedge P2 \wedge \neg P2)} \vee (P1 \wedge \neg P2)$

$\vee(P1 \wedge P2)$ with the help of distributive law. is equivalent to

$P1 \wedge (\neg P2 \vee P2)$

$P1 \wedge T \Rightarrow P1$

So the treasure is in trunk 1

Inverse: If it will not snow tonight, then I will not stay at home.

Contrapositive: If I will not stay at home, then it will not snow tonight.

q whenever p

b) I go to the beach whenever it is a sunny summer day. If p then q.

Converse: If I go to the beach, then it is a sunny summer day.
Inverse: If it is not a sunny summer day, then I will not go to the beach.
Contrapositive: If I will not go to the beach, then it is not a sunny summer day.

q is necessary for p

c) When I stay up late, it is necessary that I sleep until noon. If p then q.

If I stay up late then I sleep until noon.

Converse: If I sleep until noon, then I will stay up late.

If I sleep until noon then I stay up late.

Inverse: If I will not stay up late, then I will not sleep until noon.
Contrapositive: If I will not sleep until noon, then I will not stay up late.

no need of description prove it with T.V

Q6. Explain, without using a truth table, why $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$ is true when p, q, and r have the same truth value and it is false otherwise. 2|2

Ans. There is disjunction inside the brackets and conjunction outside the brackets. So, when p, q & r have same values i.e T or F, all the disjunctions gives True value hence the conjunction of all True is True.

Solution:- p, q, r is true.

$(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$

$(T \vee F) \wedge (T \vee F) \wedge (T \vee F)$

$T \wedge T \wedge T$.

Q7. a). $r \rightarrow \neg p$. ✓ 4|4

b). $(p \wedge r) \rightarrow q$. ✓

c). $\neg r \rightarrow \neg q$. ✓ ✓

d). $(\neg p \wedge r) \rightarrow q$.

Q8. Suppose that in Example 7, the inscriptions on Trunks 1, 2, and 3 are "The treasure is in Trunk 3," "The treasure is in Trunk 1," and "This trunk is empty." For each of these statements, determine whether the Queen who never lies could state this, and if so, which trunk the treasure is in.

Q10- Ans.

butler : b is the proposition butler is telling truth. ($\neg b$ is telling lying)

cook : c is the proposition cook is telling truth. ($\neg c$)

gardner: g is the proposition gardner is telling the truth. ($\neg g$)

handy man: h is - - - - . ($\neg h$)

Now consider statements.

If the butler is telling the truth then so is the cook.

2° $\neg(c \wedge g)$

3° $\neg(\neg g \wedge \neg h)$ — It mean both are lying.

4° If $h \to \neg c$

how can we identify each of them telling truth or lying. take 1 assumption based on some deductive reasoning conclusion.

$b \to c$ — $\neg c \vee \neg g$ — $g \vee h$ — $h \to \neg c$

$b \to c$ — $F \vee T$ — $F \vee T$ — $T \to F$

$T \to T$

~~Now~~

T — T — T — F

So we are not able to make last one as true. It mean there some contradiction

case 2:: b is false.

$b \to c$ — $\neg c \vee \neg g$ — $g \vee h$ — $h \to \neg c$

$F \to T/F$ — $F \vee T$ — $F \vee T$ — $T \to F$

F Again contradict

It mean c is also not true.

Now consider c is false

$b \to c$ — $\neg c \vee \neg g$ — $g \vee h$ — $h \to \neg c$

F F — $T \vee T$ — $F \vee T$ — $T \to T$

T — T — T — T

b= F
c= F
g= F
h= T

comes to Collusion

b=F	b=F	b=F
c=F	c=F	c=F
g=F	g=T	g=T
h=T	h=T	h=F

Now what should be final conclusion and we have find b and c is proved liar of g and h.

Conclusion: butler and cook are lying and we cannot say any thing about g

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a) "All the inscriptions are false."

Ans. False. The treasure must be in one trunk and cannot be determined.

b) "Exactly one of the inscriptions is true."

Ans.True. The inscription of trunk 1 and trunk 3 can be true.

c) "Exactly two of the inscriptions are true." One of either the first inscription is false, or the second inscription is false, or the third inscription is false.

Ans. True. The hidden treasure cannot be found or determined due to lack of context. (+) (+)

d) "All three inscriptions are true."

Ans. False. The hidden treasure can't be determined.

Q9. **Use De Morgan's laws to find the negation of each of the Following statements.** 4/4.

a). Jan is not rich or not happy. $\neg(p \wedge q) \Rightarrow \neg p \vee \neg q)$

b). Carlos will not bicycle and not run tomorrow. $\neg(p \vee \neg q) \Rightarrow \neg(p \wedge q) \Rightarrow \neg p \wedge \neg q.$

c). Mei does not walk and does not take the bus to class.

d). Ibrahim is not smart or not hard working.

