FINAL REVESION.

Physics 1

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Units and Dimensions

Q 1.

Which of the following quantities has the same dimensions as Force ?

Note: $[a] = [g] = LT^{-2}$; $[h] = L$ and $[v] = LT^{-1}$.

A. ma
B. mvt
C. mgh
D. mgt

Section 1
Slide 7

Derived quantity	Formula	Dimension	SI unit
Area	= length × length	$L \times L = L^2$	m^2
Volume	= length × length × length	$L \times L \times L = L^3$	m^3
Density	= Mass / volume	$M/L^3 = ML^{-3}$	$Kg.m^{-3}$
Velocity	= displacement / time	$L/T = LT^{-1}$	$m.s^{-1}$
Acceleration	= velocity / time	$L/T^2 = LT^{-2}$	$m.s^{-2}$
Force	= mass × acceleration	$M \times LT^{-2} = MLT^{-2}$	$Kg.m.s^{-2}$
Pressure	= force / area	$MLT^{-2}/ L^2 = ML^{-1}T^{-2}$	$Kg.m^{-1}.s^{-2}$
Work	= force × displacement	$MLT^{-2} \times L = ML^2T^{-2}$	$Kg.m^2.s^{-2}$
Power	= work / time	$ML^2T^{-2}/T = ML^2T^{-3}$	$Kg.m^2.s^{-3}$

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C. mgh

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Q 2.

The equation for the change of position of a train starting at $x = 2$ m is given by $x=0.5at^2 +bt^5$. The dimension of b is :-

A. LT^{-5}
B. T^{-3}
C. LT^{-4}
D. LT^{-3}

Solution :

$x = 0.5at^2 + bt^5$

$$[L] = a\,[T^2] + b\,[T^5]$$

** WE REMOVED THE CONSTANT "0.5" because CONSTANTS ARE DIMENSIONLESS

For both quantities "$a\,[T^2]$" and "$b\,[T^5]$" must have the same dimension which is "$[L]$"

$$\therefore a\,[T^2] = [L]$$

$$a = \frac{[L]}{[T^2]} = [LT^{-2}]$$

Section 1
Slide 3

2- Derived units

The derived dimensions for any physical quantities must apply the following principle.

1- Derived quantity = (Length)^a × (Mass)^b × (Time)^c

2- L.H.S = R.H.S "**same dimension**"

3- Physical quantities added or subtracted must have **same (dimention and unit)**.

$$\mathbf{A = B + C}$$

Units of A, B and C must be the same

Solution :

$x=0.5at^2 + bt^5$

$$[L] = a\,[T^2] + b\,[T^5]$$

$$\therefore b\,[T^5] = [L]$$

$$a = \frac{[L]}{[T^5]} = [LT^{-5}]$$

Section 1
Slide 3

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A. LT^{-5}
B. T^{-3}
C. LT^{-4}
D. LT^{-3}

Q 3.

The standard exam page is 8.50 inches by 11.0 inches. Its area in cm^2 is :

A. 93.5
B. 19.5
C. 603
D. 237.5

Section 1
Slide 19

1- convert 21.5 inches (in) into cm :

Conversion factor: 1 in = 2.54 cm = 2.54 * 10^{-2} m

8.50 inches = 8.5 x 1 inch
= 8.5 x 2.54 cm
= 21.59 cm

11.00 inches = 11 x 1 inch
= 11 x 2.54 cm
= 27.94 cm

Area = 21.59 x 27.94 = 603.2246 cm^2

Q 3.

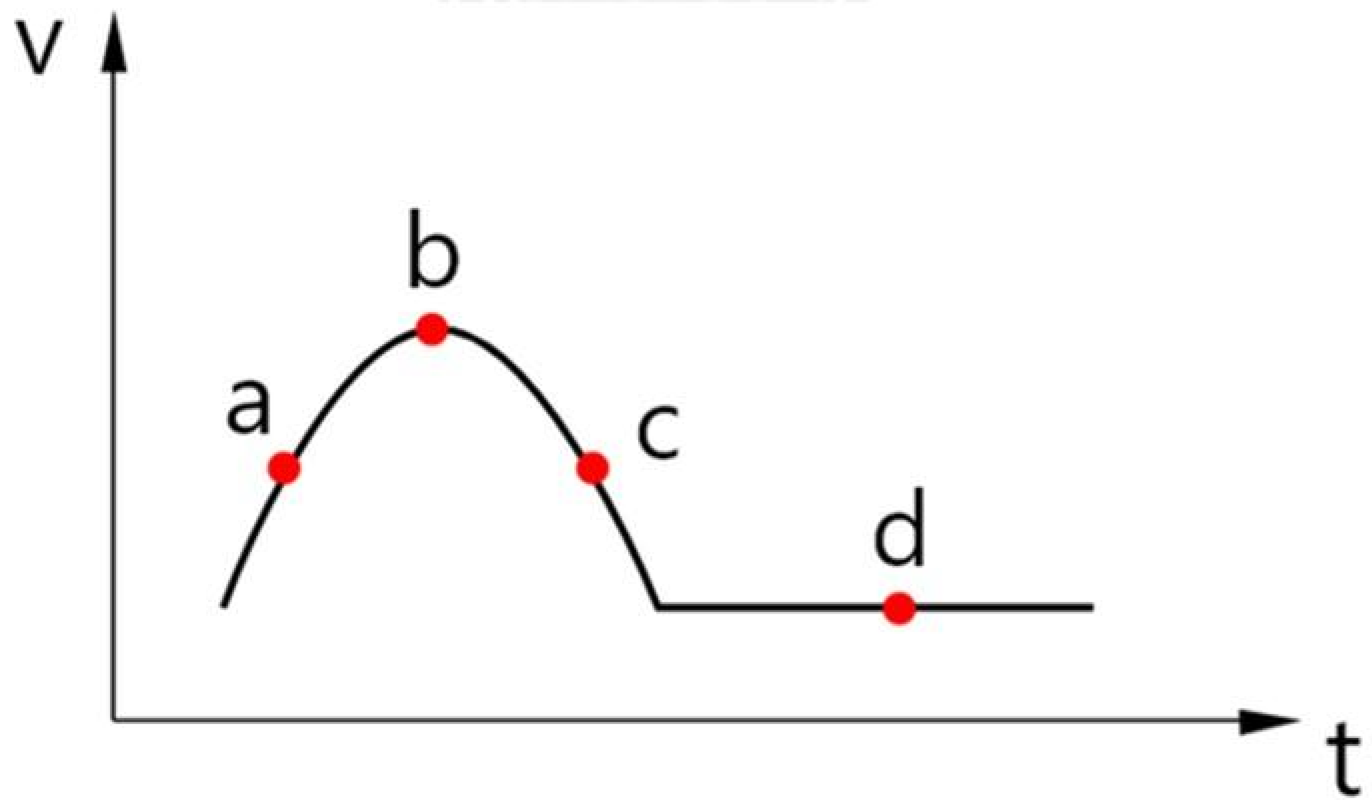
The standard exam page is 8.50 inches by 11.0 inches. Its area in cm^2 is :

A. 93.5
B. 19.5
C. 603
D. 237.5

Motion

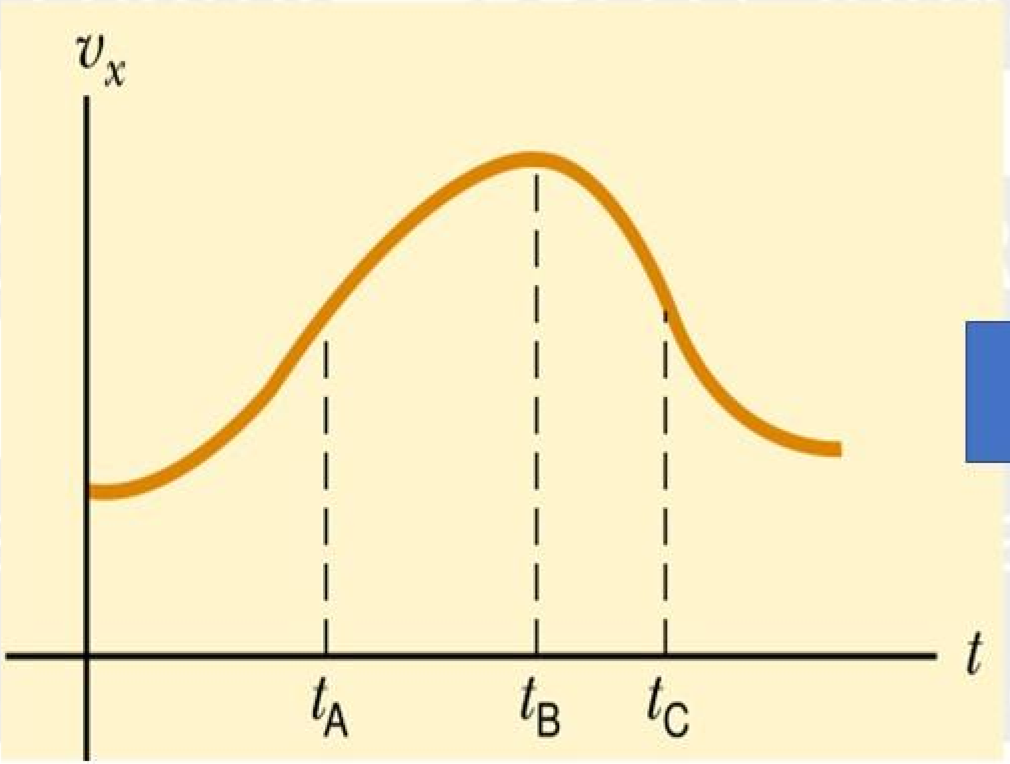
Q 4.

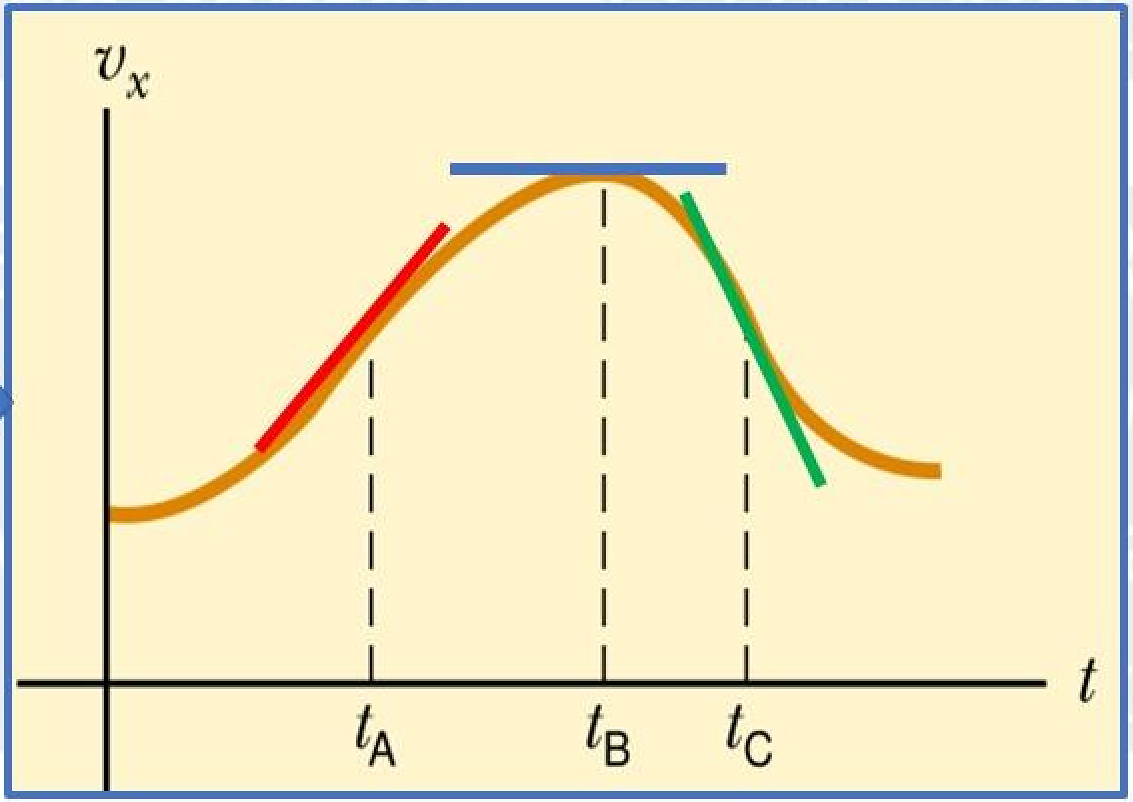
According to the given graph, acceleration is **zero** at which point ?	
According to the given graph, acceleration is **positive** at which point ?	
According to the given graph, acceleration is **negative** at which point ?	



1st Method :-

The **acceleration** at any time is the slope of the velocity–time graph at that time.



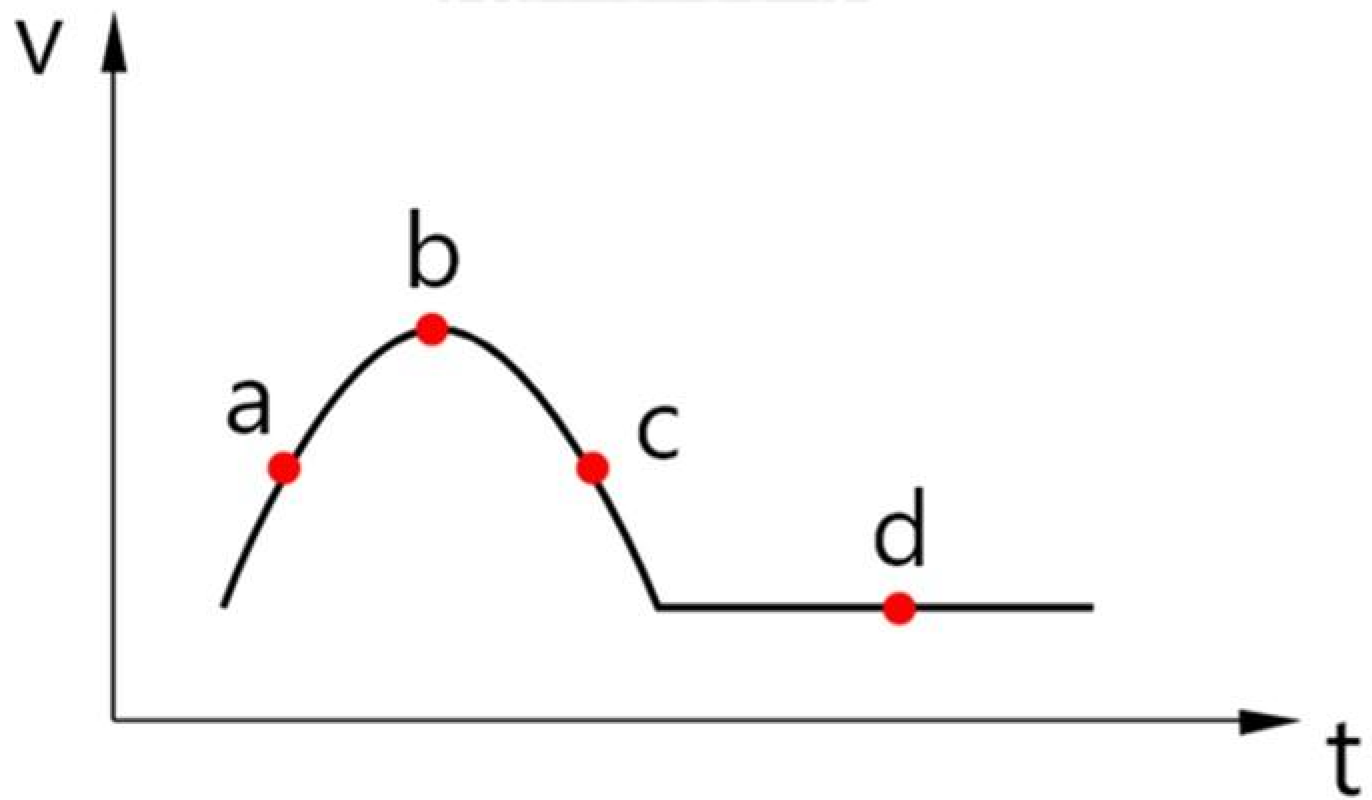


The Four Different Types of Slopes for Directions			
Positive Slope	Negative Slope	Zero Slope Horizontal Line	Undefined Slope Vertical Line

Slope at t_A is +ve ---- ∴ acceleration's sign is +ve
Slope at t_B is 0 ---- ∴ acceleration is 0
Slope at t_C is -ve ---- ∴ acceleration's sign is -ve

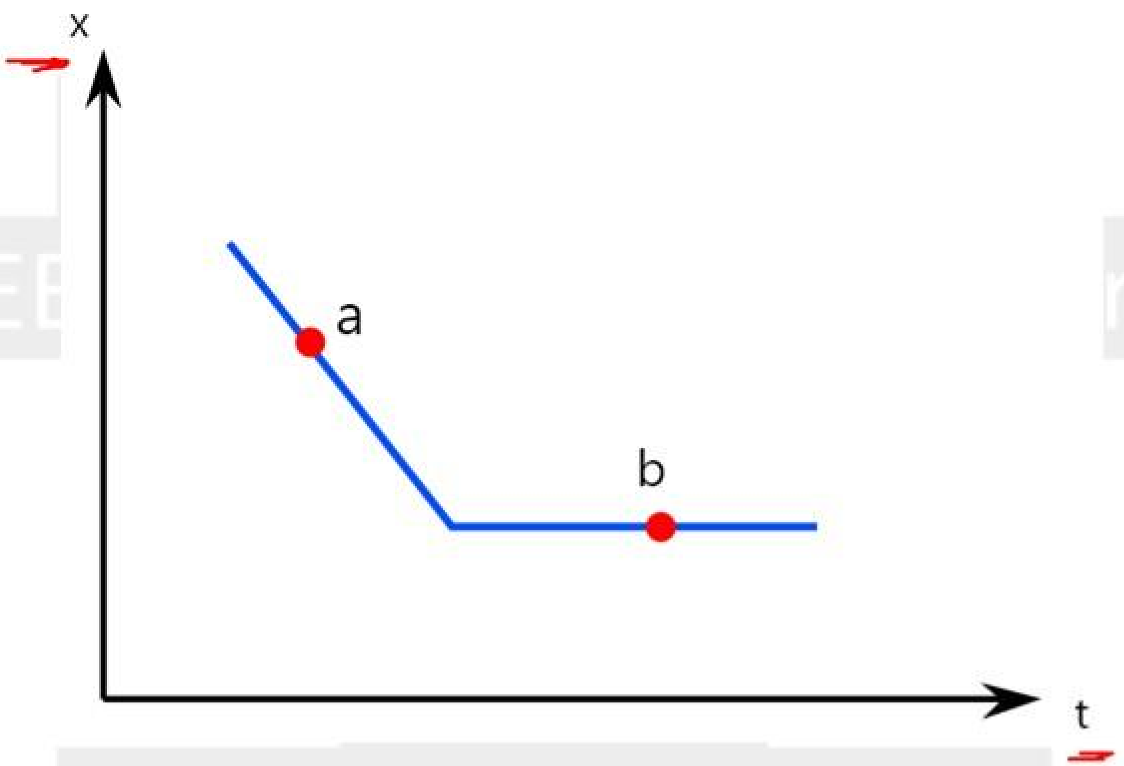
Q 4.

According to the given graph, acceleration is **zero** at which point ?	b & d
According to the given graph, acceleration is **positive** at which point ?	a
According to the given graph, acceleration is **negative** at which point ?	c

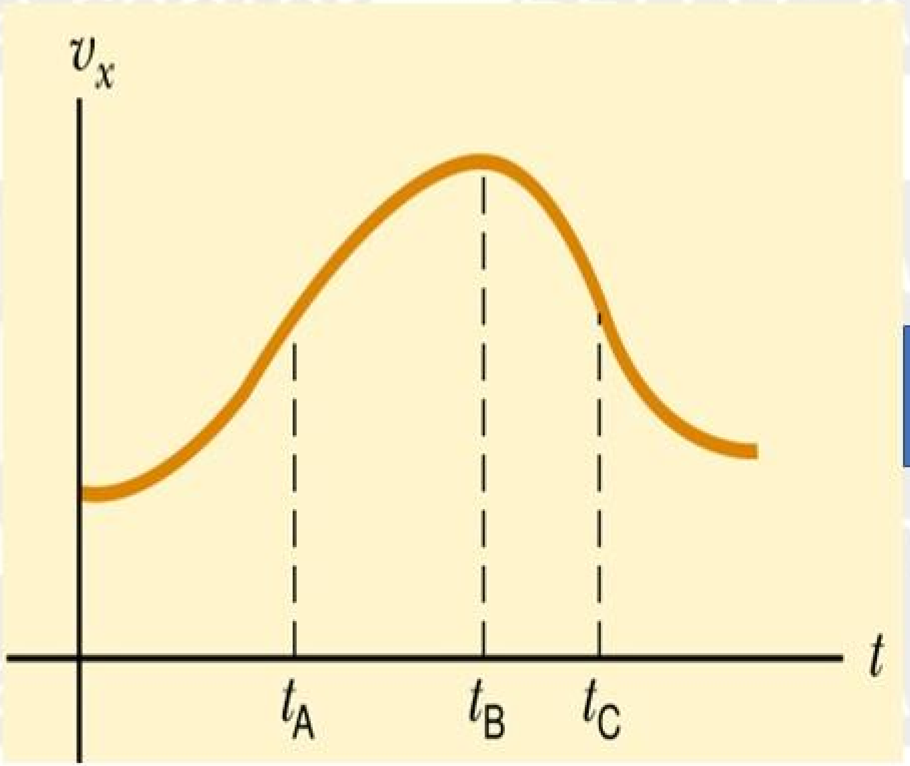


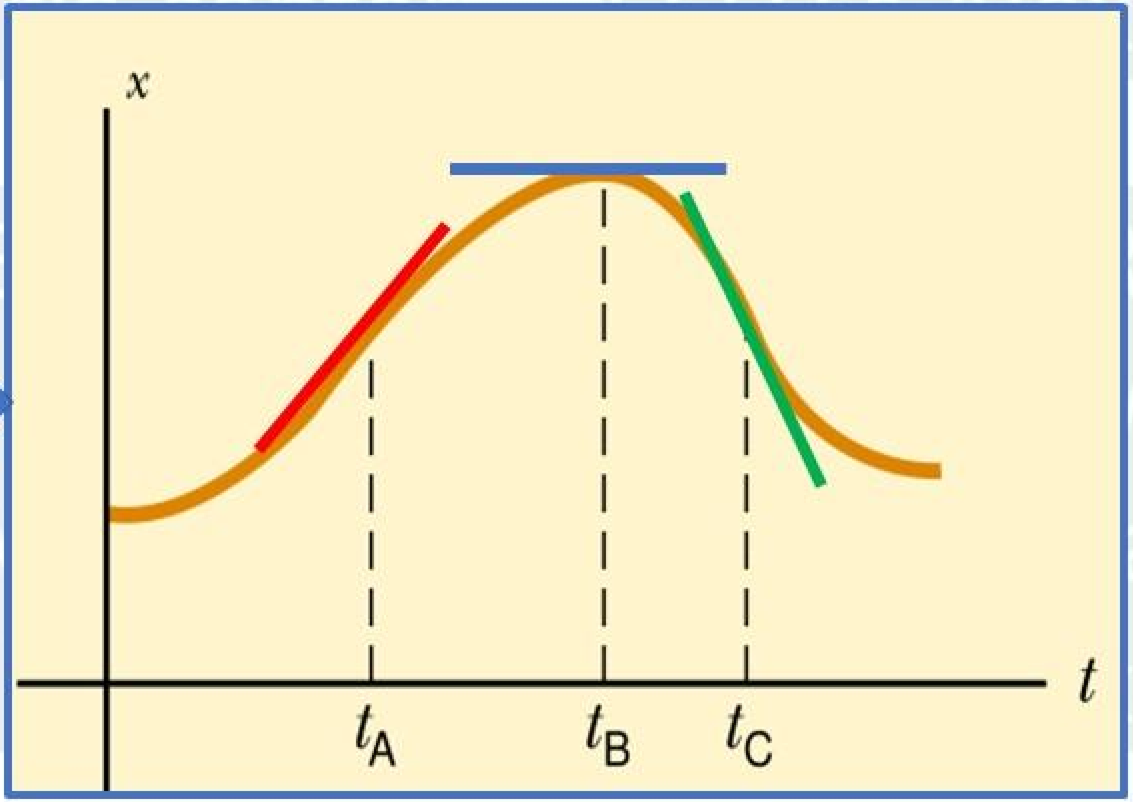
Q 5.

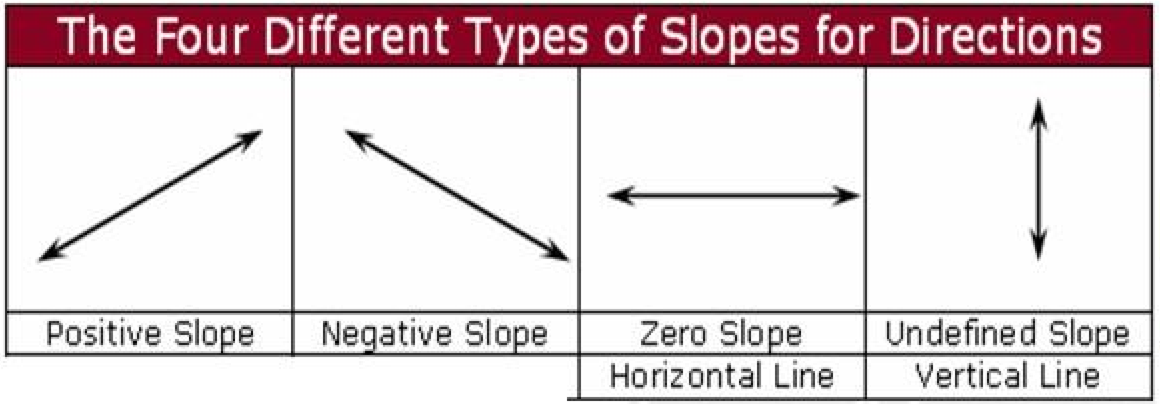
According to the given graph, which of the following points the velocity is **negative** ?	
According to the given graph, which of the following points the velocity is **zero** ?	



The **velocity** at any time is the slope of the position–time graph at that time.







