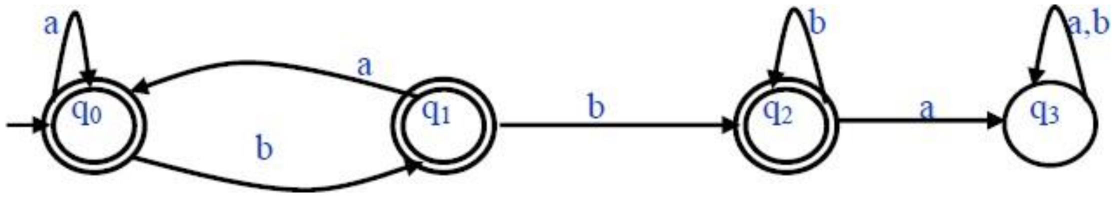
Solution Sheet 2

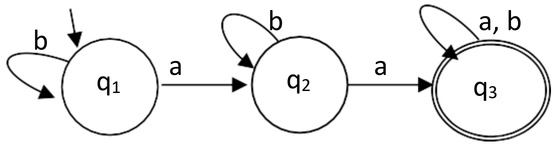
I. Choose the correct answer:

1.



a. L = {w є {a, b}*: w contains all strings that begin with a}.
b. L = {w є {a, b}*: w contains all strings that end with abb or ab}.
c. L = {w є {a, b}*: w contains all strings that do not contain bba }.
d. L = {w є {a, b}*: w contains all strings that do not contain abbb }.

2. What is the language defined by the following DFA:



a. {babab}

b. $\{b^n a b^n a b^n, n \geq 0\}$

c. $\{b^n a b^m a, m,n \geq 0\}$

d. $\{b^n a b^m a b^k, m,n,k \geq 0\}$

3. If A = {a, ba}, which of the following strings is NOT in A*:

a. bb b. λ c. aa d. baa

4. If L = {a, aa, aaa…} and M = { λ, b, bb, bbb, …}, which of the following strings is in L.M:

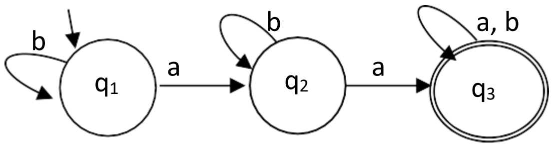
a. ab b. abba c. baa d. λ

5. If Σ = {ab, c}, u = abcc and v = cab, then , $|u^2 v|$ equals

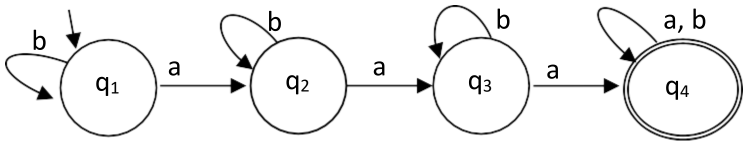
a. 11 b. 8 c. 7 d. 5

IV.

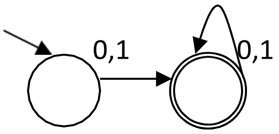
1.



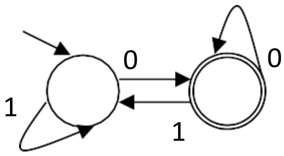
2.


b
q1
a
b
q2
a
b
q3
a
a, b
q4

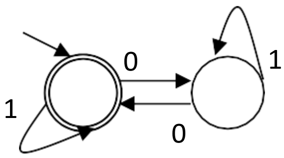
V.


0,1
0,1

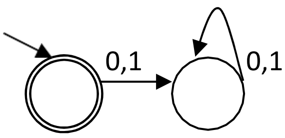
1.


0
0
1
1

2.

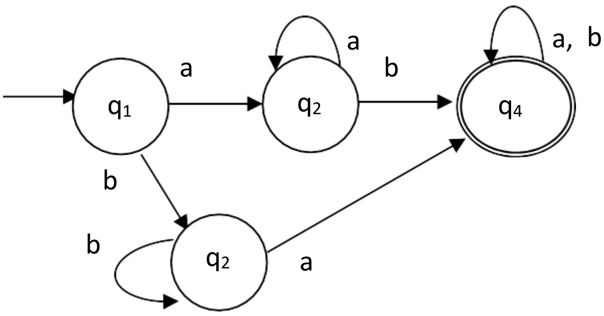

0
1
1
0

3.

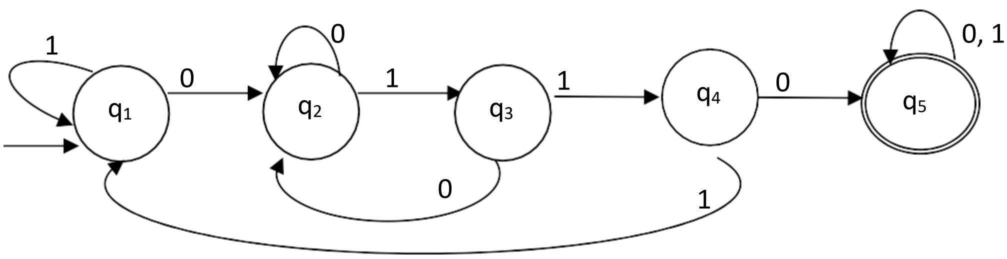

0,1
0,1

VI.

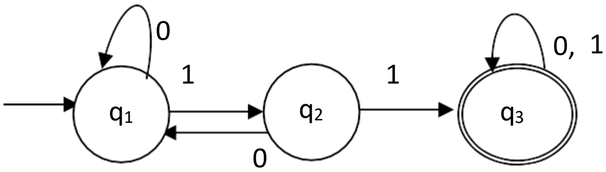
1.


a
a
b
a, b
q1
q2
q4
b
b
q2
a

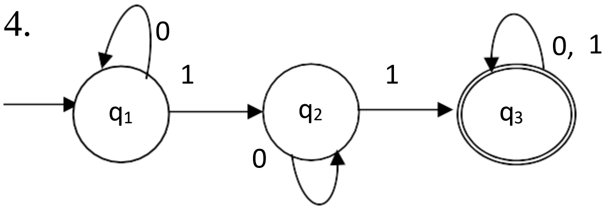
2.


1
0
0
1
1
0
0, 1
q1
q2
q3
q4
q5
0
1

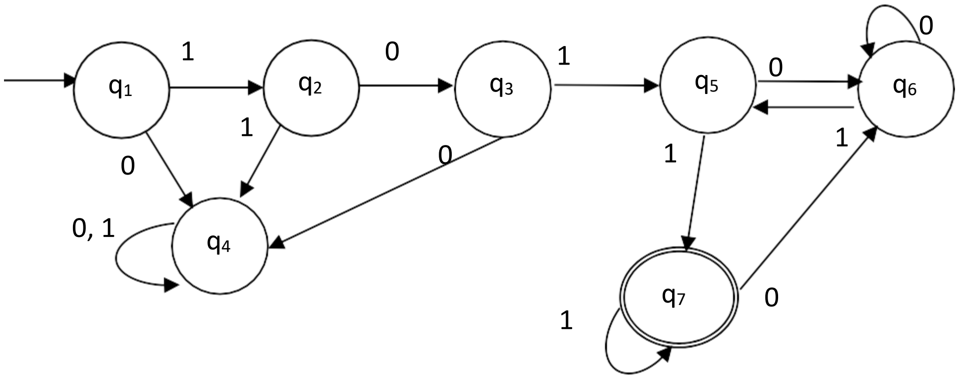
3.


q1
q2
q3
0
1
1
0
0, 1

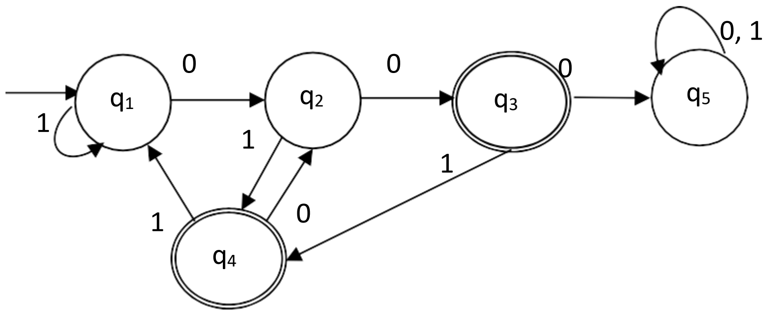
4.


q1
q2
q3
0
1
1
0
0, 1

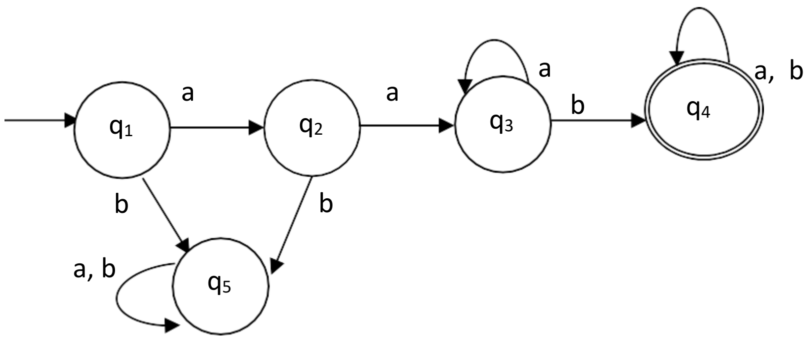
5.


q1
q2
q3
q4
q5
q6
q7
1
0
1
0
1
0
0
0, 1
1
1
0
1
0

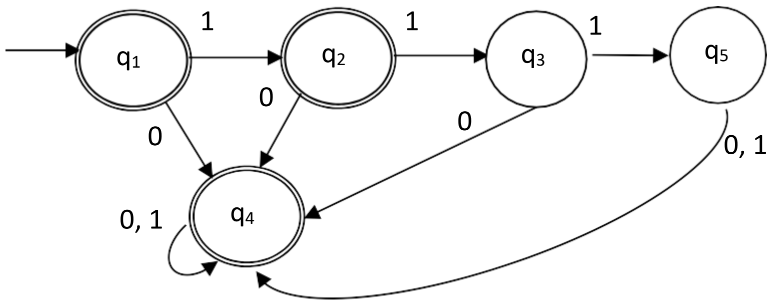
6.


q1
q2
q3
q4
q5
0
0
0
0, 1
1
1
1
0
1

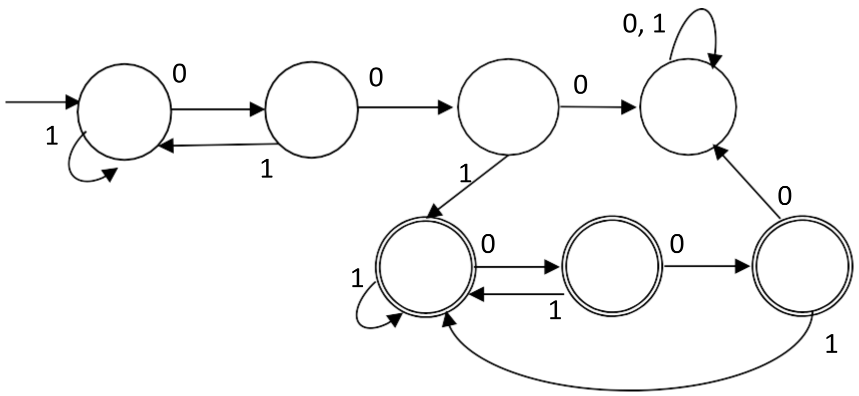
7.


q1
q2
q3
q4
q5
a
a
a
b
a, b
b
b
a, b

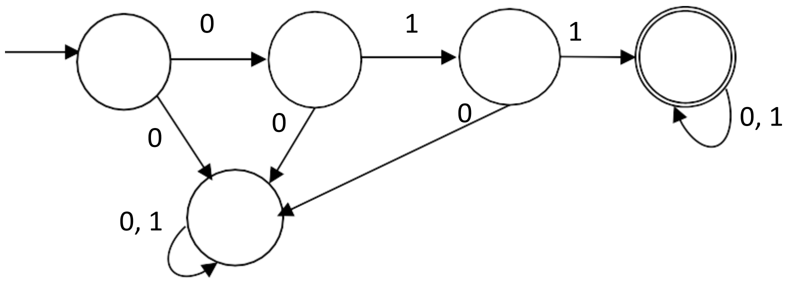
8.


q1
q2
q3
q5
q4
1
1
1
0
0
0
0, 1
0, 1

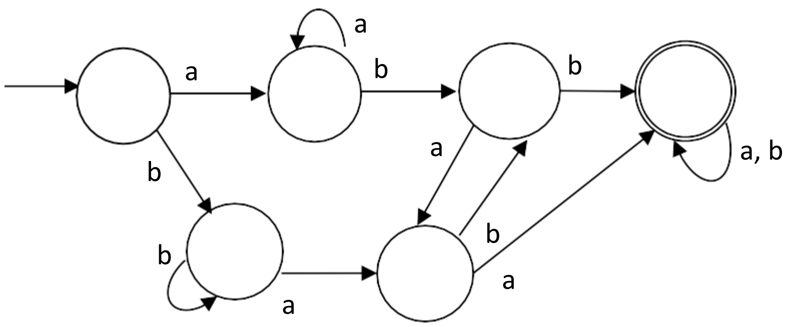
9.


0
0
0
0, 1
1
1
1
0
1
0
0
1
1

10.


0
1
1
0
0
0
0, 1
0, 1

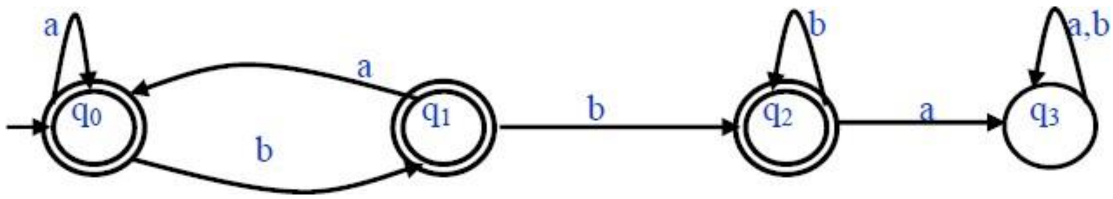
11.


a
a
b
b
b
a
a, b
b
a
b
a

Solution Sheet 2

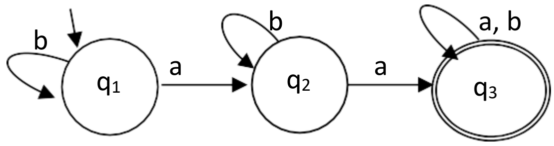
I. Choose the correct answer:

1.



a. L = {w ϵ {a, b}*: w contains all strings that begin with a}.
b. L = {w ϵ {a, b}*: w contains all strings that end with abb or ab}.
c. L = {w ϵ {a, b}*: w contains all strings that do not contain bba }.
d. L = {w ϵ {a, b}*: w contains all strings that do not contain abbb }.

2. What is the language defined by the following DFA:



a. {babab}
b. $\{b^n a b^n a b^n, n \geq 0\}$
c. $\{b^n a b^m a, m,n \geq 0\}$
d. $\{b^n a b^m a b^k, m,n,k \geq 0\}$

3. If A = {a, ba}, which of the following strings is NOT in A*:

a. bb b. λ c. aa d. baa

4. If L = {a, aa, aaa…} and M = { λ, b, bb, bbb, …}, which of the following strings is in L.M:

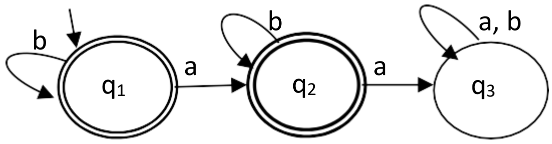
a. ab b. abba c. baa d. λ

5. If Σ = {ab, c}, u = abcc and v = cab, then , $|u^2 v|$ equals

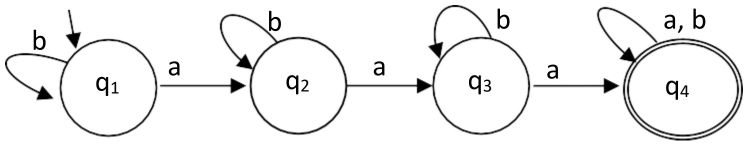
a. 11 b. 8 c. 7 d. 5

IV.

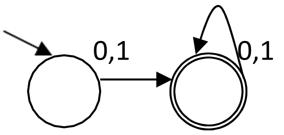
1.



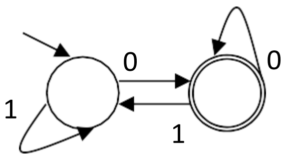
2.



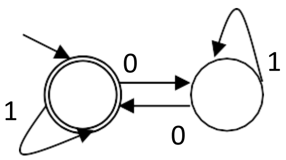
V.



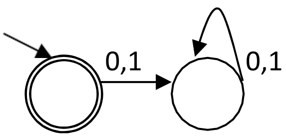
1.



2.

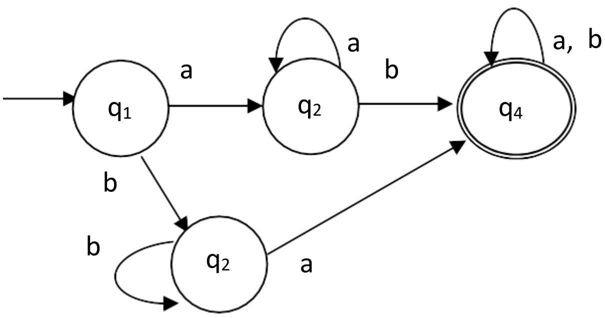


3.

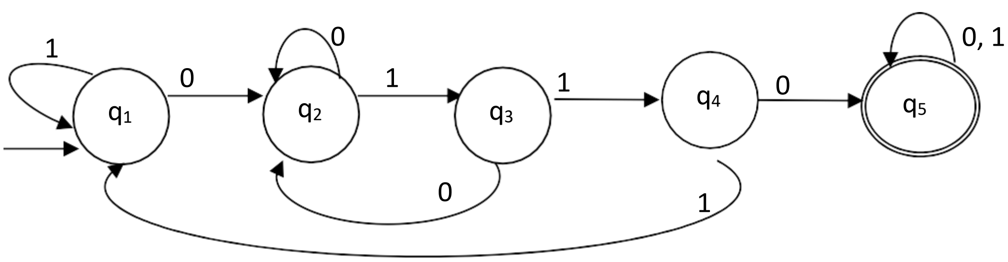


VI.

1.

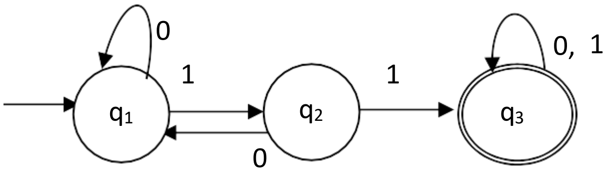


2.

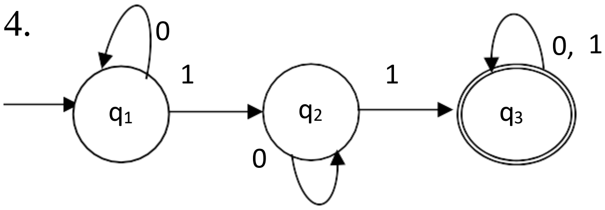


2-1

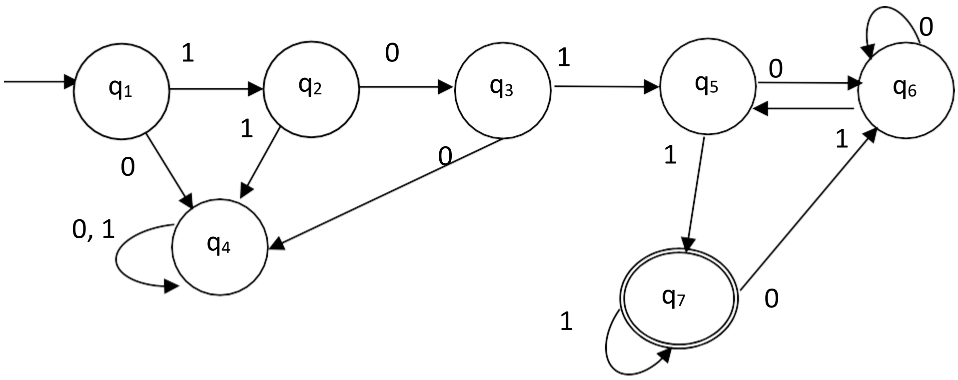
3.


0
1
1
0, 1
q1
q2
q3
0

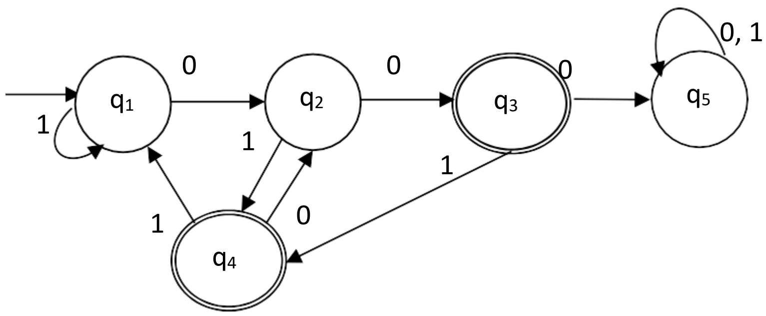
4.


0
1
1
0, 1
q1
q2
q3
0

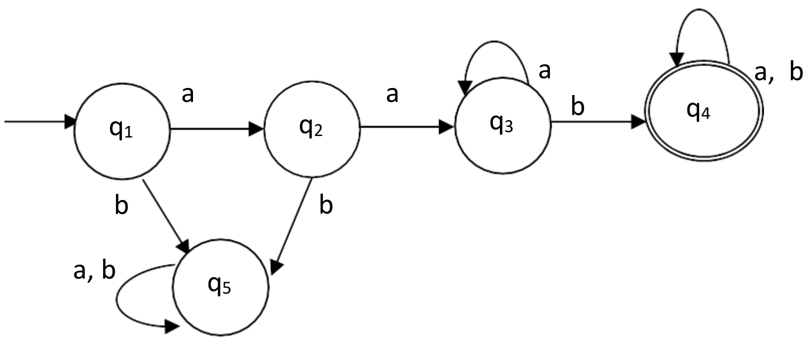
5.


1
0
1
0
0
q1
q2
q3
q5
q6
1
0
0
1
1
0, 1
q4
q7
0
1

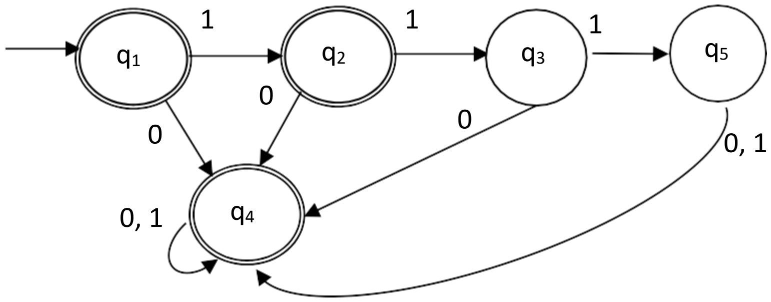
6.