Not Explanation required. Weak solution 1.5/5

Q10. There is no evidence whether the gardener and the handy man are lying or not but they both cannot be lying at the same time.Also, the cook and the butler are proved to be lying.

Q11. **Determine whether $(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$ is a tautology.** p q 3/3.

Ans. $((\neg p \wedge p) \rightarrow (\neg p \wedge q)) \rightarrow \neg q$

F → T, T → F ⇒ F.

$= (F \rightarrow (\neg p \wedge q)) \rightarrow \neg q$

$= T \rightarrow \neg q$

F: $\neg p = T$

$p \rightarrow q$: $F \rightarrow F = T.$

$\neg q = T$ (F)

This is **NOT** a tautology because if q is True, then the answer will be False. Hence, Not a tautology.

Q12. Show that ¬(p ⊕ q) and p ↔ q is logically equivalent. (Exclusive.) 5/5

Ans.

p	q	(p ⊕q)	¬(p ⊕q)	p ↔ q
F	F	F	T	T
F	T	T	F	F
T	F	T	F	F
T	T	F	T	T

Q13. Show that (p → q) ∧ (p → r) andp → (q ∧ r) are logically equivalent. 3/5

Ans. We apply the distributive law on the left side. So, p → (q ∧ r) = p → (q ∧ r).

Proper Explanation with proof required.

$(\neg p \vee q) \wedge (\neg p \vee r) \Rightarrow \neg p \vee \underbrace{(q \wedge r)}_{s} \Rightarrow \neg p \vee s \Rightarrow p \to s \Rightarrow p \to (q \wedge r)$

Q14. Show that ¬p → (q → r) andq → (p V r) are logically equivalent. 5/5

P	Q	R	¬p	(q → r)	(p V r)	¬p → (q → r)	q → (p V r)
T	T	T	F	T	T	T	T
T	T	F	F	F	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	F	F
F	F	T	T	T	T	T	T
F	F	F	T	T	F	T	T

Solution: $\neg p \to (q \to r)$

$\neg p \to q = p \vee q.$

$p \vee (q \to r) \Rightarrow \neg p \cancel{\to} q$

$\neg p \to$

Solution:
$(p \to q) \wedge (q \to p)$

Q15. Show that p ↔ q and ¬p ↔ ¬q are logically equivalent.

$(\neg q \to \neg p) \wedge (\neg p \to \neg q)$
$(\neg p \to \neg q) \wedge (\neg q \to \neg p)$
$(s \to r) \wedge (r \to s) = s \Leftrightarrow r$
$\neg p \Leftrightarrow \neg q$

Ans.

p	q	¬p	¬q	p ↔ q	¬p ↔ ¬q
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

Q16. Show that (p V q) Λ (¬p V r) → (q V r) is a tautology.

Solution: $(p \vee q) \wedge (\neg p \vee r) \to (q \vee r)$
F F ∧ T → F V F → F = F
F → F = F is a tautology.

4/5

Ans.

P	Q	R	¬P	(¬p Vr)	(p V q)	(q V r)	(p V q) Λ (¬p V r)	(p V q) Λ (¬p V r) → (q V r)
T	T	T	F	T	T	T	T	T
T	T	F	F	F	T	T	T	T
T	F	T	F	T	T	T	T	T
T	F	F	F	F	T	F	T	T→F = T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	T	T	T	F	T	F	T
F	F	F	T	T	F	F	F	T

Q17. Show that (p Λ q) → r and (p → r) Λ (q → r) are not logically Equivalent.

5/5

Ans.

P	Q	R	(p Λq)	(p→r)	(q →r)	(p Λ q) → r	(p → r) Λ (q → r)
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	F	T	T	T	T
T	F	F	F	F	T	T	F
F	T	T	F	T	T	T	T
F	T	F	F	T	F	T	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

Hence, the truth table shows that both are not logically equivalent.

Q18. Let N(x) be the statement "x has visited North Dakota," where the domain consists of the students in your school. Express each of these quantifications in English.

a)$\exists xN(x)$ b) $\forall xN(x)$ c) $\neg\exists xN(x)$ d) $\exists x\neg N(x)$

e) $\neg\forall xN(x)$ f) $\forall x\neg N(x)$

6/6

a). There exists a student in your school who has visited North Dakota.

b). All the students in your school have visited North Dakota.

c). There does not exist a student in your school who has visited North Dakota.

d). There exists a student in your school who has not visited North Dakota.

e). Not all the students in your school have visited North Dakota.

f). All the students in your school have not visited North Dakota.