Slope at t_A is +ve ---- ∴ velocity's sign is +ve
Slope at t_B is 0 ---- ∴ velocity is 0
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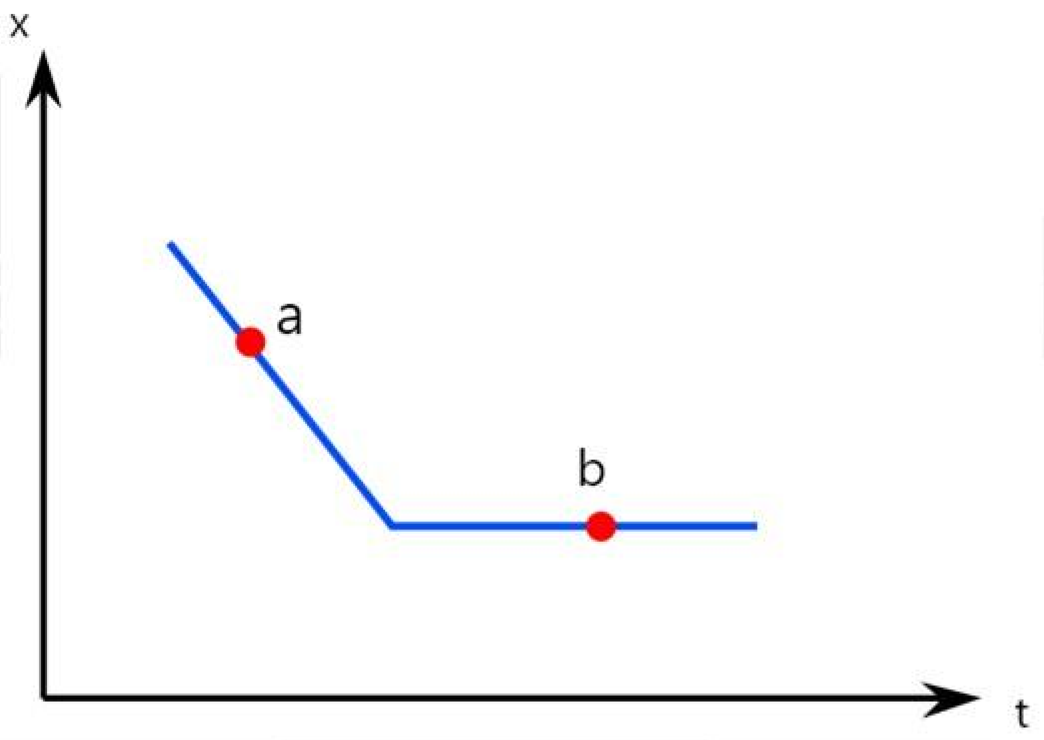
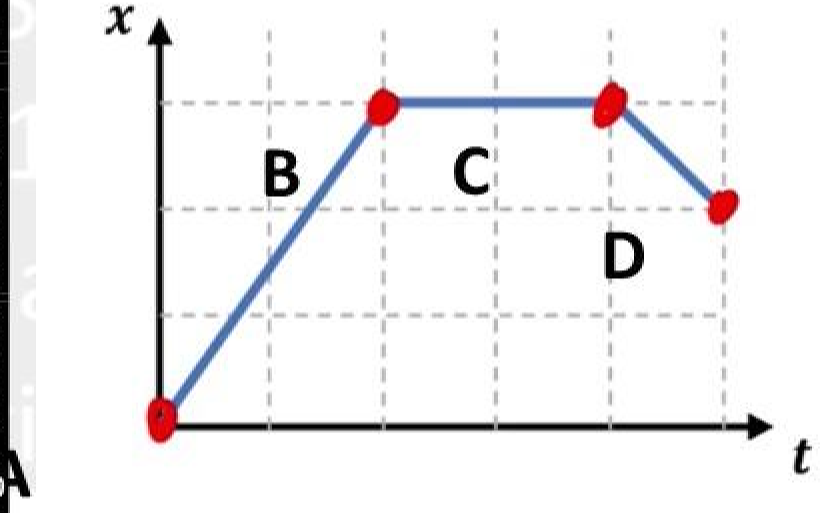
Q 5.

According to the given graph, which of the following points the velocity is **negative** ?	a
According to the given graph, which of the following points the velocity is **zero** ?	b



VIP...

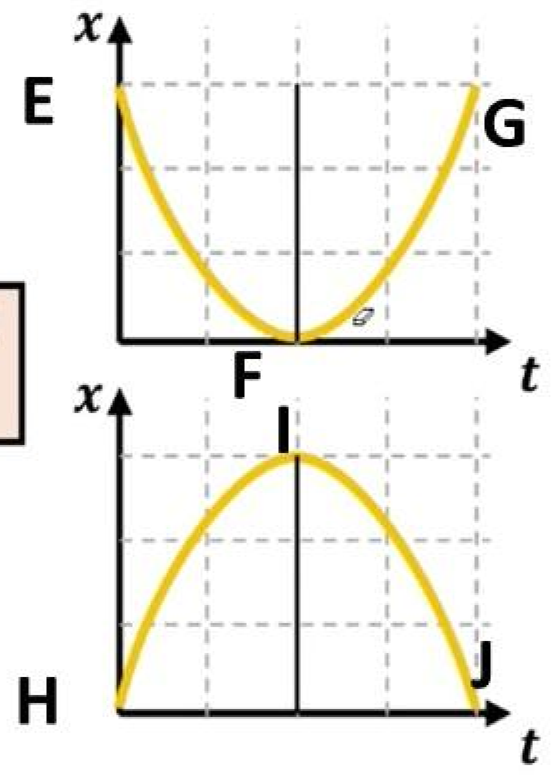
STRAIGHT POSITION GRAPH



- Straight lines → v = constant, $a = 0$

CURVED POSITION GRAPH

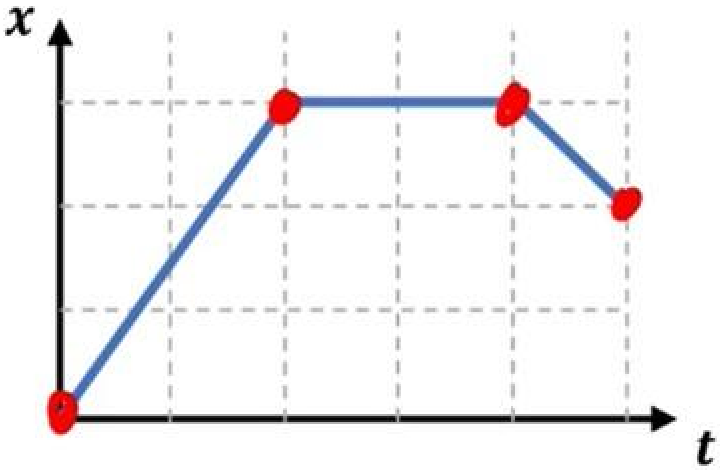
Left Side: Object ____________



Right Side: Object ____________

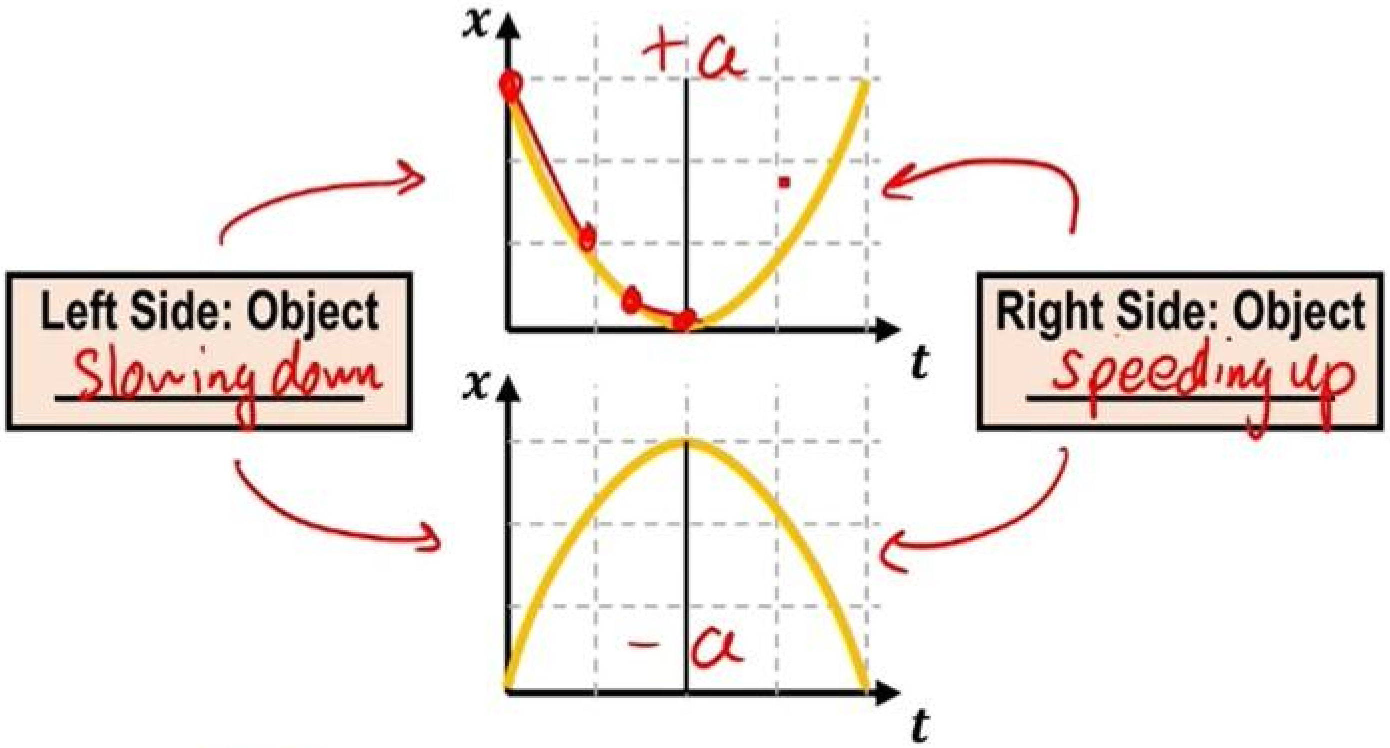
- Curving **UP** (Smiley ☺) → **[POSITIVE | NEGATIVE]** acceleration
- Curving **DOWN** (Frowny ☹) → **[POSITIVE | NEGATIVE]** acceleration

STRAIGHT POSITION GRAPH



- Straight lines → v = constant, $a = 0$

CURVED POSITION GRAPH



- Curving **UP** (Smiley ☺) → **[POSITIVE | NEGATIVE]** acceleration
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Q 6.

The coordinate-time graph of an object is a straight line with a positive slope. The object has:

A. constant displacement
B. steadily increasing acceleration
C. steadily decreasing acceleration
D. constant velocity
E. steadily increasing velocity

Q 6.

x-t

The coordinate-time graph of an object is a straight line with a positive slope. The object has:

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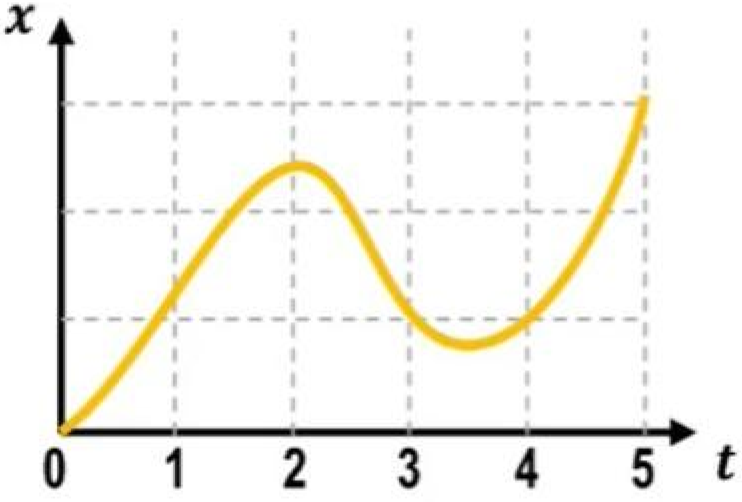
CONCEPT: POSITION-TIME GRAPHS & INSTANTANEOUS VELOCITY

- There are 2 different types of velocity you'll need to calculate in position-time graphs.

AVERAGE Velocity

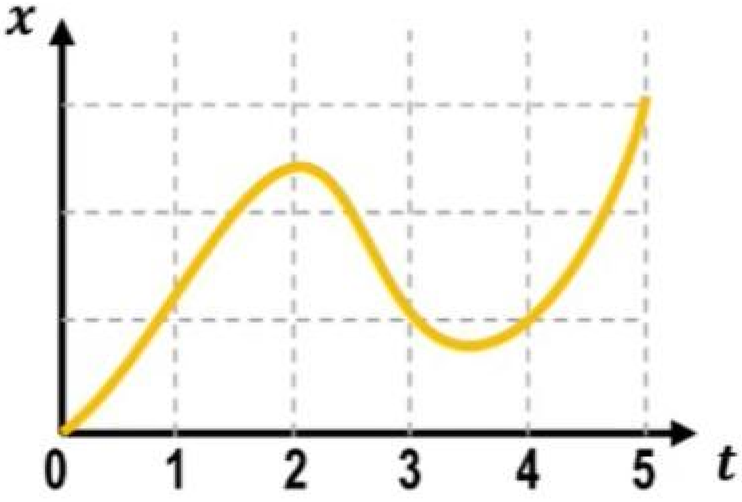
→ between *TWO* points



- $\vec{v}_{avg} = \frac{\Delta x}{\Delta t}$ = slope of line between 2 points

INSTANTANEOUS Velocity

→ at *ONE* point (instant)



- $\vec{v}$ = slope of **tangent line** ⇒ line touches graph once at 1 point
  - Use an approximated line (best guess) if not given.

Q 7.

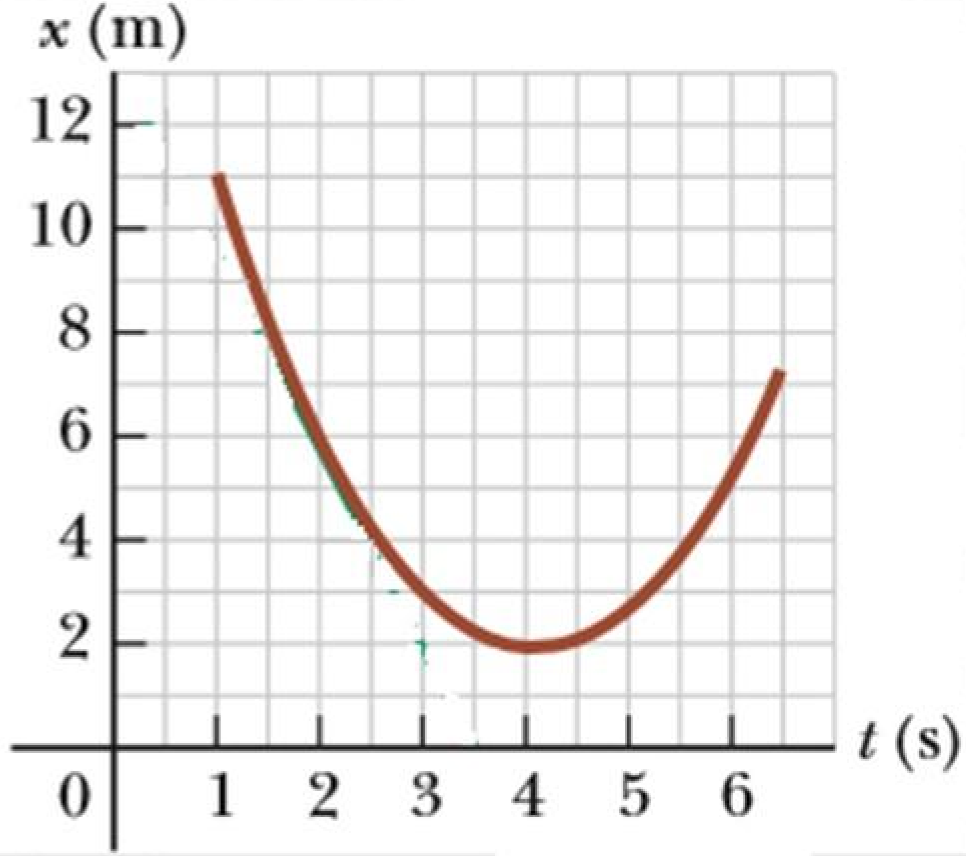
A position–time graph for a particle moving along the x axis is shown in the above figure the **average velocity** in the time interval **t=2s** to **t= 3s** is

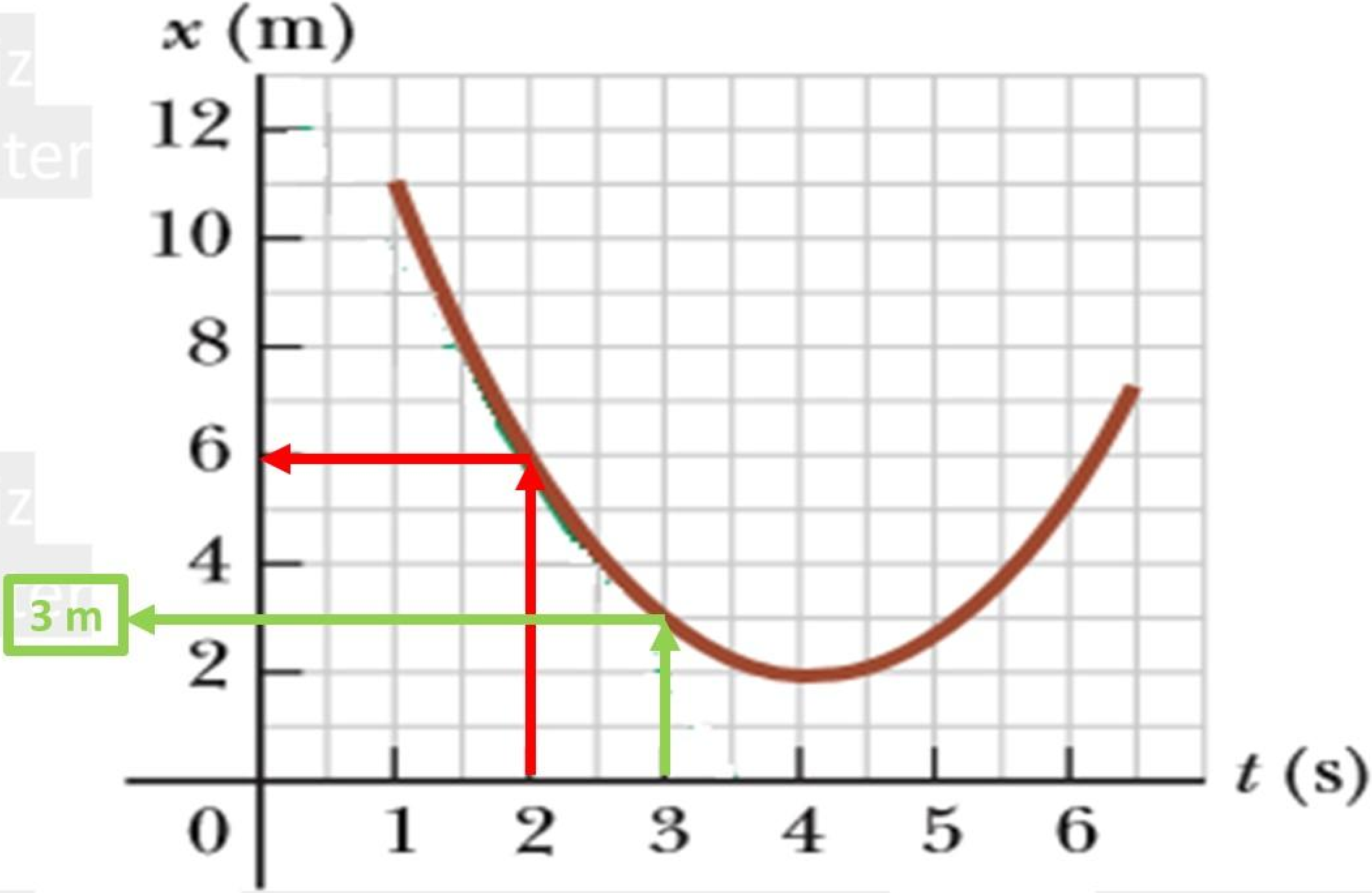
A. 3 m/s

B. -3m/s

C. -2.75m/s

D. -1.5m/s




x (m)
12
10
8
6
4
2
0
1
2
3
4
5
6
t (s)
3 m

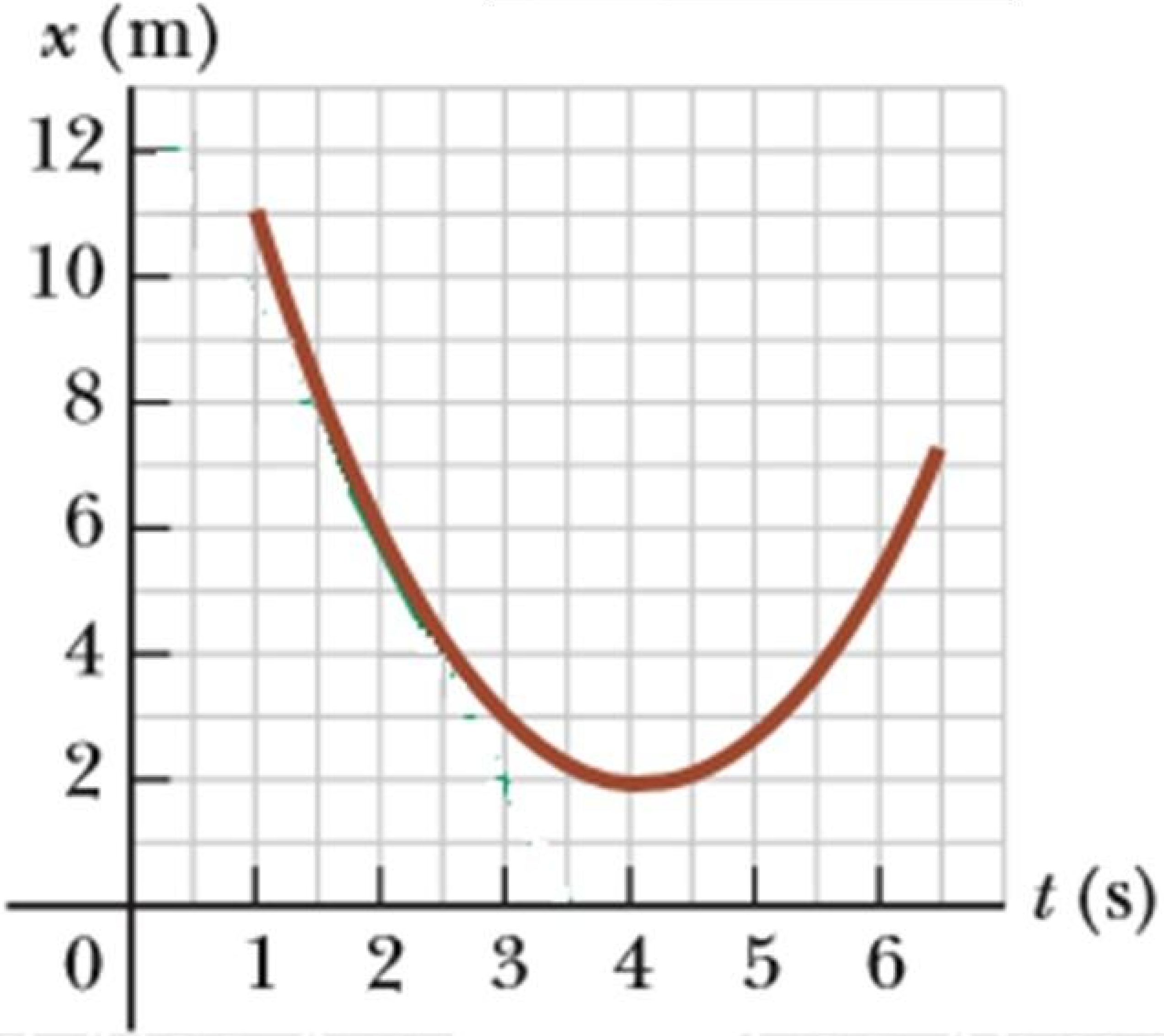
Section 2
Slide 20 , 21

Q 7.

$$\bar{v}_{average} = \frac{change\ in\ position}{change\ in\ time}$$

$$= \frac{\Delta x}{\Delta t} = \frac{x_{t=3} - x_{t=2}}{3-2}$$

$$= \frac{3\ m - 6\ m}{3\ sec - 2\ sec} = -3\ m/sec$$



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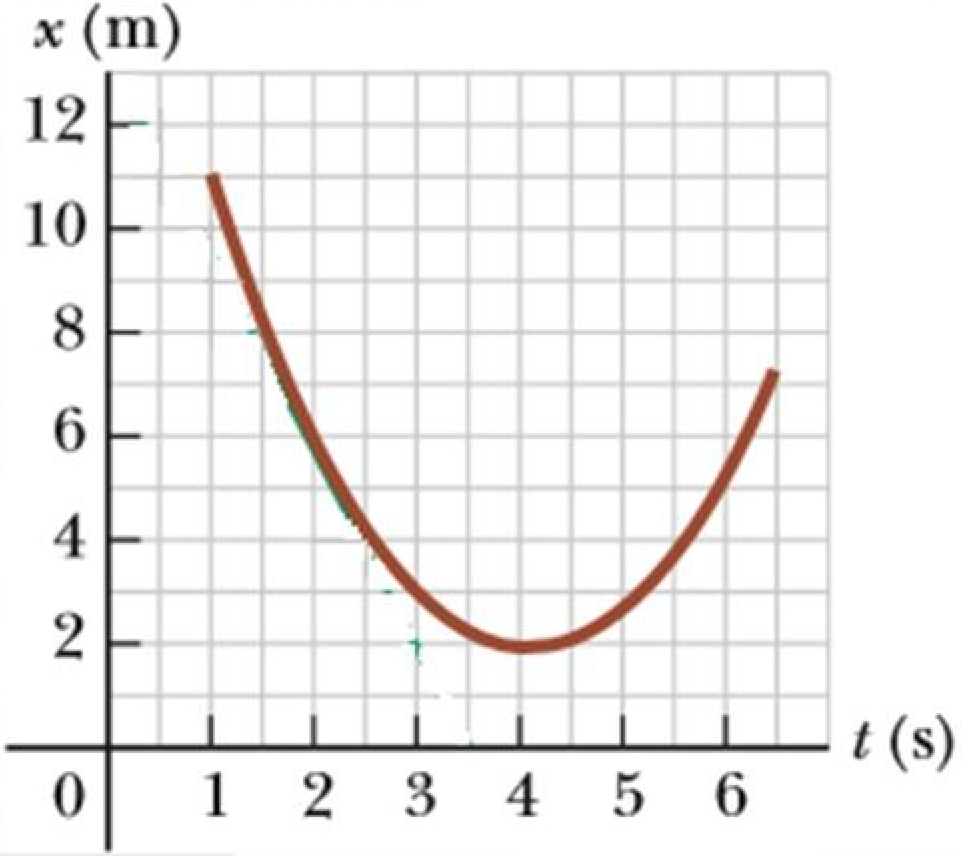


Q 8.