0, 1
0
0
0
q1
q2
q3
q5
1
1
1
1
0
q4

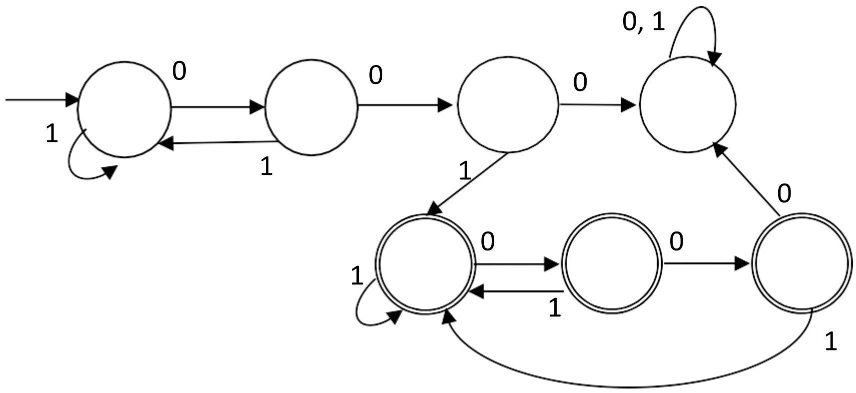
7.


a
a, b
a
a
b
q1
q2
q3
q4
b
b
a, b
q5

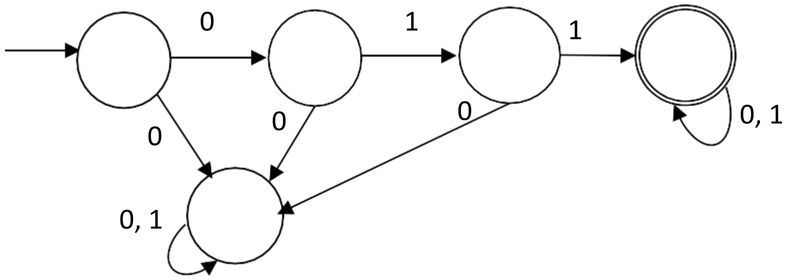
8.


1
1
1
q1
q2
q3
q5
0
0
0
0, 1
0, 1
q4

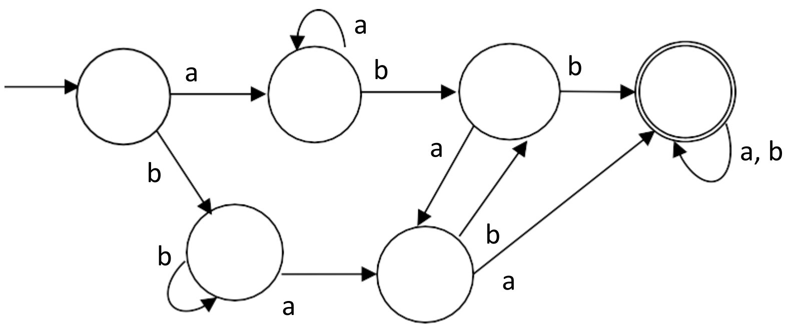
9.


0, 1
0
0
0
1
1
1
0
1
0
0
1
1

10.

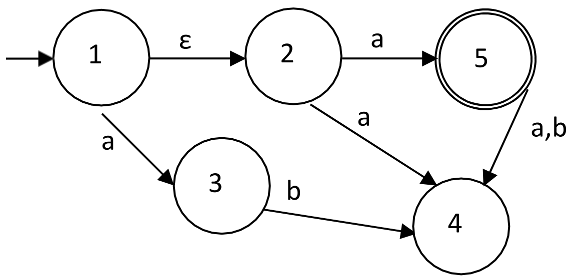

0
1
1
0
0
0
0, 1
0, 1

11.


a
a
b
b
b
a
a, b
b
b
a
a

Solution Sheet 2-2

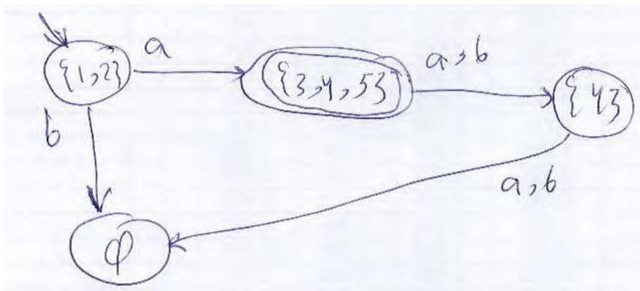
I. Consider the following NFA



1. Find its formal description.
2. Convert the above NFA into DFA.

	a	b	ε
1	{3}	ϕ	{2}
2	{4,5}	ϕ	ϕ
3	ϕ	{4}	ϕ
4	ϕ	ϕ	ϕ
5	{4}	{4}	ϕ

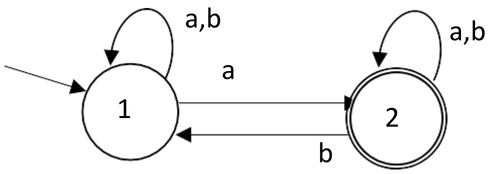
	a	b
{1,2}	{3,4,5}	ϕ
{3,4,5}	{4}	{4}
{4}	ϕ	ϕ



II. For the following NFA find

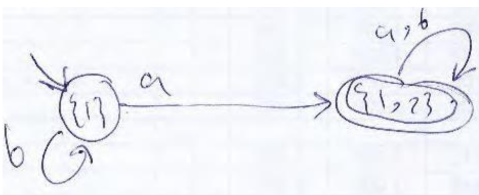
1. The formal description of the NFA.
2. Convert NFA into DFA

i.

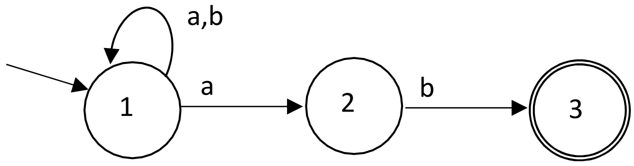


	a	b
1	{1,2}	{1}
2	{2}	{1,2}

	a	b
{1}	{1,2}	{1}
{1,2}	{1,2}	{1,2}

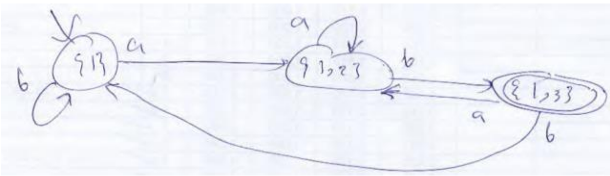


ii.

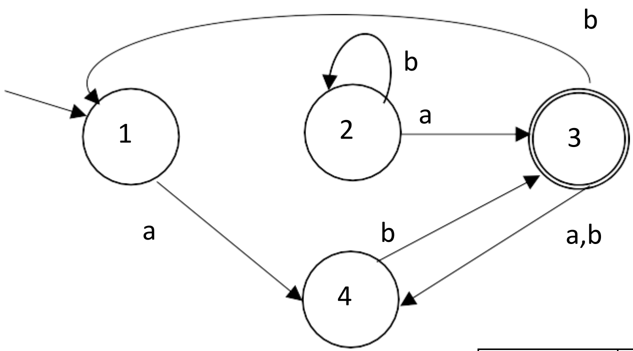


	a	b
1	{1,2}	{1}
2	ϕ	{3}
3	ϕ	ϕ

	a	b
{1}	{1,2}	{1}
{1,2}	{1,2}	{1,3}
{1,3}	{1,2}	{1}

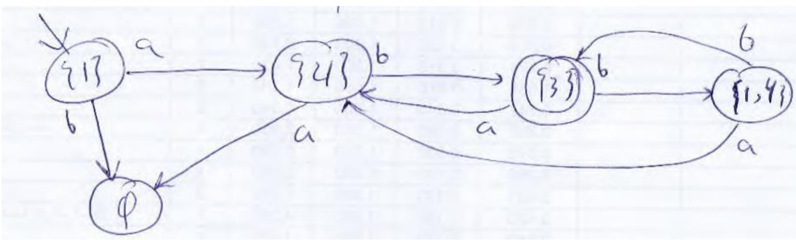


iii.



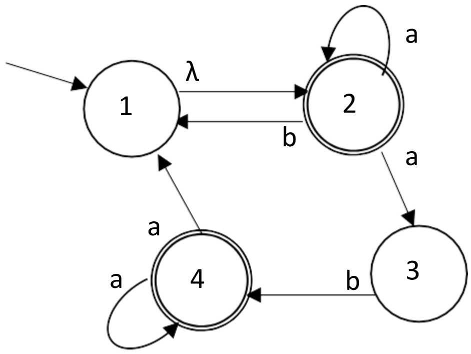
	a	b
1	{4}	ϕ
2	{3}	{2}
3	{4}	{1,4}
4	ϕ	{3}

	a	b
{1}	{4}	ϕ
{4}	ϕ	{3}
{3}	{4}	{1,4}
{1,4}	{4}	{3}



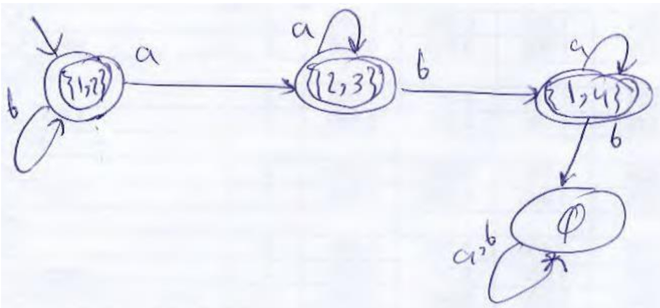
ϕ must have a cycle of 0 and 1

iv.

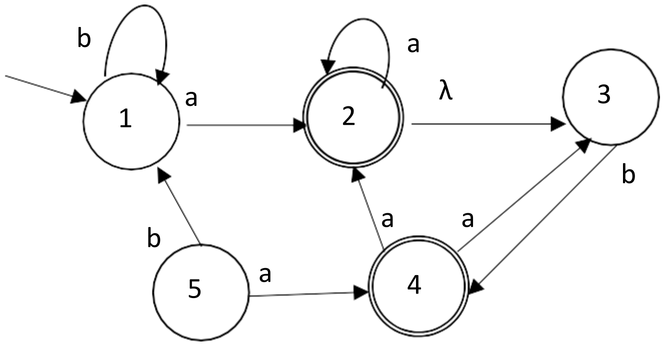


	a	b	λ
1	ϕ	ϕ	{2}
2	{2,3}	{1}	ϕ
3	ϕ	{4}	ϕ
4	{1,4}	ϕ	ϕ

	a	b
{1,2}	{2,3}	{1}
{2,3}	{2,3}	{1,4}
{1,4}	{1,4}	ϕ

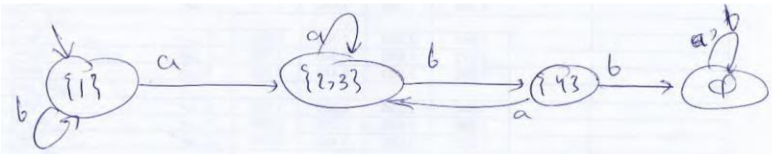


v.



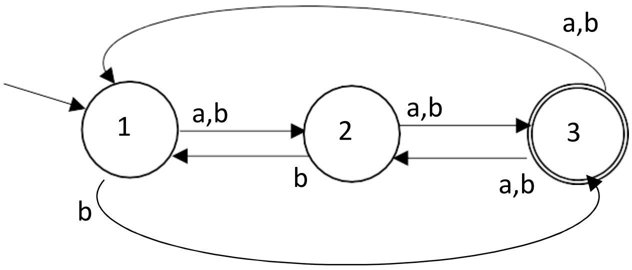
	a	b	λ
1	{2}	{1}	ϕ
2	{2}	ϕ	{3}
3	ϕ	{4}	ϕ
4	{2,3}	ϕ	ϕ
5	{4}	{1}	ϕ

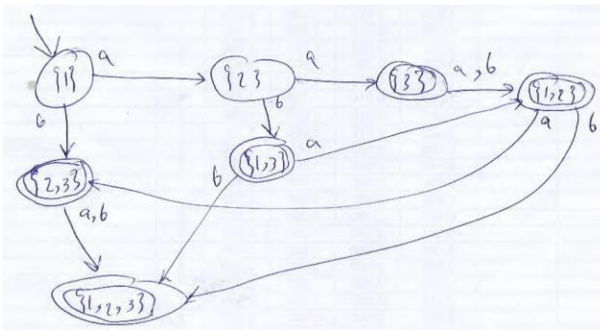
	a	b
{1}	{2}	{1}
{2,3}	{2}	{4}
{4}	{2,3}	ϕ



Final states are {2,3} and {4}

vi.





Solution Sheet 3_1

1. Give regular expression for the following languages:
   a. { w ϵ {0,1}*: w has three consecutive 0's or three consecutive 1's}
   (0+1)*(000+111)(0+1)*
   b. { w ϵ {0,1}*: w has three consecutive 0's and three consecutive 1's}
   (0+1)* (000(0+1)*111 + 111(0+1)*000) (0+1)*
   c. { w ϵ {0,1}*: w begin with 011 or 110}

   (011+110)(0+1)*

   d. { w ϵ {0,1}*: w end with 001 or 010}

   (0+1)*(001+010)

   e. { w ϵ {0,1}*: w begin with three consecutive 1's and end with 111 or 001}

   111 (0+1)* (111+001)

   f. L = {w ϵ {0, 1}*: w contains all strings that begin with 101 and end with 11}.

   101 (0+1)* 11

   g. L = {w ϵ {a, b}*: w begin with two consecutive a's and contain ab or bb}.

   aa (a+b)* (ab + bb)(a+b)*

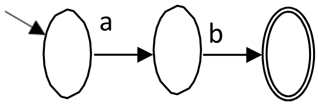
   h. L = {w ϵ {a, b}*: w contains all strings that contain two consecutive a's or three consecutive b's and end with ab and ba}.

   (a+b)*(aa + bbb) (a+b)* (abba+baab)

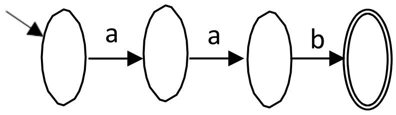
   i. L = {w ϵ {a, b}*: w contains all strings that begin with three consecutive a's and exactly one b and end with abb or baa}.

   aaab a^+ (a+b)* (abb + baa)

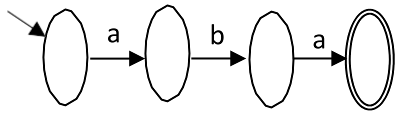
2. Let L be the language given by the regular expression (ab ∪ aab ∪ aba)*
   Design an NFA for L.



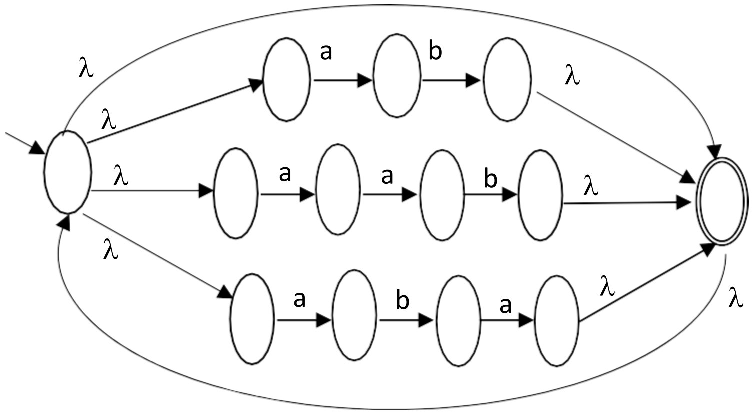
ab



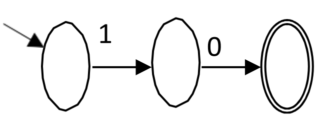
aab



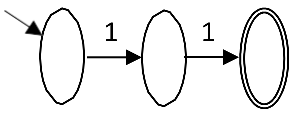
aba



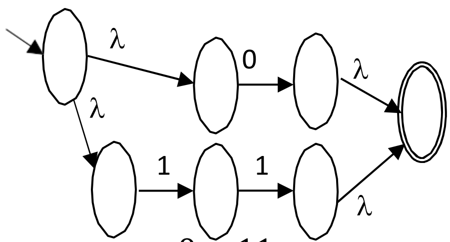
3. Construct NFA equivalent to the following regular expression (show all steps clearly):
   a. 10 + (0 + 11) 0* 1



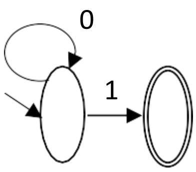
10



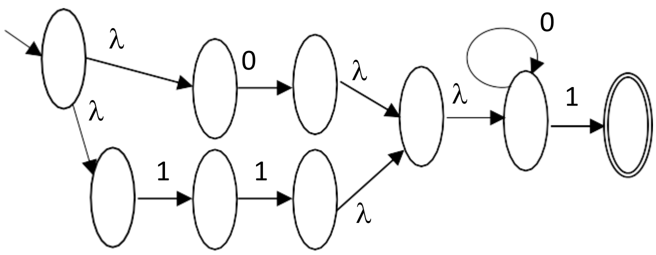
11



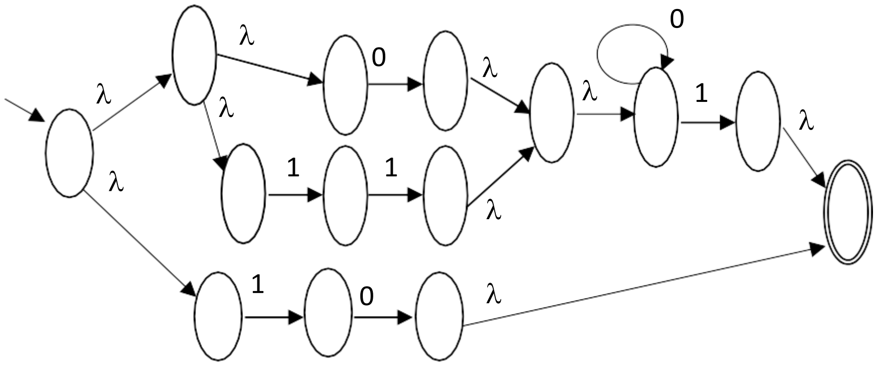
0 + 11



0*1

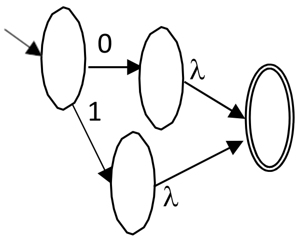


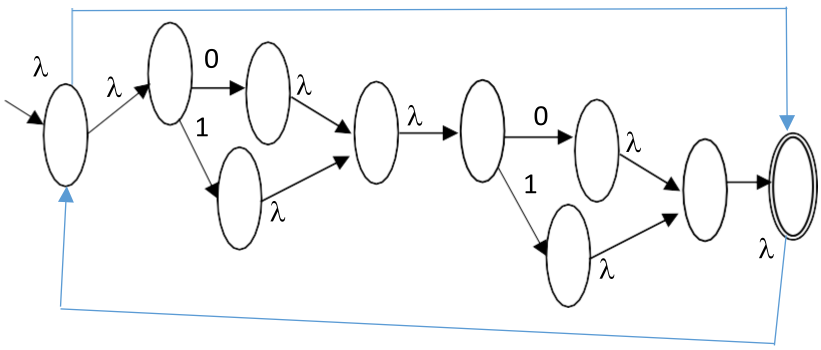
(0 + 11) 0* 1



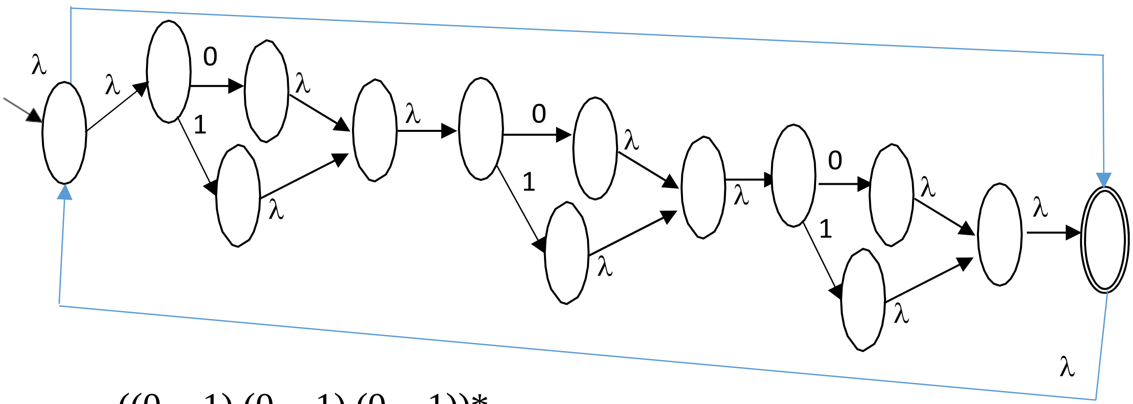
10 + (0 + 11) 0* 1

b. ((0 ∪ 1) (0 ∪ 1))* ∪ ((0 ∪ 1) (0 ∪ 1) (0 ∪ 1))*





((0 ∪ 1) (0 ∪ 1))*



((0 ∪ 1) (0 ∪ 1) (0 ∪ 1))*

c. ab*a (a+b)* + (aba* +(ab)*)*

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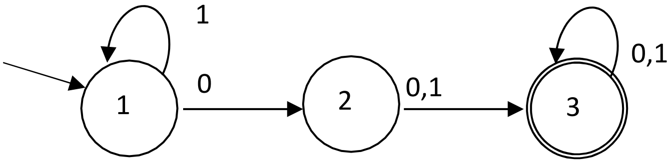
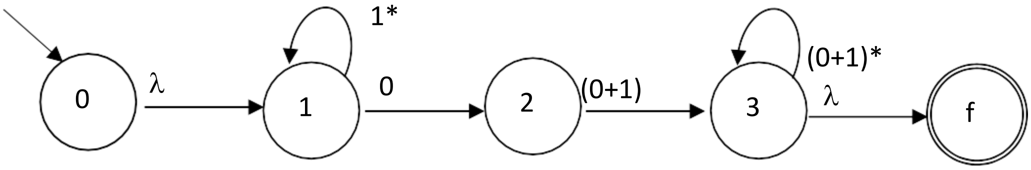
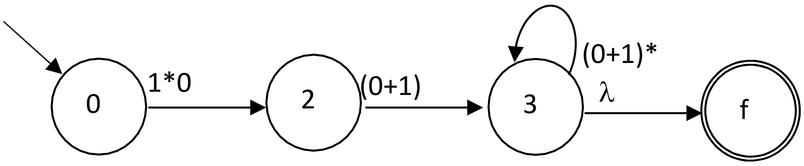
Solution Sheet 3_2

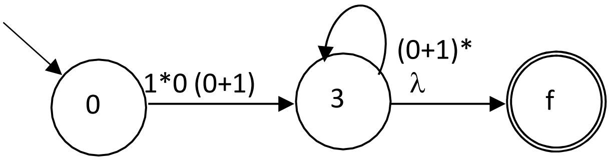
I. Convert the following DFA into regular expression:

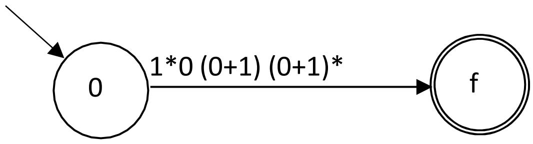
a.



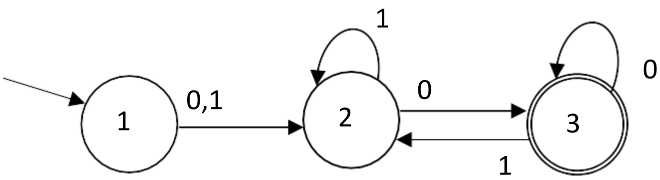


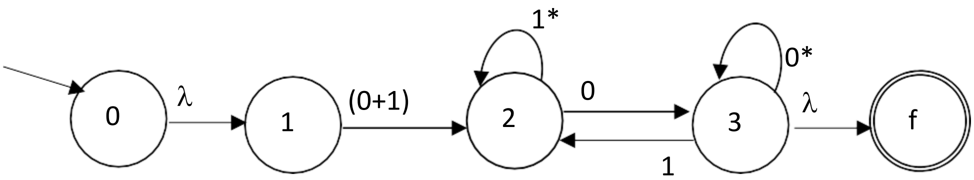


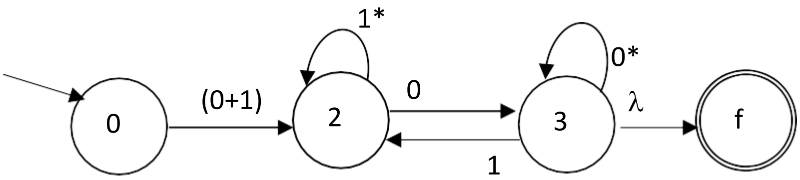


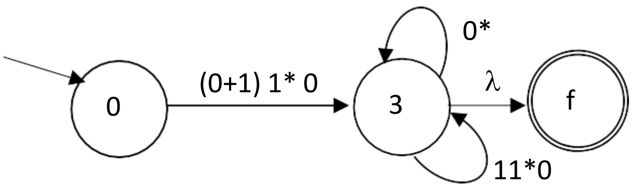


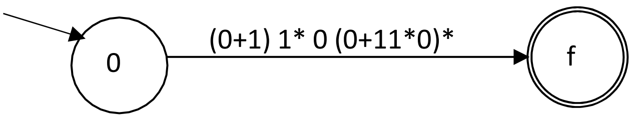
b.