Q19. Translate these statements into English, where R(x) is "x is a rabbit" and H(x) is "x hops" and the domain consists of all animals. 3.5/4

$\neg(\forall x)$

Ans. a). All rabbits hop. $\forall x\,(R(x) \to H(x))$ ~~the animal is a rabbit~~ then animal is hop.

No Rabbit

For every animals, if the animal

b). All animals are hopping rabbits. $\forall x\,(R(x) \wedge H(x))$ rabbit and animal x is hop

for every animals, All the animal x is a rabbit and animal x is hop

c). There exists an animal that, if it is a rabbit, then it hops. $\exists x\,(R(x) \to H(x))$

d). There exist some rabbits which hop. $\exists x\,(R(x) \wedge H(x))$

Not properly written

Q20. Let C(x) be the statement "x has a cat," let D(x) be the statement "x has a dog," and let F(x) be the statement "x has a ferret." Express each of these statements in terms of C(x), D(x), F(x), quantifiers, and logical connectives. Let the domain consist of all students in your class.

a) A student in your class has a cat, a dog, and a ferret.

Ans. $\exists x(C(x) \wedge D(x) \wedge F(x))$.

b) All students in your class have a cat, a dog, or a ferret.

Ans. $\forall x(C(x) \vee D(x) \vee F(x))$.

c) Some student in your class has a cat and a ferret, but not a dog.

Ans. $\exists x(C(x) \wedge F(x) \wedge \neg D(x))$.

d) No student in your class has a cat, a dog, and a ferret.

Ans. $\neg\exists x(C(x) \wedge D(x) \wedge F(x))$.

e) For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet.

Ans. $(\exists x(C(x)) \wedge (\exists x D(x)) \wedge (\exists x F(x))$.

5/5

$p \to q$

$F \to T \Rightarrow T$

Q21. Let Q(x) be the statement "x + 1 > 2x." If the domain consists of all integers, what are these truth values?

5/6

Ans. a). T $0+1>2(0)$

b). T

c). F

d). T

e). F

f). T

g). T

It is not case that there is student who is taking course in java.

Q22. Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.

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Ans. a). T

b). F

c). T

d). F

Q23. **Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people.**

5/5

a) Everyone in your class has a cellular phone.

Ans.Let, P(x) = x is a student in your class & C(x) = x has a cellular phone.

i). $\forall x(C(x))$

ii). $\forall x(P(x) \rightarrow C(x))$

b) Somebody in your class has seen a foreign movie.

Ans. Let, P(x) = x is a student in your class & M(x) = x has seen a foreign movie.

i). $\exists x(M(x))$

ii). $\exists x(P(x) \wedge M(x))$

c) There is a person in your class who cannot swim.

Ans. Let, P(x) = x is a student in your class & S(x) = x can swim.

i). $\exists x(\neg S(x))$

ii). $\exists x(P(x) \wedge \neg S(x))$

d) All students in your class can solve quadratic equations.

Ans. Let, P(x) = x is a student in your class & Q(x) = x can solve quadratic equation.

i). $\forall x(Q(x))$

ii). $\forall x(P(x) \rightarrow Q(x))$

e) Some student in your class does not want to be rich.

Ans. Let, $P(x)$ = x is a student in your class & $R(x)$ = x wants to be rich.

i). $\exists x(\neg R(x))$

ii). $\exists x(P(x) \land \neg R(x))$

Q24: Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

a) Something is not in the correct place.

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Ans. Let, $P(x)$ = x is in the correct place.

Then, $\exists x(P(x))$.

b) All tools are in the correct place and are in excellent condition.

Ans. Let, $P(x)$ = x is in the correct place& $C(x)$ = x is in excellent condition.

Then, $\forall x(P(x) \land C(x))$.

c) Everything is in the correct place and in excellent condition.

Ans. Let, $P(x)$ = x is in the correct place & $C(x)$ = x is in excellent condition.

Then, $\forall x(P(x) \land C(x))$.

d) Nothing is in the correct place and is in excellent condition.

Ans. Let, $P(x)$ = x is in the correct place & $C(x)$ = x is in excellent condition.

Then, $\neg\exists x(P(x) \land C(x))$.

e) One of your tools is not in the correct place, but it is in excellent condition.

Ans. Let, $P(x)$ = x is in the correct place & $C(x)$ = x is in excellent condition.

Then, $\exists x(\neg P(x) \land C(x))$.

Q25: Express each of these statements using quantifiers. Then form the negation of the statement so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the phrase "It is not the case that.")

a) All dogs have fleas.

Let, P(x) = x has fleas. Then, $\forall x(P(x)$.

5/5

For negation, $\neg(\forall x(P(x)) \equiv \exists x \neg P(x)$.