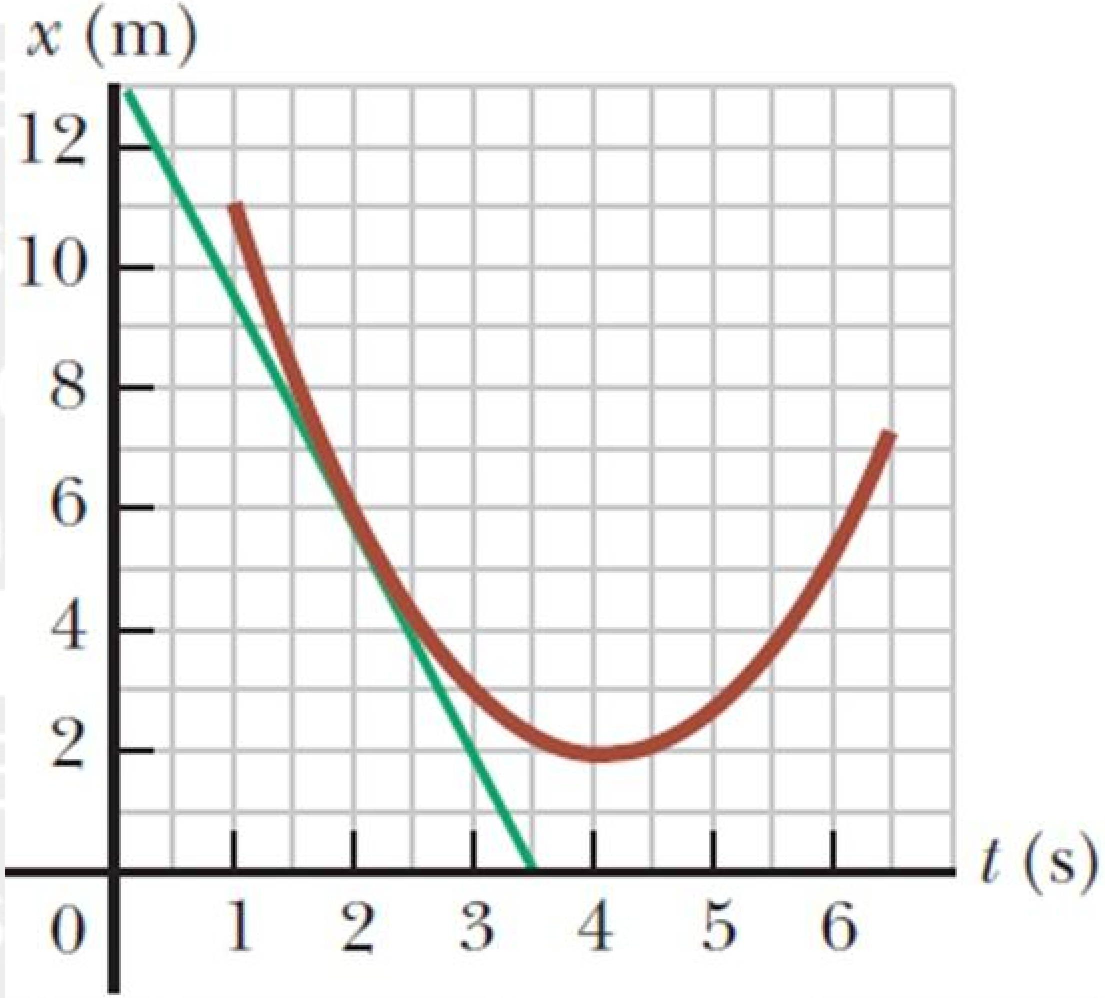
According to the graph , the **instantaneous velocity** at **t=2s** is :

A. 2 m/s

B. -3.71m/s

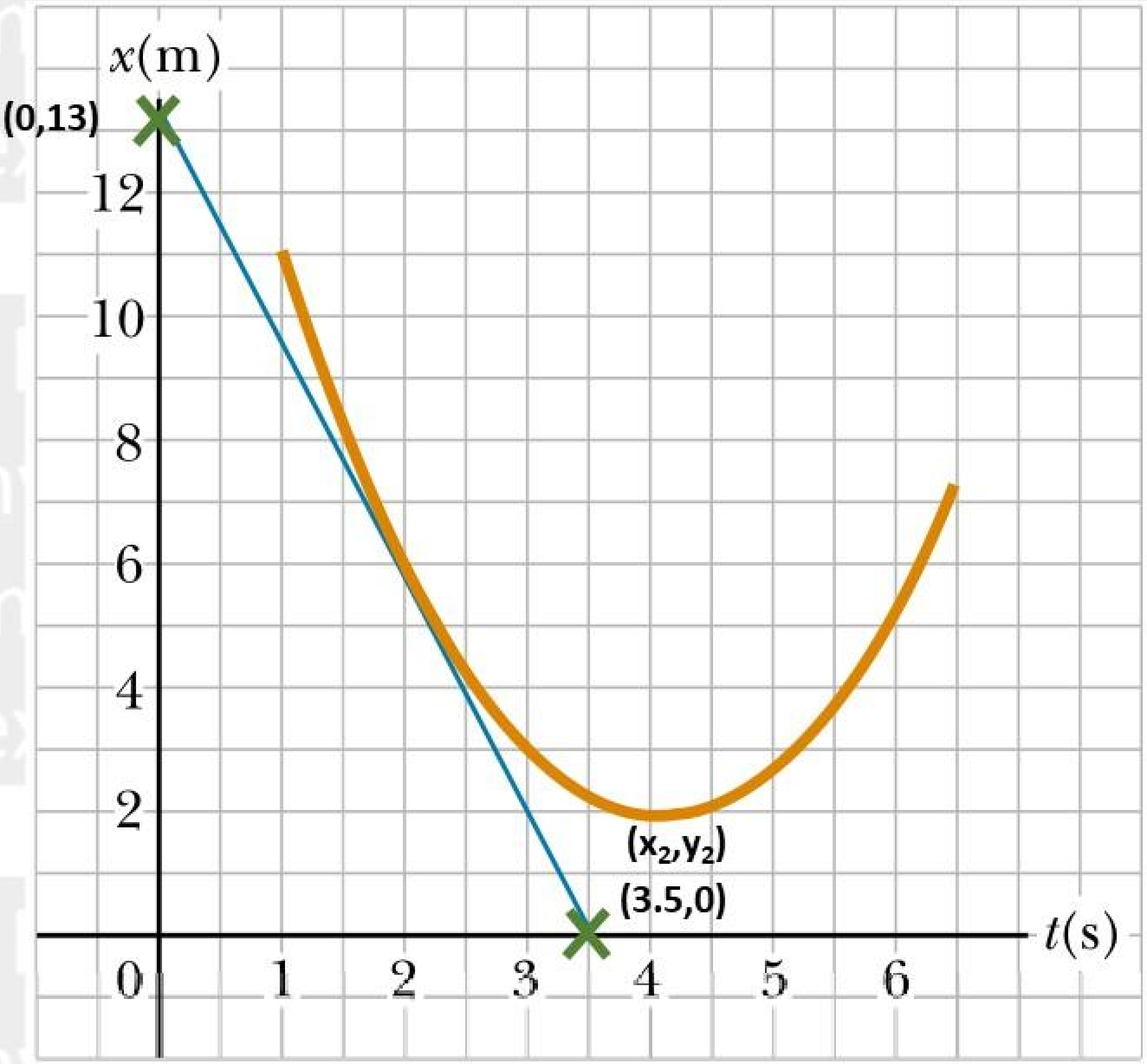
C. -2.75m/s

D. +1.5m/s



$$v_{t=2} = slope = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{0 - 13}{3.5 - 0} = -3.71\ m/sec$$



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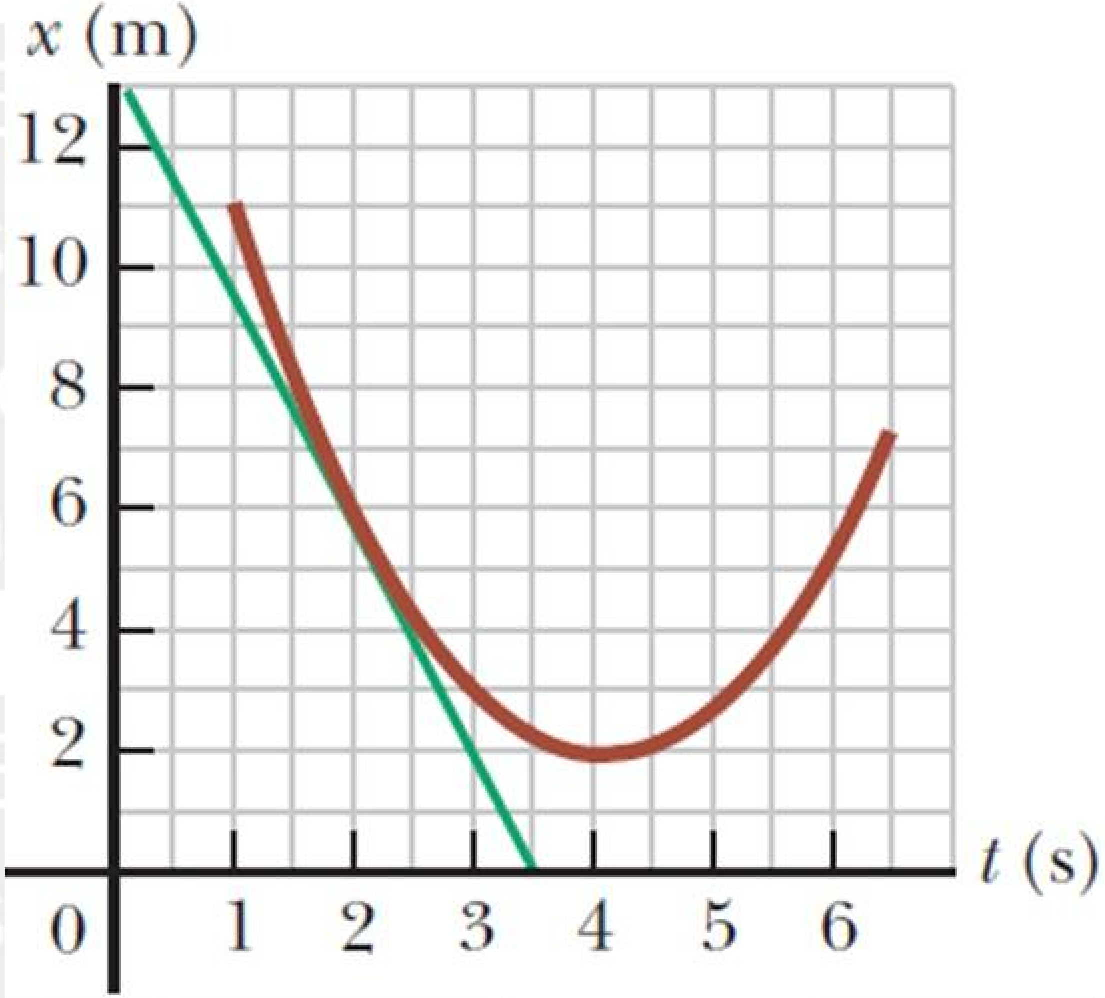
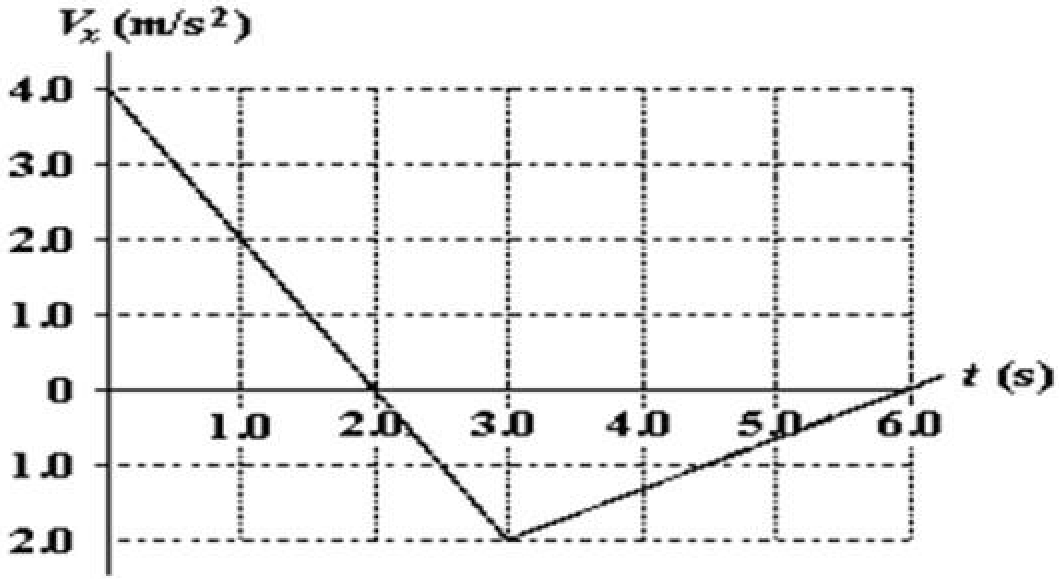
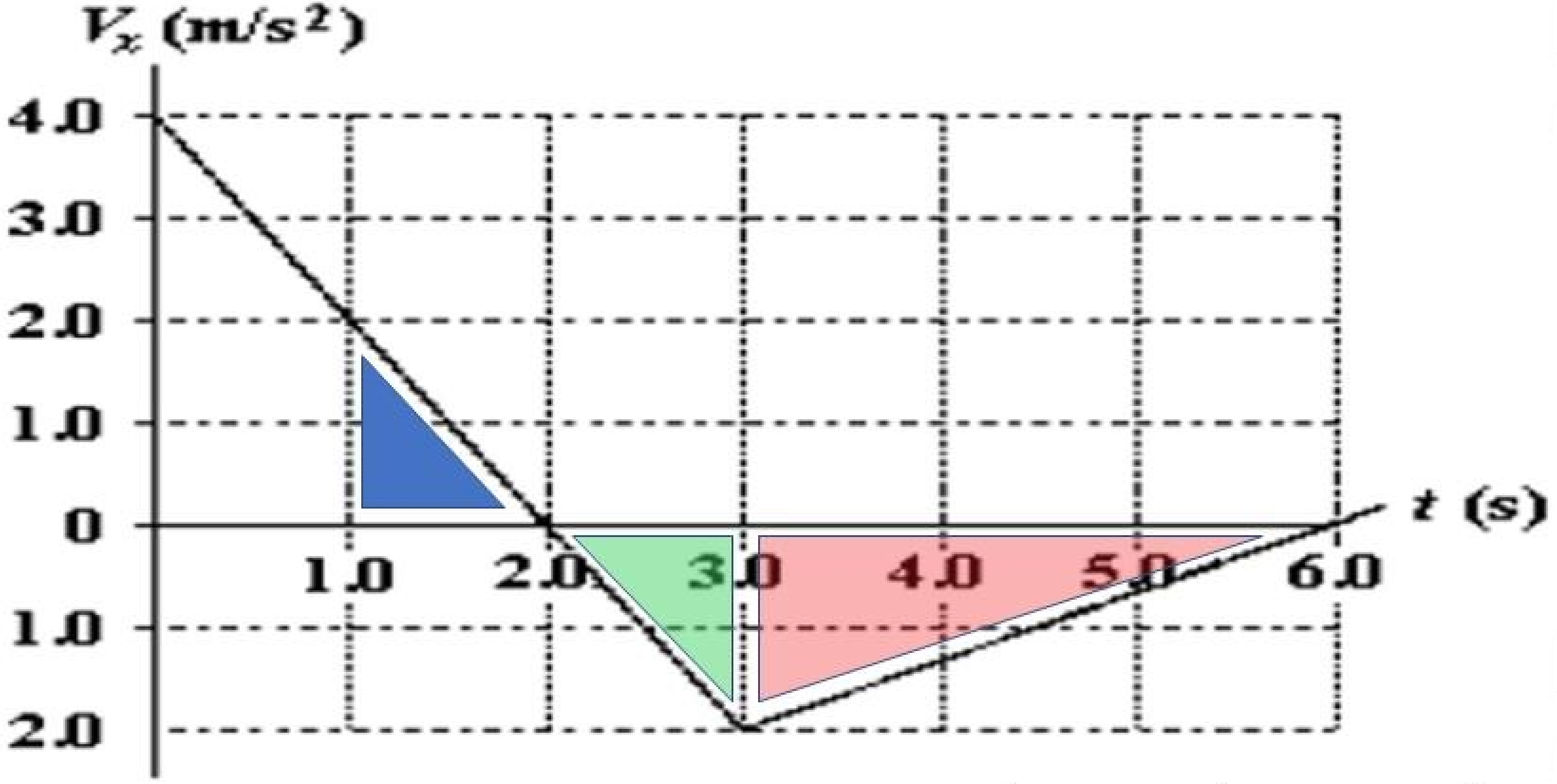


Q 9.

V_x is the velocity of a particle moving along the x axis as shown. If $x = 2.0$ m at $t = 1.0$ s, what is the position of the particle at $t = 6.0$ s?



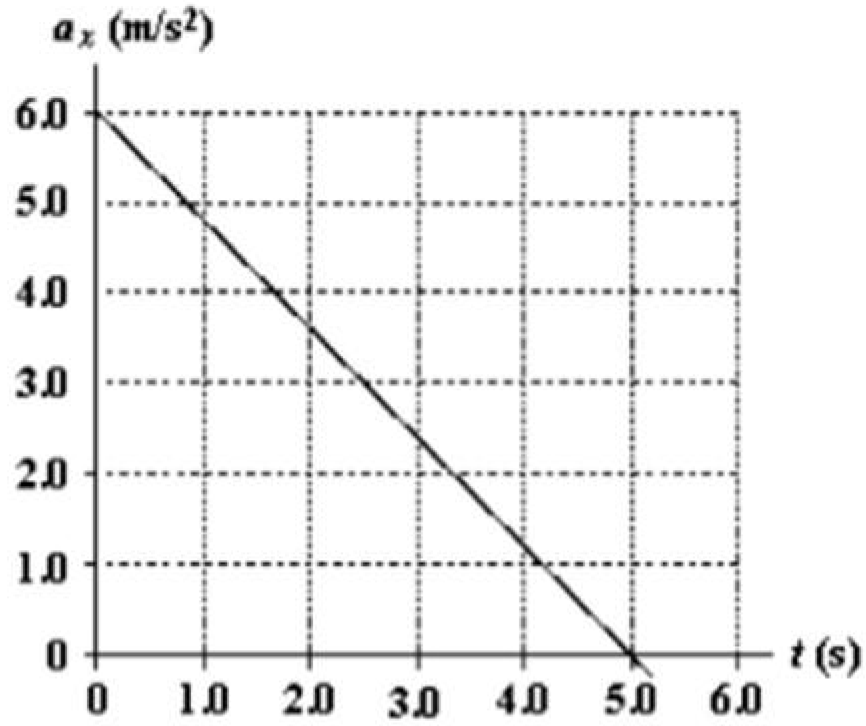
a. −2.0 m
b. +2.0 m
c. +1.0 m
d. ==−1.0 m==
e. 6.0 m

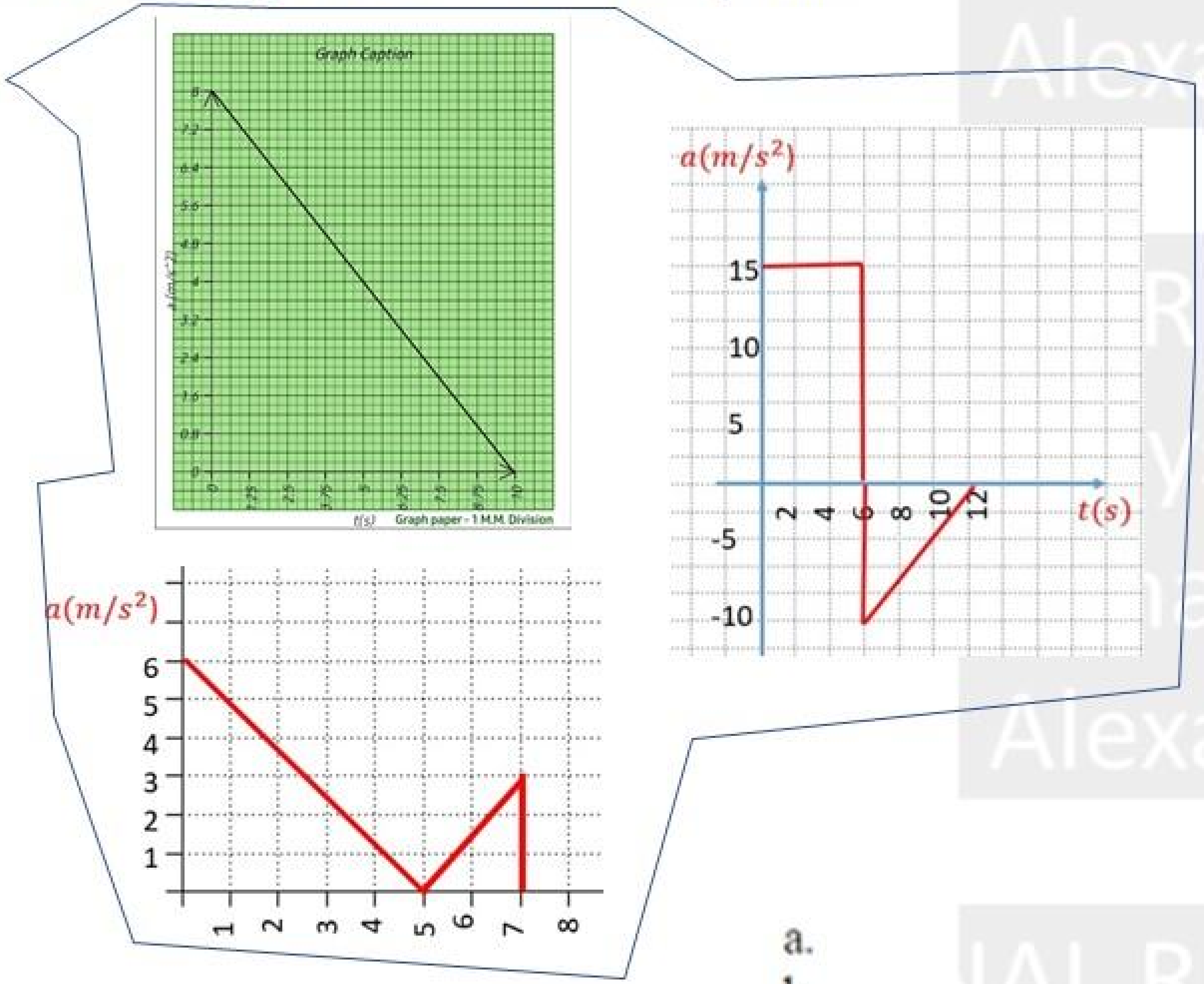


$$x_{t=6} - x_{t=1} = \frac{1}{2}(1)(2) + \frac{1}{2}(1)(-2) + \frac{1}{2}(3)(-2)$$
$$x_{t=6} - 2 = -3$$
$$x_{t=6} = -1\ m$$

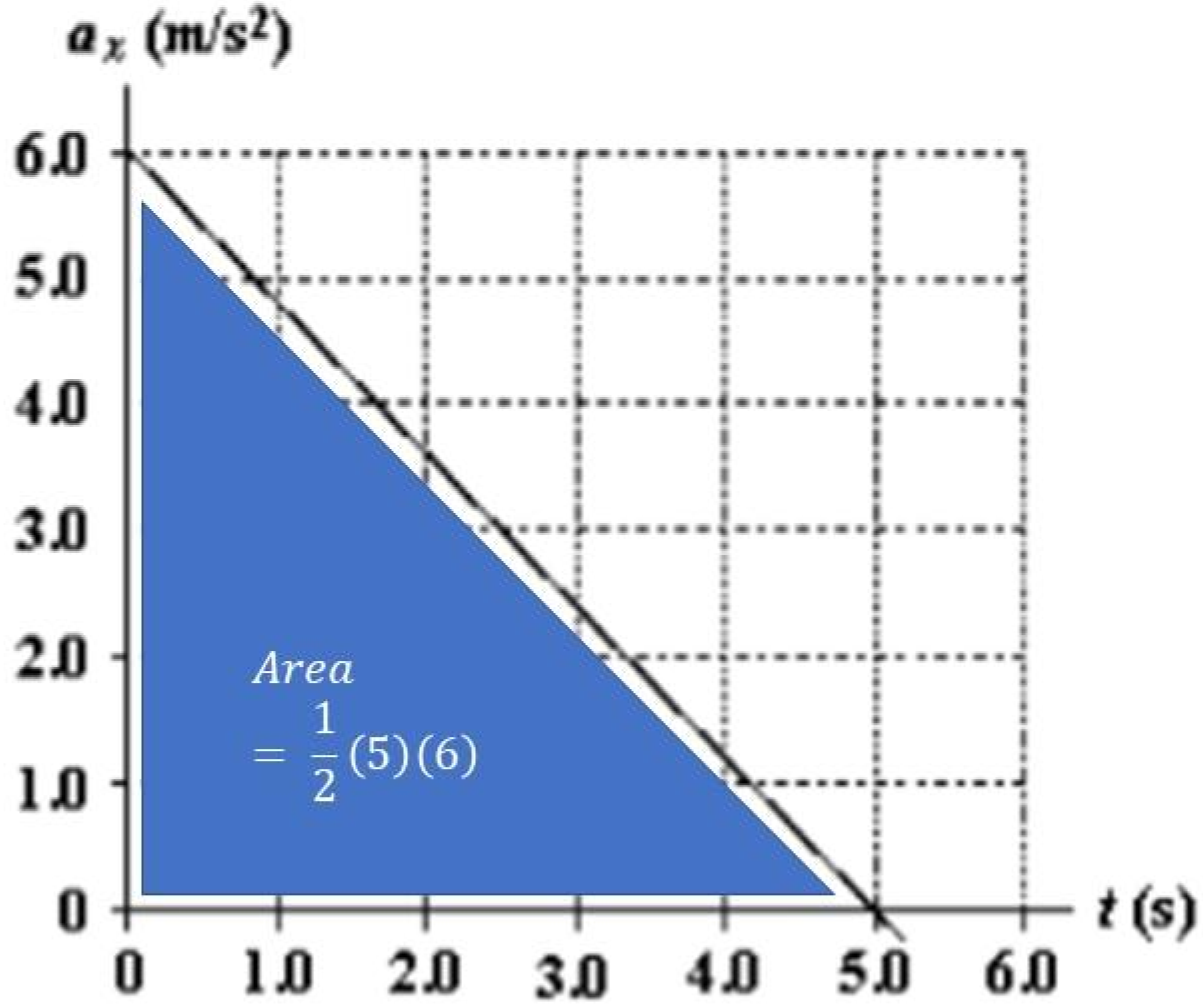
Q 10.

At $t = 0$, a particle is located at $x = 25$ m and has a velocity of 15 m/s in the positive x direction. The acceleration of the particle varies with time as shown in the diagram. What is the velocity of the particle at $t = 5.0$ s?





a. +15 m/s
b. −15 m/s
c. +30 m/s
d. 0
e. −1.2 m/s

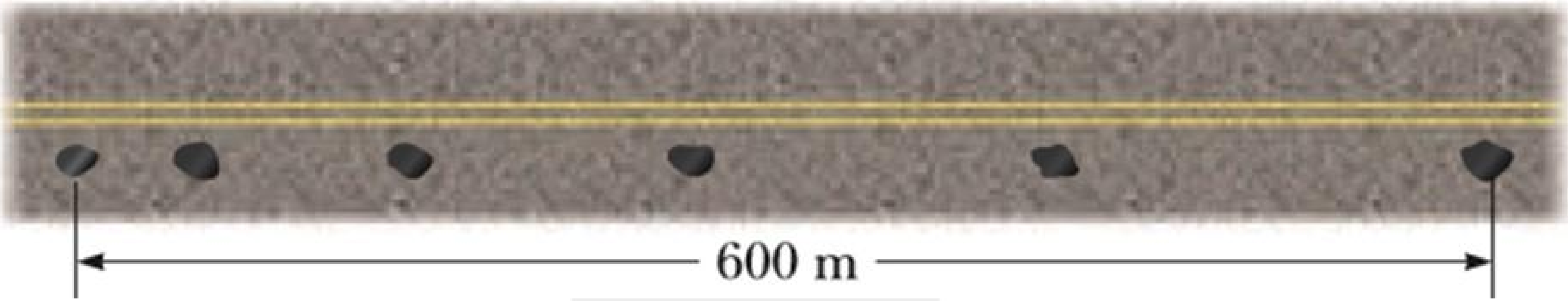


$$v_{t=5} - v_{t=0} = \frac{1}{2}(5)(6)$$

$$v_{t=5} - 15 = \frac{1}{2}(5)(6)$$

$$v_{t=5} = \frac{1}{2}(5)(6) + 15 = 30\ m/s$$

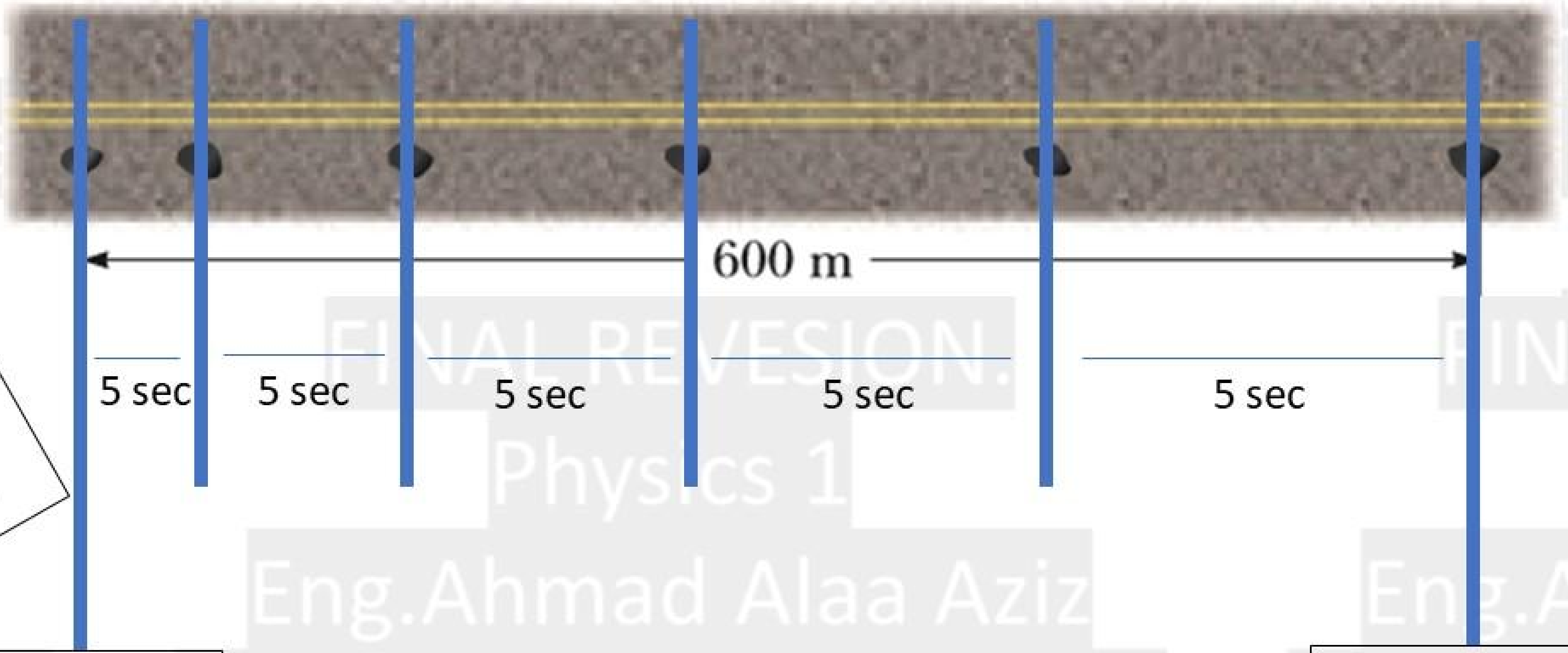
Q 11.