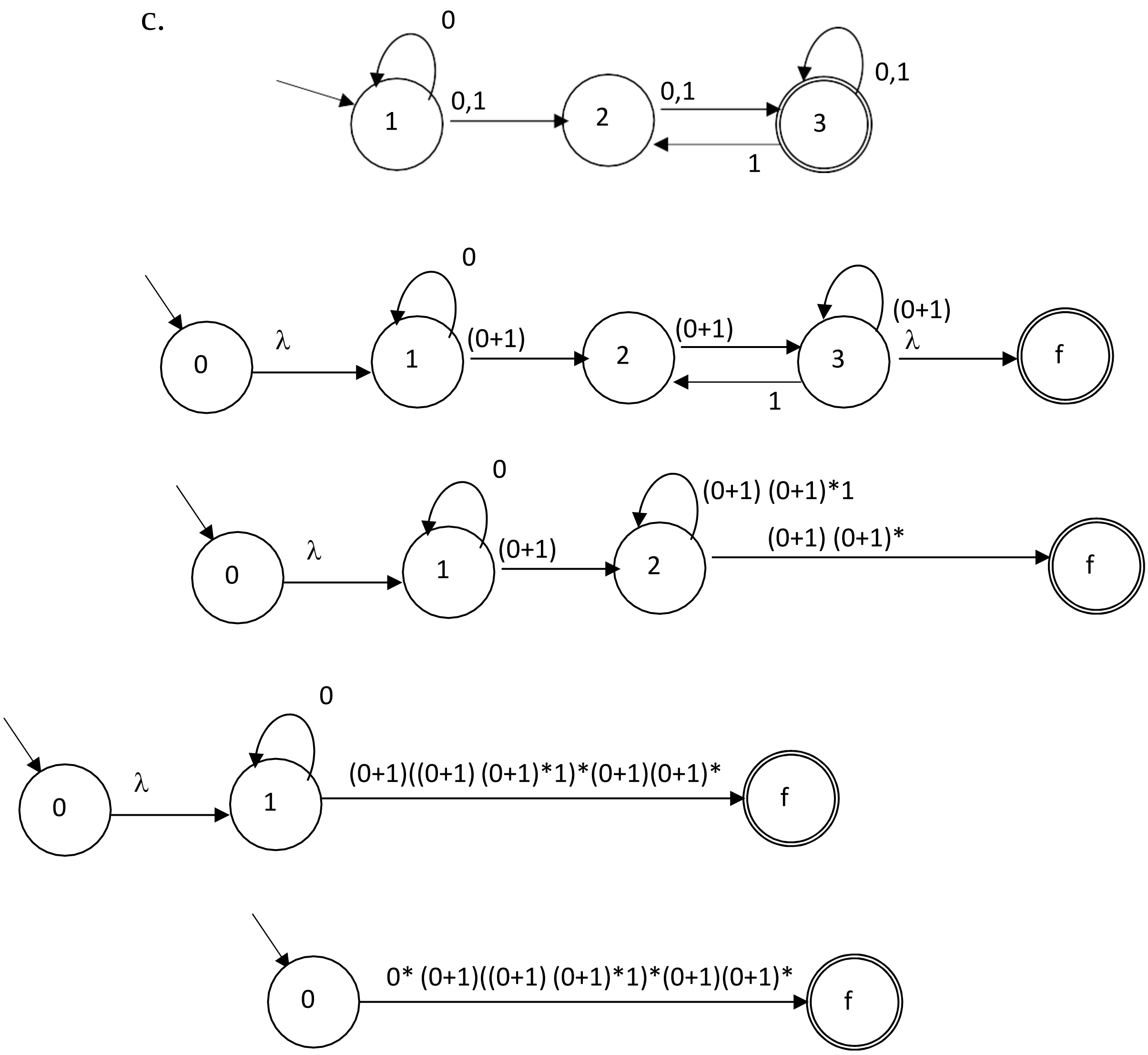




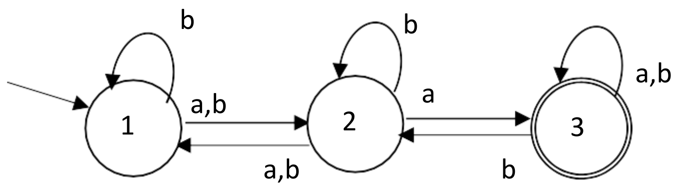




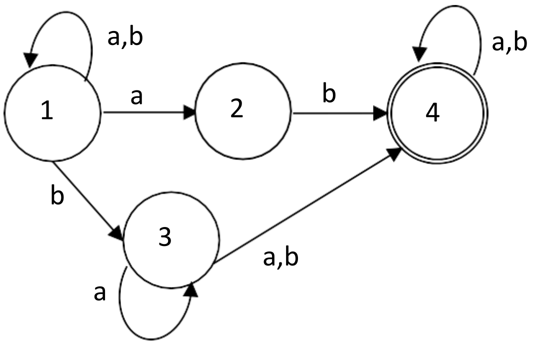
c.


0
0,1
0,1
0,1
1
1
2
3
0
λ
(0+1)
(0+1)
(0+1)
λ
1
0
1
2
3
f
0
λ
(0+1)
(0+1) (0+1)*1
(0+1) (0+1)*
0
1
2
f
0
λ
(0+1)((0+1) (0+1)*1)*(0+1)(0+1)*
0
1
f
0* (0+1)((0+1) (0+1)*1)*(0+1)(0+1)*
0
f

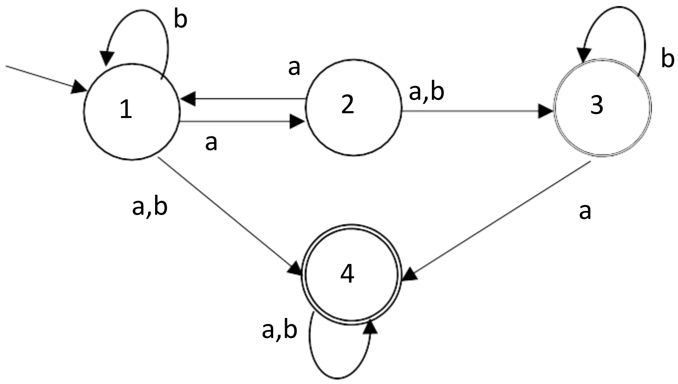
d.


b
b
a,b
a,b
a
a,b
b
1
2
3

e.



f.



Solution Sheet 3_3

1. Choose the correct answer

i. If we select a string w such that w ∈ L, and w = xyz. Which of the following portions cannot be an empty string?

a. x   b. y   c. z   d. all of the mentioned

ii. There exists a language L. We define a string w such that w ∈ L and w = xyz and |w| >= n for some constant integer n. What can be the maximum length of the substring xy i.e. |xy|<=?

a. n   b. |y| c. |x| d. none of the mentioned

iii. Answer in accordance to the pumping lemma: For all_________xy^iz ∈L

a. i>0   b. i<0   c. i<=0   d. i>=0

2. Prove that $L = \{a^nba^n: n \geq 0\}$ is not regular.

Assume L is regular, then there exist a DFA with m states that recognize L.

$\forall$ w $\in$ L where $w = a^nba^n$ and $|w| \geq n+1 > m$.

$\exists$ x, y, z $\in \sum^*$ where w = xyz, $|y| \geq 1$ and $|xy| \leq m < n+1$, $\forall\, i \geq 0$, $\exists\, xy^iz \in L$

Let i = 0, |y| = 1, y = a

$\therefore xy^iz = a^{n-1}ba^n \notin L$ because "number of a before b ≠ number of a after b"

$\therefore$ L is not regular

3. Consider the following three languages:

$L1 = \{a^nb^n: 0 \leq n \leq 100\}$

L1 is a regular language because it is a finite language.

$L2 = \{a^nb^n : n \geq 0\}$

L2 is not a regular language prove using the pumping lemma as stated in the lecture.

$L3 = \{a^nb^m : n, m \geq 0\}$

L3 is a regular language because it can be represented by the regular expression (a*b*).

Determine which language is regular and which is not regular. (prove your answer).

4. Given $\sum = \{a, b\}$, prove that $L = \{ww^R: w \in \sum^*\}$ is not regular.

Assume L is regular, then there exist a DFA with m states that recognize L.

$\forall\ w \in L$, assume $w = a^n bba^n$ and $|w| \geq n+1 > m$.

$\exists\ x, y, z \in \sum^*$ where $w = xyz$, $|y| \geq 1$ and $|xy| \leq m < n+1$, $\forall\ i \geq 0$, $\exists\ xy^iz \in L$

Let $i = 0$, $|y| = 1$, $y = a$

$\therefore xy^iz = a^{n-1}bba^n \neq ww^R$ $\qquad \therefore xy^iz \notin L$

$\therefore$ L is not regular

5. Prove that $L = \{1^n: n \text{ is square}\}$ is not regular.

Assume L is regular, then there exist a DFA with m states that recognize L.

$\forall\ w \in L$ where $w = 1^n$, n is square, and $|w| = n$.

$\exists\ x, y, z \in \sum^*$ where $w = xyz$, $|y| \geq 1$ and $|xy| \leq m < n$.

$\forall\ i \geq 0$ $\quad \exists\ xy^iz \in L$ $\qquad |xy^iz| = n$ and n is a square number.

Let $i = 0$, $|y| = 1$ $\quad$, $|xy^iz| = n - 1$

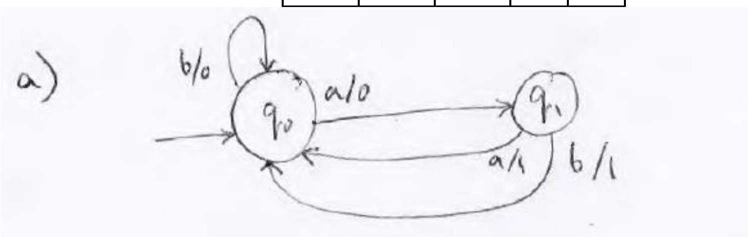
If n is a square number then n-1 is not a square number, thus the resultant string does not belong to L. Therefore L is not regular.

Solution Sheet 4

I. Draw the transition diagram of the finite state machine ($\sum, O, Q, f, g, q0$)

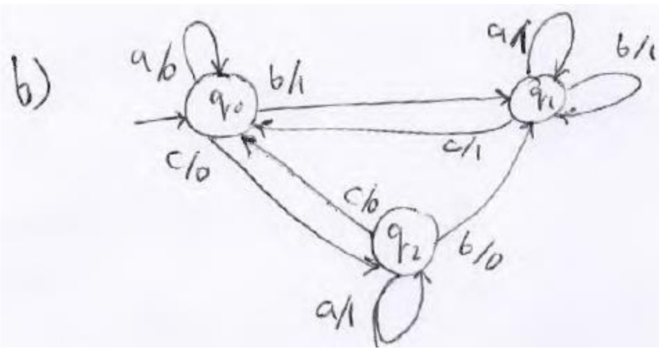
a. $\sum$ = {a,b} , O = {0,1} , Q= {q0, q1}

	f		G	
$\sum$ / Q	a	b	a	b
q0	q1	q0	0	0
q1	q0	q0	1	1



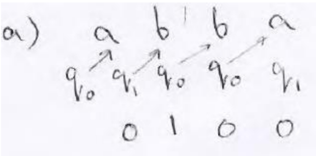
b. $\sum$ = {a,b,c} , O = {0,1} , Q= {q0, q1,q2}

	f			g		
$\sum$ / Q	a	b	c	A	b	c
q0	q0	q1	q2	0	1	0
q1	q1	q1	q0	1	1	1
q2	q2	q1	q0	1	0	0

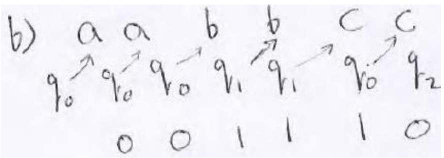


II. Find the output string of the previous finite state machines and the following input strings:

a. abba (exercise (I-a))

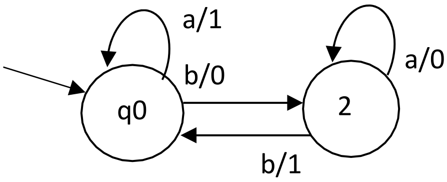


b. aabbcc (exercise (I-b))

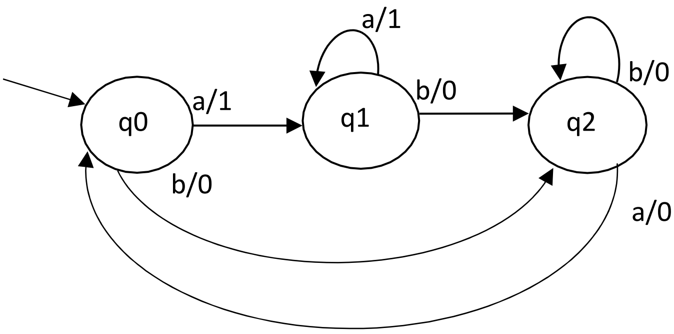


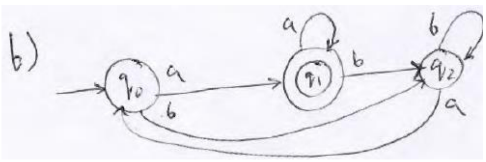
III. Show the DFA of the following FSM and state its formal description.

a.



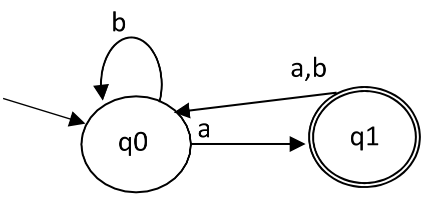
b.

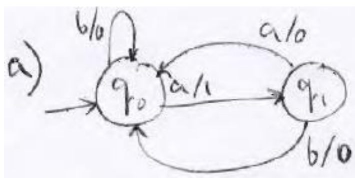




IV. Draw the transition diagram of the DFA as a finite state machine and state its formal description.

a.





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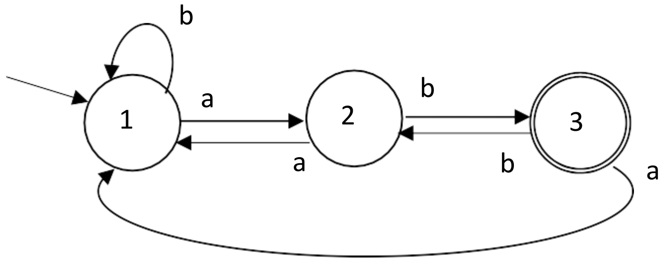
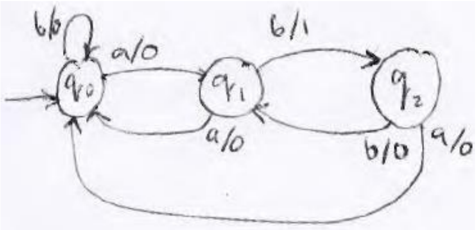
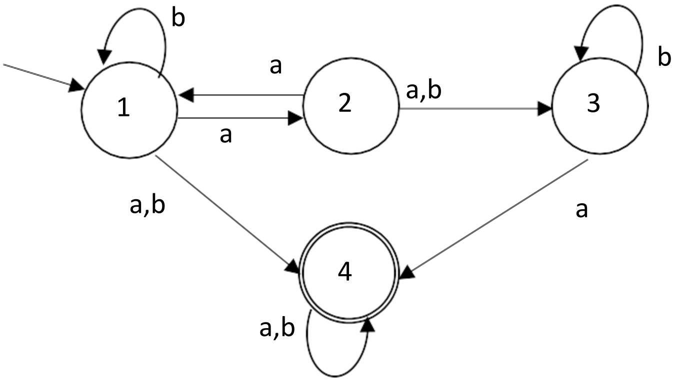
b.





c.



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Solution Sheet 5

1. Choose the correct answer:
   i. Which of the expression is appropriate? For production p: a → b where a ∈ V and b ∈________
      a. V b. S c. (V+∑)* d. V+ ∑
   ii. Which of the following statement is correct?
      a. All Regular grammar are context free but not vice versa
      b. All context free grammar are regular grammar but not vice versa
      c. Regular grammar and context free grammar are the same entity
      d. None of the mentioned
   iii. For S→0S1 | λ for ∑ = {0, 1}, which of the following is wrong for the language produced?
      a. Non regular language
      b. $0^n1^n \mid n\geq1$
      c. $0^n1^n \mid n\geq0$
      d. Context free language
   iv. Which of the following CFG's can't be simulated by a DFA?
      a. S → Sa | b
      b. S → abX, X → cY, Y → d | aX
      c. S → aSb | ab
      d. None of these
   v. The set $\{a^nb^n \mid n = 1, 2, 3 \ldots\}$ can be generated by the CFG
      a. S → ab | aSb
      b. S → ab | aSb | λ
      c. S → aaSbb | abS
      d. S → aaSbb | abS | λ

2. Find the CFG that generates the language L = {x∈{0,1}*: n0(x) ≠ n1(x)}
   G = ({S, A, B}, {0, 1}, S, P)

   P = { S → A, S → B, A → 0, B → 1, A → 0A , A → 1AA, A → A1A, A → AA1, B → 1B, B → 0BB, S → B0B, B → BB0}

3. For the following CFG, say what language is generated:
   a. S → aSa, S → bSb, S → a, S → b

   $L = \{w\,(a+b)\,w^R : w \in \{a, b\}^*\}$

   b. S → aS, S → bS, S → a

   $L = \{w\,a : w \in \{a, b\}^*\}$

   c. S → bbAab, A → aAa, A → bAb, A→ λ.

   $L = \{bb\,ww^R\,ab : w \in \{a, b\}^*\}$

   d. S → 1S1S0S , S → 1S0S1S , S → 0S1S1S, S → λ

$L = \{n_1(w) = 2n_0(w) \ : w \in \{0, 1\}^* \}$

e. $S \to aA,\ A \to bS,\ S \to \lambda$

$L = \{(ab)^n : n >= 0 \}$

f. $S \to BaB,\ S \to B,\ B \to bB,\ B \to \lambda$

L = {All strings with no more than one a}

4. Find a CFG for the following languages:
   a. $L = \{WcW^R : w \in \sum^*\}$.

$G = (\{S\}, \{a, b, c\}, S, P = \{S \to aSa,\ S \to bSb,\ S \to c\})$

   b. $L = \{cWaW^R : w \in \sum^*\}$.

$G = (\{S, A\}, \{a, b, c\}, S, P = \{S \to cA,\ A \to aAa,\ A \to bAb,\ A \to a\})$

   c. $L = \{ na(x) > nb(x) : x \in \{a,b\}^* \}$

$G = (\{S\}, \{a, b, c\}, S, P = \{S \to a,\ S \to aS,\ S \to bSS,\ S \to SSb,\ S \to SbS\})$

5. Consider the grammar of question (2-a). Derive the string abaabaaba

$S \to aSa \to abSba \to abaSaba \to abaaSaaba \to abaabaaba$

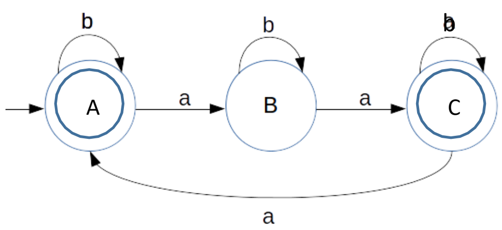
6. Consider the grammar of question (2-b). Derive the string ababbba

$S \to aS \to abS \to abaS \to ababS \to ababbS \to ababbbS \to ababbba$

7. Consider the grammar of question (2-c). Derive the string 00011011

$S \to 0S1 \to 0S011 \to 00S1011 \to 000S11011 \to 00011011$

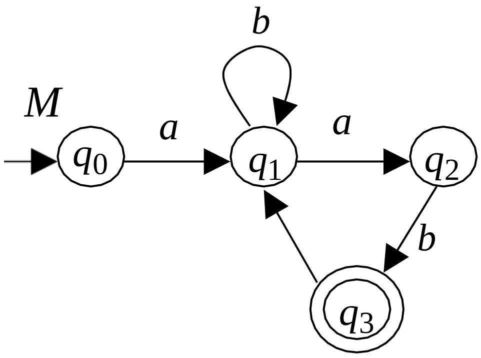
8. Convert the following DFA into regular grammar
   a.



$G = (\{A, B, C\}, \{a, b\}, A, P)$

$P = \{A \to aB,\ A \to bA,\ A \to \lambda,\ B \to bB,\ B \to aC\ ,\ C \to aA,\ C \to bC,\ C \to \lambda\}$

b.



G = ({q0, q1, q2, q3}, {a, b}, q0, P)

P = {q0 → aq1, q1 → bq1, q1 → aq2, q2 → bq3, q3 → q1 , q3 → λ}

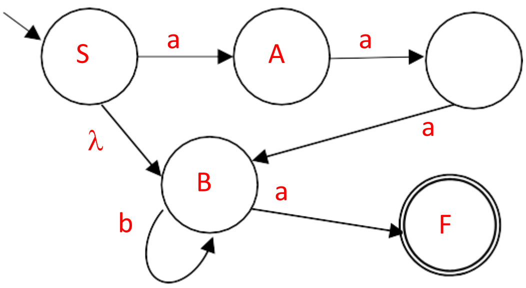
9. Convert the following grammars into equivalent NFAs:

a. G =({A,B,S}, {a,b}, S, P)

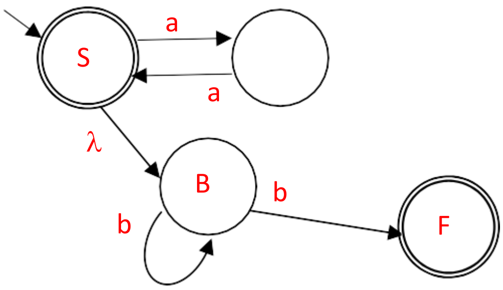
P = { $S \to aA \mid B$

$A \to aa\ B$ }

$B \to b\ B \mid a$



b. G = ({S, B}, {a, b}, S, {S → aaS, S → B, S → λ, B → bB, B → b})



c. G = ({S, A, B}, {a, b}, S, { S → abA, A → baB, B → aA, B → bb})

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Solution Sheet 6

1. Consider the following pushdown automata for the language: $L = \{xx^R: x \in \{a,b\}^*\}$,
   $M = (Q, \sum, \Gamma, \delta, q0, z, F)$, where $\sum = \{a,b\}$ and $F = \{q2\}$

$\delta(q0, a, z) = (q0, az)$	$\delta(q0, b, z) = (q0, bz)$
$\delta(q0, a, a) = (q0, aa)$	$\delta(q0, b, a) = (q0, ba)$
$\delta(q0, a, b) = (q0, ab)$	$\delta(q0, b, b) = (q0, bb)$
$\delta(q0, \lambda, z) = (q1, z)$	$\delta(q0, \lambda, a) = (q1, a)$
$\delta(q0, \lambda, b) = (q1, b)$	$\delta(q1, a, a) = (q1, \lambda)$
$\delta(q1, b, b) = (q1, \lambda)$	$\delta(q1, \lambda, z) = (q2, z)$

   Trace the execution of the PDA for the strings:

   a. aabaabaa   b. ababbbaba

   a. (q0, aabaabaa, z) ⊢ (q0, abaabaa, az) ⊢ (q0, baabaa, aaz) ⊢ (q0, aabaa, baaz) ⊢ (q0, abaa, abaaz) ⊢ (q0, baa, aabaaz) ⊢ (q0, aa, baabaaz) ⊢ (q0, a, abaabaaz) ⊢ (q0, λ, aabaabaaz) ⊢ (q1, λ, aabaabaaz)

   q1 is not a final state, the string is not accepted.