Therefore,

Ans. There exists a dog which does not have fleas.

b) There is a horse that can add.

Let, P(x) = ~~x~~ horse can add. Then, $\exists x P(x)$.

For negation, $\neg(\exists x(P(x)) \equiv \forall x \neg P(x)$.

Therefore,

Ans. ~~All~~ horses cannot add.

c) Every koala can climb.

Let, P(x) = x can climb. Then, $\forall x(P(x)$.

For negation, $\neg(\forall x(P(x)) \equiv \exists x \neg P(x)$.

Therefore,

Ans. There exists a koala which cannot climb.

d) No monkey can speak French.

Let, P(x) = x can speak French. Then, $\forall x(\neg P(x))$.

For negation, $\neg(\forall x(\neg P(x)) \equiv \exists x \neg(\neg P(x)) \equiv \exists x P(x)$.

Therefore,

Ans. There is a monkey which can speak French.

e) There exists a pig that can swim and catch fish.

Let, $P(x)$ = x can swim & $C(x)$ = x can catch fish.

Then, $\exists x(P(x) \land C(x))$.

For negation, $\neg(\exists x(P(x) \land C(x))) \equiv \forall x \neg(P(x) \land C(x)) \equiv \forall x (\neg P(x) \lor \neg C(x))$.

Therefore,

Ans. All pigs cannot swim or not catch fish.

Q26: Express the negation of these propositions using quantifiers, and then express the negation in English.

4/4

a) Some drivers do not obey the speed limit.

Ans. Let, $P(x)$ = x obeys the speed limit. Then, $\exists x \neg P(x)$.

For negation, $\neg(\exists x(\neg P(x)) \equiv \forall x \neg(\neg P(x))$.

Therefore,

Ans. All drivers obey the speed limit.

b) All Swedish movies are serious.

Ans. Let, $P(x)$ = x is serious. Then, $\forall x\, P(x)$.

For negation, $\neg(\forall x\, P(x) \equiv \exists x \neg P(x)$.

Therefore,

Ans. There is a Swedish Movie which is not serious.

c) No one can keep a secret.

Let, $P(x)$ = x can keep a secret. Then, $\forall x(\neg P(x))$.

For negation, $\neg(\forall x(\neg P(x)) \equiv \exists x \neg(\neg P(x)) \equiv \exists x P(x)$.

Therefore,

Ans. Someone can keep a secret.

d) There is someone in this class who does not have a good attitude.

Ans. Let, P(x) = x has a good attitude. Then, $\exists x \neg P(x)$.

For negation, $\neg(\exists x(\neg P(x)) \equiv \forall x \neg(\neg P(x))$.

Therefore,

Ans. Everybody in this class have a good attitude.

Q27: Translate these system specifications into English, where the predicate S(x, y) is "x is in state y" and where the domain for x and y consists of all systems and all possible states, respectively.

5/5

a) $\exists x S(x, \text{open})$

Ans. There exists a system x that is in open state.

b) $\forall x(S(x, \text{malfunctioning}) \vee S(x, \text{diagnostic}))$

Ans. All the systems are malfunctioning or in diagnostic state.

is in state malfunctioning

c) $\exists x S(x, \text{open}) \vee \exists x S(x, \text{diagnostic})$

Ans. There exists a system that is in open state or there exists a system that is in diagnostic state.

d) $\exists x \neg S(x, \text{available})$

Ans. There is a system that is not in available state.

e) $\forall x \neg S(x, \text{working})$

Ans. All the systems are not in working state.

Q28. Express each of these system specifications using predicates, quantifiers, and logical connectives.

a) Every user has access to an electronic mailbox.

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Let, P(x) = x has access to an electronic mailbox.

Ans. $\forall x\, P(x)$.

b) The system mailbox can be accessed by everyone in the group if the file system is locked.

Let, P(x) = The system mailbox can be accessed by everyone in the group x& F(x) = File system in x group is locked.

Ans. $\forall x (F(x) \rightarrow P(x))$.

c) The firewall is in a diagnostic state only if the proxy server is in a diagnostic state.

Let, P(x) = The firewall of group x is in diagnostic state &Q(x) = The proxy server group x is in diagnostic state.

Ans. $\forall x\, (Q(x) \rightarrow P(x))$.

q → p

d) At least one router is functioning normally if the throughput is between 100 kbps and 500 kbps and the proxy server is not in diagnostic mode.

Let, P(y) = The router y functions normally & Q(x) = The throughput ofgroup x is between 100 kbps and 500 kbps& R(x) = The proxy server group x is in diagnostic state.

Ans. $\exists x(Q(x) \land \neg R(x) \rightarrow \exists y P(y))$.