One drop of oil falls straight down onto the road from the engine of a moving car **every 5 s**. this figure shows the pattern of the drops left behind on the pavement. What is the **average speed** of the car over this section of its motion?

A. 0 m/s
B. -24 m/s
C. 120 m/s
D. 24 m/s

Q 11.



Section 2
Slide 8

$t_{initial} = 0$

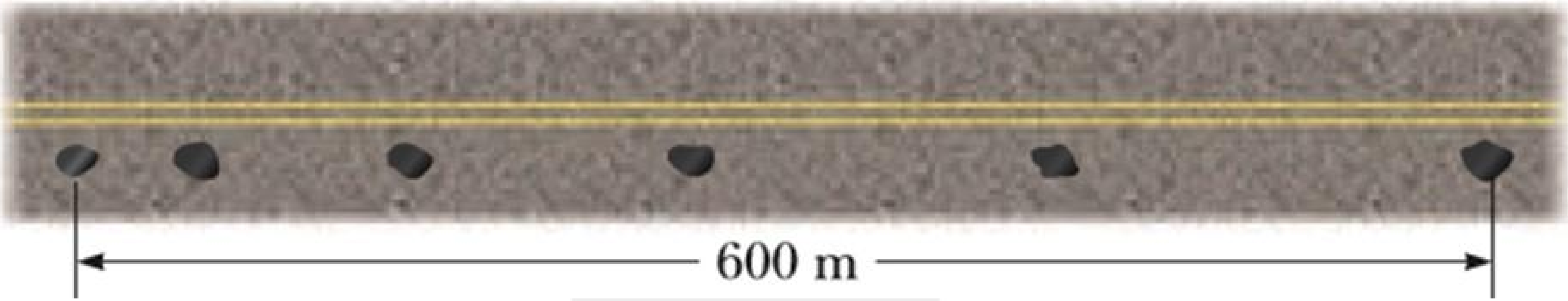
$x_{initial} = 0$

$t_{final} = 25$ sec

$x_{final} = 600$

$$v_{average} = \frac{x_{final} - x_{initial}}{t_{final} - t_{initial}} = \frac{600 - 0}{25 - 0} = 24\ m/sec$$

Q 11.



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C. 120 m/s
D. 24 m/s

Q 12.

The average speed of a moving object during a given interval of time is always:

A. the magnitude of its average velocity over the interval
B. the distance covered during the time interval divided by the time interval
C. one-half its speed at the end of the interval

Q 12.

The average speed of a moving object during a given interval of time is always:

A. the magnitude of its average velocity over the interval

B. the distance covered during the time interval divided by the time interval

C. one-half its speed at the end of the interval

Section 2
Slide 8

Average Speed

$$\text{Average Speed} = \frac{\text{Total distance travelled}}{\text{Total time}}$$

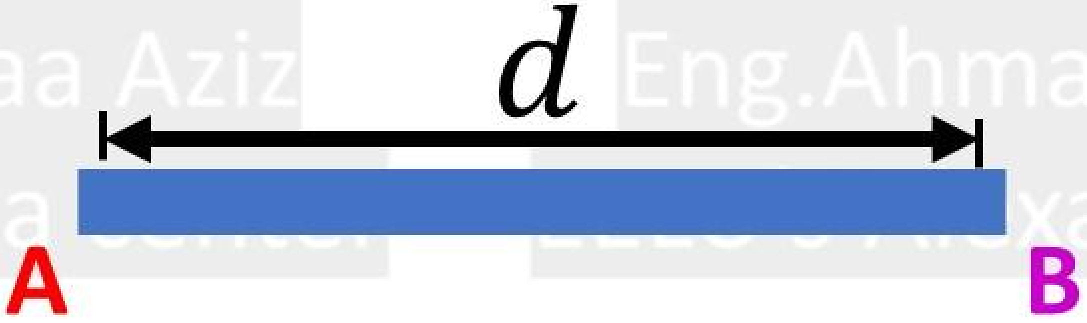
Average speed is a **scalar quantity** (has no direction).

Q 13.

A person walks first at a constant speed of **5.00 m/s** along a straight line from point **A** to point **B** and then back along the line from **B** to **A** at a constant speed of **3.00 m/s** What is his **average speed** over the entire trip?

A. 0
B. 4 m/s
C. 3.75 m/s
D. 8 m/s

Section 2
Slide 8



$$Time\ taken\ to\ travel\ from\ A\ to\ B = \frac{d}{5}$$

$$Time\ taken\ to\ travel\ from\ B\ to\ A = \frac{d}{3}$$

$$Total\ time\ taken = \frac{d}{5} + \frac{d}{3} = d\,\frac{8}{15}$$

$$Average\ speed = \frac{Total\ distance}{Total\ time\ taken} = \frac{2d}{\frac{8}{15}d} = \frac{2}{\left(\frac{8}{15}\right)} = 3.75\ m/s$$

Average Speed

$$\text{Average Speed} = \frac{\text{Total distance travelled}}{\text{Total time}}$$

Average speed is a **scalar quantity** (has no direction).

Q 13.

A person walks first at a constant speed of **5.00 m/s** along a straight line from point **A** to point **B** and then back along the line from **B** to **A** at a constant speed of **3.00 m/s** What is his **average speed** over the entire trip?

A. 0

B. 4 m/s

C. 3.75 m/s

D. 8 m/s

Q 14.

A particle confined to motion along the x axis moves with constant acceleration from **x = 2.0 m** to **x = 8.0 m** during a **2.5-s** time interval. The **velocity** of the particle **at x = 8.0 m** is **2.8 m/s**. **What is the acceleration during this time interval?**

A. 0.8 m/s^2
B. 0.64 m/s^2
C. 0.32 m/s^2
D. 0.48 m/s^2

$Given: -$

$t = 2.5\ sec$

$\because x_{initial} = 2\ m$

$\because x_{final} = 8\ m$

$\therefore \Delta x = x_{final} - x_{initial} = 6\ m$

$v_{final} = 2.8\ m/s$

$a = ?$

$v_{initial} = \ ?$

$$\because \Delta x = \left(\frac{v_{final} + v_{initial}}{2}\right) t$$

$$\therefore 6 = \left(\frac{2.8 + v_{initial}}{2}\right) 2.5$$

$\therefore v_{initial} = 2\ m/s$

$\because v_{final} = v_{initial} + at$

$\therefore 2.8 = 2 + a(2.5)$

$\therefore a = 0.32\ m/s^2$

Section 2
Slide 31

1. $v = v_0 + at$
2. $\Delta x = (\frac{v + v_0}{2})t$
3. $\Delta x = v_0 t + \frac{1}{2}at^2$
4. $v^2 = v_0^2 + 2a\Delta x$

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B. $0.64\ m/s^2$

C. $0.32\ m/s^2$

D. $0.48\ m/s^2$

Q 15.

The position of a particle as it moves along the x axis is given for $t > 0$ by $\mathbf{x = (t^3 - 3t^2 + 6t)\ m}$, where t is in s. **Where is the particle** when it achieves its **minimum speed** (after $t = 0$) and what is that **speed** ?

A. 7 m & 3 m/s
B. 1 m & 8 m/s
C. 4 m & 3 m/s
D. 3 m & 4 m/s

$Given : x = t^3 - 3t^2 + 6t$

Step 1 : *Find the velocity function* : $v = \dfrac{dx}{dt} = 3t^2 - 6t + 6$

Step 2 : Find the critical points of the velocity function : $v = 3t^2 - 6t + 6$

To get the max. or minimum points of a certain function (which is in our case the velocity function) we first must find the critical points of that function.

$$\frac{dv}{dt} = 0$$

$$6t - 6 = 0$$

$\underline{t = 1 \text{ sec} \text{ --------- } \textit{is the only critical point}}$

To find out whether it is a maximum or minimum point we perform the second derivative test.

$$\frac{d^2v}{dt^2} = 6 > 0 \quad \text{------- } \textit{therefore minimum point}$$

$$v_{minimum} = v\Big|_{t=1\,sec} = 3 - 6 + 6 = 3\ \frac{m}{s}$$

$$\textit{position of minimum velocity} = x\Big|_{t=1\,sec} = 1 - 3 + 6 = 4\ m$$

Q 15.

The position of a particle as it moves along the x axis is given for $t > 0$ by $\mathbf{x = (t^3 - 3t^2 + 6t)\ m}$, where t is in s. **Where is the particle** when it achieves its **minimum speed** (after $t = 0$) and what is that **speed** ?

A. 7 m & 2 m/s

B. 1 m & 8 m/s

C. 4 m & 3 m/s

D. 3 m & 4 m/s

Q 16.

101. A particle is initially at rest. Beginning at $t = 0$, it begins moving, with an acceleration given by $a = a_o\{1 - [t^2/(4.0\ \ )]\}$ for $0 \le t \le 2$ s and $a = 0$ thereafter. The initial value is $a_o = 20\ m/s^2$. What is the particle's velocity after 1.0 s? How far has the particle traveled after 2.0 s?

Answer:

$$v = \int a\,dt = \int a_o\left\{1 - \left[\frac{t^2}{(4.0\ \ )}\right]\right\}dt$$

$$v = a_o\left\{t - \left[\frac{t^3}{(12\ \ )}\right]\right\}$$

t = 1 sec $\quad v = 20\left\{1 - \left[\frac{1}{(12)}\right]\right\} = 18.3\,\frac{m}{s}$

t = 2 sec $\quad v = 20\left\{2 - \left[\frac{8}{(12)}\right]\right\} = 26.7\ m/s$

$$x = \int v dt = \int a_o\left\{t - \left[\frac{t^3}{(12\ \ )}\right]\right\}dt = a_o\left\{\frac{t^2}{2} - \left[\frac{t^4}{(48\ \ )}\right]\right\}$$

At $t = 2$ s:

$$x = 20\left\{\frac{2^2}{2} - \left[\frac{2^4}{(48)}\right]\right\} = 33.3\ m$$

Free Fall Motion

Free Fall Motion

First rule :

the acceleration due to gravity: $g = 9.8\, m/s^2$ is always directed downwards regardless of the direction of motion of the object

Second rule :

Always assume that the {+ve direction} upon which all the signs of your quantities rely on is the direction of motion of your object.

Third rule :

Apply the rules of Uniform variable motion

1. $v = v_0 + at$
2. $\Delta x = (\frac{v + v_0}{2})t$
3. $\Delta x = v_0 t + \frac{1}{2}at^2$
4. $v^2 = v_0^2 + 2a\Delta x$

Q 17.

A baseball is thrown vertically into the air. The acceleration of the ball at its highest point is:

A. zero
B. g, down
C. g, up
D. $2g$, down
E. $2g$, up

Q 17.

A baseball is thrown vertically into the air. The acceleration of the ball at its highest point is:

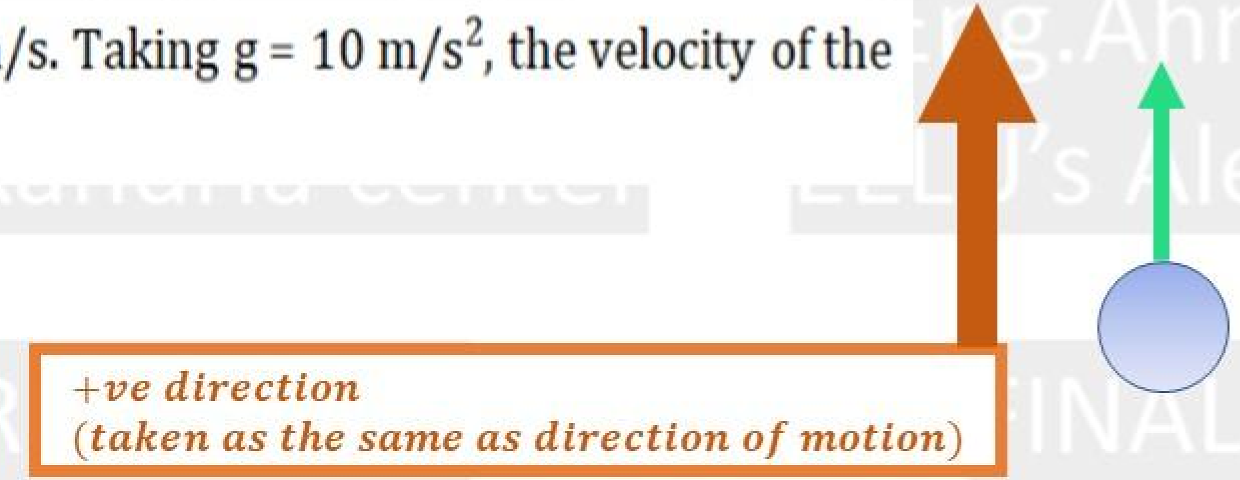
A. zero
B. g, down
C. g, up
D. $2g$, down
E. $2g$, up

Q 18.

An object is thrown vertically upward at 35 m/s. Taking g = 10 m/s^2, the velocity of the object 5 s later is:

A. 7.0 m/s up
B. 15 m/s down
C. 15 m/s up
D. 85 m/s down
E. 85 m/s up

An object is thrown vertically upward at 35 m/s. Taking g = 10 m/s^2, the velocity of the object 5 s later is:



$\because g = -10\ m/s^2$

** the negative sign because g is downwards and our positive direction is upwards so it's opposite to our +ve direction

$\because v_{initial} = +35\ m/s$

** the positive sign because "$v_{initial}$" is upwards and our positive direction is also upwards.

$t = 5\ sec$

$v_{final} = ??$

$\because v_{final} = v_{initial} + gt$

$\therefore v_{final} = 35 + (-10)(5)$

$\therefore v_{final} = -15\ m/s$

** the negative sign means that this velocity is opposite to our positive direction(upwards) so it's pointing down

$\therefore v_{final} = +15\ m/s$ *downwards*

1. $v = v_0 + at$
2. $\Delta x = (\frac{v + v_0}{2})t$
3. $\Delta x = v_0 t + \frac{1}{2}at^2$
4. $v^2 = v_0^2 + 2a\Delta x$

Q 18.

An object is thrown vertically upward at 35 m/s. Taking g = 10 m/s^2, the velocity of the object 5 s later is:

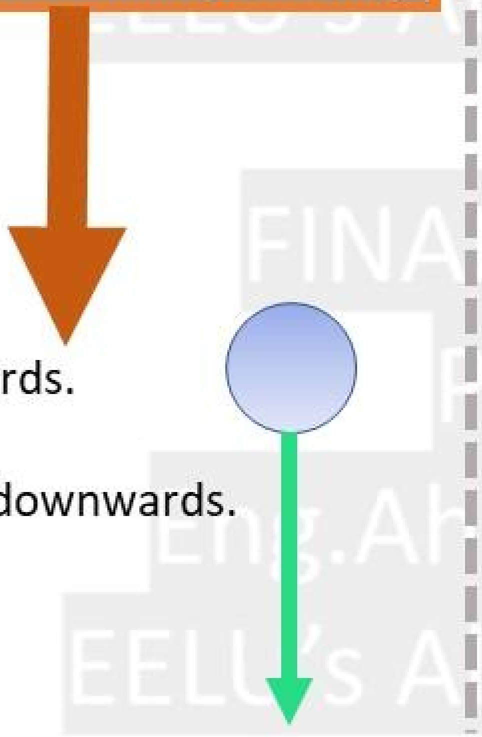
A. 7.0 m/s up
B. 15 m/s down
C. 15 m/s up
D. 85 m/s down
E. 85 m/s up

Q 19.

An object is thrown vertically downward with an initial speed 1 m/s. After 5 s the object will have travelled:

Q 19.

$+ve$ *direction*
(*taken as the same as direction of motion*)



$\because g = +9.8\ m/s^2$
** the positive sign because g is downwards and our positive direction is also downwards.
$\because v_{initial} = +1\ m/s$
** the positive sign because "$v_{initial}$" is downwards and our positive direction is also downwards.
$t = 5\ sec$
$\Delta y = ??$, we used "Δy" because the motion is along the y-axis.

$$\because \Delta y = (v_{initial})t + \frac{1}{2}gt^2$$

$$\therefore \Delta y = (1)5 + \frac{1}{2}(9.8)(5)^2 = 127.5\ m$$

$$\therefore \Delta y = +\ 127.5\ m$$

** the positive sign means that this position is in the *same direction as* our positive directi
so it's pointing downwards

$$\therefore \Delta y = +\ 127.5\ m\ \ downwards$$

1. $v = v_0 + at$
2. $\Delta x = (\frac{v + v_0}{2})t$
3. $\Delta x = v_0 t + \frac{1}{2}at^2$ ✓
4. $v^2 = v_0^2 + 2a\Delta x$

Q 20.

A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The building is 50.0 m high, and the stone just misses the edge of the roof on its way down, as shown in Figure 2.14. Using $t_A = 0$ as the time the stone leaves the thrower's hand at position Ⓐ, determine

(a) the time at which the stone reaches its maximum height,

(b) the maximum height,

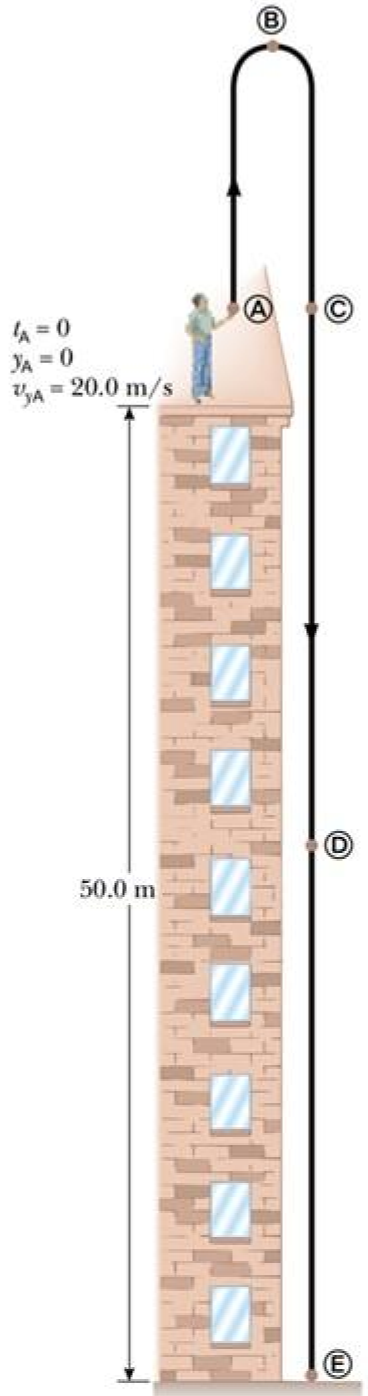
(c) the time at which the stone re-turns to the height from which it was thrown

(d) the velocity of the stone at this instant.

(e) the velocity and position of the stone at $t = 5.00$ s.

(f) the velocity of the stone just before it hits the ground at Ⓔ

(g) the total time the stone is in the air.



(a) the time at which the stone reaches its maximum height,

+ve direction (taken as the same as direction of motion)

From (A) to (B) taking the positive direction upwards:

$\therefore g = -9.8\ m/s^2$

** the negative sign because g is downwards and our positive direction is upwards so it's opposite to our +ve direction

$\because v_{initial-at-A} = +\,20\ m/s$

** the positive sign because "$v_{initial-at-A}$" is upwards and our positive direction is also upwards.

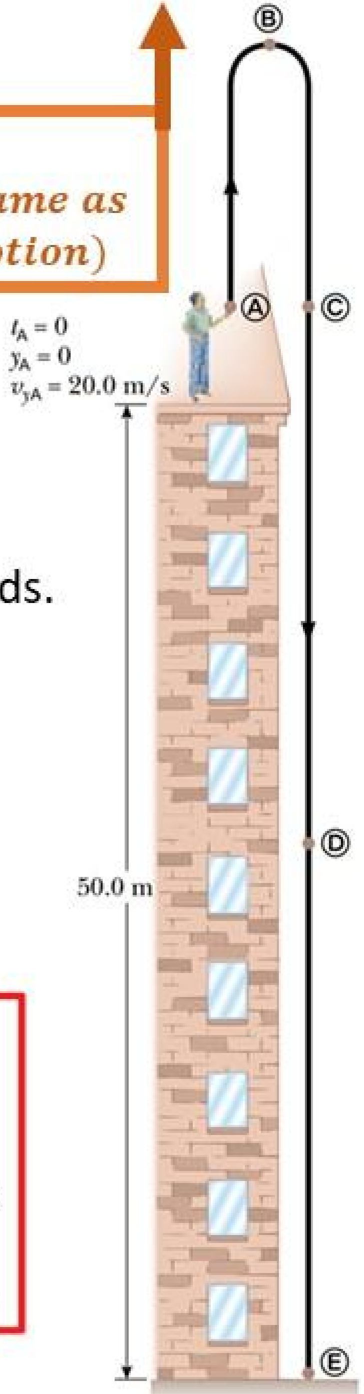
$v_{final-at-B} = 0 \quad --\ \textit{because as the stone goes upwards it's under the effect of the}$ gravity which causes the ball to reduces it's speed till it reaches the point of maximum height at which it stops momentarily in air and the instantaneous velocity becomes zero

$\because v_{final-at-B} = v_{initial-at-A} + g t_B$

$\therefore 0 = 20 + (-9.8) t_B$

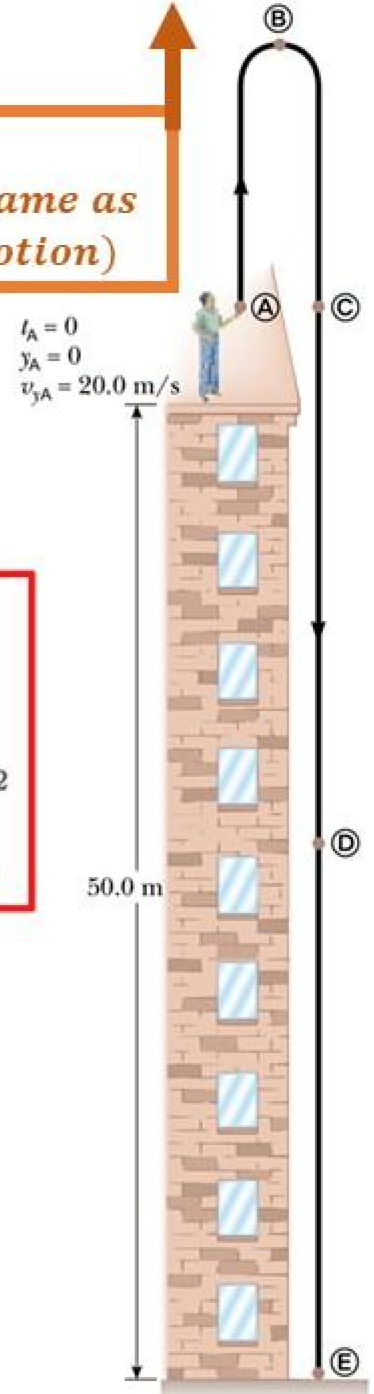
$\therefore t_B = 2.0408\ sec$ which is the time required to reach point B

1. $v = v_0 + at$
2. $\Delta x = (\frac{v + v_0}{2})t$
3. $\Delta x = v_0 t + \frac{1}{2}at^2$
4. $v^2 = v_0^2 + 2a\Delta x$



(b) the maximum height,

+ve direction (taken as the same as direction of motion)



1. $v = v_0 + at$
2. $\Delta x = (\frac{v + v_0}{2})t$
3. $\Delta x = v_0 t + \frac{1}{2}at^2$
4. $v^2 = v_0^2 + 2a\Delta x$

To get the maximum height :

$\because \Delta y_{(A-to-B)} = \left(\frac{v_{final-at-B} + v_{initial-at-A}}{2}\right) t_B$, we used "$\Delta y$" because the motion is along the y-axis.

$$\therefore \Delta y_{(A-to-B)} = \left(\frac{0 + 20}{2}\right)(2.0408) = 20.408\ m$$

$$\therefore (y_B - y_A) = 20.408\ m$$

$\because y_A = 0$ <u>*as it is the initial position of the whole journey and is considered the reference point to all position measurements*</u>

$$\therefore y_B = 20.408\ m$$

A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The building is 50.0 m high, and the stone just misses the edge of the roof on its way down, as shown in Figure 2.14. Using $t_A = 0$ as the time the stone leaves the thrower's hand at position Ⓐ, determine

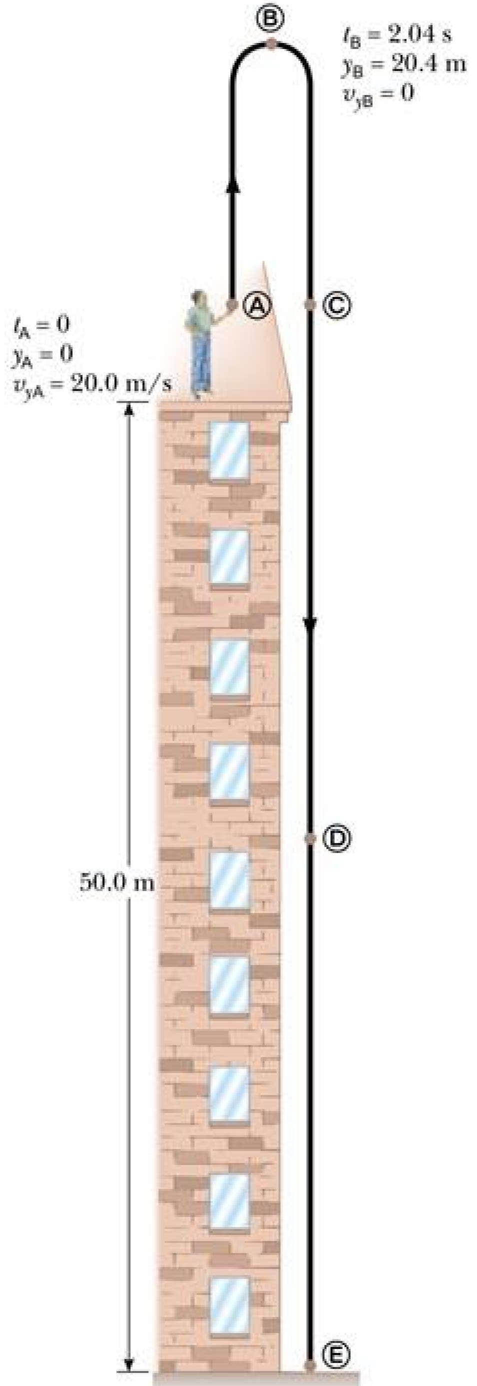
(a) ~~the time at which the stone reaches its maximum height,~~

(b) ~~the maximum height,~~

(c) the time at which the stone re-turns to the height from which it was thrown

(d) the velocity of the stone at this instant.