   Another move

   (q0, aabaabaa, z) ⊢ (q0, abaabaa, az) ⊢ (q0, baabaa, aaz) ⊢ (q0, aabaa, baaz) ⊢ (q0, abaa, abaaz) ⊢ (q1, abaa, abaaz) ⊢ (q1, baa, baaz) ⊢ (q1, aa, aaz) ⊢ (q1, a, az) ⊢ (q1, λ, z) ⊢ (q2, λ, z)

   q2 is the final state, the string is accepted

   b. (q0, ababbbaba, z) ⊢ (q0, babbbaba, az) ⊢ (q0, abbbaba, baz) ⊢ (q0, bbbaba, abaz) ⊢ (q0, bbaba, babaz) ⊢ (q0, baba, bbaabaz) ⊢ (q1, baba, bbaabaz) ⊢ (q1, aba, baabaz)

   q1 is not a final state, the string has not been finished, the string is not accepted.

2. Consider the following PDA with initial state q0, accepting state q2 and initial stack symbol z.

$\delta(q0, a, z) = (q1, az)$	$\delta(q0, b, z) = (q1, bz)$
$\delta(q1, a, a) = \{(q1, a), (q2,a)\}$	$\delta(q1, b, a) = (q1, a)$
$\delta(q1, a, b) = (q1, b)$	$\delta(q1, b, b) = (q1, b)$
$\delta(q2, \lambda, a) = (q2, \lambda)$	$\delta(q2, \lambda, z) = (q2, \lambda)$

Trace the execution of the PDA for the strings:

a. aabbabba   b. babbbabb

a. $(q0, aabbabba, z) \vdash (q1, abbabba, az) \vdash (q1, bbabba, az) \vdash (q1, babba, az) \vdash (q1, abba, az) \vdash (q1, bba, az) \vdash (q1, ba, az) \vdash (q1, a, az) \vdash (q2, \lambda, az) \vdash (q2, \lambda, z) \vdash (q2, \lambda, \lambda)$
q2 is the final state, the string is accepted.

b. $(q0, babbbabb, z) \vdash (q1, abbbabb, bz) \vdash (q1, bbbabb, bz) \vdash (q1, bbabb, bz) \vdash (q1, babb, bz) \vdash (q1, abb, bz) \vdash (q1, bb, bz) \vdash (q1, b, bz) \vdash (q1, \lambda, bz)$
q1 is not a final state, the string is not accepted

3. For each grammar, state the formal description for the equivalent PDA:
a. $P = \{S \to S1, S1 \to AS1, S1 \to \lambda, A \to aA, A \to b\}$

Start: $\delta(q0, \lambda, z) = (q1, Sz)$

$\delta(q1, \lambda, S) = (q1, S1)$ $\delta(q1, \lambda, S1) = (q1, AS1)$

$\delta(q1, a, A) = (q1, A)$ $\delta(q1, b, A) = (q1, \lambda)$

Finish: $\delta(q1, \lambda, z) = (qf, \lambda)$

$M = (\{q0, q1, qf\}, \{a,b\}, \{S, S1, A, z\}, \delta, q0, z, \{qf\})$

b. $P = \{S \to S1, S1 \to aA, A \to aA, A \to bA, A \to \lambda\}$

Start: $\delta(q0, \lambda, z) = (q1, Sz)$

$\delta(q1, \lambda, S) = (q1, S1)$ $\delta(q1, a, S1) = (q1, A)$

$\delta(q1, a, A) = (q1, A)$ $\delta(q1, b, A) = (q1, A)$

$\delta(q1, \lambda, A) = (q1, \lambda)$

Finish: $\delta(q1, \lambda, z) = (qf, \lambda)$

$M = (\{q0, q1, qf\}, \{a,b\}, \{S, S1, A, z\}, \delta, q0, z, \{qf\})$

Solution Quiz 1 (Model A)

Choose the correct answer: **[10 marks]**

1. Let f and g be the functions from the set of integers to the set of integers defined by f (x) = 4x + 3 and g(x) = 2x + 2. What is the composition of f and g?
   a. (f o g)(x) = 8x + 9.
   b. (f o g)(x) = 8x + 11.
   c. (f o g)(x) = 6x + 7.
   d. (f o g)(x) = 8x + 8.

2. Let R be the relation in the set {1, 2, 3} is given by:

   i. R = {(1, 2), (2, 3), (1, 3)}.
      a. R is reflexive, symmetric, but not transitive
      b. R is reflexive, transitive, but not symmetric
      c. R is transitive, but not reflexive and not symmetric
      d. R is an equivalence relation.

   ii. R = {(1, 1) (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)}
      a. R is reflexive, transitive, but not symmetric
      b. R is reflexive, symmetric, but not transitive
      c. R is reflexive, but not transitive and not symmetric
      d. R is an equivalence relation.

3. Let A = {1, 2, 3}. Which relation is symmetric but neither reflexive nor transitive.
   a. R = {(1, 2), (2, 2)}.
   b. R = {(1, 2), (2, 1)}.
   c. R = {(1, 1), (2, 1), (3,3)}.
   d. R = {(1, 2), (1, 3), (2,3)}.

4. A function is said to be_______________if and only if f(a) = f(b) implies that a = b for all a and b in the domain of f.
   a. Onto
   b. Bijective
   c. One-to-one
   d. neither one-one nor onto

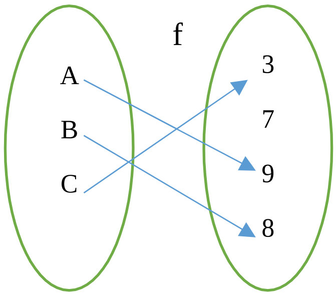
5. Let A = {1,2,3}, B = {8,10,15}, consider a function f: A → B, such that f = {(1,10), (2,8), (3,15)}, then f is
   a. Onto function and not One-to-one function
   b. One-to-one function and not onto function
   c. Bijective function
   d. neither one-to-one nor onto

6. By the pigeonhole principle, if 40 coins are distributed among 7 boxes, then there must be some box with at least____coins.
   a. 8
   b. 7
   c. 5
   d. 6

7. For any alphabet $\sum=\{0,1\}$, $\sum^0$ denotes
   a. {0, 1}.
   b. {0}
   c. { λ }.
   d. none of the above.

8. If L = {a, ab} and M = { λ, b, ba}, the language L.M is:
   a. {λ , a, ab, b , ba}
   b. {a, ab, abb, aba, abba}
   c. { a, ab, b , ba}
   d. {λ, a, ab, abb, aba, abba}

9. Given the following arrow diagrams, f



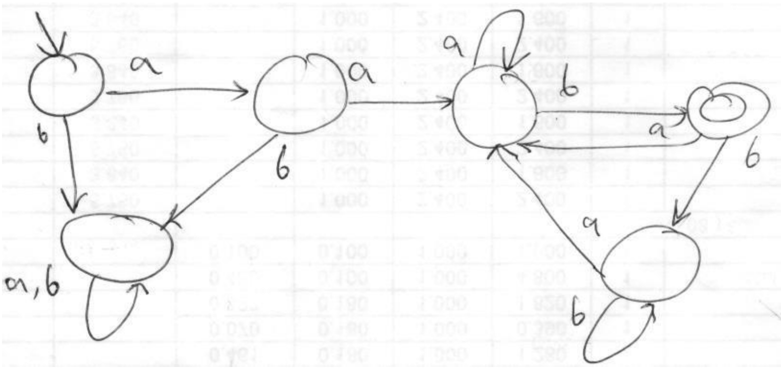
   a. Is not a function.
   b. Is a bijective function.
   c. Is an Onto function.
   d. Has not an inverse function.

10. Which of the following statements is correct
   a. $\lambda \in \Sigma^{+}$
   b. $\phi \in \Sigma^{+}$
   c. $\lambda \in \Sigma^{*}$
   d. $\phi \in \Sigma^{*}$

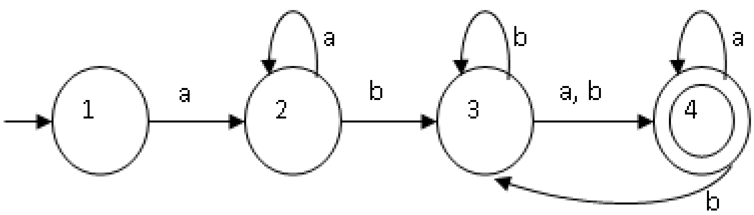
Quiz 2 (Model A)

1. Design DFA that accept the following language: **[5 marks]**

L = {w ϵ {a, b}*: w begin with two consecutive a's and end with exactly one b}.



2. Convert the following NFA into an equivalent DFA. **[5 marks]**



2)

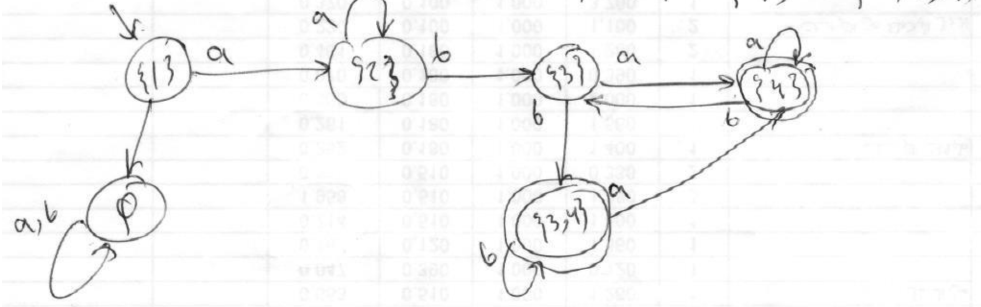
NFA

	a	b
1	{2}	φ
2	{2}	{3}
3	{4}	{3,4}
4	{4}	{3}

⟹

DFA

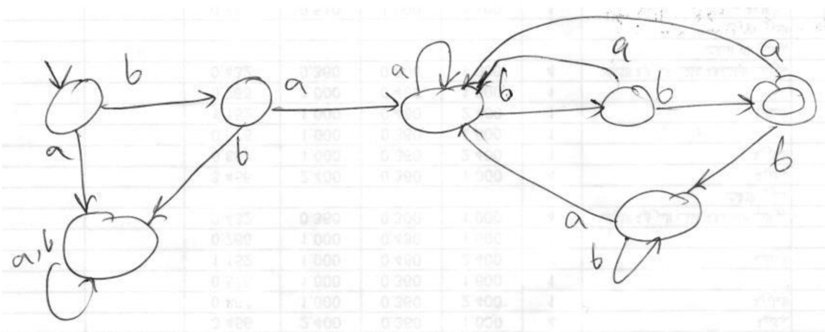
	a	b
{1}	{2}	φ
{2}	{2}	{3}
{3}	{4}	{3,4}
{4}	{4}	{3}
{3,4}	{4}	{3,4}



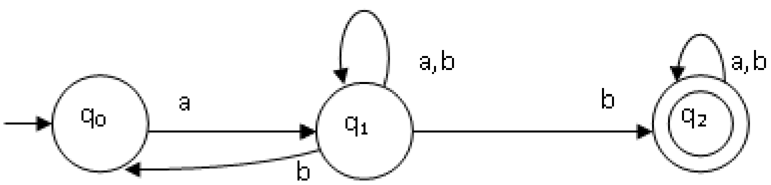
Quiz 2 (Model B)

1. Design DFA that accept the following language: [5 marks]

L = {w ϵ {a, b}*: w begin with ba and end with exactly two consecutive b's}.



2. Convert the following NFA into an equivalent DFA. [5 marks]



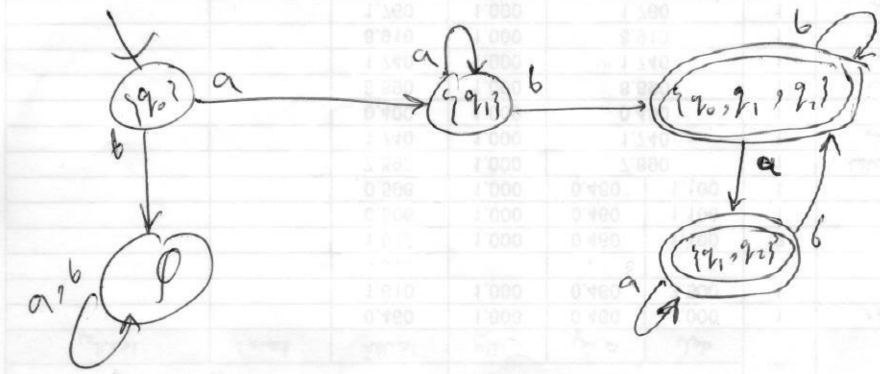
2

NFA

	1	2
q_0	$\{q_1\}$	φ
q_1	$\{q_1\}$	$\{q_0, q_1, q_2\}$
q_2	$\{q_2\}$	$\{q_2\}$

DFA

	a	b
$\{q_0\}$	$\{q_1\}$	φ
$\Rightarrow \{q_1\}$	$\{q_1\}$	$\{q_0, q_1, q_2\}$
$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_0, q_1, q_2\}$
$\{q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_0, q_1, q_2\}$

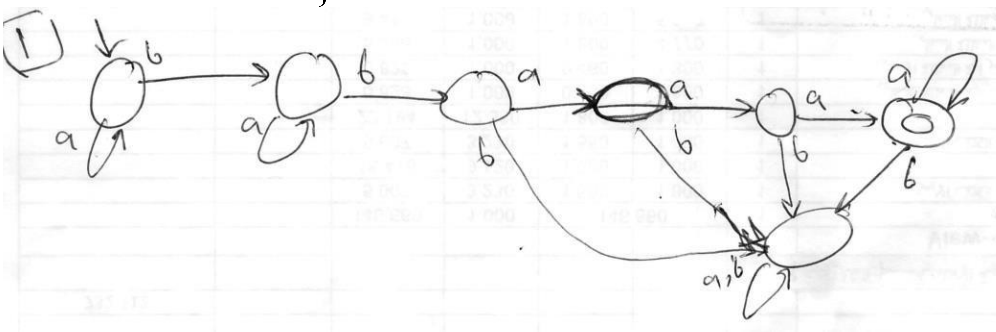


Quiz 2 (Model C)

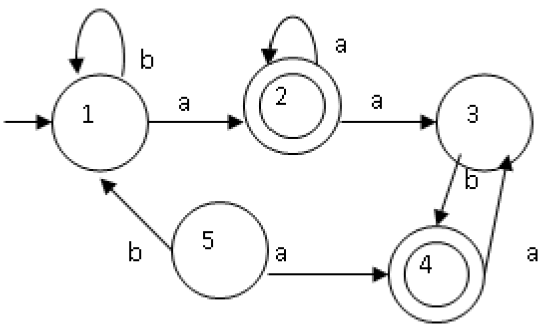
1. Design DFA that accept the following language: [5 marks]

L = {w ϵ {a, b}*: w contains all strings that contain exactly two b's and end with three consecutive a's}.

1



2. Convert the following NFA into an equivalent DFA. [5 marks]



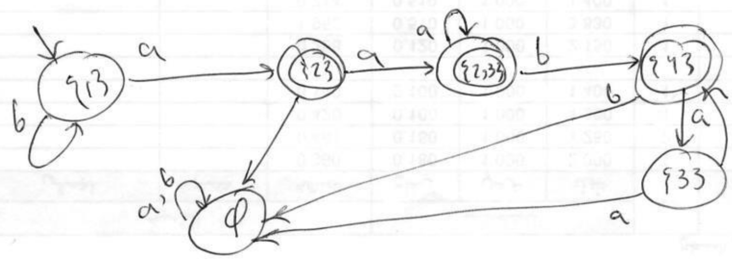
2

NFA

	a	b
1	{2}	{1}
2	{2,3}	φ
3	φ	{4}
4	{3}	φ
5	{4}	{1}

DFA

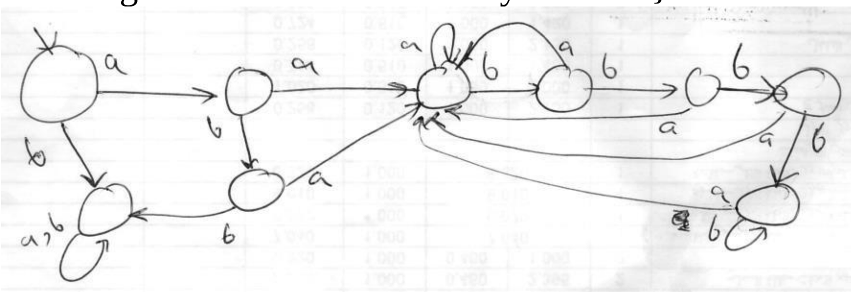
	a	b
{1}	{2}	{1}
{2}	{2,3}	φ
{2,3}	{2,3}	{4}
{4}	{3}	φ
{3}	φ	{4}



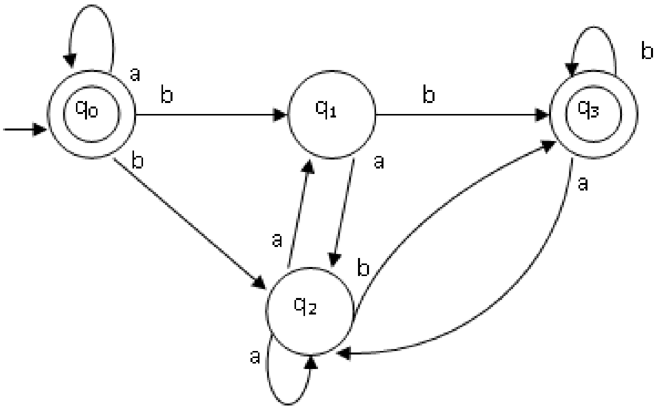
Quiz 2 (Model D)

1. Design DFA that accept the following language: [5 marks]

L = {w ϵ {a, b}*: w contains all strings that begin with two consecutive a's or the substring aba and end with exactly three b's}.



2. Convert the following NFA into an equivalent DFA. [5 marks]



2

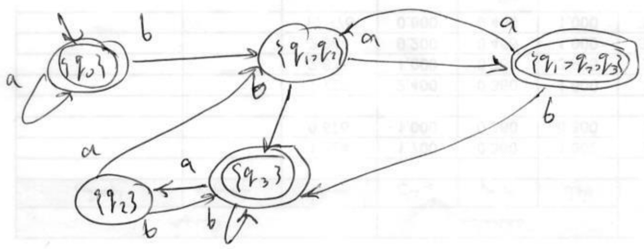
NFA

	a	b
q_0	$\{q_0\}$	$\{q_1, q_2\}$
q_1	$\{q_2\}$	$\{q_3\}$
q_2	$\{q_1, q_2\}$	$\{q_3\}$
q_3	$\{q_2\}$	$\{q_3\}$

⟹

DFA

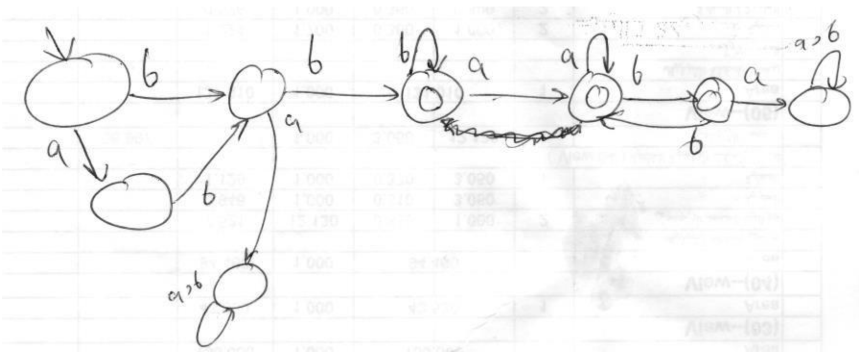
	a	b
$\{q_0\}$	$\{q_0\}$	$\{q_1, q_2\}$
$\{q_1, q_2\}$	$\{q_1, q_2, q_3\}$	$\{q_3\}$
$\{q_1, q_2, q_3\}$	$\{q_1, q_2\}$	$\{q_3\}$
$\{q_3\}$	$\{q_2\}$	$\{q_3\}$
$\{q_2\}$	$\{q_1, q_2\}$	$\{q_3\}$



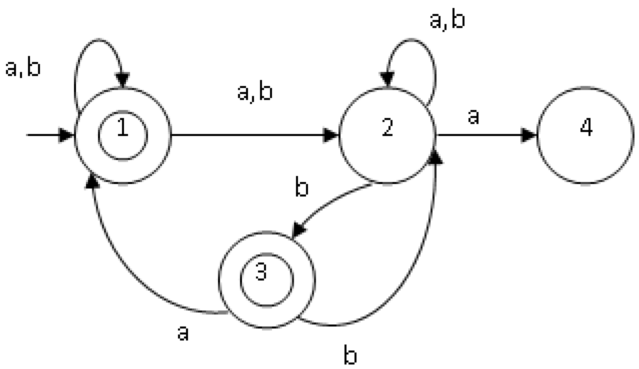
Quiz 2 (Model E)

1. Design DFA that accept the following language: [5 marks]

L = {w ϵ {a, b}*: w contains all strings that begin with two consecutive b’s or the substring abb and do not contain aba}.



2. Convert the following NFA into an equivalent DFA. [5 marks]



2

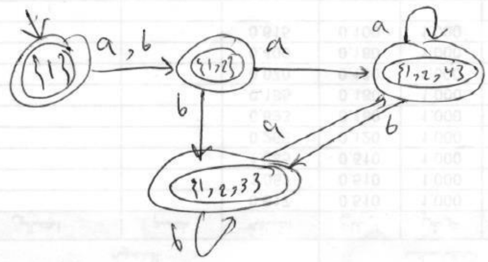
NFA

	a	b
1	{1,2}	{1,2}
2	{2,4}	{2,3}
3	{1}	{2}
4	φ	φ

⟹

DFA

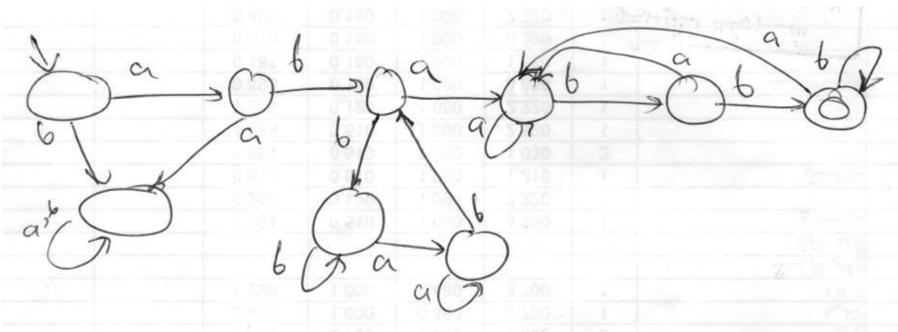
	a	b
{1}	{1,2}	{1,2}
{1,2}	{1,2,4}	{1,2,3}
{1,2,4}	{1,2,4}	{1,2,3}
{1,2,3}	{1,2,4}	{1,2,3}



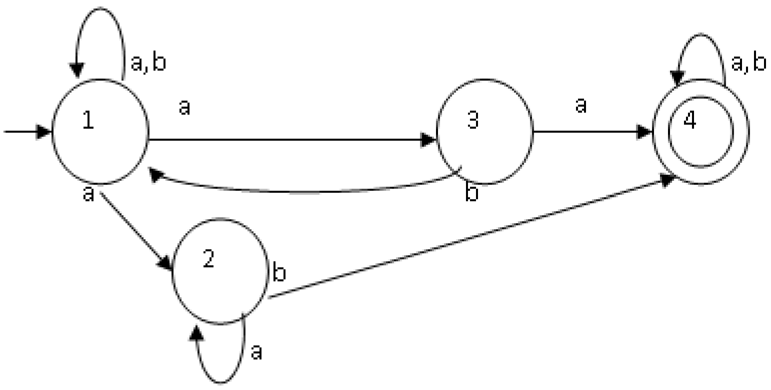
Quiz 2 (Model F)

1. Design DFA that accept the following language: [5 marks]

L = {w ϵ {a, b}*: w contains all strings that begin with ab and contain aba and end with the substring bb}.