(e) the velocity and position of the stone at $t = 5.00$ s.



Find (f) the velocity of the stone just before it hits the ground at Ⓔ

(g) the total time the stone is in the air.

(c) the time at which the stone re-turns to the height from which it was thrown

(d) the velocity of the stone at this instant.

From (B) to (C) taking the positive direction downwards:

$\therefore g = +9.8\ m/s^2$

** the positive sign because g is downwards and our positive direction is also downwards.

$\because v_{initial-at-B} = 0$

$\because$ magnitude of " $v_{final-at-C}$ " = magnitude of $v_{initial-at-A}$

<u>but direction of</u> " $v_{final-at-C}$ " <u>is downwards</u>

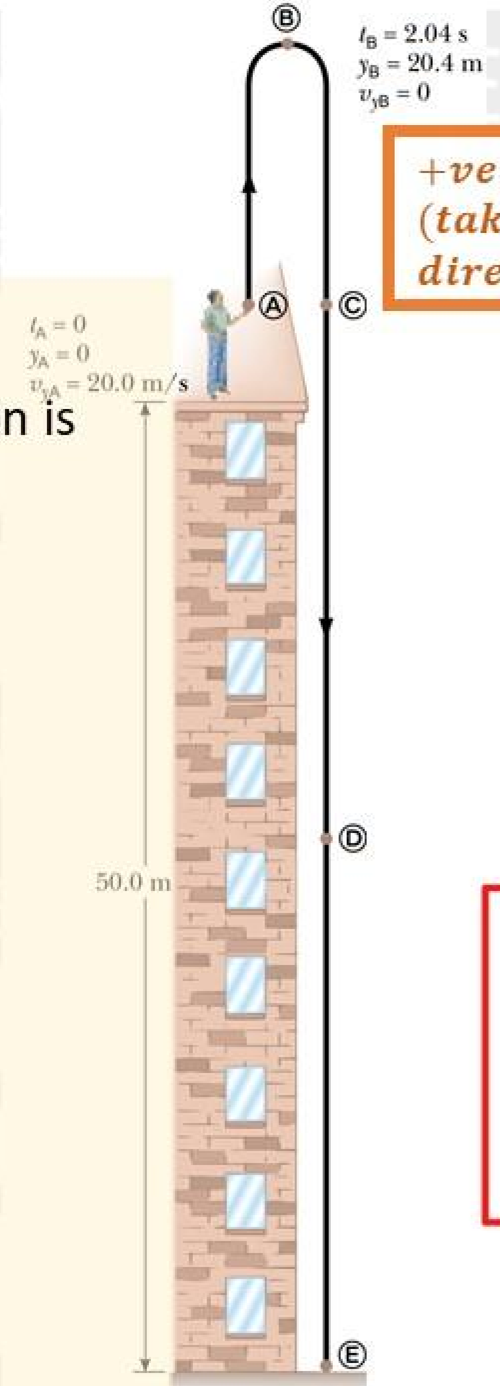
$\because v_{final-at-C} = v_{initial-at-B} + g t_C$

$\therefore 20 = 0 + (9.8) t_C$

$\therefore t_C = 2.0408\ sec$ which is the time required to reach point C

$\therefore t_{A-to-B-to-C} = t_B + t_C = 2.0408 + 2.0408 = 4.08163\ sec$

Notice that : $y_C = 0$ <u>as the ball returned to the reference point A and point C coincides on point A</u>



+ve direction (taken as the same as direction of motion)

1. $v = v_0 + at$
2. $\Delta x = (\frac{v + v_0}{2})t$
3. $\Delta x = v_0 t + \frac{1}{2}at^2$
4. $v^2 = v_0^2 + 2a\Delta x$

A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The building is 50.0 m high, and the stone just misses the edge of the roof on its way down, as shown in Figure 2.14. Using $t_A = 0$ as the time the stone leaves the thrower's hand at position Ⓐ, determine

(a) ~~the time at which the stone reaches its maximum height,~~

(b) ~~the maximum height,~~

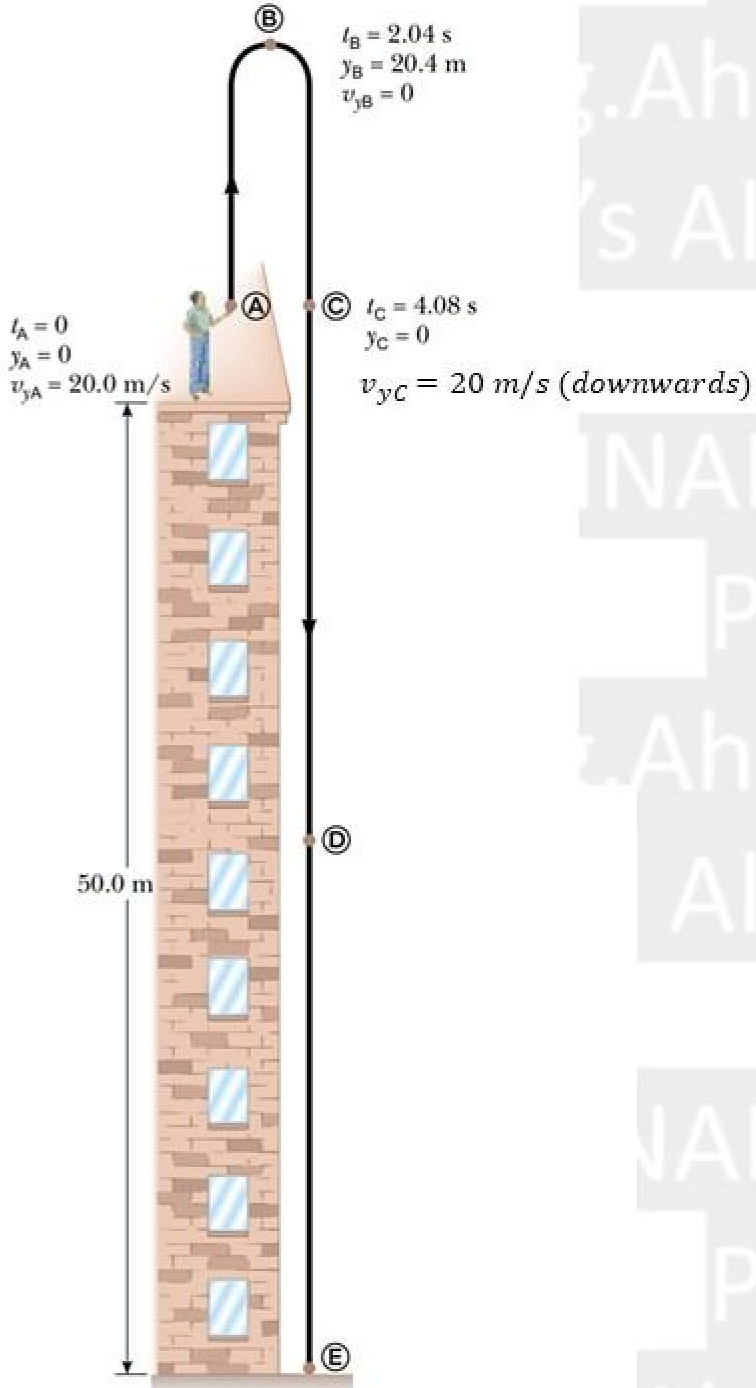
(c) ~~the time at which the stone re-turns to the height from which it was thrown~~

(d) ~~the velocity of the stone at this instant~~.

(e) the velocity and position of the stone at $t = 5.00$ s.

Find (f) the velocity of the stone just before it hits the ground at Ⓔ

(g) the total time the stone is in the air.



(e) the velocity and position of the stone at $t = 5.00$ s.

Assume that point D is at which t = 5 sec

From (C) to (D) taking the positive direction downwards:

$\therefore g = +9.8\ m/s^2$

** the positive sign because g is downwards and our positive direction is also downwards.

$\because v_{initial-at-c} = 20\ m/s$

$\because v_{final-at-D} = v_{initial-at-C} + g(t_{from\ C-to-D})$

$\therefore v_{final-at-D} = 20 + (9.8)(t_D - t_C) = 20 + (9.8)(5 - 4.08)$

$\therefore v_{final-at-D} = 29.016\ m/s$ (downwards)

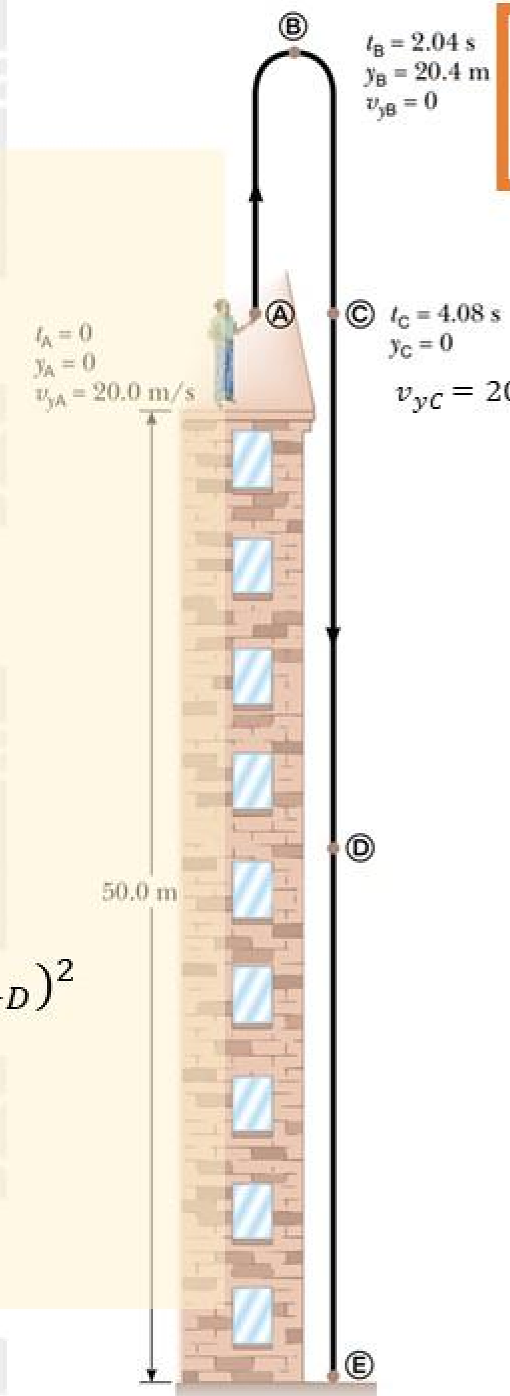
$\because \Delta y_{(C-to-D)} = (v_{initial-at-C})(t_{from\ C-to-D}) + \frac{1}{2} g (t_{from\ C-to-D})^2$

$\therefore (y_D - y_C) = (20)(5 - 4.08) + \frac{1}{2}(9.8)(5 - 4.08)^2$

$\therefore (y_D - y_C) = 22.54736$

$\therefore (y_D - 0) = 22.54736$

$\therefore y_D = 22.54736\ m$ *(downwards)*



$v_{yC} = 20\ m/s\ (downwards)$

+ve direction (taken as the same as direction of motion)

1. $v = v_0 + at$
2. $\Delta x = (\frac{v + v_0}{2})t$
3. $\Delta x = v_0 t + \frac{1}{2}at^2$
4. $v^2 = v_0^2 + 2a\Delta x$

A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The building is 50.0 m high, and the stone just misses the edge of the roof on its way down, as shown in Figure 2.14. Using $t_A = 0$ as the time the stone leaves the thrower's hand at position Ⓐ, determine

(a) ~~the time at which the stone reaches its maximum height,~~

(b) ~~the maximum height,~~

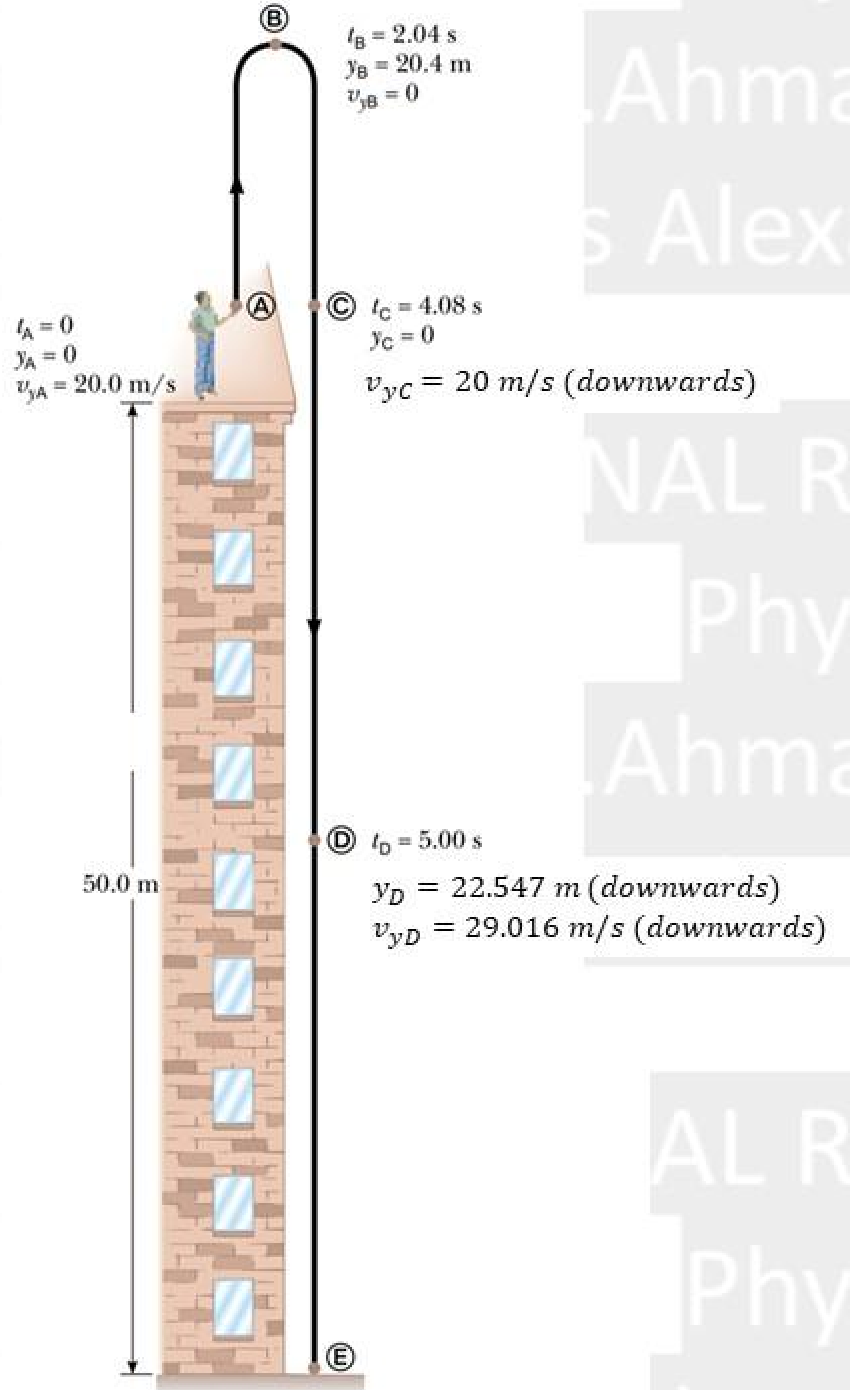
(c) ~~the time at which the stone re-turns to the height from which it was thrown~~

(d) ~~the velocity of the stone at this instant~~.

(e) t~~he velocity and position of the stone at $t = 5.00$~~ s.

Find (f) the velocity of the stone just before it hits the ground at Ⓔ

(g) the total time the stone is in the air.



Find (f) the velocity of the stone just before it hits the ground at Ⓔ

From (D) to (E) taking the positive direction downwards:
$\therefore g = +9.8\ m/s^2$
** the positive sign because g is downwards and our positive direction is also downwards.

$\because v_{initial-at-D} = 29.016\ \ m/s$

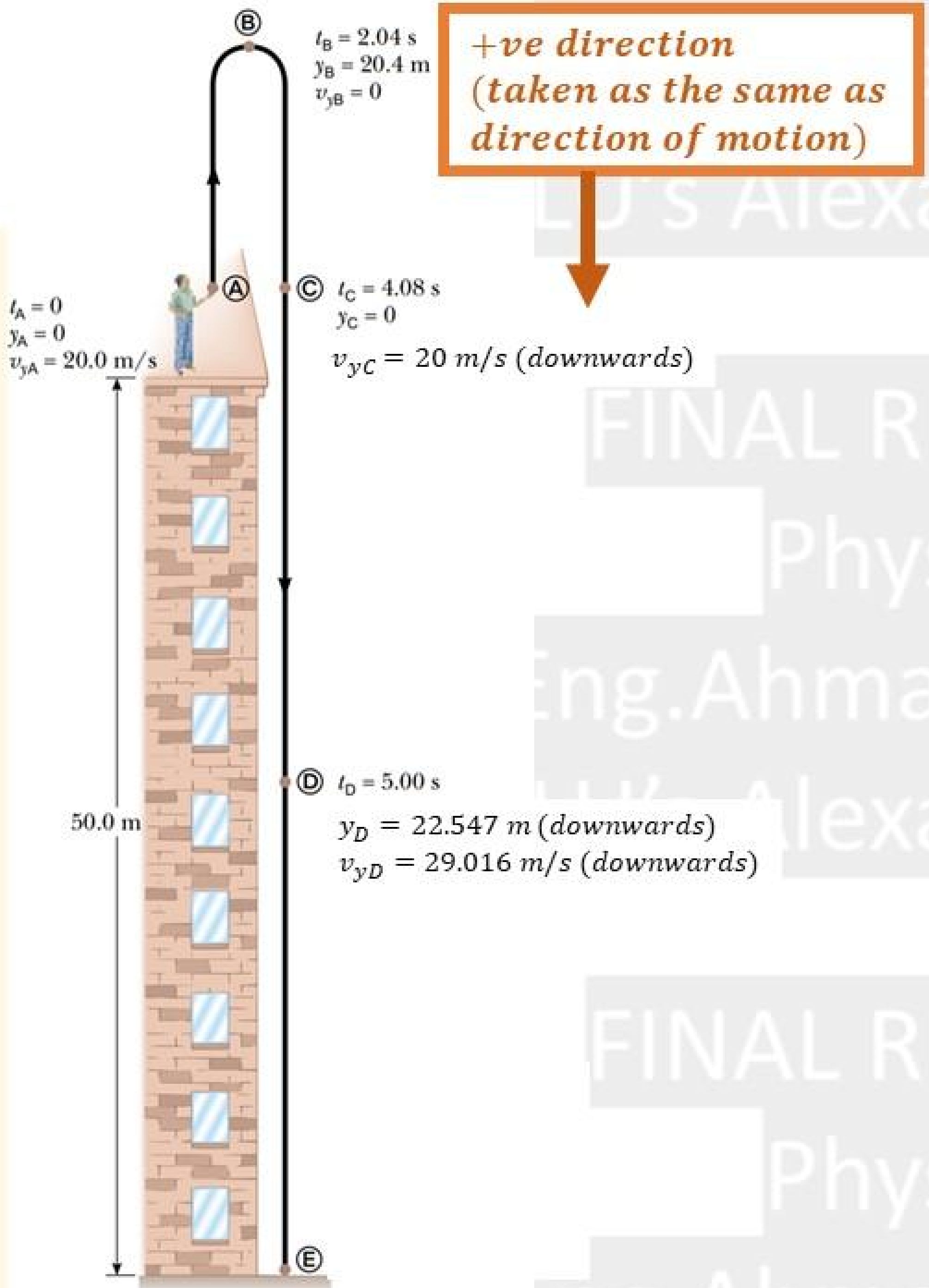
$\because (v_{final-at-E})^2 = (v_{initial-at-D})^2 + 2g\Delta y_{(D-to-E)}$

$\therefore (v_{final-at-E})^2 = (v_{initial-at-D})^2 + 2g(y_E - y_D)$

$\because y_E = 50\ m$ (downwards)
**obtained from the figure "height of the building" as point "E" is at the base of the building while point "A" which is our reference is at the top of the building.

$\therefore (v_{final-at-E})^2 = (29.016)^2 + 2(9.8)(50 - 22.547)$

$\therefore v_{final-at-E} = 37.14845\ m/s$ (downwards)



A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The building is 50.0 m high, and the stone just misses the edge of the roof on its way down, as shown in Figure 2.14. Using $t_A = 0$ as the time the stone leaves the thrower's hand at position Ⓐ, determine

(a) ~~the time at which the stone reaches its maximum height,~~

(b) ~~the maximum height,~~

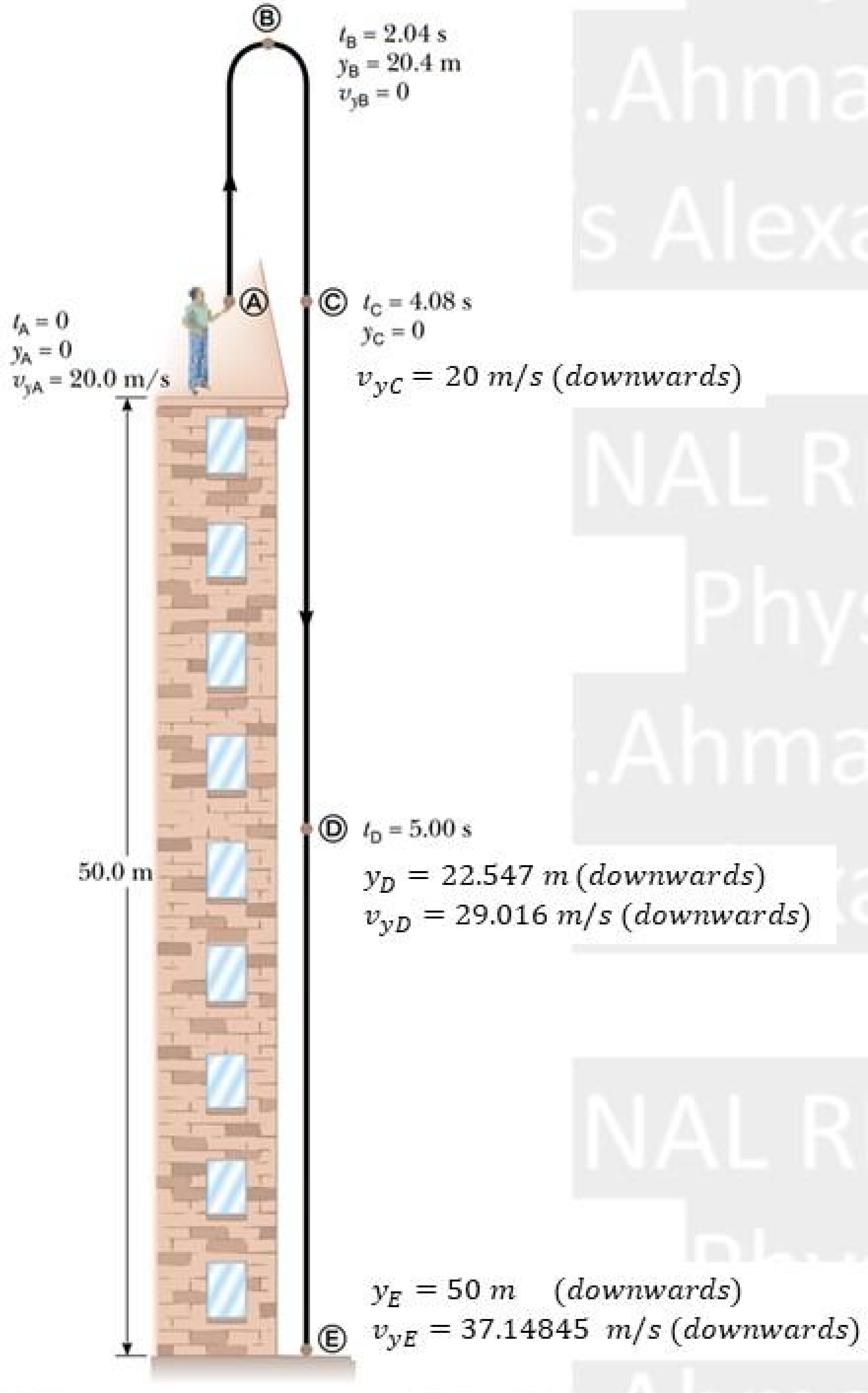
(c) ~~the time at which the stone re-turns to the height from which it was thrown~~

(d) ~~the velocity of the stone at this instant~~.

(e) ~~the velocity and position of the stone at $t = 5.00$~~ s.

Find ~~(f) the velocity of the stone just before it hits the ground at Ⓔ~~

(g) the total time the stone is in the air.



(g) the total time the stone is in the air.

From (D) to (E) taking the positive direction downwards:

$\therefore g = +9.8\ m/s^2$

* The positive sign because g is downwards and our positive direction is also downwards.

$\because v_{initial-at-D} = 29.016\ m/s$

$\because v_{final-at-E} = 37.14845\ m/s$

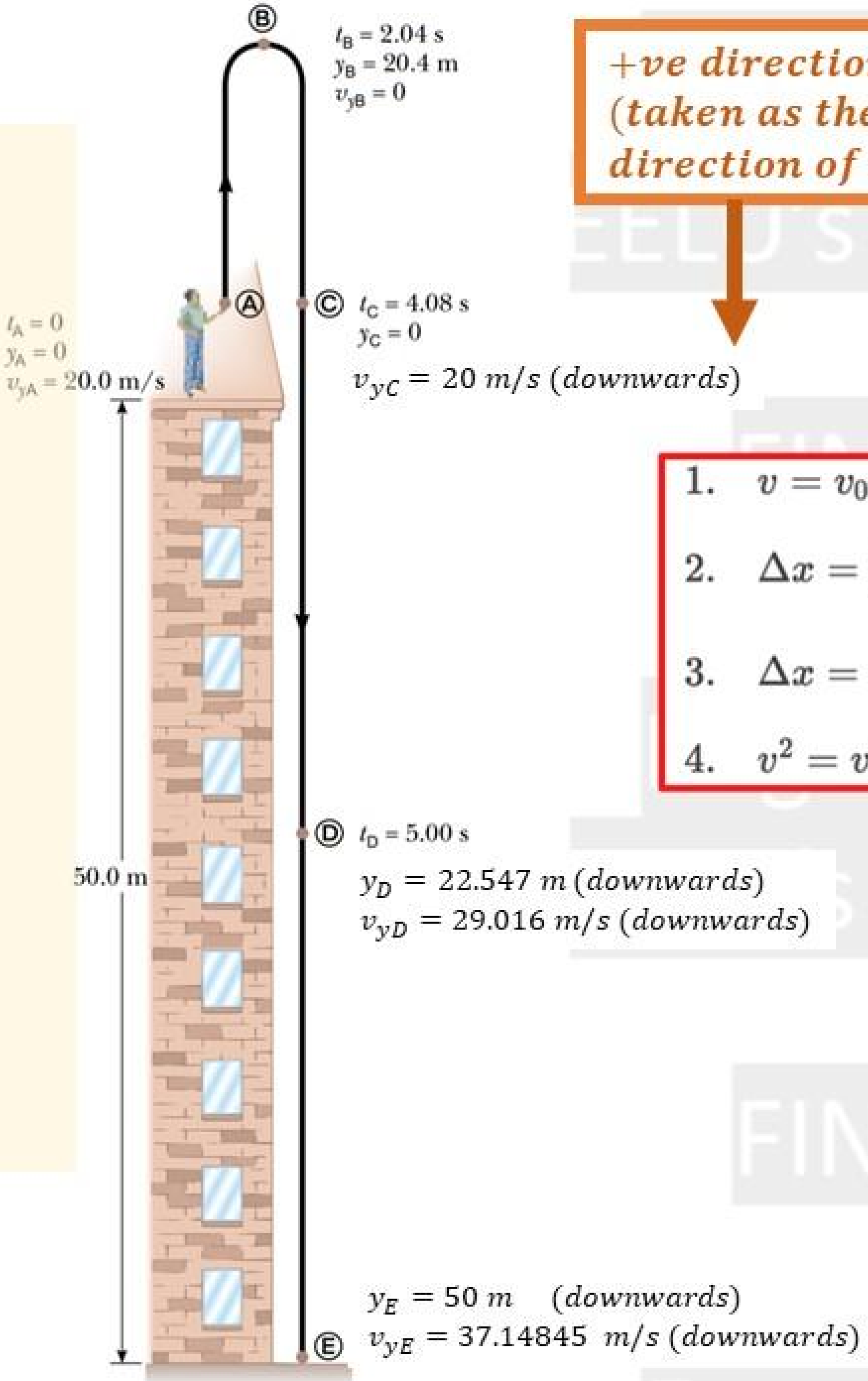
$\because v_{final-at-E} = v_{initial-at-D} + g(t_{from\,D-to-E})$

$\therefore 37.14845 = 29.016 + (9.8)(t_E - t_D)$

$\because t_D = 5\ sec$

$\therefore 37.14845 = 29.016 + (9.8)(t_E - 5)$

$\therefore t_E = 5.829841\ \text{sec} = TOTAL\ TIME\ OF\ STONE\ IN\ AIR$



+ve direction (taken as the same as direction of motion)

1. $v = v_0 + at$
2. $\Delta x = (\frac{v + v_0}{2})t$
3. $\Delta x = v_0 t + \frac{1}{2}at^2$
4. $v^2 = v_0^2 + 2a\Delta x$

Q 21.

You throw a ball vertically upward so that it leaves the ground with velocity 15.00 m/s
what is the acceleration just before it reaches maximum altitude ? considering the positive coordinate is pointing upward.

A. $-9.8\ m/s^2$

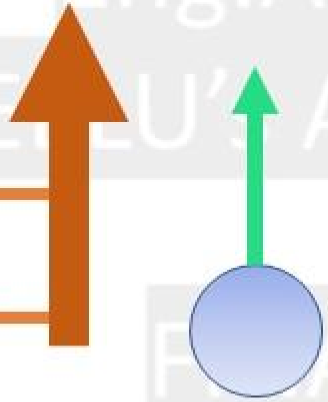
B. $9.8\ m/s^2$

C. None

D. $0\ m/s^2$

You throw a ball vertically upward so that it leaves the ground with velocity 15.00 m/s what is the acceleration just before it reaches maximum altitude ? considering the positive coordinate is pointing upward.

$+ve\ direction$
$(taken\ as\ indicated\ in\ question)$



$\because g = -9.8\ m/s^2$

** the negative sign because g is downwards and our positive direction is upwards so it's opposite to our +ve direction

$\because v_{initial} = +\,15\ m/s$

1. $v = v_0 + at$
2. $\Delta x = (\frac{v + v_0}{2})t$
3. $\Delta x = v_0 t + \frac{1}{2}at^2$
4. $v^2 = v_0^2 + 2a\Delta x$

Q 21.

You throw a ball vertically upward so that it leaves the ground with velocity 15.00 m/s what is the acceleration just before it reaches maximum altitude ? considering the positive coordinate is pointing upward.

A. $-9.8\ m/s^2$

B. $9.8\ m/s^2$

C. None

D. $0\ m/s^2$

Q 22.

A ball is thrown straight up in the air. For which situation is the instantaneous velocity is zero?

A ball is thrown straight up in the air. For which situation is the instantaneous acceleration is zero?

A ball is thrown straight up in the air. For which situation are both the instantaneous velocity and the acceleration zero?

Q 22.

*A ball is thrown straight up in the air. For which situation is the instantaneous velocity is zero? At the top of its flight path (when it had reached it's maximum amplitude)

*A ball is thrown straight up in the air. For which situation is the instantaneous acceleration is zero? No way

*A ball is thrown straight up in the air. For which situation are both the instantaneous velocity and the acceleration zero? No way

*Kinematic equations can be used only when the acceleration is zero **False**

*Kinematic equations can be used only when the acceleration is constant **True**

Projectile Motion

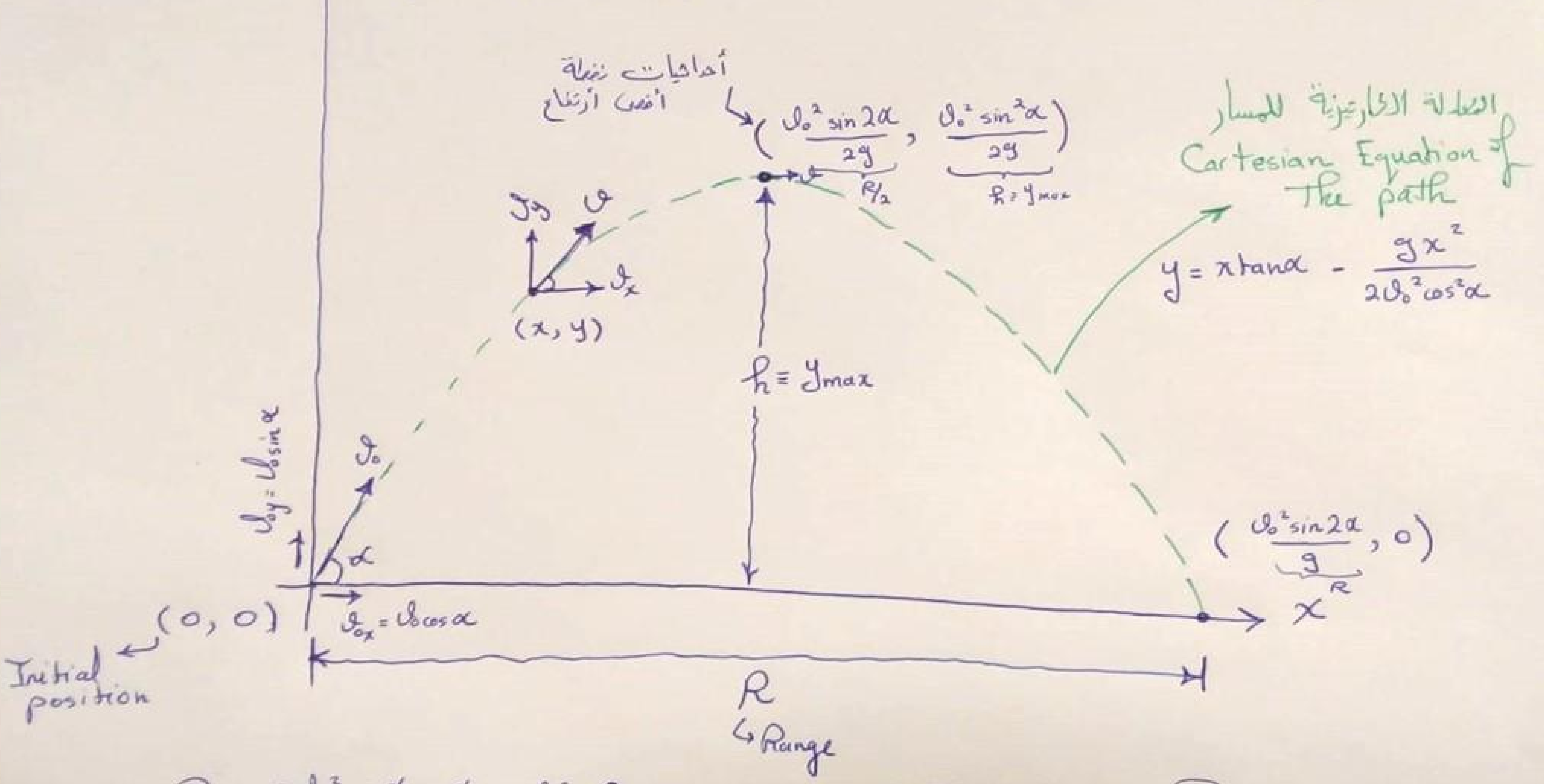
Q 23.

A ball is thrown horizontally from the top of a building 0.10 km high. The ball strikes the ground at a point 65 m horizontally away from and below of the point of release. What is the speed of the ball just before it strikes the ground?

A. 43 m/s
B. 47 m/s
C. 39 m/s
D. 36 m/s

$$(x, y) = \left((v_0\cos\alpha)t\ ,\ (v_0\sin\alpha)t - \tfrac{1}{2}gt^2\right)$$

$$\frac{d}{dt}\downarrow\ (v_x, v_y) = \left(v_0\cos\alpha\ ,\ (v_0\sin\alpha) - gt\right) \Rightarrow v = \sqrt{v_x^2 + v_y^2}$$



$$* \ R = \frac{2v_0^2\sin\alpha\cos\alpha}{g} = \frac{v_0^2\sin 2\alpha}{g} \longrightarrow \text{To get max. Range} \Rightarrow \sin 2\alpha = 1$$

$$2\alpha = 90$$

$$\alpha = 45^\circ = \frac{\pi}{4}$$

Notice that since the ball was thrown horizontally it means that the inclination angle of projectile "α" is equal to zero , i.e. $(\alpha = 0)$

Assume that we have two points:
Point 1 "A": Is the ball's initial position
Point 2 "B": Is the ball's final position

From the figure notice that B=(65,-100) which means:

$\underline{\therefore \boldsymbol{x_B = 65}}$

$\therefore (v_o cos\alpha)t_B = 65$

$\because \alpha = 0$

$\therefore v_o t_B = 65$ --- (1)

$\underline{\therefore \boldsymbol{y_B = -100}}$

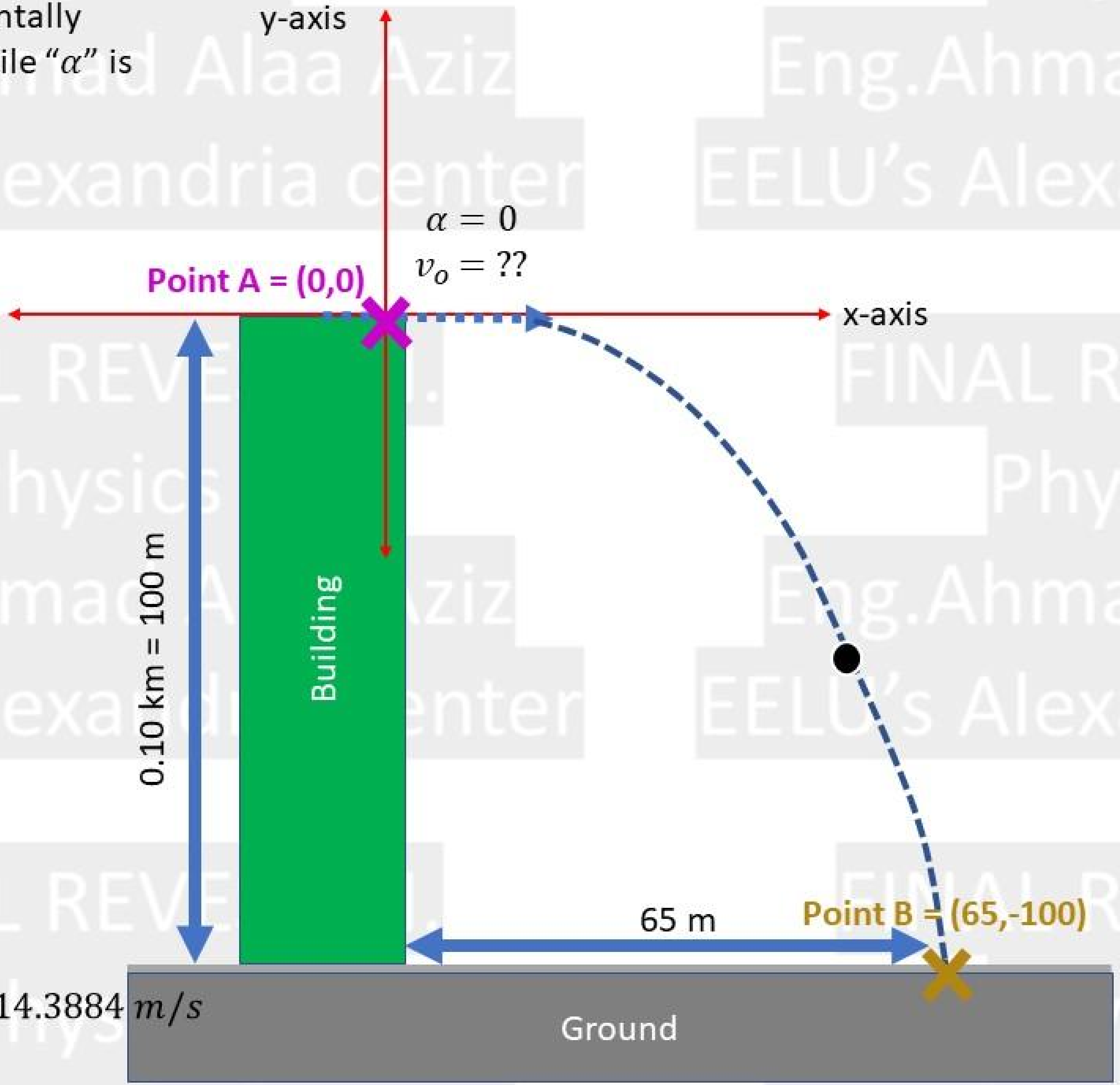
$$\therefore (v_o sin\alpha)t_B - \frac{1}{2}g{t_B}^2 = -100$$

$\because \alpha = 0$

$$\therefore -\frac{1}{2}g{t_B}^2 = -100$$

$$\therefore g{t_B}^2 = 200$$

$$\therefore t_B = +\sqrt{\frac{200}{g}} - -in\ (1)\ \therefore v_o = \frac{65}{\sqrt{\frac{200}{g}}} = 14.3884\ m/s$$



$\because v_{x_B} = (v_o cos\alpha)$

$\because \alpha = 0$

$\therefore v_{x_B} = v_o$ --- (1)

$\therefore v_{x_B} = 14.3884\ m/s$

$\because v_{y_B} = (v_o sin\alpha) - gt_B$

$\because \alpha = 0$

$\therefore v_{y_B} = -gt_B$

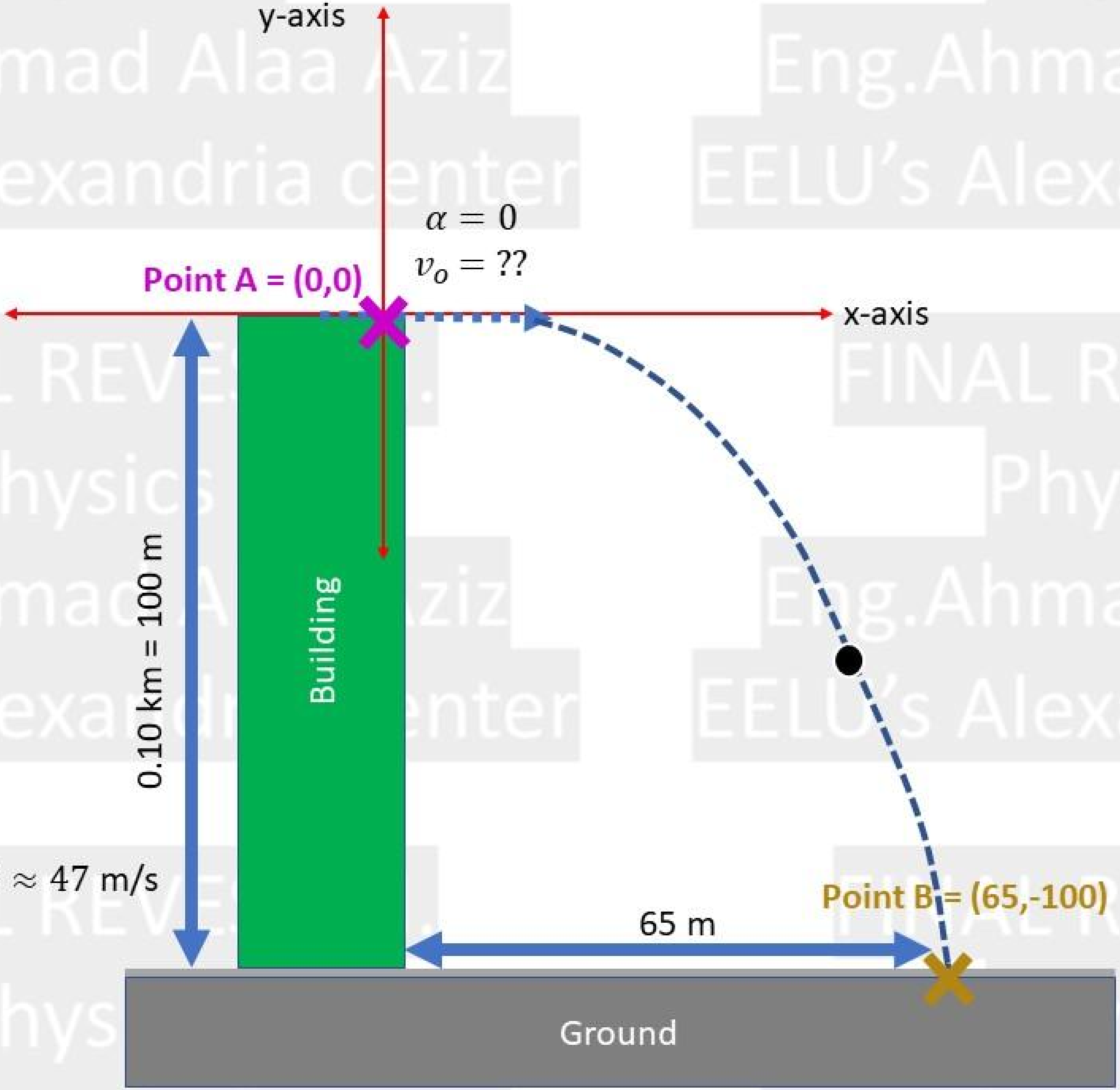
$$\therefore v_{y_B} = -g\sqrt{\frac{200}{g}} = -\sqrt{200g}$$

$$\because v_B = \sqrt{{v_{x_B}}^2 + {v_{y_B}}^2}$$

$$\therefore v_B = \sqrt{{v_o}^2 + (-\sqrt{200g})^2}$$

$$\therefore v_B = \sqrt{{v_o}^2 + 200g}$$

$$\therefore v_B = \sqrt{(14.3884)^2 + 200g} = 46.5513 \approx 47 \text{ m/s}$$



Q 23.

A ball is thrown horizontally from the top of a building 0.10 km high. The ball strikes the ground at a point 65 m horizontally away from and below of the point of release. What is the speed of the ball just before it strikes the ground?

A. 43 m/s

B. 47 m/s

C. 39 m/s

D. 36 m/s

Vectors

Q 24.

A vector, **B** , when added to the vector **C = 3i^ + 4j^** yields a resultant vector which is in the positive y direction and has a magnitude equal to that of **C**. What is the magnitude of **B** ?

A. 18
B. 6.3
C. 9.5
D. 3.2

A vector, **B** , when added to the vector **C = 3i^ + 4j^** yields a resultant vector which is in the positive y direction and has a magnitude equal to that of **C**. What is the magnitude of **B** ?

$$\boxed{\vec{B}+\vec{C}} = \boxed{|\vec{C}|}\ \boxed{\hat{j}}$$

$$Assume : \vec{B} = x_B\hat{\imath} + y_B\hat{\jmath}$$

$$\therefore x_B\hat{\imath} + y_B\hat{\jmath} + 3\hat{\imath} + 4\hat{\jmath} = \sqrt{3^2+4^2}\,\hat{\jmath}$$

$$\therefore (x_B+3)\hat{\imath} + (y_B+4)\hat{\jmath} = (0)\hat{\imath} + 5\hat{\jmath}$$

$$\therefore (x_B+3) = 0 \qquad \& \qquad (y_B+4) = 5$$

$$\therefore x_B = -3 \qquad \& \qquad y_B = 1$$

$$\therefore \vec{B} = -3\hat{\imath} + \hat{\jmath} \qquad \therefore |\vec{B}| = \sqrt{(-3)^2+(1)^2} = \sqrt{10} \approx 3.162$$

Q 24.

A vector, **B** , when added to the vector **C = 3i^ + 4j^** yields a resultant vector which is in the positive y direction and has a magnitude equal to that of **C**. What is the magnitude of **B** ?

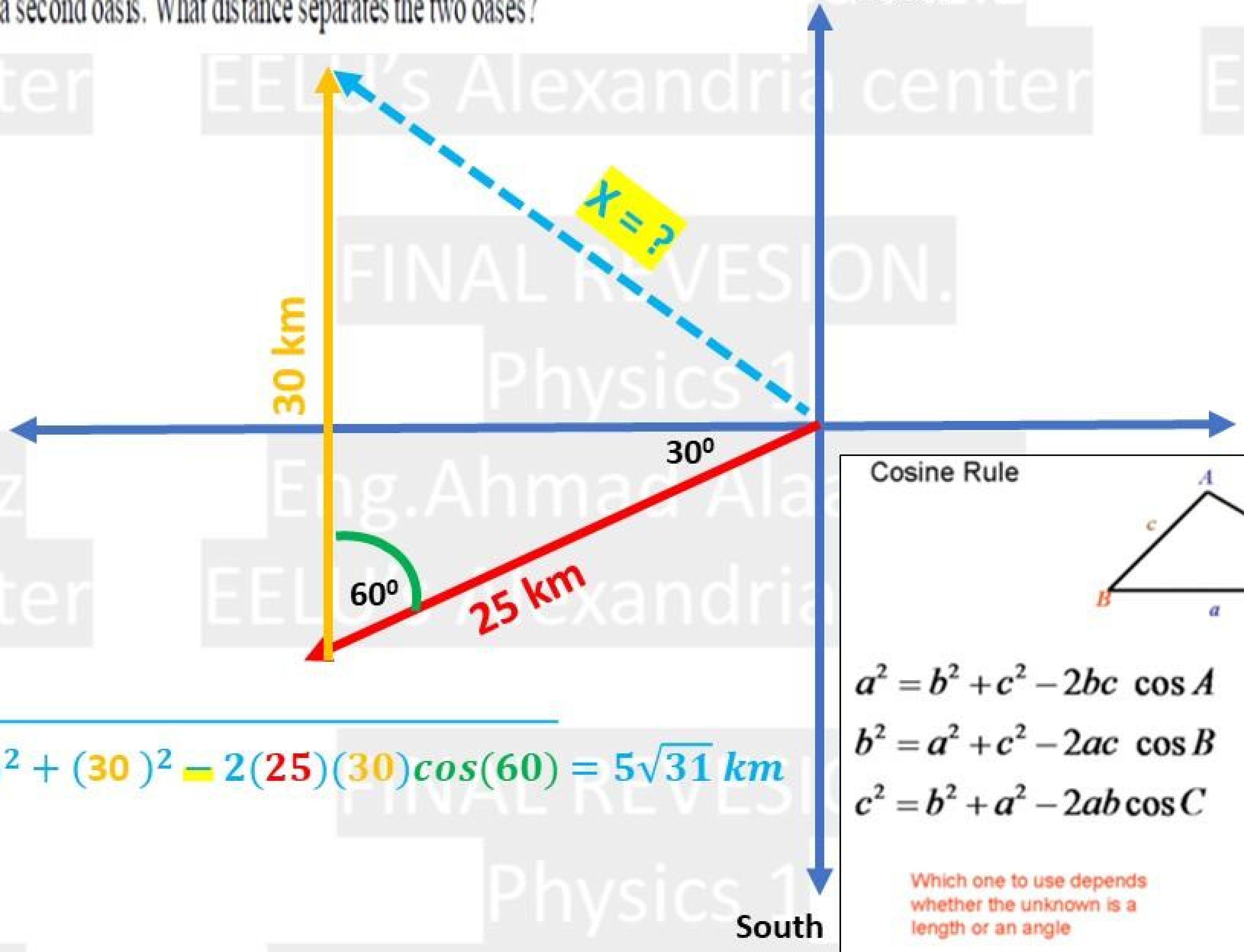
A. 18
B. 6.3
C. 9.5
D. 3.2

Q 25.

Starting from one oasis, a camel walks 25 km in a direction 30° south of west and then walks 30 km toward the north to a second oasis. What distance separates the two oases?

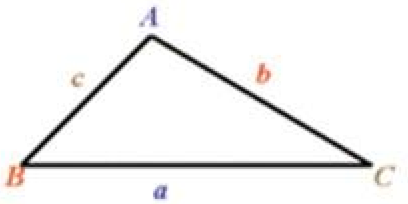
15 km	a.
48 km	b.
28 km	c.
53 km	d.
55 km	e.