2. Convert the following NFA into an equivalent DFA. [5 marks]



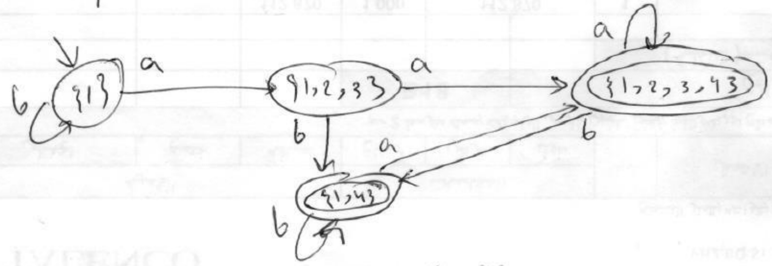
2

NFA

	a	b
1	{1,2,3}	{1}
2	{2}	{4}
3	{4}	{1}
4	{4}	{4}

DFA

	a	b
{1}	{1,2,3}	{1}
{1,2,3}	{1,2,3,4}	{1,4}
{1,2,3,4}	{1,2,3,4}	{1,4}
{1,4}	{1,2,3,4}	{1,4}



Ahmed

Year:
Course Name:
Course Code:
Instructor: Dr. Heba Ahmed Abdelsalam Elnemr

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Date:
Time Allowed:
Mark: 20
Test Code: 04

I. Section 1

1- Let R be the relation in the set {a, b, c} is given by R = {(a, b), (a, c), (b, c)}.

- Ⓐ R is reflexive, symmetric, but not transitive
- Ⓑ R is reflexive, transitive, but not symmetric
- Ⓒ R is ~~transitive, but not reflexive and not symmetric~~
- Ⓓ R is an equivalence relation.

2- Given the language L = {ab, aa, baa}, which of the following strings are in L*? 1) abaabaaab 2) aaaabaa 3). baaaaabaab 4) baaaaab

- Ⓐ 1, 2 and 3
- Ⓑ 1, 3 and 4
- Ⓒ 2, 3 and 4
- Ⓓ ~~1, 2 and 4~~

3- The reverse of the string "BcAbed" defined over = {Ab, Bc, d, e} is

- Ⓐ debABc
- Ⓑ deAbcB
- Ⓒ deAbBc
- Ⓓ debAcB

4- Suppose there are 75 people in a room. Then, at least how many people must have their birthday in the same month?

- Ⓐ 75/12
- Ⓑ ~~7~~
- Ⓒ 6
- Ⓓ 5

$\lceil 75/12 \rceil$

5- Given f(x)=x^2+1 and g(x)=2x, (g o f)(x) is:

- Ⓐ 4x^2 + 1
- Ⓑ 4x^2 + 2
- Ⓒ 2x^2 + 1
- Ⓓ ~~2x^2 + 2~~

$g(f(x)) = g(x^2+1) = 2(x^2+1) = 2x^2+2$

6- NFA has a transition function

- Ⓐ Q X σ U {λ} → Q
- Ⓑ ~~Q X σ U {λ} → 2^Q~~
- Ⓒ Q X σ → Q
- Ⓓ Q X σ → 2^Q

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7- Regular expression for all strings starts with ab and ends with ba is.

A. aba*b*ba

B. ab (a+b)*ba

C. ab(ab)*ba

D. All of the mentioned

8- Which one of the following languages over the alphabet {0,1} is described by the regular expression: (0+1)*0(0+1)*0(0+1)*?

A. The set of all strings containing the substring 00.

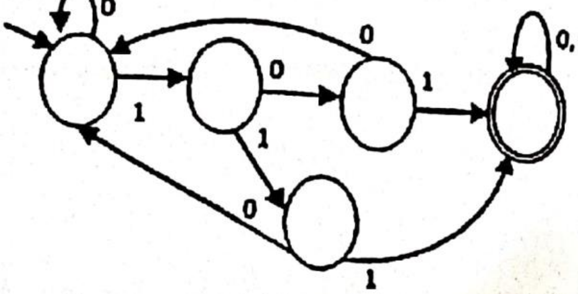
B. The set of all strings containing at most two 0's.

C. The set of all strings containing at least two 0's.

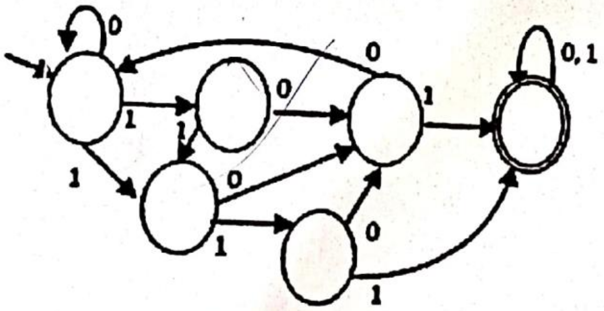
D. The set of all strings that begin and end with either 0 or 1.

9- The DFA that accepts the language L = {w ϵ {0, 1}*: w contains either 111 or 101 as a substring} is:

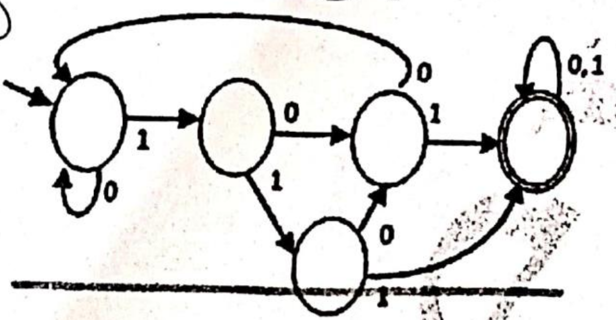
A.



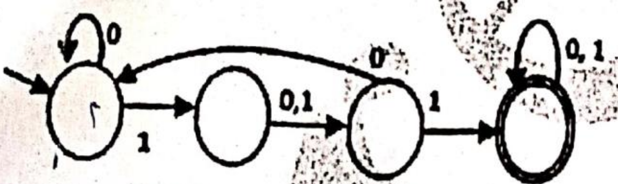
B.



C.



D.



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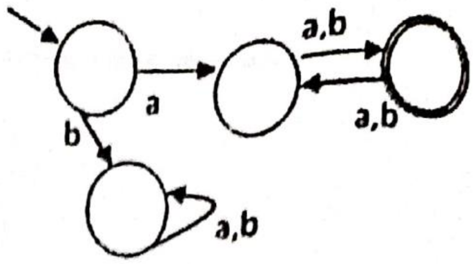
10- The following DFA accepts all strings that

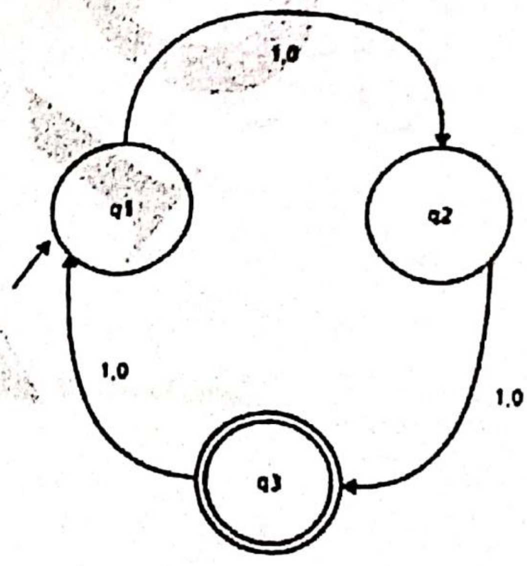


ⓐ Starts with "a" and have odd length.
ⓑ Starts with "a" or have odd length.
ⓒ ~~Starts with "a" and have even length~~
ⓓ Starts with "a" or have even length.

11- The regular expression for the language L= { w ∈ {0, 1}*: w has odd number of 1's} is:
ⓐ ~~0*(10*10*)*10*~~
ⓑ 0*(10*1)*10*
ⓒ 0*(10*10*1)*0*
ⓓ 0*10*10*10*

12- Consider the following regular expression "a(a+b)b*" All of the following strings are accepted except ___
ⓐ ~~aaa~~
ⓑ abb
ⓒ aa
ⓓ aab

13- Which of the following strings will the given DFA not accept?



ⓐ λ
ⓑ 10001010
ⓒ 11010
ⓓ 10111

14- Find the correct language described by the regular expression $a+(ab)^*$

Ⓐ {λ , a, ab, abab, ababab, ...}

Ⓑ {a, aab, aabab, aababab, ...}

Ⓒ {λ , a, b, aa, ab, ba, bb, ...}

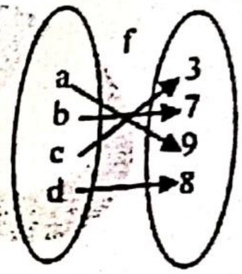
Ⓓ {a, b, aa, ab, ba, bb, ...}

II. Section 2

1- A function F is bijective if, and only if, for all elements x1 and x2 in the domain, if F(x1) = F(x2), then x1 = x2.

Ⓐ True
Ⓑ False

2- The function f represented by the following arrow diagram has an inverse function

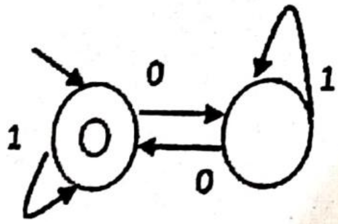


Ⓐ ~~True~~
Ⓑ False

3- A regular language is any language defined by the DFA only.

Ⓐ True
Ⓑ False

4- The following DFA accepts the language L = {w ∈ {0,1}*: w has even number of zeros}.



Ⓐ True
Ⓑ False

5- The regular expression $(0+\varepsilon)(1+\varepsilon)$ denotes the language $\{\varepsilon,0,1,01\}$.

Ⓐ True
Ⓑ False

Scanned with CamScanner

6- A regular expression for L = {w ϵ {a, b}*: w contains all strings that begin with three consecutive a's and end with substrings aab and bba} is "aaa (a+b)* (aab+bba)".

Ⓐ True

Ⓑ False

Scanned with CamScanner

Answer the following questions:

Question one: (12 Marks)

a. State true or false:

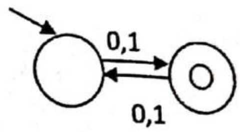
(3 Marks)

i. Every finite state machine can be converted into an equivalent finite state acceptor and every finite state acceptor can be converted into finite state machine.

ii. G = ({S, A, B}, {a, b}, S, {S→AB, A→aA, A→λ, B→bB, B→λ}) is a right linear grammar.

iii. The domain of the transition function of the nondeterministic pushdown automaton is Q x (∑ ∪ {λ}) x Γ and the range is Q x Γ *.

iv. The following DFA accepts the language L = {w Є {0, 1}$^+$}.



v. Every regular language is a context free language but not every context free language is a regular language.

vi. The regular expression for the language {w ∈ {a, b}*: w contains at least one a} is a (a + b)*.

b. Consider the following DFA:

(9 Marks)

$M = (\Sigma, Q, \delta, q_0, \{q_0, q_3\})$, where $Q = \{q_0, q_1, q_2, q_3\}$ and $\Sigma = \{a, b\}$

$\delta(q_0, a) = q_0$ $\qquad$ $\delta(q_0, b) = q_1$

$\delta(q_1, a) = q_2$ $\qquad$ $\delta(q_1, b) = q_3$

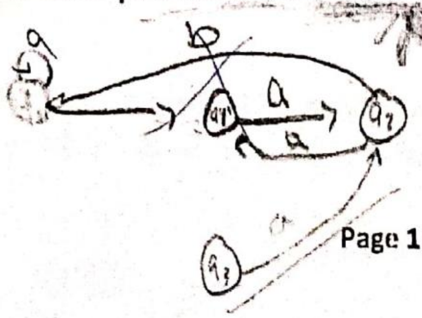
$\delta(q_2, a) = q_1$ $\qquad$ $\delta(q_2, b) = q_0$

$\delta(q_3, a) = q_2$ $\qquad$ $\delta(q_3, b) = q_3$

i. Convert this DFA into finite state machine and state its formal description.

ii. Find the output string of the previous finite state machine if the input string was abbaba.

iii. Convert the previous DFA into regular grammar.



Ahmy

Question two (17 Marks):

a) (10 Marks)

i. Design DFA that accept the language L = {w ϵ {0, 1}*: w contains all strings that do not contain the substrings 110 and end with the substring 11}.

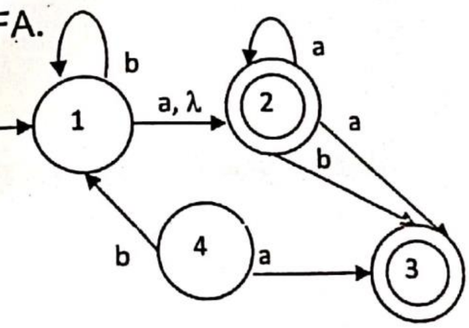
ii. Design DFA that accept the language L = {w ϵ {0, 1}*: w contains all strings that begin with two consecutive 1's or the substring 01 and end with three consecutive 0's}.

iii. Give regular expression for L = {w ϵ {a, b}*: w contains all strings that contain three consecutive a's and end with substrings aa and bb}.

b) (7 Marks).

i. Convert the following regular expression "(ab+ba) aba* + (aba+ab)*ab*" into an equivalent NFA.

ii. Convert the following NFA into an equivalent DFA.



Question three: (7 Marks):

a) Choose the correct answer: **(4 Marks)**

i. Pumping lemma is generally used for proving
   a. a given language is regular c. a given grammar is regular
   b. a given language is not regular d. a given grammar is not regular

ii. regular expression (a*+b) denotes the language
   a. {λ, a, b} c. {λ, a, b, aa, ab, ba, bb,.....}
   b. {λ, a, b, aa, aaa, ...} d. {a, b, aa, ab, ba, bb,.....}

iii. The CFG: s → as | bs | a | b generates the language
   a. {a, b} c. Σ^+
   b. {aa, ab, ba, bb} d. None of these

iv. Which of the following language is regular?
   a. $\{a^i b^i : i>=0\}$ c. $\{a^i b^i : 0<i<5\}$
   b. $\{a^i b^i : i>=1\}$ d. None of the mentioned

b) For the following CFG say what language is generated: **(3 Marks)**

i. S → aSa , S → ab , S → abb.

ii. $S \rightarrow aSb$, $S \rightarrow c$.
iii. $S \rightarrow bbAab$, $A \rightarrow aAa$, $A \rightarrow bAb$, $A \rightarrow \lambda$.

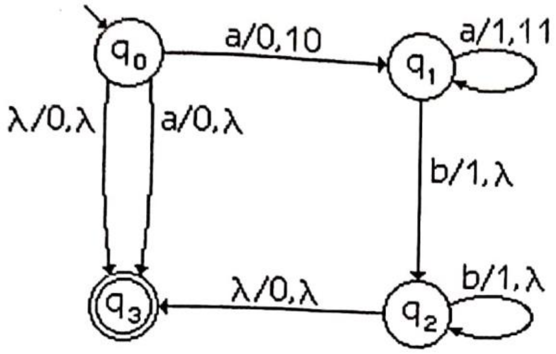
Question four: (8 Marks):

a) (3 Marks)

For the following grammar G = ({S, B}, {a, b}, S, {S→abS, S→B, B→bB, B→b}), get the formal description for the equivalent NPDA.

b) (5 Marks)

Consider the following NPDA ({q0, q1, q2, q3}, {a, b}, {0, 1}, δ, q0, z=0, {q3})



Trace the operation of the NPDA on the following input string and decide whether it is accepted or rejected: aaaabbbb.

Question five: (6 Marks):

a) (2 Marks)

Consider the following grammar:
G = ({S, B}, {a, b}, S, {S→aSb, S→B, B→bB, B→b}). Drive the string a^2b^4.

b) (4 Marks)

For the following Turing machine T = (Q, Σ, Γ, δ, q0, #, {qf}), the read/write head is positioned over the leftmost input symbol.

$$\delta(q_1,a)=(q_1,a,R) \quad \delta(q_1,b)=(q_2,b,R) \quad \delta(q_1,c)=(q_3,c,R)$$
$$\delta(q_2,a)=(q_4,a,R) \quad \delta(q_2,b)=(q_2,b,R) \quad \delta(q_2,c)=(q_f,c,R)$$
$$\delta(q_3,a)=(q_4,a,R) \quad \delta(q_3,b)=(q_4,b,R) \quad \delta(q_3,c)=(q_3,c,R)$$

Get the output of the following input string: abcabc

$(q_1, a) = (q_1, aR)$

Best wished,
Dr. Heba Elnemr

final 2020 Automata Answer

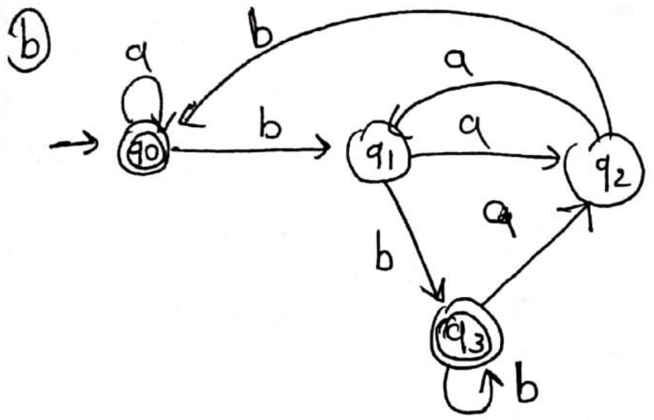
Question 1

(a) True or false

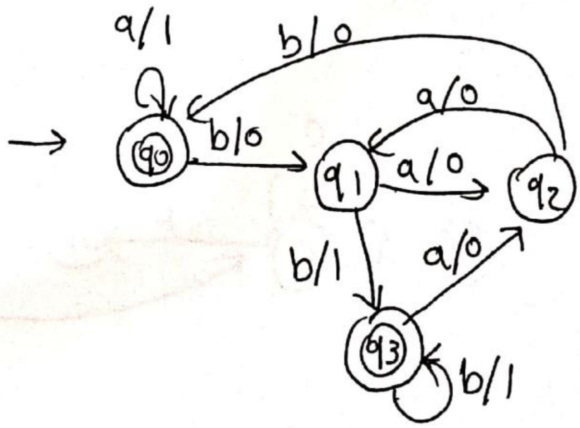
(i) true (ii) ~~False~~ true (iii) True

(iv) False (v) True (vi) False

(b)



(i) Finit state machine →



Formal Description (1) set of state {q0, q1, q2, q3} Q

(2) Σ input symbol {a, b} (3) O output symbol {0, 1}

(4) q0 start state

(5) f : next state function

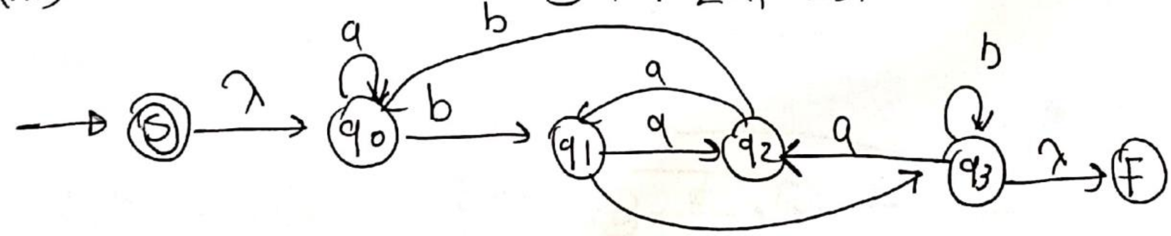
(6) g : output function.

	f		g	
Σ / q	a	b	a	b
q_0	q_0	q_1	1	0
q_1	q_2	q_3	0	1
q_2	q_1	q_0	0	1
q_3	q_2	q_3	0	1

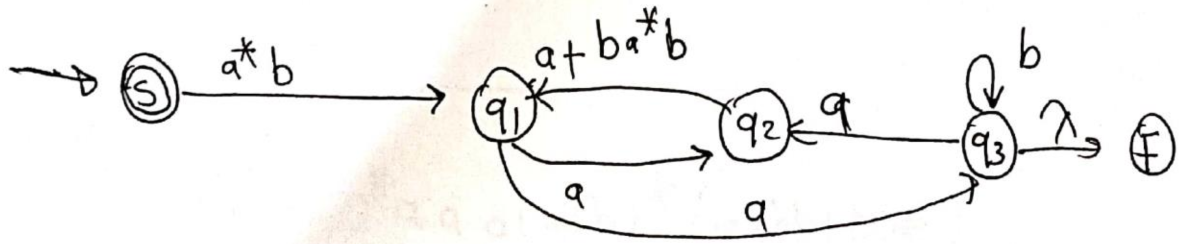
(ii)

input	a	b	b	a	b	a	
state	q_0	q_0	q_1	q_3	q_2	q_0	q_0
output	1	0	1	0	1	1	

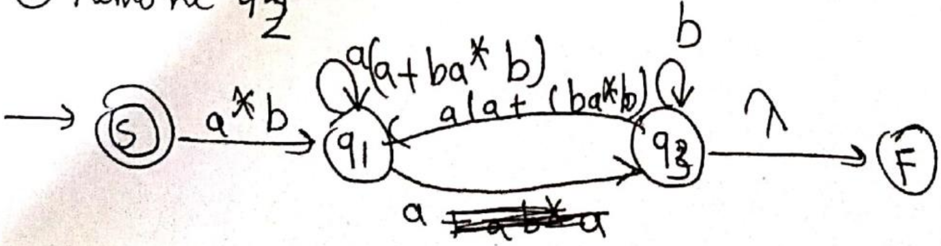
(iii) From DFA into Regular Expression



① Remove q_0

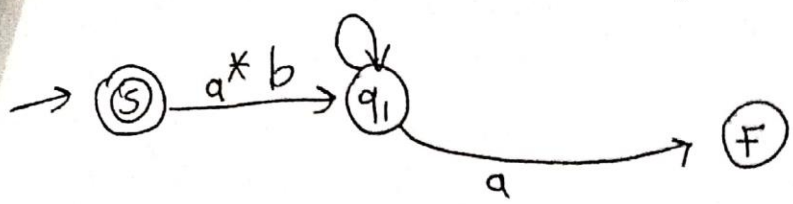


② Remove q_2



Remove q_3

$$a(a+ba^*b)+aba(a+(ba^*b))$$



(4) Remove q_1

$$S \xrightarrow{a^*b(a(a+ba^*b)+aba(a+(ba^*b)))^*a} F$$

From DFA into Regular grammer

$q_0 \to aq_0 \mid bq_1 \mid \lambda$

$q_1 \to aq_2 \mid bq_3$

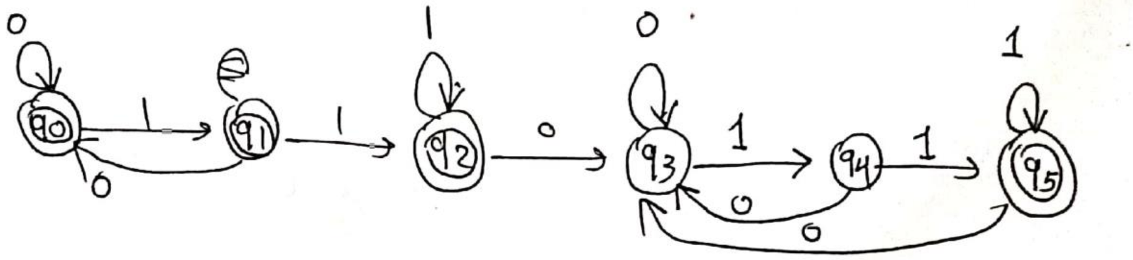
$q_2 \to aq_1 \mid bq_0$

$q_3 \to aq_2 \mid bq_3 \mid \lambda$

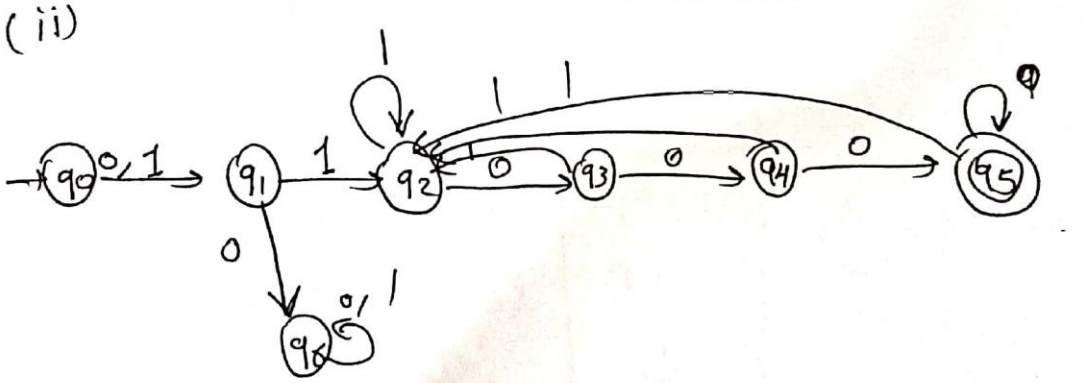
Question 2

(a)