Starting from one oasis, a camel walks 25 km in a direction 30° south of west and then walks 30 km toward the north to a second oasis. What distance separates the two oases?



$$X = \sqrt{(25)^2 + (30)^2 - 2(25)(30)\cos(60)} = 5\sqrt{31}\ km$$

Cosine Rule



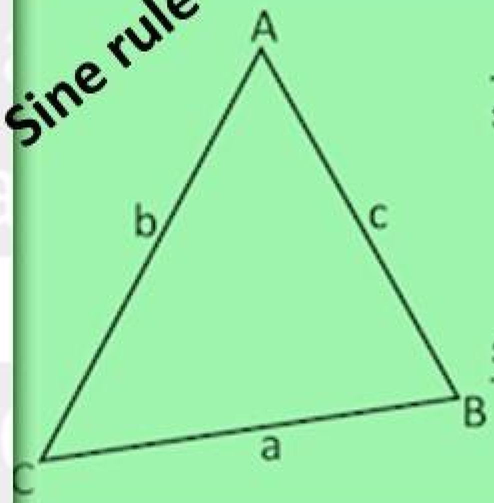
$$a^2 = b^2 + c^2 - 2bc\ \cos A$$
$$b^2 = a^2 + c^2 - 2ac\ \cos B$$
$$c^2 = b^2 + a^2 - 2ab\cos C$$

The formula can be rearranged to:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Which one to use depends whether the unknown is a length or an angle

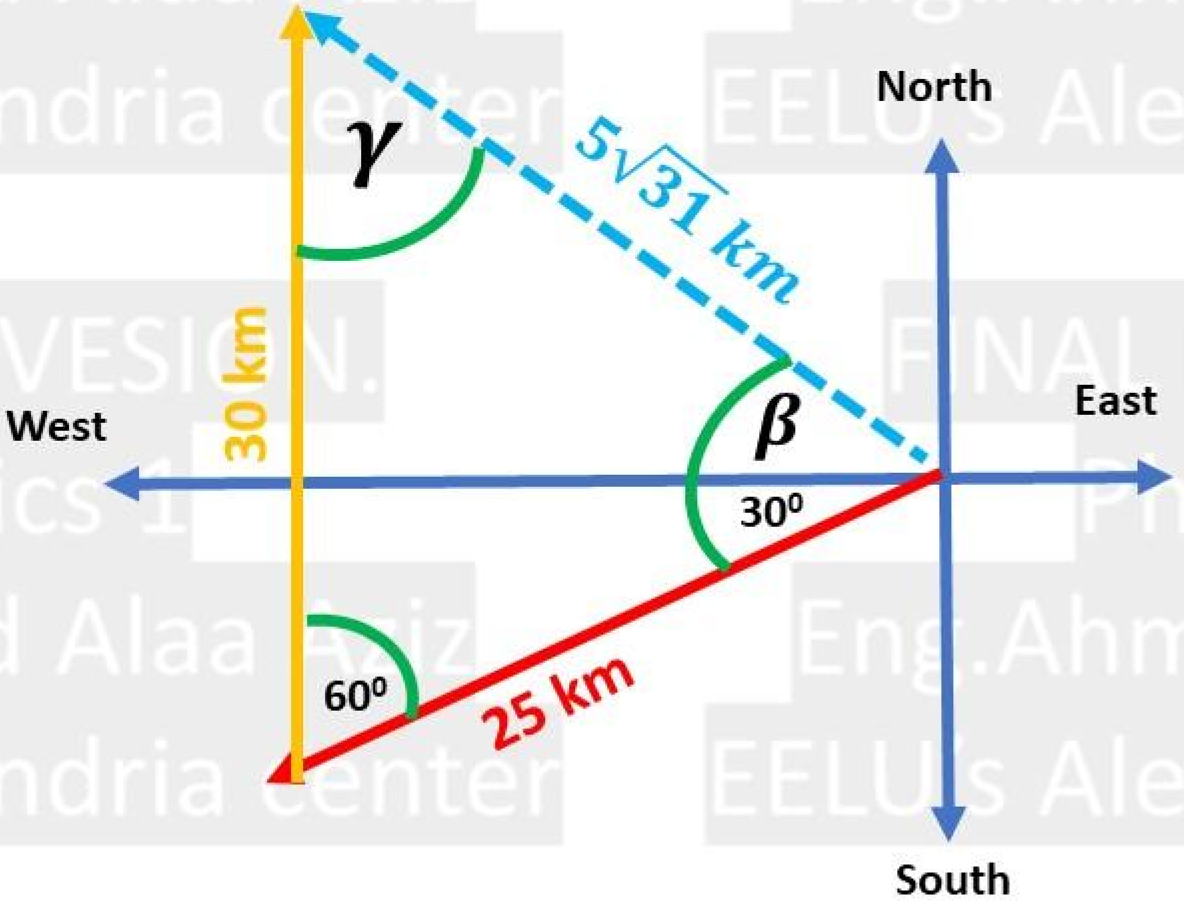
Sine rule



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

or

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



$$\because \frac{30}{\sin(30+\beta)} = \frac{25}{\sin(\gamma)} = \frac{5\sqrt{31}}{\sin(60)}$$

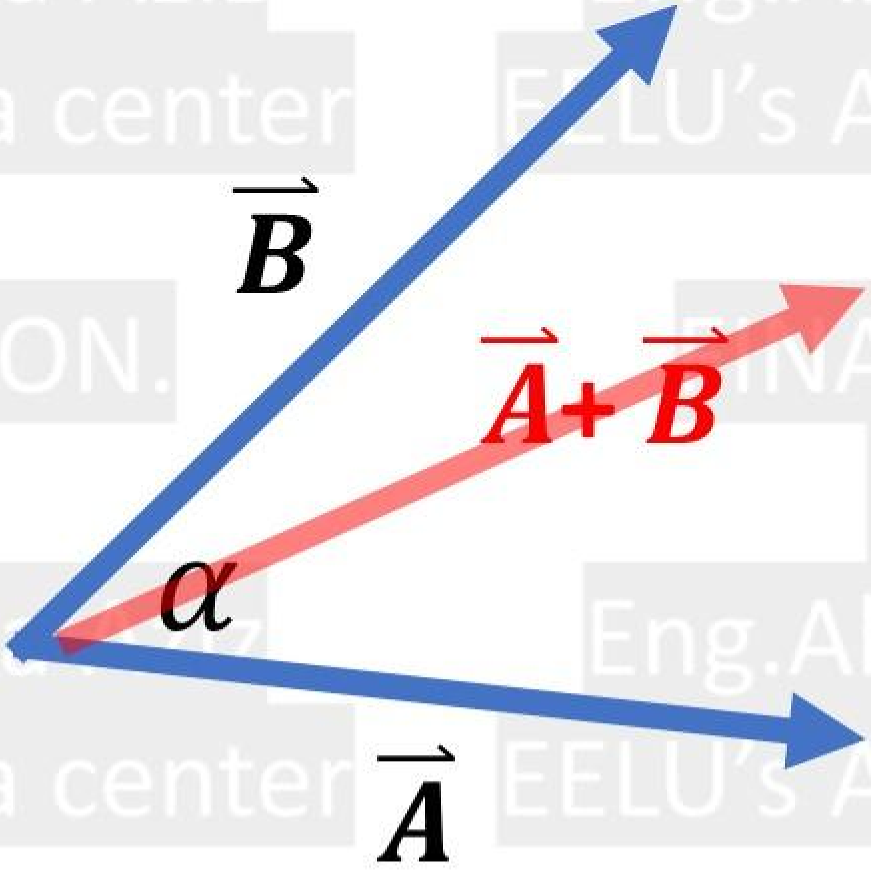
$$\therefore \sin(\gamma) = 25 * \frac{\sin(60)}{5\sqrt{31}}$$

$$\therefore \gamma = \sin^{-1}\left(25 * \frac{\sin(60)}{5\sqrt{31}}\right) = 51.0517°$$

$$\therefore \sin(30+\beta) = 30 * \frac{\sin(60)}{5\sqrt{31}}$$

$$\therefore (30+\beta) = \sin^{-1}\left(30 * \frac{\sin(60)}{5\sqrt{31}}\right) = 68.94° \quad \therefore \beta = 38.94827°$$

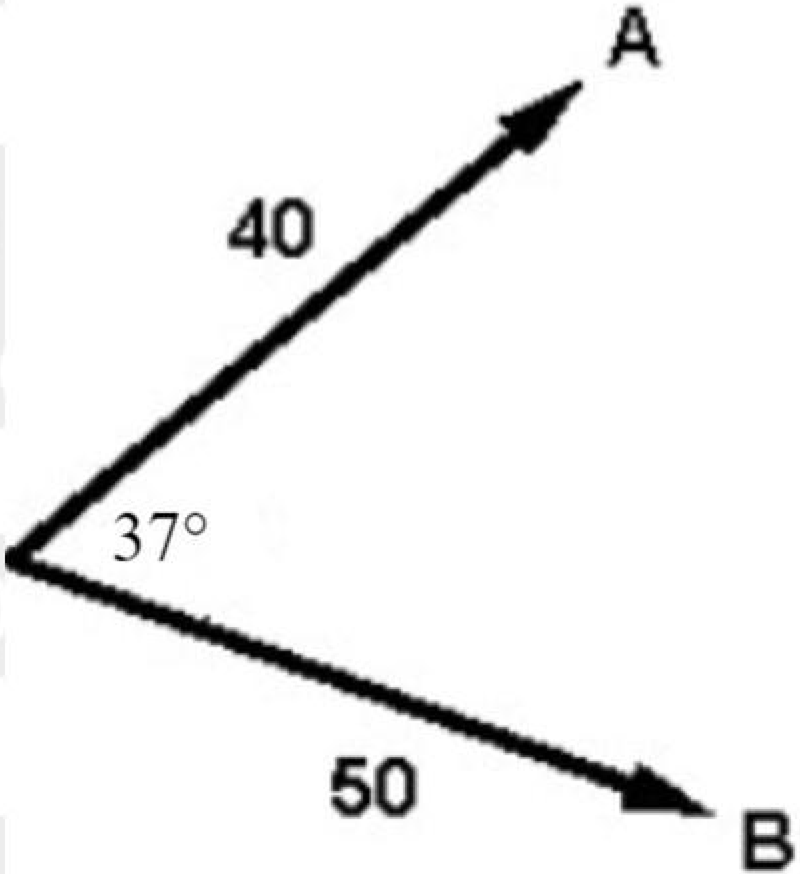
General Rule



$$\left|\vec{A} + \vec{B}\right| = \sqrt{\left|\vec{A}\right|^2 + \left|\vec{B}\right|^2 + 2\left|\vec{A}\right|\left|\vec{B}\right| cos\alpha}$$

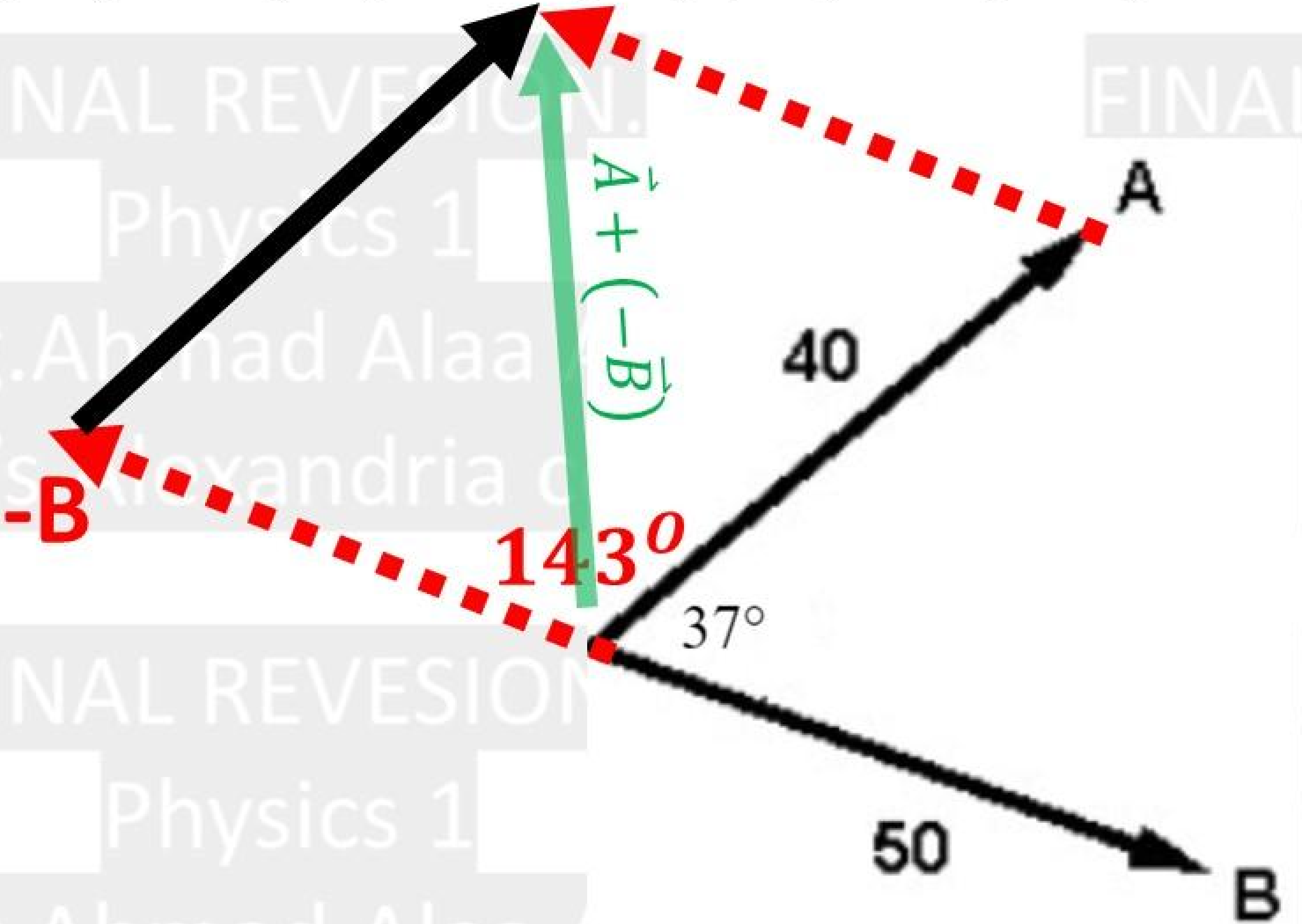
Q 26.

Vectors **A** and **B** are shown**.** What is the magnitude of a vector **C if C = A-B ?**



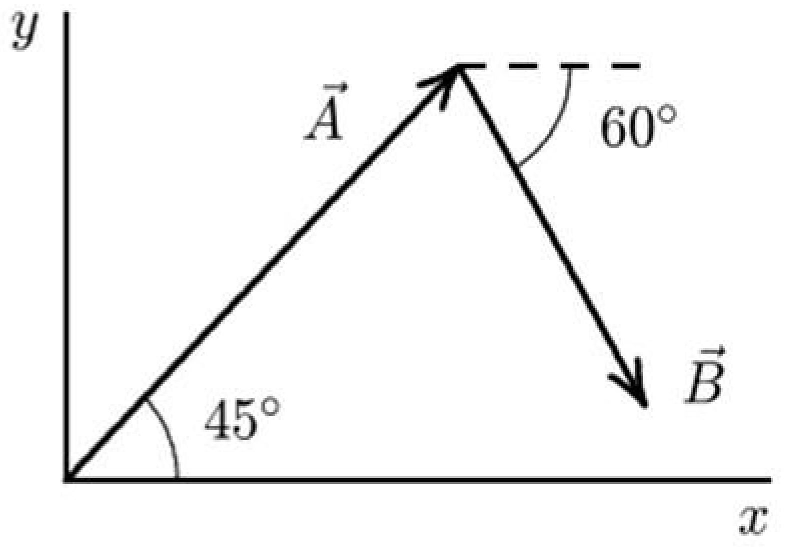
$$\left|\vec{A}+\left(-\vec{B}\right)\right|=\sqrt{\left|\vec{A}\right|^{2}+\left|-\vec{B}\right|^{2}+2\left|\vec{A}\right|\left|-\vec{B}\right|cos\alpha}$$

$$\left|\vec{A}+\left(-\vec{B}\right)\right|=\sqrt{(40)^{2}+(50)^{2}+2(40)(50)\cos(143)}=30.1$$

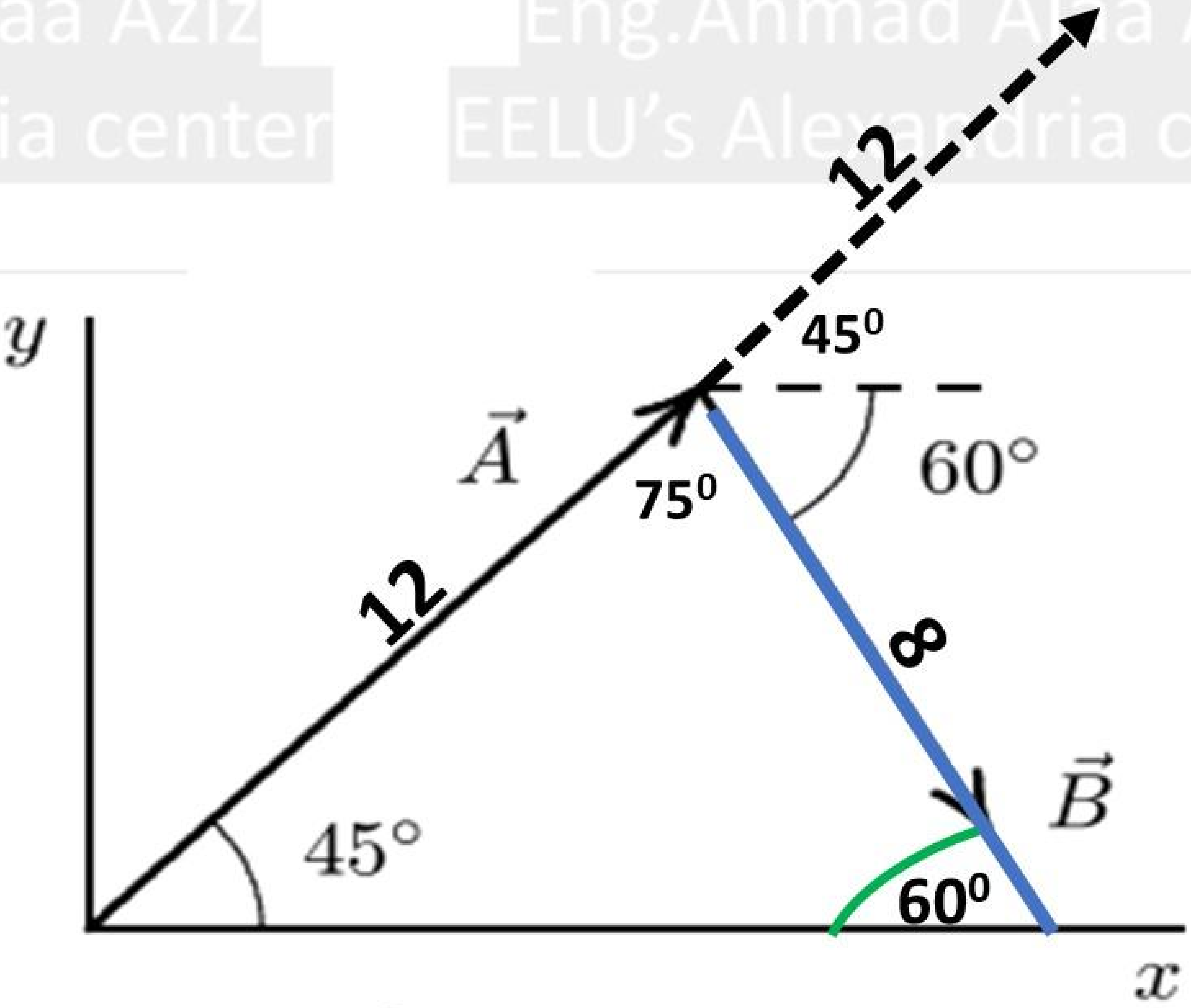


Q 27

In the diagram, $\vec{A}$ has magnitude 12 m and $\vec{B}$ has magnitude 8 m. The x component of $\vec{A} + \vec{B}$ is about:



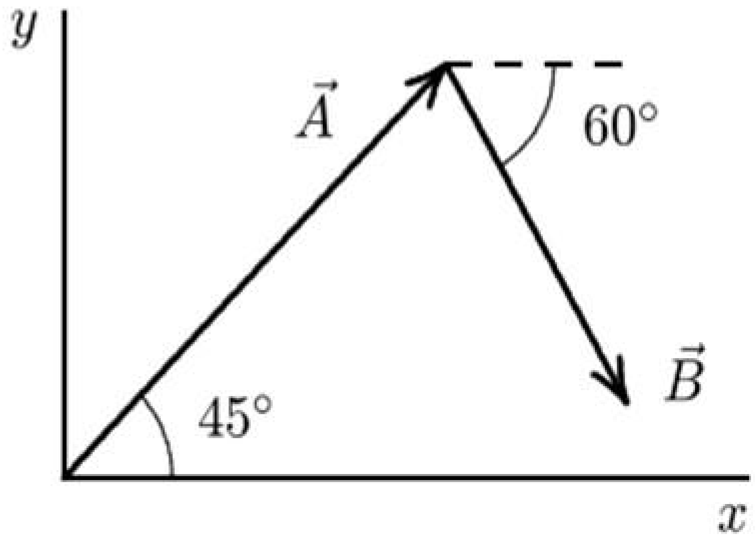
A. 5.5 m
B. 7.6 m
C. 12 m
D. 14 m
E. 15 m



$$\vec{A}+\vec{B}=\sqrt{(12)^2+(8)^2+2(12)(8)\cos(45+60)}=12.5$$

Q 27

In the diagram, $\vec{A}$ has magnitude 12 m and $\vec{B}$ has magnitude 8 m. Find |A+B| is about:



A. 5.5 m
B. 7.6 m
C. 12 m
D. 14 m
E. 15 m

Q 28.

$$\alpha = \cos^{-1}\left(\frac{\boldsymbol{A}.\boldsymbol{B}}{\boldsymbol{AB}}\right)$$, **What is the range of "α" ?**

Inverse function	Domain	Range
$\theta = \sin^{-1}x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$
$\theta = \cos^{-1}x$	$-1 \le x \le 1$	$0 \le \theta \le \pi$
$\theta = \tan^{-1}x$	Real numbers	$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$
$\theta = \operatorname{cosec}^{-1}x$	$x \ge 1$ or $x \le -1$	$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2},\ \theta \ne 0$
$\theta = \sec^{-1}x$	$x \ge 1$ or $x \le -1$	$0 < \theta \le \pi,\ \theta \ne \frac{\pi}{2}$
$\theta = \cot^{-1}x$	Real numbers	$0 < \theta < \pi$

Q 29.

if A × B = A . B
Then the angle between A and B vectors will be:

A. 30
B. 45
C. 60
D. 90

if A × B = A . B

$$|\vec{A}||\vec{B}|sin\theta = |\vec{A}||\vec{B}|cos\theta$$
$$sin\theta = cos\theta$$
$$\therefore \theta + \theta = 90$$
$$\therefore 2\theta = 90$$
$$\therefore \theta = 45$$

Q 29.

if A × B = A . B
Then the angle between A and B vectors will be:

A. 30
B. 45
C. 60
D. 90

The kinematic equations thus become:

Velocity vector as a function of time

$$v_{xf} = v_{xi} + a_x t$$

$$v_{yf} = v_{yi} + a_y t$$

$$\vec{v}_f = \vec{v}_i + \vec{a}t$$

Position vector as a function of time

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

Q 30.

At $t = 0$, a particle leaves the origin with a velocity of 8.0 m/s in the positive *y-direction* and moves in the *XY* plane with a constant acceleration of (4.0**i** + 2.0**j**) m/s^2. At the instant the *x* coordinate of the particle is 29 m, what is the speed of the particle?

A. 12.4 m/s

B. 21.8 m/s

C. 19.6 m/s

D. 26.1 m/s

At $t = 0$, a particle leaves the origin with a velocity of 8.0 m/s in the positive *y-direction* and moves in the *XY* plane with a constant acceleration of (4.0**i** + 2.0**j**) m/s². At the instant the *x* coordinate of the particle is 29 m, what is the speed of the particle?

since: $x_i = 0 \ \ and \ \ y_i = 0$ **leaves the origin**

$\because \ \vec{r_i} = x_i\,\mathbf{i} + y_i\,\mathbf{j} = 0$

$\because \ \vec{v_i} = v_{xi}\,\mathbf{i} + v_{yi}\,\mathbf{j} = \mathbf{8j}$

$\because \ \vec{a} = 4\,\mathbf{i} + 2\mathbf{j}$

$\because \vec{r_f} = \vec{r_i} + \vec{v_i}\,t + \frac{1}{2}\vec{a}\,t^2$

$\therefore \vec{r_f} = \vec{v_i}\,t + \frac{1}{2}\vec{a}\,t^2 = (8\mathbf{j})\mathrm{t} + \frac{1}{2}\,(4\,\mathbf{i} + 2\mathbf{j})t^2$

$\therefore \vec{r_f} = (8\mathbf{j})\mathrm{t} + (\mathbf{2i} + \mathbf{j})t^2$

$\therefore \vec{r_f} = (\mathbf{2}t^2)\boldsymbol{i} + (t^2 + 8t)\mathbf{j}$

$(\mathbf{2}t^2) = \mathbf{29}$

$t = \mathbf{3.807}\ sec$

$\because \ \vec{v_i} = \mathbf{8j}$

$\because \ \vec{a} = 4\,\mathbf{i} + 2\mathbf{j}$

$\because \vec{v_f} = \vec{v_i} + \vec{a}\,t$

$\therefore \vec{v_f} = \mathbf{8j} + (4\,\mathbf{i} + 2\mathbf{j})t$

$\therefore \vec{v_f} = 4t\,\mathbf{i} + (8 + 2\mathbf{t})j$

$\therefore \vec{v_f}\Big|_{t=3.8} = 4(3.8)\mathbf{i} + \big(8 + 2(3.8)\big)j$

$\therefore \vec{v_f}\Big|_{t=3.8} = 2\sqrt{58}\,\mathbf{i} + \big(8 + \sqrt{58}\big)j$

$\therefore \left|\vec{v_f}\Big|_{t=3.8}\right| = \sqrt{\left(2\sqrt{58}\right)^2 + \left(8 + \sqrt{58}\right)^2}$

$= \mathbf{21.814}\ \boldsymbol{m/sec}$

Q 30.

At $t = 0$, a particle leaves the origin with a velocity of 8.0 m/s in the positive *y-direction* and moves in the *XY* plane with a constant acceleration of $(4.0\mathbf{i} + 2.0\mathbf{j})\ m/s^2$. At the instant the *x* coordinate of the particle is 29 m, what is the speed of the particle?

A. 12.4 m/s

B. 21.8 m/s

C. 19.6 m/s

D. 26.1 m/s

Q 31.

A particle starts from the origin at $t = 0$ with a velocity of 6.0i m/s and moves in the *XY* plane with a constant acceleration of (-2.0i + 4.0j) m/s^2. At the instant, the particle achieves its maximum positive *x* coordinate, how far is it from the origin?

A. 20 m/s
B. 21.8 m/s
C. 36 m/s
D. 26.1 m/s

A particle starts from the origin at $t = 0$ with a velocity of 6.0i m/s and moves in the XY plane with a constant acceleration of (-2.0i + 4.0j) m/s². At the instant, the particle achieves its maximum positive x coordinate, how far is it from the origin?

since: $x_i = 0 \;\; and \;\; y_i = 0$ <u>leaves the origin</u>

$\because \; \vec{r_i} = x_i\,\mathbf{i} + y_i\,\mathbf{j} = 0$

$\because \; \vec{v_i} = \mathbf{6i}$

$\because \; \vec{a} = -2\,\mathbf{i} + 4\,\mathbf{j}$

$\because \vec{r_f} = \vec{r_i} + \vec{v_i}\,t + \frac{1}{2}\vec{a}\,t^2$

$\therefore \vec{r_f} = \vec{v_i}\,t + \frac{1}{2}\vec{a}\,t^2 = (6\mathbf{i})\mathrm{t} + \frac{1}{2}\,(-2\,\mathbf{i} + 4\mathbf{j})t^2$

$\therefore \vec{r_f} = (6\mathbf{i})\mathrm{t} + (-\mathbf{i} + 2\mathbf{j})t^2$

$\therefore \vec{r_f} = (6t - t^2)\boldsymbol{i} + (2t^2)\mathbf{j}$

Getting the maximum positive x-coordinate

$x_{r_f} = 6t - t^2$

$\frac{dx_{r_f}}{dt} = (6 - 2t) = 0$

$t = 3\ sec \quad ---critical\ point$

$\frac{d^2x_{r_f}}{dt^2} = -2 \quad ----maximum\ \ point$

$\therefore \vec{r_f}\Big|_{t=3} = (6(3) - (3)^2)\boldsymbol{i} + (2(3)^2)\mathbf{j}$

$\therefore \vec{r_f}\Big|_{t=3} = 9\,\boldsymbol{i} + 18\,\mathbf{j}$

$\therefore \left|\vec{r_f}\Big|_{t=3}\right| = \sqrt{(9)^2 + (18)^2} = 9\sqrt{5}\,m = 20.1246\,m$

Q 31.

A particle starts from the origin at $t = 0$ with a velocity of 6.0i m/s and moves in the *XY* plane with a constant acceleration of (-2.0i + 4.0j) m/s^2.
At the instant, the particle achieves its maximum positive *x* coordinate, how far is it from the origin?

A. 20 m/s
B. 21.8 m/s
C. 36 m/s
D. 26.1 m/s

Q 32.

If $\vec{C} = [10 \text{ m}, 30°]$ and $\vec{D} = [25 \text{ m}, 130°]$,
what is the direction & magnitude
of the sum of these two vectors?