(i)



(ii)

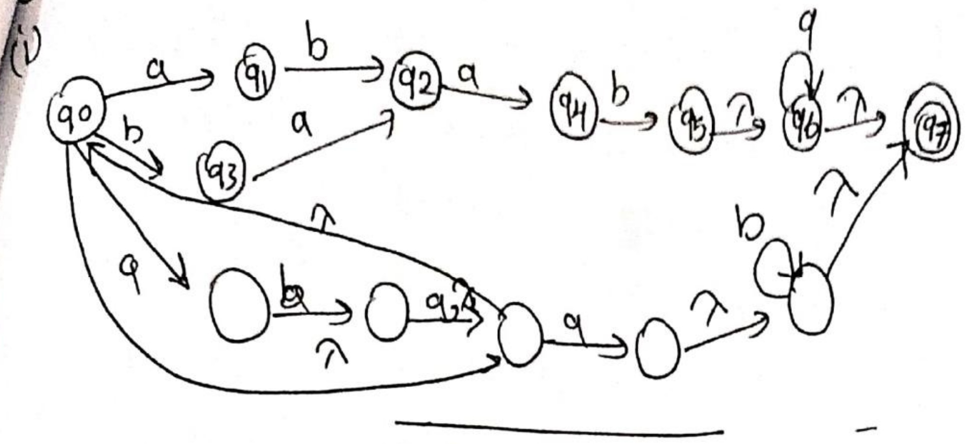


(iii)

$$(a+b)^*aaa(a+b)^*(aabb+bbaa)$$

b)

(i)



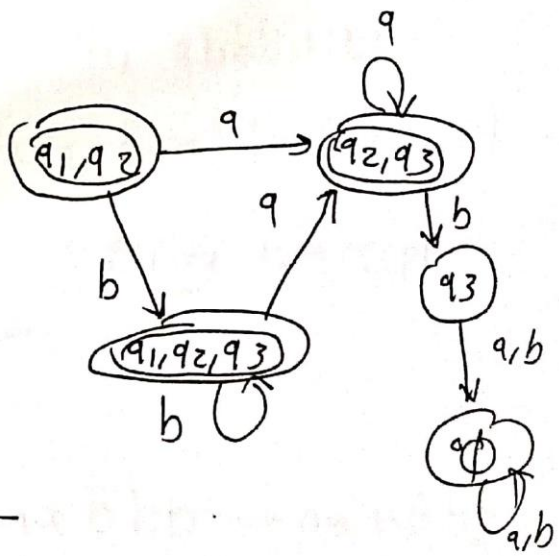
(ii)

	a	b	λ
q_1	q_2	q_1	q_2
q_2	$\{q_2,q_3\}$	q_3	ϕ
q_3	ϕ	ϕ	ϕ
q_4	q_3	q_3	ϕ

+

	λ
q_1	$\{q_1,q_2\}$
q_2	q_2
q_3	q_3
q_4	q_4

	a	b
$\{q_1,q_2\}$	q_2,q_3	$\{q_1,q_2,q_3\}$
$\{q_2,q_3\}$	$\{q_2,q_3\}$	q_3
q_3	ϕ	ϕ
ϕ	ϕ	ϕ
q_1,q_2,q_3	$\{q_2,q_3\}$	$\{q_1,q_2,q_3\}$



(Question3)

(a) [b] (b) [b] (c) [C] (d) [C]

(b)

(i) $L=\{a^n(ab+abb)a^n : n\geq 0\}$ (ii) $L=\{a^n c b^n ; n\geq 0\}$

(iii) $L=\{w a^n w^R (ab)^n : n\geq 0 ; w\in\{a,b\}^*\}$

Question 4

(a)

(start)	$\delta(q_0, \lambda, z) = \{(q_1, Sz)\}$
$S \to abS$	$\delta(q_1, ab, S) = \{(q_1, S)\}$
$S \to B$	$\delta(q_1, \lambda, S) = \{(q_1, B)\}$
$B \to bB$	$\delta(q_1, b, B) = \{(q_1, B)\}$
$B \to b$	$\delta(q_1, b, B) = \{(q_1, \lambda)\}$
(finish)	$\delta(q_1, \lambda, z) = \{(q_F, z)\}$

Formal Description

① Q : set of state (q_0, q_1, q_F) ② Σ : input $\{a, b, \lambda\}$

③ stack alphabit $\Gamma = \{S, B, \lambda, z\}$ ④ δ transition function

⑤ $q_0 \in Q$ initial state ⑥ $Z \in \Gamma$ initial stacksymbol

⑦ $q_F \in Q$ Final state.

ⓑ $(q_0, aaabbb, 0) \vdash (q_1, aabbb10) \vdash (q_1, abbb, 110)$

$\vdash (q_1, bbb, 1110) \vdash (q_2, bb, 110) \vdash (q_2, b, 10) \vdash (q_2, \lambda, 0)$

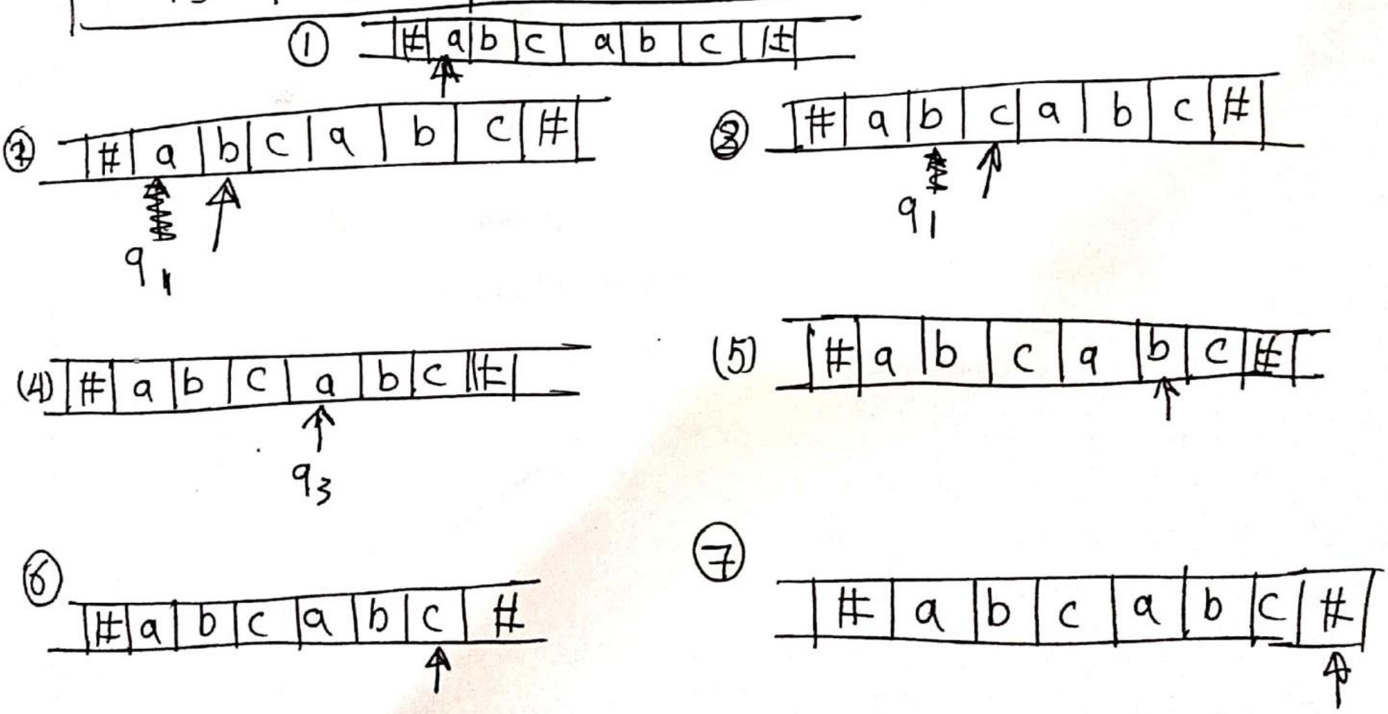
$\vdash (q_3, \lambda, \lambda)$ $\because q_3$ is final state $\therefore$ string is accept

Question 5

$S \to aSb \to aaSbb \to aaBbb \to aabBbb$

$\to$ aabbbb

current state	symbol Read	symbol written	Direction	next state
q_1	a	a	R	q_1
q_1	b	b	R	q_2
q_1	c	c	R	q_3
q_2	a	a	R	q_4
q_2	b	b	R	q_2
q_2	c	c	R	q_f
q_3	a	a	R	q_4
q_3	b	b	R	q_4
q_3	c	c	R	q_3



∴ output is abcabc

The transition function for an NPDA (Q, Σ, Γ, δ, q_0, z, F) has the form

$\delta : Q \times (\Sigma \cup \{\lambda\}) \times \Gamma \rightarrow$ finite subsets of $Q \times \Gamma^*$

$\delta : Q \times \Sigma \times (\Gamma \cup \{\lambda\}) \rightarrow$ finite subsets of $Q \times \Gamma^*$

$\delta : Q \times (\Sigma \cup \{\lambda\}) \times (\Gamma \cup \{\lambda\}) \rightarrow$ finite subsets of $Q \times \Gamma^*$

$\delta : Q \times (\Sigma \cup \{\lambda\}) \times \Gamma^* \rightarrow$ finite subsets of $Q \times \Gamma^*$

Regular grammar generate

context free language

regular language but not context free language

context free language but not regular language

none of the mentioned

The CFG ($S \rightarrow abSab$, $S \rightarrow a$, $S \rightarrow bb$) generates the language

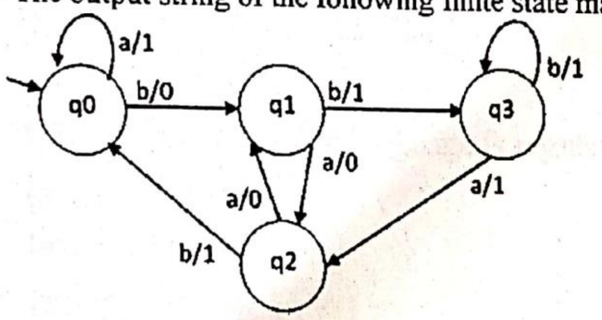
(ab)*(a+bb)(ab)*

{(ab)^n (a+bb) (ab)^n, n>=0}

a*b*(abb+bba)a*b*

{(ab)^n (abb+bba) (ab)^n, n>=0}

The output string of the following finite state machine for an input string “abbaba” is



101100

101110

101111

1011111

- The regular expression for the language L = {w ε {a, b}*: w contains all strings that begin with the substring'abb and substring bba and contain the substring aba} is

(abb+bba)(a+b)*aba(a+b)*

(abbbba+bbaabb)(a+b)*aba(a+b)*

abbbba (a+b)*aba(a+b)*

(abbbba+bbaabb)(a*b*)aba(a*b*)

Which one of the following grammars generates the language L = {$abww^Rb$: w î {a, b}* }

{S → abAb, A → aAb, A → bAa}

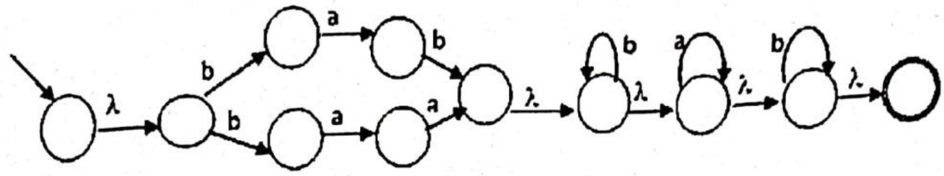
{S → abA, S → Ab, A → aAa, A → bAb, A → λ }

{S → abA, A → aAa, A → bAb, A → b}

{S → abAb, A → aAa, A → bAb, A → λ }

. The regular expression (bab+baa) (bab)* is equivalent to

)



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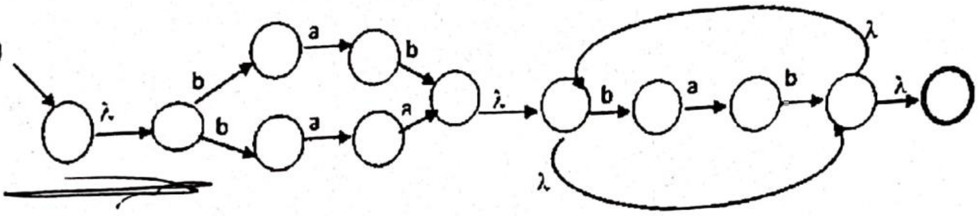

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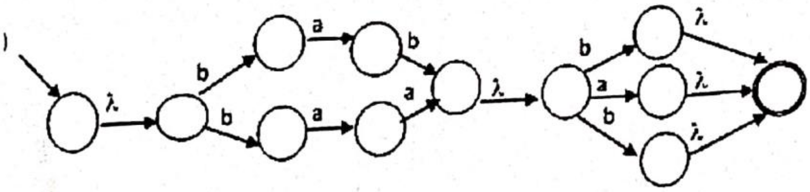

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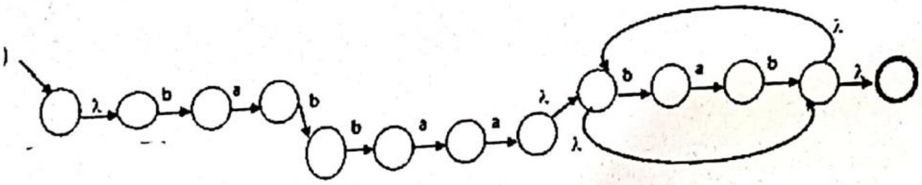


. Which of the following language is regular?

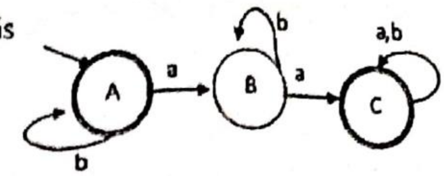
{a^i b^i : i>=0}

{a^i b^j :i >j, i >= 100}

{a^i b^i : 100 >= i >=0}

none of the mentioned

Consider the following DFA, the equivalent regular grammar is



G = ({A, B, C}, {a, b}, A, P), P = {A → Ba, A → bA, B → Bb, B → Ca , C → Ca, C → bC}

G = ({A, B, C}, {a, b}, A, P), P = {A → Ba, A → bA,
A → λ ,B → Bb, B → Ca , C → Ca, C → bC,
C → λ }

G = ({A, B, C}, {a, b}, A, P), P = {A → aB, A → bA, B → bB, B → aC , C → aC, C → bC}

G = ({A, B, C}, {a, b}, A, P), P = {A → aB, A → bA, A → λ , B → bB, B → aC , C → aC, C → bC, C → λ }

The regular expression (a +c b*) represents the language:

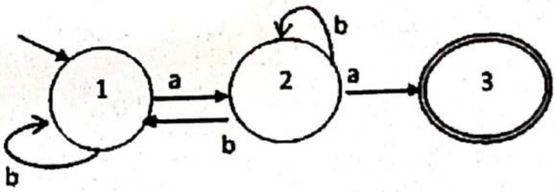
) {a,b,c,cb,cbb,cbbb,....}

) {a,c,cb,cbb,cbbb....}

) {a, λ ,cb,cbcb,cbcbcb,....}

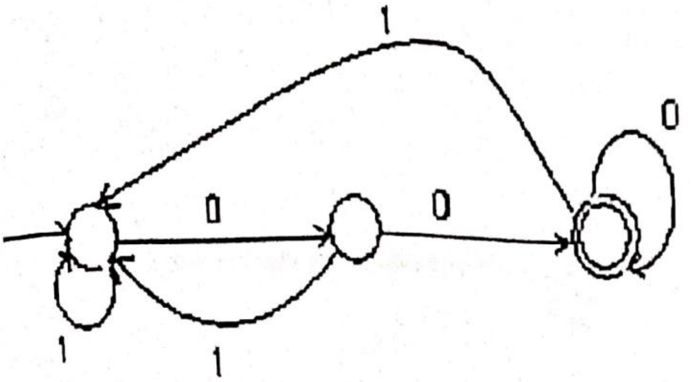
) {λ ,acb,acbacb,acbacbacb,...}

The following NFA is equivalent to the regular expression



b*ab*bb*a b*a
b*ab*(bb*a+b)*a
b*a(bb*a+b)*a
none of the mentioned

The DFA shown below accepts the set of all strings over {0, 1} that



End with 00

End with 0

Begin either with 0 or 1

Contain the substring 00

The regular expression (a +c b*) represents the language:

{a,b,c,cb,cbb,cbbb,....}

{a,c,cb,cbb,cbbb....}

{a, λ ,cb,cbcb,cbcbcb,....}

{λ ,acb,acbacb,acbacbacb,...}

If we select a string w such that w ∈ L, and w = xyz. Which of the following portions cannot be an empty string?

x

y

z

all of the mentioned

Let A = {1,2,3}, B = {a,b,c,d} and let R = {(1,a), (1,b), (2,c), (3,d)}

R is one-to-one function

R is onto function

R is bijective function

R is not a function

Let R be the relation in the set {1, 2, 3} is given by R = {(1, 1), (1,3), (3, 3),(2,2),(3,1)}.

R is reflexive, symmetric, but not transitive

R is reflexive, transitive, but not symmetric

R is transitive, but not reflexive and not symmetric

R is an equivalence relation.

Consider the languages $L = \varphi$ and $M = \{1\}$. Which one of the following represents $L^* \cup L^* M^*$?

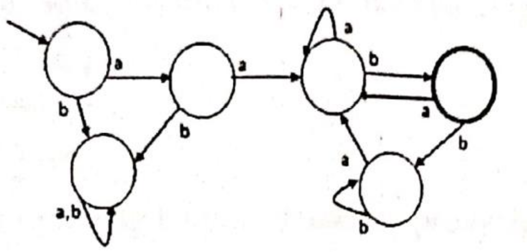
) $\{\varepsilon\}$

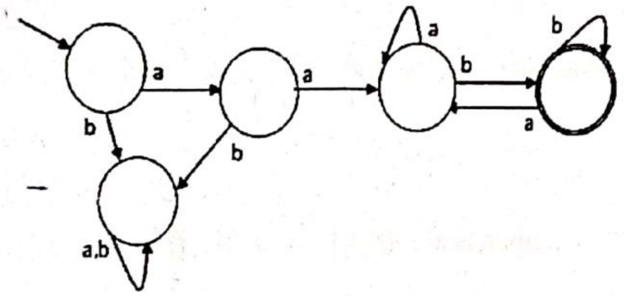
) $\{\varepsilon, 1\}$

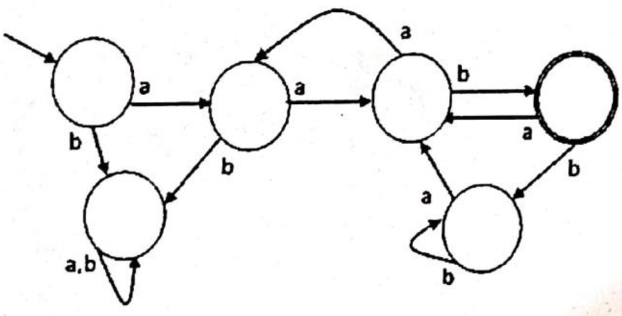
) φ

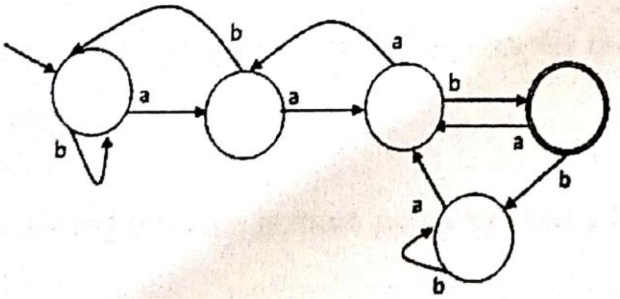
) 1^*

The DFA that accepts the language L= {w $\in$ {0,1}* | w begin with two consecutive a's and end with exactly one b}.







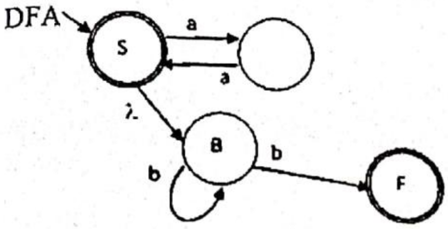


Every finite automata can be converted into finite state machine and every finite state machine can be converted into finite automata.

True

False

The grammar $G = (\{S, B\}, \{a, b\}, S, \{S \to aaS, S \to B, S \to \lambda, B \to bB, B \to b\})$ is equivalent to the following DFA



True
False

if $\Sigma = \{Ab, cD, b,a\}$, then the length of the string "aAbbacDAbb" is 10

7

True
False

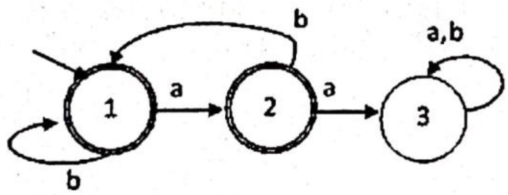
The pumping lemma is used to prove that a given language is regular.

True
False

Let $f(x) = 2x - 1$ and $g(x) = x + 2$ be functions on N then $(g \circ f)(x) = 2x+3$

True
False

The following DFA accepts the language $L = \{w \in \{0,1\}^*: w \text{ has odd number of a's}\}$.



True
False

The regular expression $[(0+1)^* \phi]$ denotes the language ϕ

True
False

The Turing machine is more powerful than a PDA and NFA.

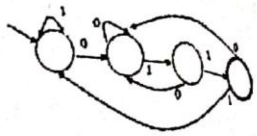
True
False

$G = (\{S, A, B\}, \{a, b\}, S, \{S \to AB, A \to aA, A \to \lambda, B \to bB, B \to \lambda\})$ is a regular grammar.

True

False

The given DFA accepts the language $L = \{w \in \Sigma^* \mid w \text{ ends with } 011\}$

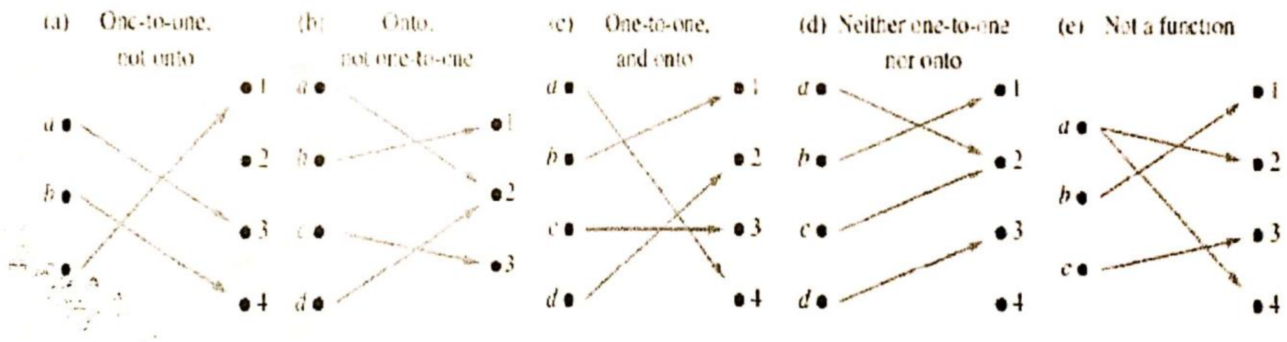
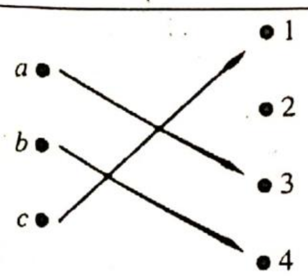


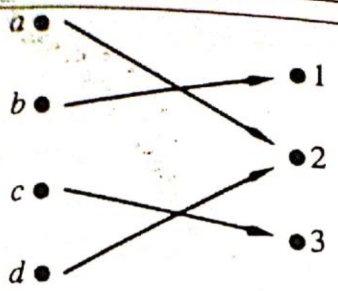
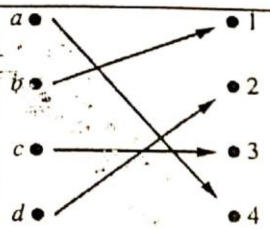
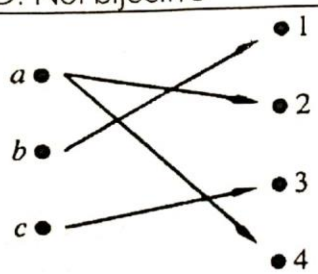
True

False

Automata - Revision (1) - 2020 - Set Operations

١	Example(1) Let: U = {a,b,c,d,e,f,g,h,i,j}, A = {a,b,c}, B = {a,f,g}, C = {h,i,f}, Find: • A ∪ B = {a,b,c,f,g} • A ∩ C = {} • A - B = {b, c} • A' ∪ B' = {d,e,f,g,h,i,j } ∪ {b,c,d,e,h,i,j} = {b,c,d,e,f,g,h,i,j } • (A ∪ B) ' = {d,e,h,i,j} • |B| = 3, |U| = 10 • P(A) = { Φ , {a}, {b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c}}, $\|P(A)\| = 8 = 23 = 2^{\|A\|}$	
٢	Example(2) Find the Cardinality for: ١. Φ = null = 0 ٢. {a, b} = 2 ٣. {1,2,3,4,5,6} = 6 ٤. {Φ} = 1 ٥. {{}} = 1	
٣	Let the set A is {1, 2, 3} and B is {2, 3, 4}. Then the number of elements in A ∪ B is? a) 4 b) 5 c) 6 d) 7	A
٤	Let the set A is {1, 2, 3} and B is {2, 3, 4}. Then number of elements in A ∩ B is? a) 1 b) 2 c) 3 d) 4	B
٥	The intersection of the sets {1, 2, 5} and {1, 2, 6} is the set ____________ a) {1, 2} b) {5, 6} c) {2, 5} d) {1, 6}	A
٦	Two sets are called disjoint if there ____________ is the empty set. a) Union b) Difference c) Intersection d) Complement	C
٧	Which of the following two sets are disjoint? a) {1, 3, 5} and {1, 3, 6} b) {1, 2, 3} and {1, 2, 3} c) {1, 3, 5} and {2, 3, 4} d) {1, 3, 5} and {2, 4, 6}	D

٨.	The difference of {1, 2, 3} and {1, 2, 5} is the set ____________ a) {1} b) {5} c) {3} d) {2}	C
٩.	The complement of the set A is _____________ a) A – B b) U – A c) A – U d) B – A	B
١٠.	What is the Cardinality of the Power set of the set {0, 1, 2}? a) 8 b) 6 c) 7	A
١١.	7. If A is {{Φ}, {Φ, {Φ}}}, then the power set of A has how many element? a) 2 b) 4 c) 6 d) 8	B
١٢.	Example (3):  	
١٣.	 a. onto b. into c. one to one d. one one onto	C

١٤.	 a. onto b. into c. one to one d. one one onto	A
١٥.	 A. bijective B. injective C. surjective D. composite function	A
١٦.	 A. bijective B. injective C. surjective D. Not bijective	D
١٧.	 A. bijective B. injective C. surjective D. Not a function	D

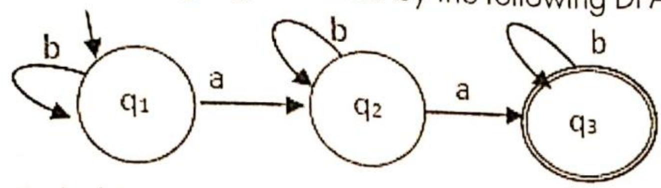
١٨	Surjective function is also called --------. A. onto B. into C. one to one D. one one onto	A
١٩	One to one onto function is also called ----------. A. bijective B. injective C. surjective D. composite function	A
٢٠	Suppose there are 50 people in a room. Then, at least how many people must have their birthday in the same month? a. 50/12 b. 12/50 c. 5 d. 4	C
٢١	Let A = {0; 1; 2; 3} and R a relation over A: R = {(0; 0); (0; 1); (0; 3); (1; 1); (1; 0); (2; 3); (3; 3)} a. R is reflexive, not symmetric, and not transitive b. R is not reflexive, not symmetric, and transitive c. R is not reflexive, symmetric, and not transitive d. R is not reflexive, not symmetric, and not transitive	B
٢٢	Let f(x) = 2x + 3 and g(x) = 3x + 2 be functions on N. What is (g o f)(x)? a. (g o f) (x) = 6x + 11. b. (g o f) (x) = 6x + 7. c. (g o f) (x) = 5x + 5. d. (g o f) (x) = 6x + 5.	A
٢٣	Let L={a,b,c} , w=abb and u=bcaa, Find wu a. abbbcaa b. bcaaabb c. abb+bcaa d. bcaa+abb	A
٢٤	Let L={a,b,c} , w=abb and u=bcaa, Find |wu| a. 7 b. 3 c. 4	A
٢٥	Let L={a,b,c} , w=abb and u=bcaa, Find $(wu)^r$ a. abbbcaa b. bcaaabb c. aacbbba d. bcaa+abb	C
٢٦	If A = {a, ba}, which of the following strings is NOT in A*: a. bb	A

b. λ
c. aa
d. baa

٢٧. If Σ = {ab, c}, u = abcc and v = cab, then , |u2 v| equals
a. 11
b. 8
c. 7
d. 5

A

٢٨. What is the language defined by the following DFA:



a. {babab}
b. {bn a bn a bn, n)0}
c. {bn a bm a, m,n)0}
d. {bn a bm a bk, m,n,k)0}

Automata – Revision (1) – 2020 – Regular Expression

٢٩. Which of the following does not represents the given language? Language: {0,01}
a) 0+01
b) {0} U {01}
c) {0} U {0}{1}
d) {0} {01}

D

٣٠. According to the given language, which among the following expressions does it correspond to? Language L={x {0,1}|x is of length 4 or less}

a)$(0+1+0+1+0+1+0+1)^4$
b)$(0+1)^4$
c)$(01)^4$
d) $(0+1+\varepsilon)^4$

D

٣١. Which among the following looks similar to the given expression? ((0+1). (0+1)) *
a) {x {0,1} *|x is all binary number with even length}
b) {x {0,1} |x is all binary number with even length}
c) {x {0,1} *|x is all binary number with odd length}
d) {x {0,1} |x is all binary number with odd length}

A

٣٢. Concatenation Operation refers to which of the following set operations:
a) Union
b) Dot
c) Kleene
d) Two of the options are correct

B

٣٣. Concatenation of R with Φ outputs:
a) R
b) Φ

B

	d) None of the mentioned	
٣٤.	RR^* can be expressed in which of the forms: a) R^+ b) R^- c) $R^+ \cup R^-$ d) R	A
٣٥.	Which among the following are incorrect regular identities? a) $\varepsilon R=R$ b) $\varepsilon^*=\varepsilon$ c) $\Phi^*=\varepsilon$ d) $R\Phi=R$	D
٣٦.	$(0+\varepsilon)(1+\varepsilon)$ represents a) {0, 1, 01, ε} b) {0, 1, ε} c) {0, 1, 01 ,11, 00, 10, ε} d) {0, 1}	A
٣٧.	Regular Expression R and the language it describes can be represented as: a) R, R(L) b) L(R), R(L) c) R, L(R) d) All of the mentioned	C
٣٨.	Let for [= {0,1} R= ([[[) *, the language of R would be a) {w | w is a string of odd length} b) {w | w is a string of length multiple of 3} c) {w | w is a string of length 3} d) All of the mentioned	B
٣٩.	If [= {0,1}, then Φ^* will result to: a) ε b) Φ c) [d) None of the mentioned	A
٤٠.	The finite automata accept the following languages: a) Context Free Languages b) Context Sensitive Languages c) Regular Languages d) All the mentioned	C
٤١.	Which of the following regular expressions represents the set of strings which do not contain a substring 'rt' if [= {r, t} a) $(rt)^*$ b) $(tr)^*$	D

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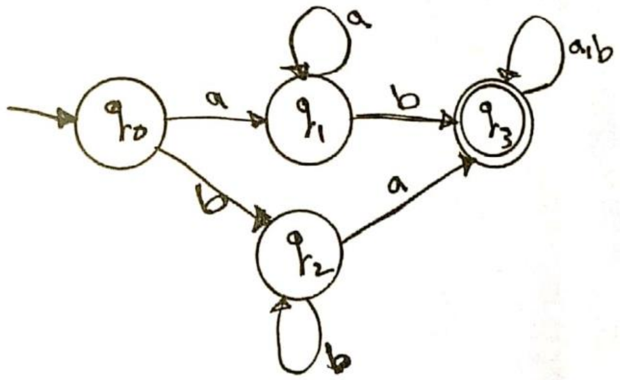
	c) (r*t*) d) (t*r*)	
٤٢.	Regular expression for all strings starts with ab and ends with bba is. a) aba*b*bba b) ab(ab)*bba c) ab(a+b)*bba d) All of the mentioned	C
٤٣.	There are ________ tuples in finite state machine. a) 4 b) 5 c) 6 d) unlimited	B

(VI) Design a DFA and state its formal description that accept the following languages :-

(1) The language that contain all strings in the alphabet $\{a,b\}^*$ which contain the substring "ab" or "ba".

Ans.

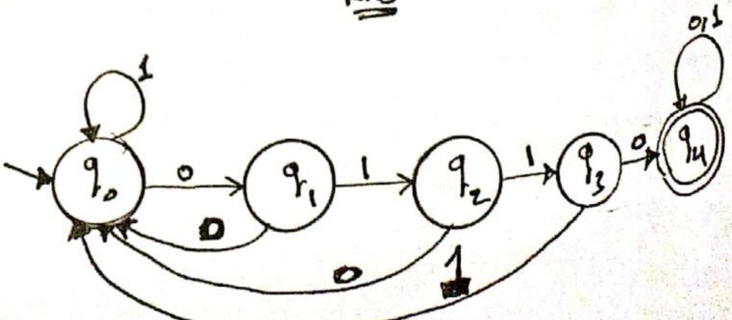


* Formal description :-

δ	a	b
q_0	q_1	q_2
q_1	q_1	q_3
q_2	q_3	q_2
q_3	q_3	q_3

(2) The language that contain all the strings in the alphabet $\{0,1\}^*$ which contains "0110" as substring.

Ans.

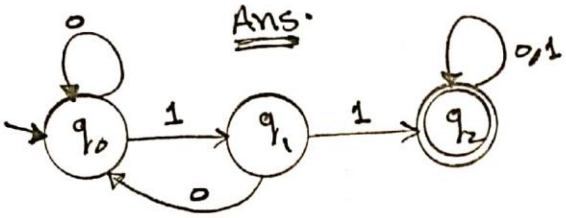


al description:-

δ	0	1
q_0	q_1	q_0
q_1	q_0	q_2
q_2	q_0	q_3
q_3	q_4	q_0
q_4	q_4	q_4

(3) The Language that Contain all the strings in the alphabet $\{0,1\}^*$ which Contain at least two Consecutive 1's.

Ans.

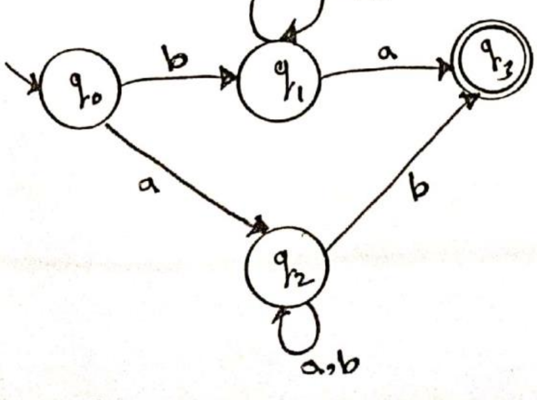


* The formal description:-

δ	0	1
q_0	q_0	q_1
q_1	q_0	q_2
q_2	q_2	q_2

NFA & Convert NFA to DFA

* EX: Convert From NFA to DFA:-



Ans.

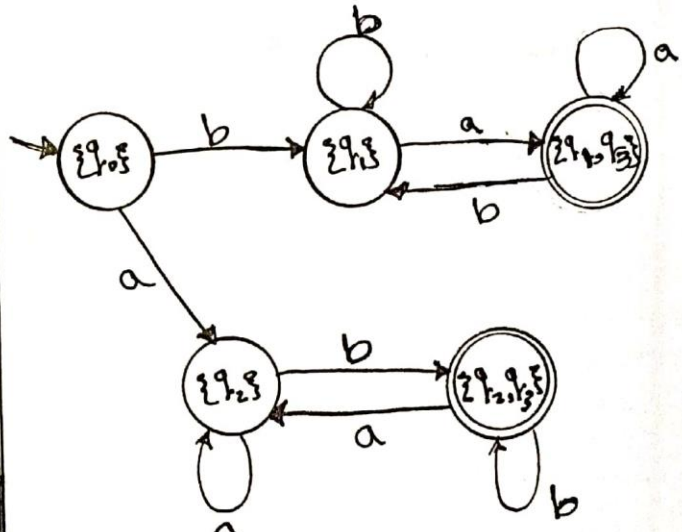
*الخطوة الأولى :-

δ	a	b
q_0	$\{q_2\}$	$\{q_1\}$
q_1	$\{q_1, q_3\}$	$\{q_1\}$
q_2	$\{q_2\}$	$\{q_2, q_3\}$
q_3	Φ	Φ

*الخطوة الثانية :-

δ	a	b
start → $\{q_0\}$	$\{q_2\}$	$\{q_1\}$
$\{q_2\}$	$\{q_2\}$	$\{q_2, q_3\}$
$\{q_1\}$	$\{q_1, q_3\}$	$\{q_1\}$
$\{q_2, q_3\}$	$\{q_2\}$	$\{q_2, q_3\}$
$\{q_1, q_3\}$	$\{q_1, q_3\}$	$\{q_1\}$

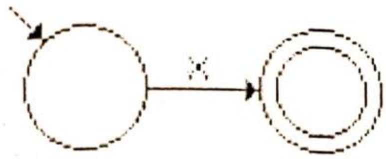
*الخطوة الثالثة :-



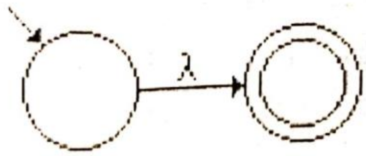
*لازم نحدد ال final state :-

اي state هيظهر فيها q_3 هتبقى Final.

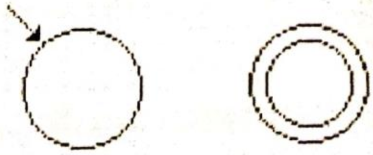
- **From Primitive Regular Expressions to NFAs**



nfa for x

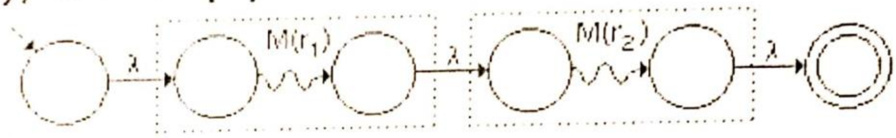


nfa for λ



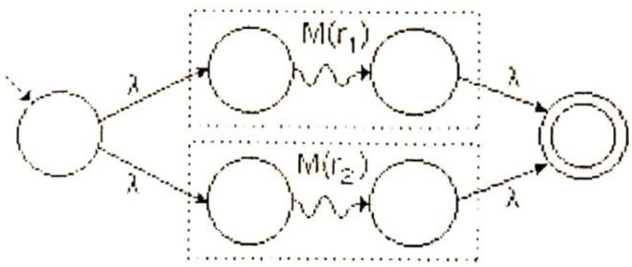
nfa for ∅

- **From Regular Expressions to NFAs**
  - For **concatenation** (strings in L(r1) followed by strings in L(r2), we simply chain the NFAs together, as shown.



nfa for $r_1 r_2$

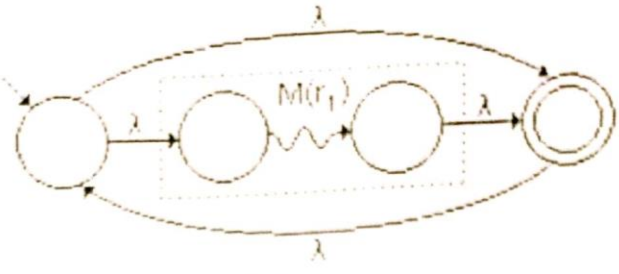
  - The **+** denotes "**or**" in a regular expression, so it makes sense that we would use an NFA with a choice of paths. (This is one of the reasons that it's easier to build an NFA than a DFA.)



nfa for $r_1 + r_2$

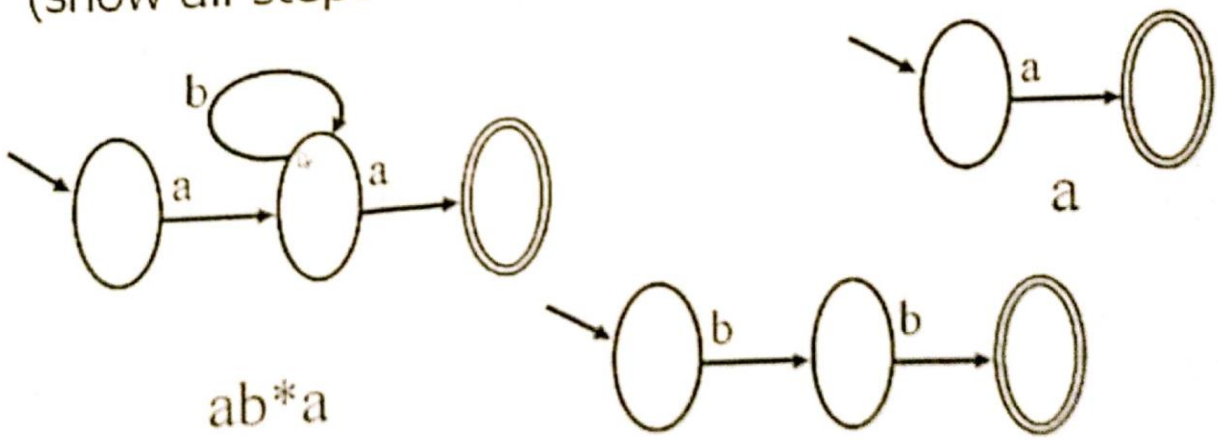
- The **star** denotes zero or more applications of the regular expression, so we need to set up a loop in the NFA.

  We can do this with a backward-pointing and a forward-pointing λ arc to bypass the NFA entirely.



nfa for r_1^*

- **Example**
- Construct an NFA equivalent to the regular expression (show all steps clearly): "ab*a + (a + bb)*"

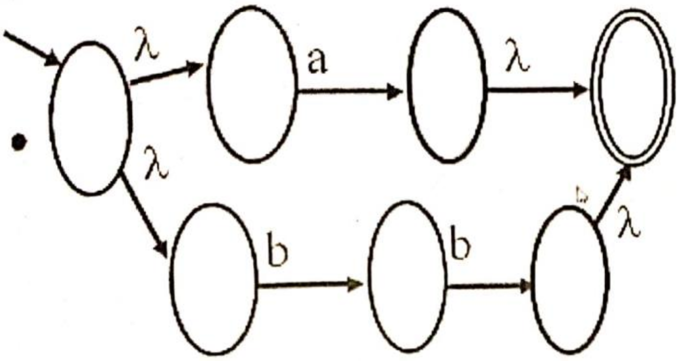


ab*a

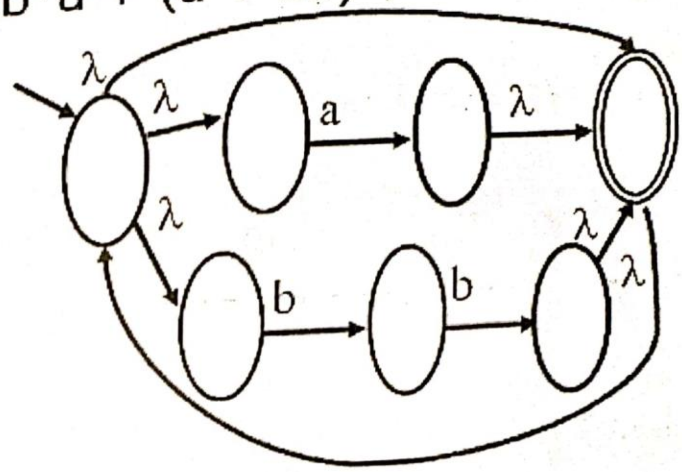
a

bb

- **Example**
- Construct an NFA equivalent to the regular expression (show all steps clearly): "ab*a + (a + bb)*"

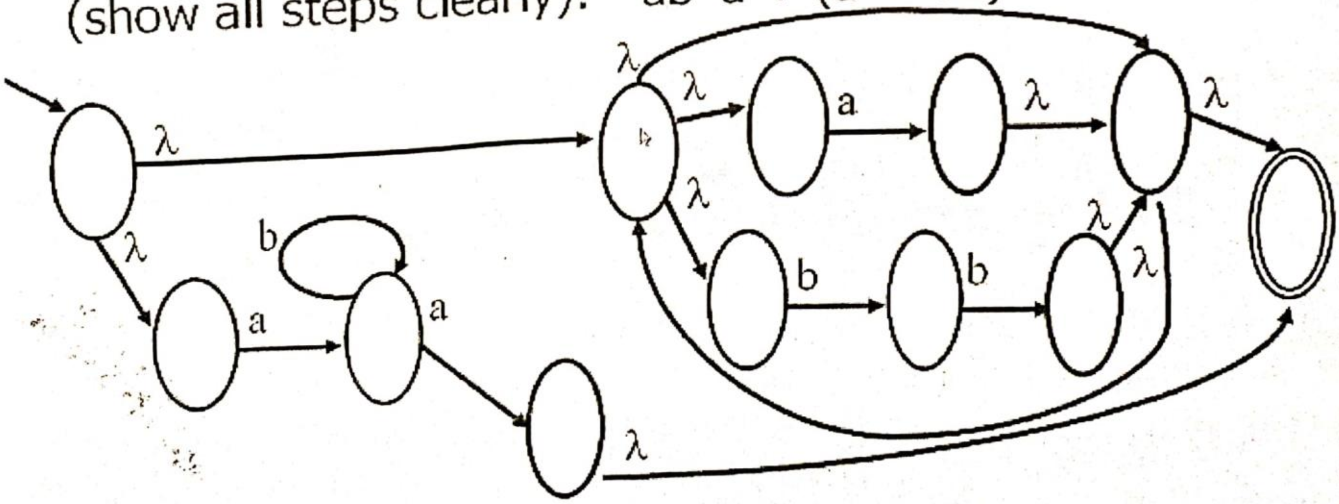


a + bb



(a + bb)*

- **Example**
- Construct an NFA equivalent to the regular expression (show all steps clearly): "ab*a + (a + bb)*"



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Automata - Final - 2011-2012 - Model Answer

Please check the answer carefully; if you have a correction, please share with your friends