If $\vec{\boldsymbol{C}} = [10\text{ m}, 30°]$ and $\vec{\boldsymbol{D}} = [25\text{ m}, 130°]$

$$\vec{\boldsymbol{C}} = 10\angle 30° = 10\cos(30)\hat{\imath} + 10\,sin(30)\hat{\jmath} = 5\sqrt{3}\hat{\imath} + 5\hat{\jmath}$$

$$\vec{\boldsymbol{D}} = 25\angle 130° = 25\cos(130)\hat{\imath} + 25\,sin(130)\hat{\jmath}$$

$$\vec{\boldsymbol{C}} + \vec{\boldsymbol{D}} = (5\sqrt{3} + 25\cos(130))\,\hat{\imath} + (5 + 25\sin(130))\,\hat{\jmath}$$

$$\vec{\boldsymbol{C}} + \vec{\boldsymbol{D}} = -7.4\,\hat{\imath} + 24.15\,\hat{\jmath}$$

$$\left|\vec{\boldsymbol{C}} + \vec{\boldsymbol{D}}\right| = \sqrt{(-7.4)^2 + (24.15)^2} = 25.26\,units\ of\ length$$

$$\theta_{(\vec{\boldsymbol{C}} + \vec{\boldsymbol{D}})} = \tan^{-1}\left(\frac{24.15}{-7.4}\right) = -72.944°$$

Newton’s Laws

Q 33.

An object that has a mass “m” is moving with acceleration a under the effect of the force “F”. If the mass of the object is increased to “2m”, then the acceleration will change by what factor?

A. 0.5
B. 2
C. 4
D. 0.25

Case 1

$F_1 = F$

$m_1 = m$

$a_1 = a$

$F_1 = m_1 a_1$ --- (1)

Case 2

$F_2 = F$

$m_2 = 2m$

$a_2 = ?$

$F_2 = m_2 a_2$ ---- (2)

Divide (1) by (2)

$$\frac{F_1}{F_2} = \frac{m_1 a_1}{m_2 a_2}$$

$$\frac{F}{F} = \frac{m a_1}{2m a_2}$$

$$1 = \frac{a_1}{2a_2}$$

$$2a_2 = a_1$$

$$a_2 = \frac{a_1}{2}$$

Q 33.

An object that has a mass m is moving with acceleration a under the effect of the force F. If the mass of the object is increased to 2m, then the acceleration will change by what factor?

A. 0.5
B. 2
C. 4
D. 0.25

Q 34.

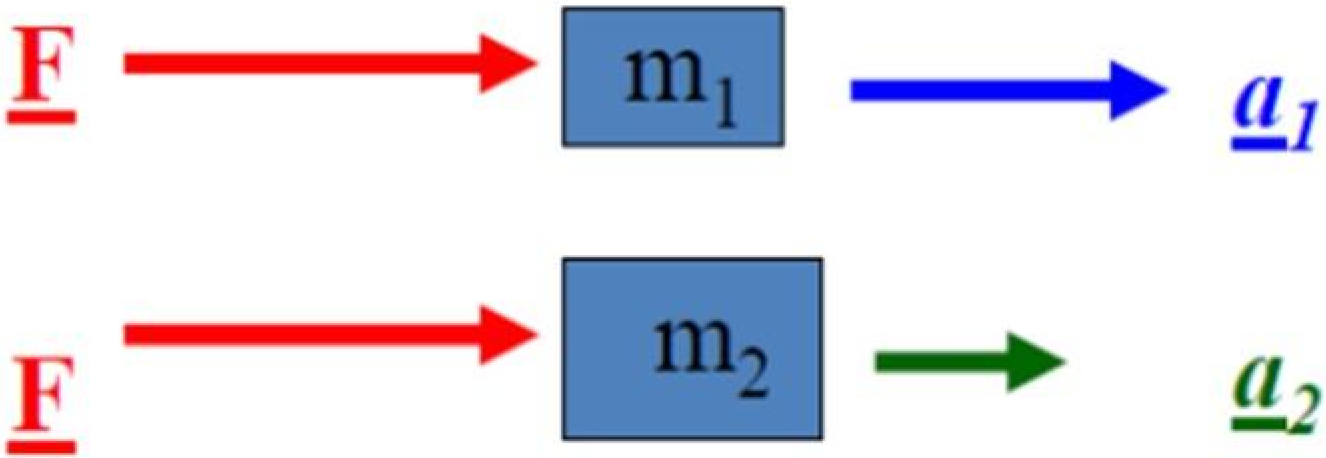
Same force is applied on two different blocks where **($m_2 > m_1$)** select all correct choices

A. $a_2 < a_1$

B. $a_2 = a_1$

C. $a_2 > a_1$

D. $a_2 = a_1 = 0$



At constant force ----- the acceleration is inversely proportional to the mass

$$a \propto \frac{1}{m}$$

in our example the mass increases so the acceleration would decrease

Q 34.

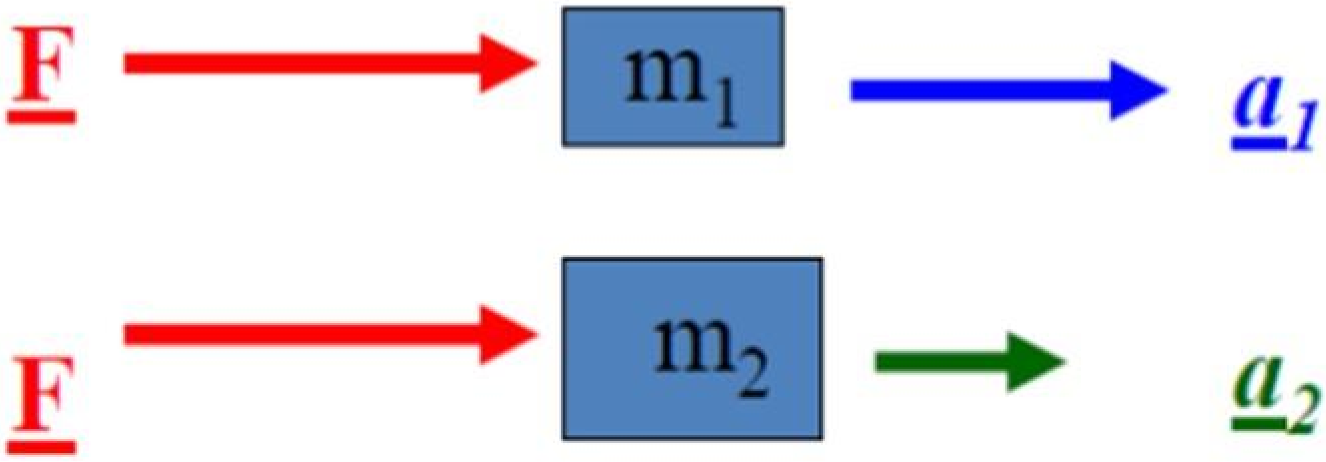
Same force is applied on two different blocks where **($m_2>m_1$)** select all correct choices

A. $a_2<a_1$

B. $a_2=a_1$

C. $a_2>a_1$

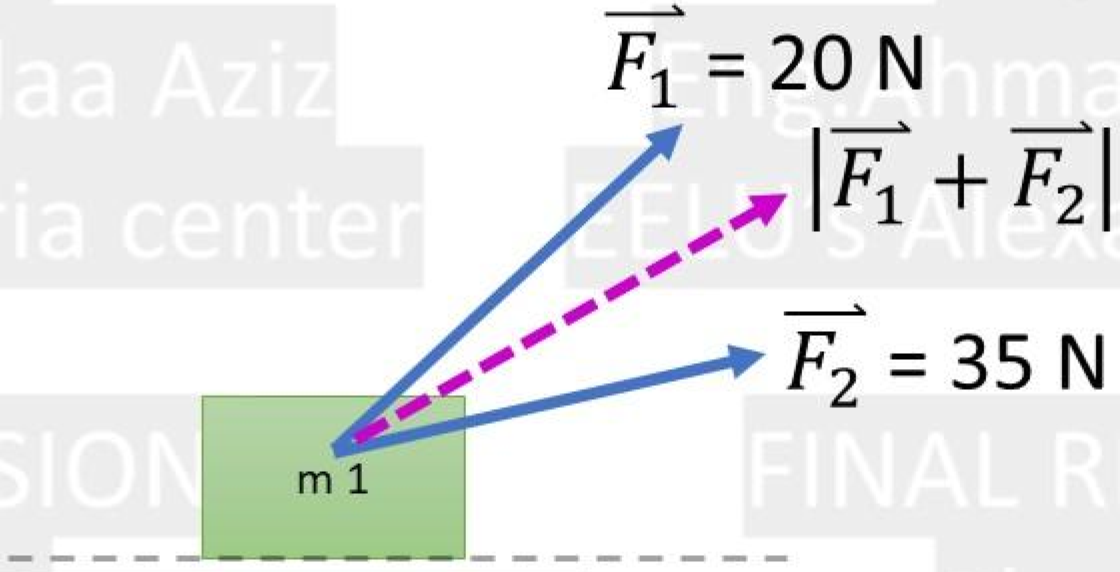
D. $a_2=a_1=0$



Q 35.

The only two forces acting on a body have magnitudes of 20 N and 35 N and directions that differ by 80°. The resulting acceleration has a magnitude of 20 m/s^2. What is the mass of the body?

2.4 kg	a.
2.2 kg	b.
2.7 kg	c.
3.1 kg	d.
1.5 kg	e.



$$\left|\overrightarrow{F_1} + \overrightarrow{F_2}\right| = \sqrt{|F_1|^2 + |F_2|^2 + 2|F_1||F_2|cos\theta}$$

$$\left|\overrightarrow{F_1} + \overrightarrow{F_2}\right| = \sqrt{(20)^2 + (35)^2 + 2(20)(35)\cos(80)}$$

$$= 43.22160\ Newtons$$

$$F = ma$$

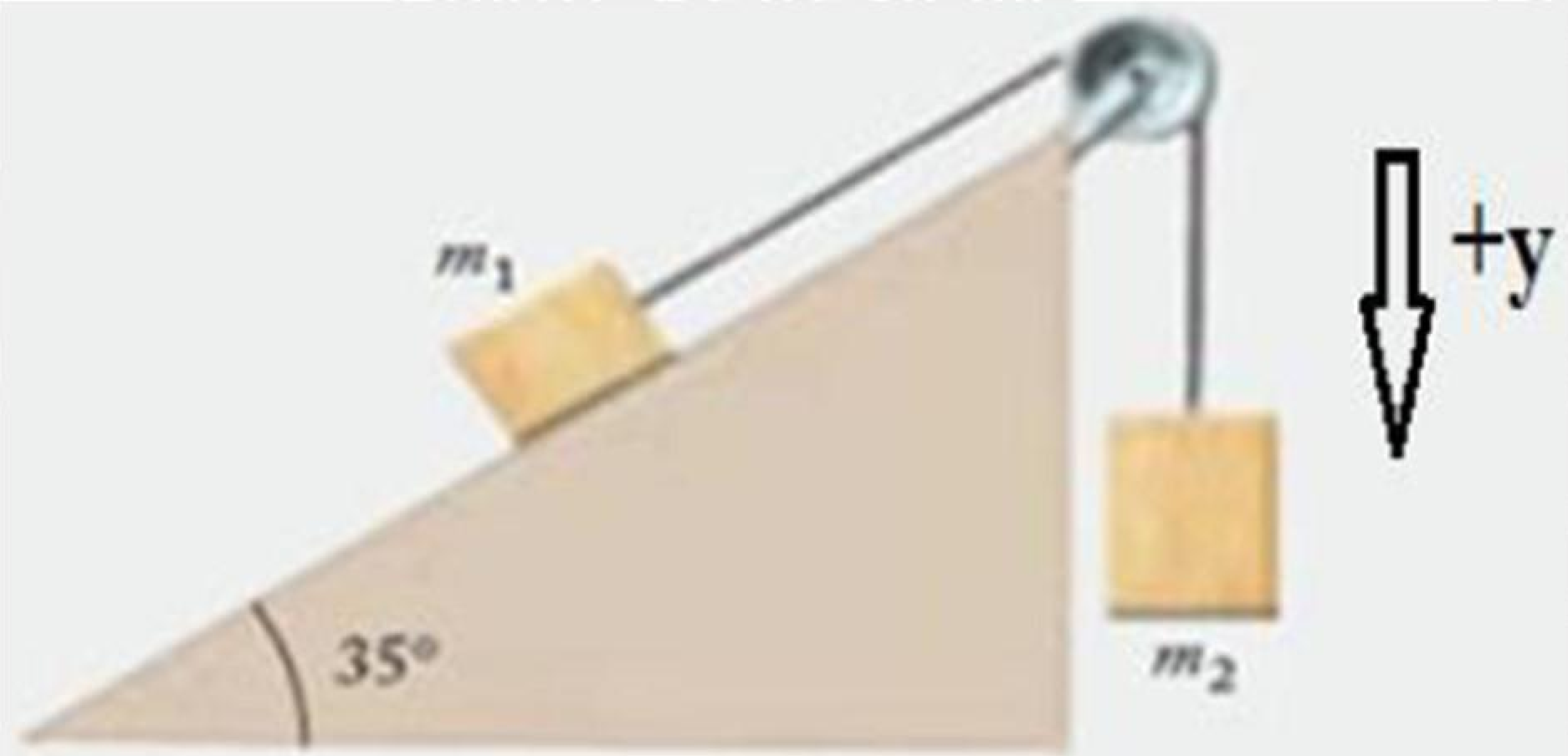
$$(43.22160) = m(20)$$

$$m = 2.16108\ kg \approx 2.2\ kg$$

Q 36.

Two masses $m_1 = 1.5$ kg and $m_2 = 3$ kg are connected by a thin string running over a massless pulley. One of the masses hangs from the string; the other mass slides on a 35° ramp with a coefficient of kinetic friction $\mu_k = 0.4$

What is the acceleration of the masses?



For "m_1"

$\because \sum F_x = m_1 a$

$\therefore T - f_k - (m_1 g)sin\theta = m_1 a$

$\because \sum F_y = 0$ (no motion)

$\therefore N - (m_1 g)cos\theta = 0$

$\therefore N = (m_1 g)cos\theta$

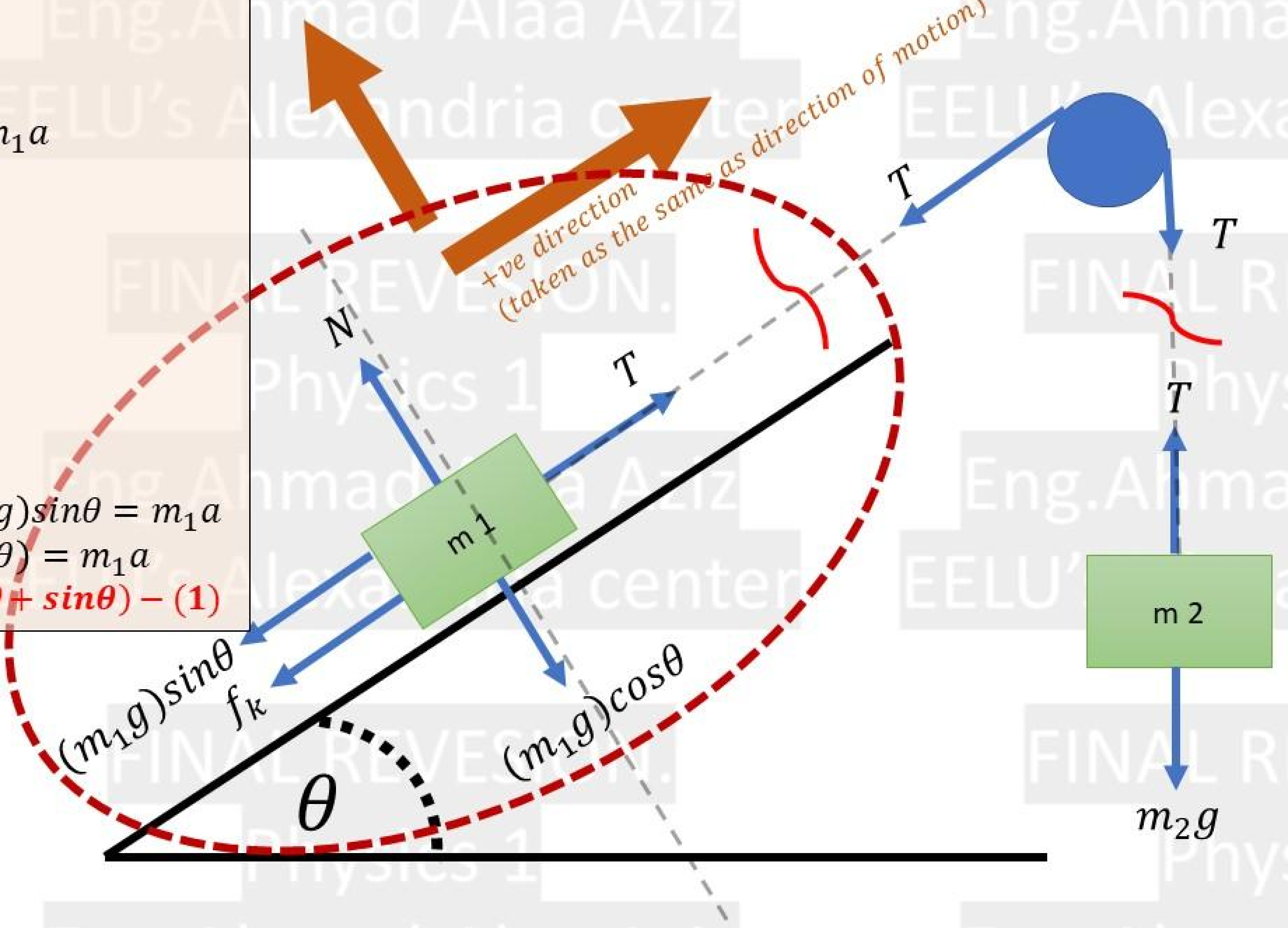
$\because f_k = \mu N$

$\therefore f_k = \mu (m_1 g)cos\theta$

$\therefore T - \mu (m_1 g)cos\theta - (m_1 g)sin\theta = m_1 a$

$\therefore T - (m_1 g)(\mu cos\theta + sin\theta) = m_1 a$

$\therefore \boldsymbol{m_1 a = T - (m_1 g)(\mu cos\theta + sin\theta) - (1)}$



For "m_2"

$\because \sum F_y = m_2 a$ (motion vertically down)

$\therefore m_2 g - T = m_2 a$

$\therefore \boldsymbol{m_2 a = m_2 g - T} \quad \textbf{(2)}$

$\because \boldsymbol{m_1 a = T - (m_1 g)(\mu cos\theta + sin\theta)} \quad \textbf{(1)}$

***Adding* (1)& (2)**

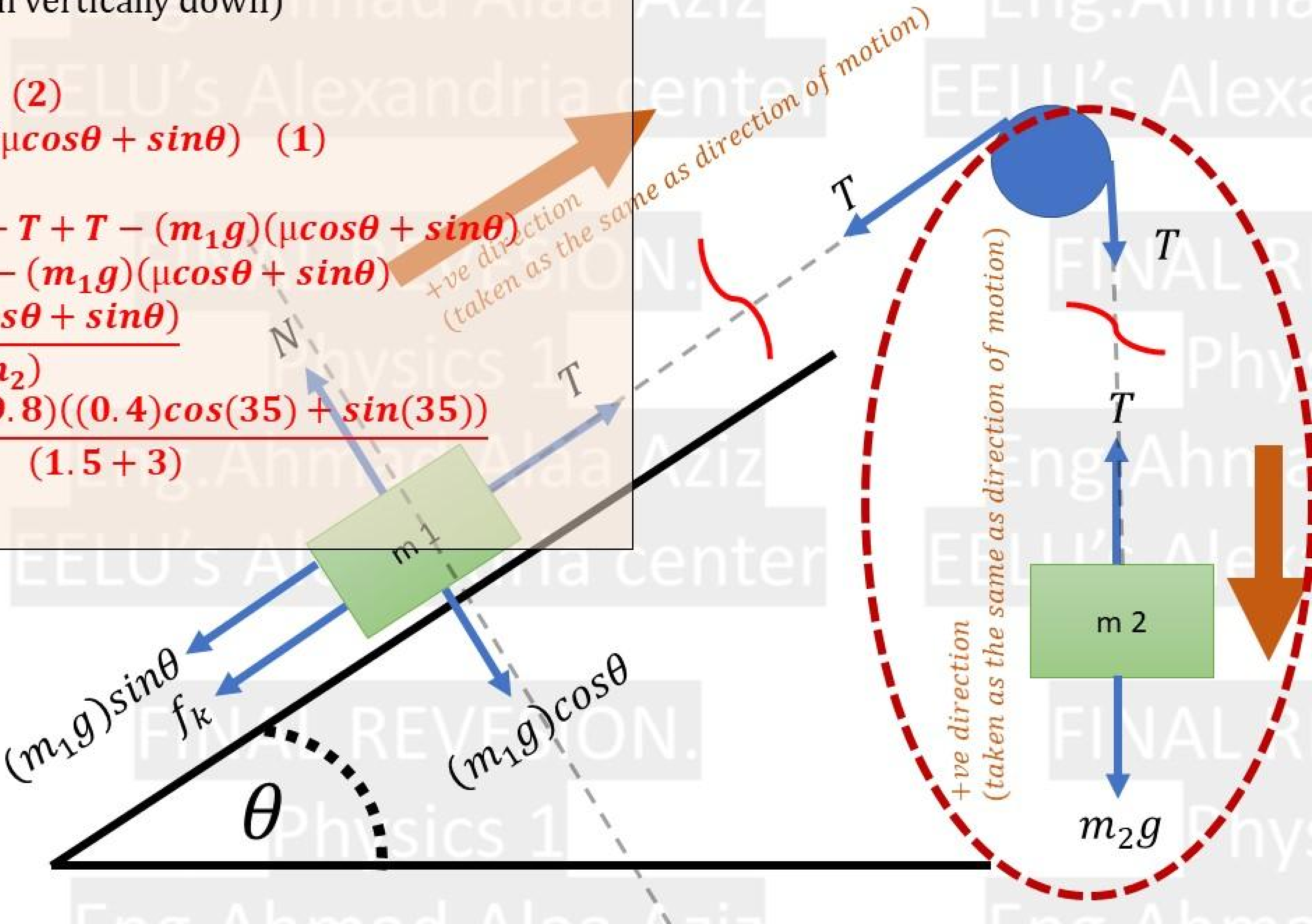
$\boldsymbol{m_2 a + m_1 a = m_2 g - T + T - (m_1 g)(\mu cos\theta + sin\theta)}$

$\boldsymbol{(m_1 + m_2)a = m_2 g - (m_1 g)(\mu cos\theta + sin\theta)}$

$$\boldsymbol{a = \frac{m_2 g - m_1 g(\mu cos\theta + sin\theta)}{(m_1 + m_2)}}$$

$$\boldsymbol{a = \frac{3(9.8) - (1.5)(9.8)((0.4)cos(35) + sin(35))}{(1.5 + 3)}}$$

$\boldsymbol{a = 3.58929\ m/s^2}$



33. Two masses $m_1 = 1.5$ kg and $m_2 = 3$ kg are connected by a thin string running over a massless pulley. One of the masses hangs from the string; the other mass slides on a 35° ramp with a coefficient of kinetic friction $\mu_k = 0.4$ (Fig. 6.33). What is the acceleration of the masses?

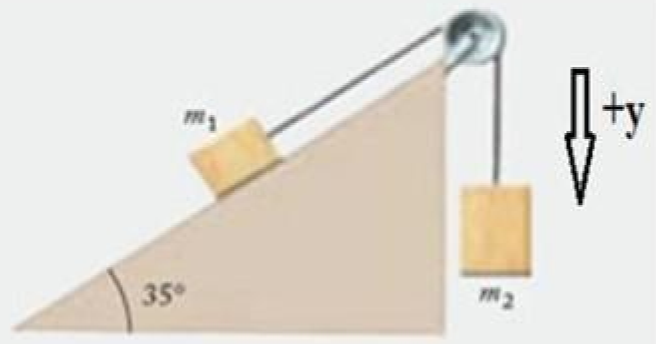


FIGURE 6.33 Two masses, an incline, and a pulley.

Answer:

Writing the equation of motion for the two masses:

m_1

$$T - m_1 g \sin\theta - f_k = m_1 a \qquad \ldots(1)$$

Where $f_k = \mu m_1 g \cos\theta$

So:

$$T - m_1 g \sin\theta - \mu m_1 g \cos\theta = m_1 a \qquad \ldots(2)$$

m_2

$$m_2 g - T = m_2 a \qquad \ldots(3)$$

By adding equations 3 and 2 and solve for the acceleration:

$$a = \frac{m_2 g - m_1 g(\sin\theta + \mu\cos\theta)}{m_1 + m_2} = 3.6\ m/s^2$$

Challenging problems:

Q.S.1

When vector $\vec{A}$ is added to vector $\vec{B}$, which has a magnitude of 5.0 , the vector representing their sum is perpendicular to $\vec{A}$ and has a magnitude that is twice that of $\vec{A}$. What is the magnitude of $\vec{A}$?

A – 2.2
B – 4.5
C – 4.5
D – 5.0
E – 7.0

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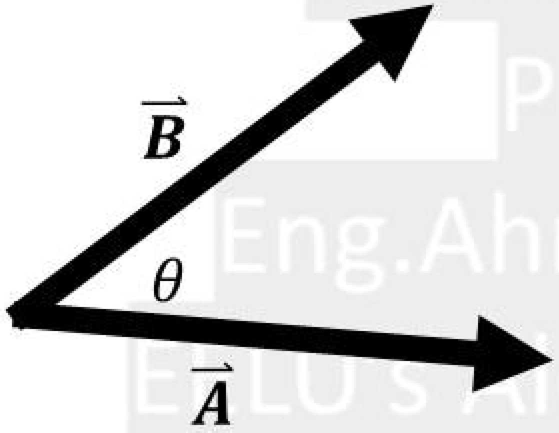
$$\boxed{|\vec{B}| = 5}$$

$$\boxed{(\vec{A}+\vec{B})} \odot \vec{A} = 0$$



$$(\vec{A}+\vec{B}) \odot \vec{A} = 0$$

$$(\vec{A}\odot \vec{A} + \vec{B}\odot \vec{A}) = 0$$

$$\left(|\vec{A}|^2 + |\vec{B}||\vec{A}|cos\theta\right) = 0 \quad \div\, both\ sides\ by |\vec{A}|$$ where "θ" is the angle between $\vec{A}$ & $\vec{B}$

$$(|\vec{A}| + |\vec{B}|cos\theta) = 0$$

$$cos\theta = \frac{-|\vec{A}|}{|\vec{B}|} \quad and\ since\ \ |\vec{B}| = 5 \quad \therefore cos\theta = \frac{-|\vec{A}|}{5}$$

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$$\boxed{|\vec{A}+\vec{B}| = 2|\vec{A}|}$$

$$\because |\vec{A}+\vec{B}| = 2|\vec{A}|$$

$$\therefore \sqrt{|\vec{A}|^2 + |\vec{B}|^2 + 2|\vec{A}||\vec{B}|cos\theta} = 2|\vec{A}|$$

$$\therefore |\vec{A}|^2 + |\vec{B}|^2 + 2|\vec{A}||\vec{B}|cos\theta = 4|\vec{A}|^2$$

$$\because |\vec{B}| = 5 \qquad \because cos\theta = \frac{-|\vec{A}|}{5}$$

$$\therefore |\vec{A}|^2 + 25 + 2|\vec{A}| * 5 * \frac{-|\vec{A}|}{5} = 4|\vec{A}|^2$$

$$\therefore |\vec{A}|^2 + 25 - 2|\vec{A}|^2 = 4|\vec{A}|^2$$

$$\therefore |\vec{A}| = \sqrt{5} \approx 2.236$$

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Thank you