Ahmed Hagag

Question 1: Choose the correct answer: **(10 Marks)**

1) Let |B| = 5, then |B x B| has:
   - $\boxed{\text{a) 25 elements}}$ b) 10 elements c) 50 elements
2) If |A| = 3, then the power set of A has:
   - a) 6 elements b) 9 elements $\boxed{\text{c) 8 elements}}$
3) Choose a correct function from A to A, where A = {a, b, c}.
   - $\boxed{\text{a) f = \{ (a, b), (b, c), (c, b)\}}}$
   - b) f = { (a, b), (b, c), (c, a), (c, b)}
   - c) f = { (a, b), (b, c), (c, b), (a, c)}
4) The function f: R → R, where f(x) = x^2 is bijective.
   - a) True $\boxed{\text{b) False}}$
5) If f: A → B is invertible, then f is onto.
   - $\boxed{\text{a) True}}$ b) False
6) Find L in : {a , aa, ab} L = {ab, aab, abb, aa, aaa, aba}
   - $\boxed{\text{a) L = \{a, b\}}}$
   - b) L = {b, ab}
   - c) L = {ba, λ}
7) Let G = < {S, B}, {a, b}, P, S> , where P is: S →λ | a S | B, B→ b | bB, then
   - $\boxed{\text{a) G is a regular grammar.}}$
   - b) G is context-free grammar
8) Let G = < {S, B}, {a, b, c}, P, S>, where P is given by: S → aBc, B →λ | b B, then
   - a) G is a regular grammar.
   - $\boxed{\text{b) G is a context-free grammar.}}$

9) Let G = < {A, S}, {a, b}, P, S>, where P is: S → b A b, A →aA | λ, then $\lambda \in L(G)$.
   a) True b) False

10) Which of the following sets is not a valid alphabet?
   a) {0, 1} b) {a} c) {0, 1, 2, 3, 4,…}

11) Two grammars are **equivalent** if they generate the same langauge.
   a) True b) False

12) Choose a correct grammar for the language $\{b, bbb, \ldots, b^{2n+1}, \ldots\}$
   a) S →λ | b | bb S b) S → b | bb S c) S → b | bS

13) The langauge $\{a^n b^n c^n : n \geq 1\}$ is context-free.
   a) True b) False

14) The langauge $\{a^n b^n : n \geq 1\}$ is
   a) Regular b) context-free

15) Finite languages are always regular.
   a) True b) False

16) The concatenation of two context-free languages is context-free.
   a) True b) False

17) Find a regular expression to describe the langauge {a, b, c}.
   a) a + b+ c b) abc c) a ∪ b ∪ c

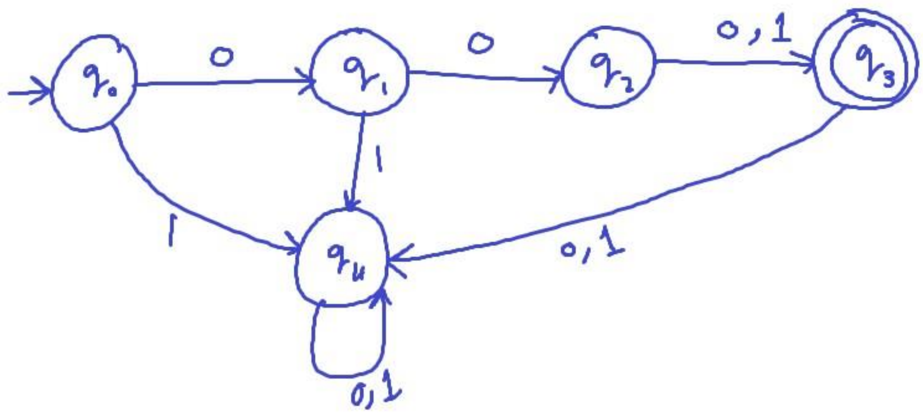
18) Find the langauge described by the regular expression $a^* b^*$.
   a) { λ, a, b, aa, ab, bb, aaa, aab, abb, …}
   b) {a, b, aa, ab, bb, aaa, aab, abb, …}
   c) {λ, ab, abab, ababab, …}

19) Find a regular expression to describe the language { λ, aa, aaaa, aaaaaa, …}
   a) a^* b) $(aa)^*$ c) aa^*

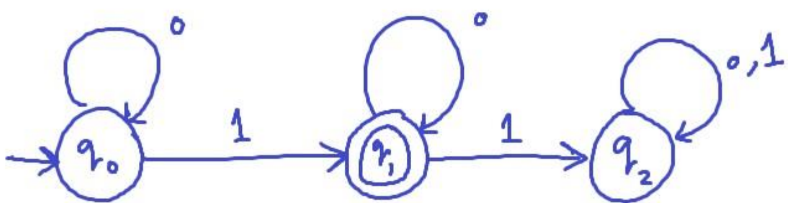
20) Find a regular expression to describe the langauge { aa, aaa, aaaa, aaaaa, ….}
   a) a^* b) $(aa)^*$ c) aaa^*

Question 2: 1) For $\Sigma = \{0, 1\}$, construct the following machines: **(8 Marks)**

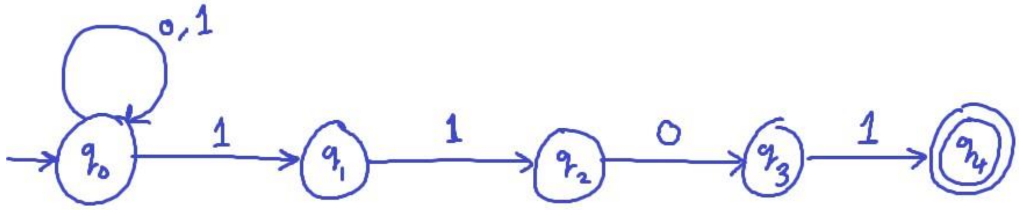
a) DFA that accepts the language $\{000, 001\}$.



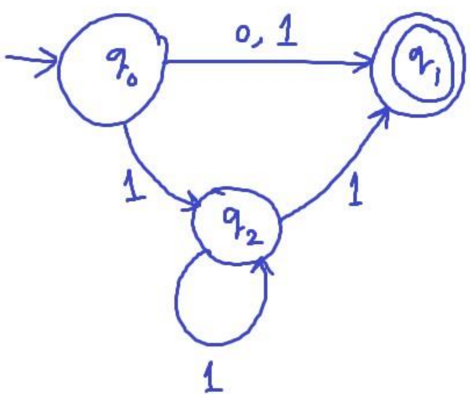
b) DFA that accepts all strings with exactly one 1.



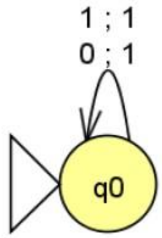
c) NFA that accepts the langauge of all strings *ending* with 1101.



d) NFA without λ-transitions and with a single final state that accepts the language $\{0\} \cup \{1^n : n \geq 1\}$.



2) Describe the parts of the following Mealy machine. **(2 Marks)**



it's output 1 for any Input's 0 or 1

The mealy machine Contains 6 Parts :

$\boxed{1}$ $Q = \{q_0\}$ is the set of states .

$\boxed{2}$ $\Sigma = \{0, 1\}$ is the set of alphabet .

$\boxed{3}$ $\Gamma = \{1\}$ is the set of output symbols .

$\boxed{4}$ $q_0 \in Q$ is the start state .

$\boxed{5}$ δ_1 is the Transition function :

$$\delta_1(q_0, 0) = q_0$$
$$\delta_1(q_0, 1) = q_0$$

$\boxed{6}$ δ_2 is the output function :

$$\delta_2(q_0, 0) = 1$$
$$\delta_2(q_0, 1) = 1$$

<u>Question 3</u>: **(10 Marks)**

1) Write regular expression to describe each of the following languages over the alphabet {a, b}

 a) The langauge of all a's and b's that have at least 2 consecutive a's.

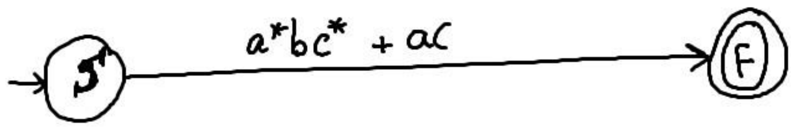
$(a+b)^* aa (a+b)^*$

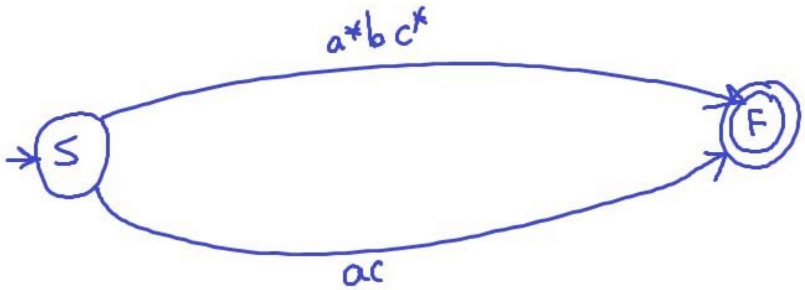
 b) The language of all strings with an even number of a's followed by an odd number of b's.

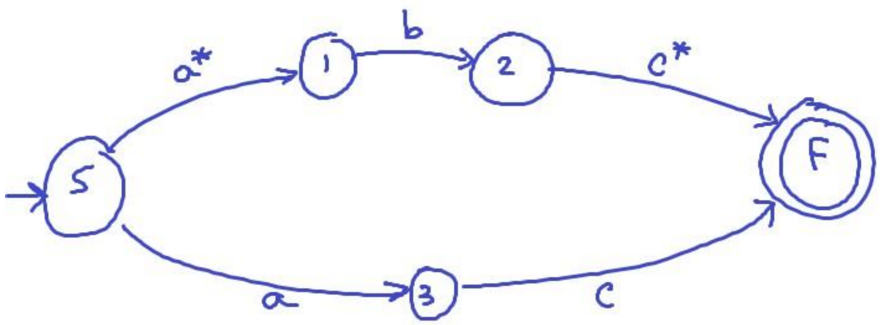
$(aa)^* b (bb)^*$

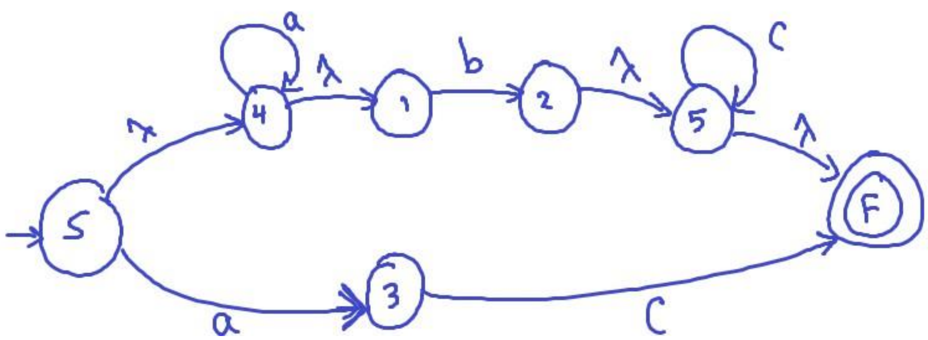
2) Construct NFA for the following regular expression using RE to FA algorithm:

$$a^*bc^* + ac$$

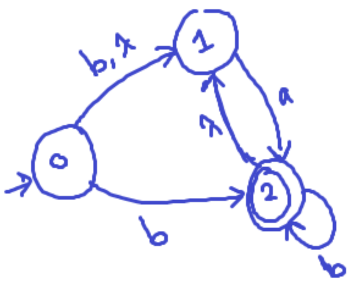








3) Consider the Lambda-NFA given by the table:



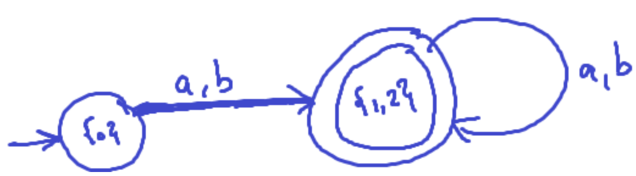
		a	b	λ
Start	0	∅	{1,2}	{1}
	1	{2}	∅	∅
Final	2	∅	{2}	{1}

a) Find the λ-closure of the states of this machine.

state	λ - closure
0	{0, 1}
1	{1}
2	{1, 2}

b) Convert the machine into DFA and draw the graph for the resulting machine.

		a	b
start	{0}	{1,2}	{1,2}
	{1}	{1,2}	∅
Final	{2}	{1,2}	{1,2}
Final	{1,2}	{1,2}	{1,2}



Question 4: **(10 Marks)**

1) Show that the langauge L = $\{a^n b^{n+1} : n \geq 1\}$ is NOT regular.

By using the Pumping Lemma:

(1) we assume that L is regular, and we can represent it by FA with # of state = m.

(2) we select $w \in L$, where $w = a^n b^{n+1}$: $n > m$

we write the string $w = xyz$, where $|xy| \leq m$, $|y| > 0$

$\therefore$ xy must contains a's
y " " "

and $xy^iz \in L$ for $i \geq 0$

(3) if we let $i = 3$

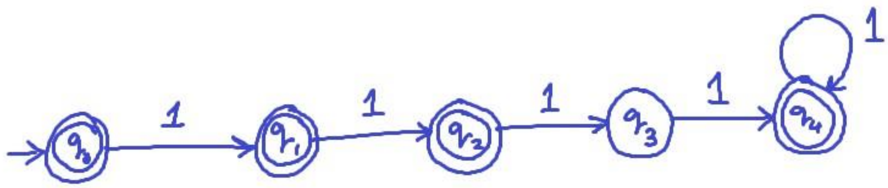
then $xy^3z \in L$, But for this case the number of a's is greater than the number of b's

(4) by Contradiction

$\therefore$ L is Not regular language.

2) Show whether or not the language $\{1^n : n \geq 0, n \neq 3\}$ is regular.

The language is regular , we show that by NFA



3) Write regular grammar to describe the set:

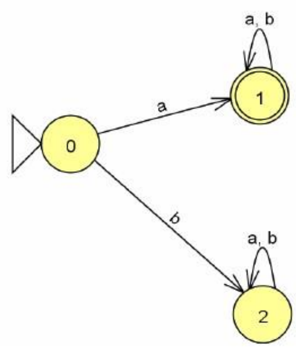
$$\{a^n b^m : m > 0, n > 0\}$$

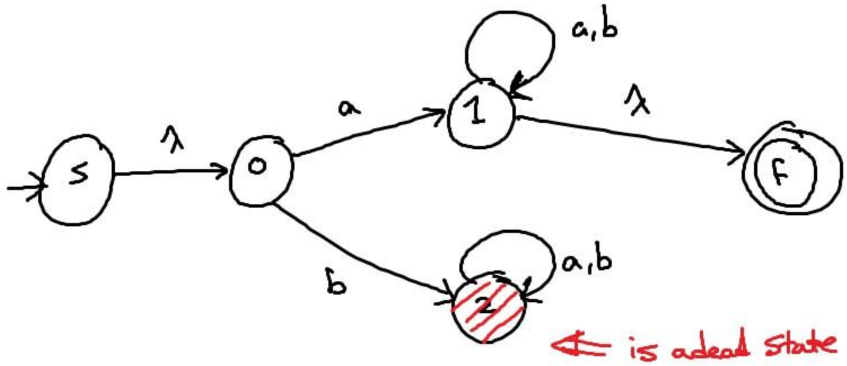
$G = \langle \{a, b\}, \{S, A\}, S, P \rangle$, where P :

$S \longrightarrow aS \mid aA$

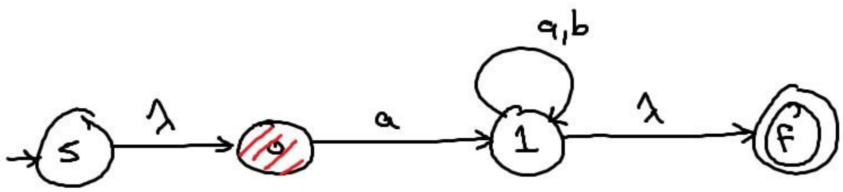
$A \longrightarrow bA \mid b$

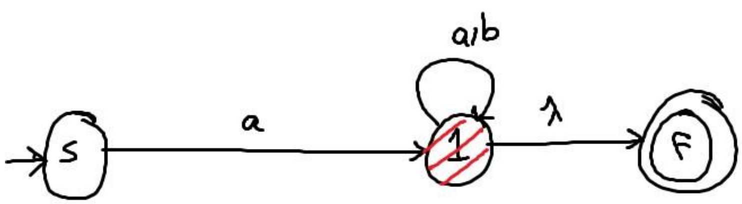
4) Step by step find a regular expression describes the langauge accepted by the following automaton using FA to RE algorithm.

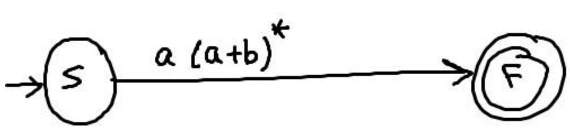




We remove the state 2, because it's a dead state.







R.E. $= a(a+b)^*$

Question 5: **(10 Marks)**

1) Show that the langauge of all palindromes over the alphabet {0, 1} is context-free.

we represent it by a Context-free grammar

$G = < \{0,1\}, \{S\}, S, P >$, where P :

$S \longrightarrow 0S0 \mid 1S1 \mid 0 \mid 1 \mid \lambda$.

2) Show that the function f(n) = n +1 , n ≥ 0 is Turing computable.

we represent the number by uniary form i.e. $\lambda \rightarrow 0$

$1 \rightarrow 1$

$11 \rightarrow 2$

$111 \rightarrow 3$

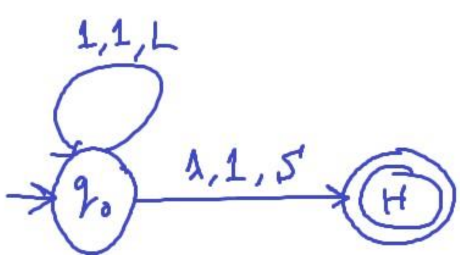
$\vdots$

We write a turing machine that input $(n) \rightarrow w$

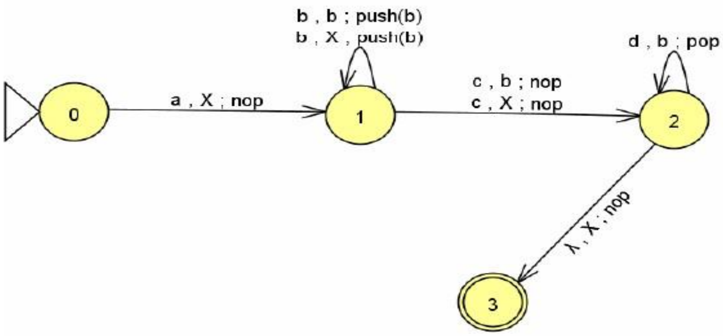
& output $(n+1) \rightarrow w'$

i.e. $f(w) = w'$

$q_0 w \vdash^* w'$



3) Consider the PDA given by the following graph:



a) Write the 7 parts of this PDA.
b) Describe the language accepted by this machine. $L = \{ab^n c d^n : n \geq 0\}$

$\boxed{1}$ $Q = \{0, 1, 2, 3\}$, states.

$\boxed{2}$ $\Sigma = \{a, b, c, d\}$, Alphabet.

$\boxed{3}$ $\Gamma = \{X, b\}$, stack alphabet.

$\boxed{4}$ $0 \in Q$ Start State.

$\boxed{5}$ $X \in \Gamma$ start stack symbol.

$\boxed{6}$ $F = \{3\}$ final set of states.

$\boxed{7}$ I

$\langle 0, a, X, nop, 1 \rangle$

$\langle 1, b, X, push(b), 1 \rangle$

$\langle 1, b, b, Push(b), 1 \rangle$

$\langle 1, c, X, nop, 2 \rangle$

$\langle 1, c, b, nop, 2 \rangle$

$\langle 2, d, b, Pop, 2 \rangle$

$\langle 2, \lambda, X, nop, 3 \rangle$

Made by : Eng/General& Eng/Alaa Nassar

Automata - Final - 2012-2013 - Model Answer

Please check the answer carefully; if you have a correction, please share with your friends

Ahmed Hagag

Faculty of Computers and Information Technology
Course Name: Automata Models
Course code: CAS 205
Instructor: Dr. Azza Taha

EELU
الجامعة المصرية للتعلم الإلكتروني
Egyptian E-Learning University

Year:2012-2013 (Fall semester)
Final Exam. (20-1-2013)
Time allowed: 3 hrs.
Marks: 50

Answer the following questions:

Question 1: Choose the correct answer: **(10 Marks)**

1. Find a correct ONTO function from f: A ⟶ A, where A = {a, b, c}
   a. f = { (a, b), (b, c), (c, c)}
   b. f = { (a, a), (b, c), (c, a)}
   c. f = { (a, c), (b, a), (c, b)}
   d. f = { (b, c), (a, c), (b, c)}
2. If |A| = 3 and |B| = 2, then |A x B| equals
   a. 6 b. 9 c. 4 d. 8
3. The function $f: R \longrightarrow R$, where $f(x) = 3x - 5$ is invertible.
   a. True b. False
4. The function $f: R \longrightarrow R$, where $f(x) = x^2$ is bijective.
   a. True b. False
5. If L = {ab, bb}, which of the following strings is NOT in L^*:
   a. aa b. ab c. abbb d. bb
6. Find L in {λ, a, ab}. L = {b, ab, ba, aba, abb, abba}
   a. L = {b, ba} c. L = { ab, ba}
   b. L = {b, ab} d. L = {λ, b, ba}
7. Let G = < {D,S} , {0,1,2,…,9} , P , S >, where P is: $S \longrightarrow D \mid DS,\ D \longrightarrow 0 \mid 1 \mid ... \mid 9$, which of the following strings is NOT in L(G):
   a. 2123 b. 23abb c. 10110 d. 2013

8. Let G = < {D,S} , {0,1,2,…,9} , P , S > , where P is:

   S $\longrightarrow$ D 1 | D3 | D5 | D7 | D9

   D $\longrightarrow \lambda$ |D0|D 1|D2|D3|D4|D5|D7| D8| D9

   Which of the following strings is NOT in L(G):

   a. 21 b. 22 c. 23 d. 27

9. Choose a *correct* grammar generating the language { λ, ab, abab, …}
   a. S $\longrightarrow$ ab | ab S
   b. S $\longrightarrow \lambda$ | ab S
   c. S $\longrightarrow$ ab | b S a
   d. S $\longrightarrow$ ab S

10. Let G = < {D,S} , {0,1,2,…,9} , P , S > , where P is: $S \longrightarrow D \mid DS,\ D \longrightarrow 0 \mid 1 \mid ... \mid 9,$ then G is a regular grammar.

    a. True b. False

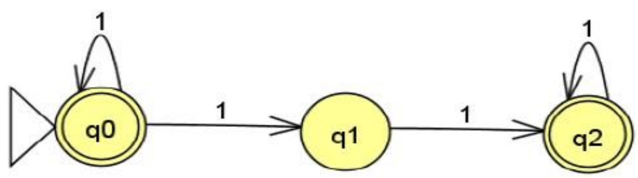
11. The complement of any finite language is a regular language.

    a. True b. False

12. The language $\{a^n b^n : n \geq 0\}$ is context free language.

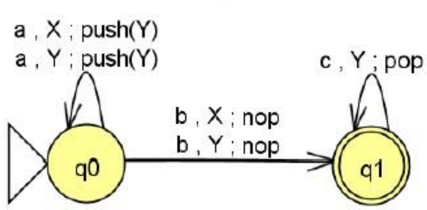
    a. True b. False

13. The language accepted by the following automaton is:



   a. $\{1^n : n \geq 0\}$
   b. $\{1^n : n \geq 0, n \neq 1\}$
   c. $\{1, 11, 111, 1111, \ldots\}$
   d. $\{1^n : n \geq 0\}$

14. The language accepted by the following PDA is:



   a. {abc}
   b. $\{a^n b^n c^n : n \geq 0\}$
   c. $\{a^n b\, c^n : n \geq 0\}$
   d. $\{a^n b c^n : n \geq 1\}$

15. Regular Expressions are algebraic notations used to describe:

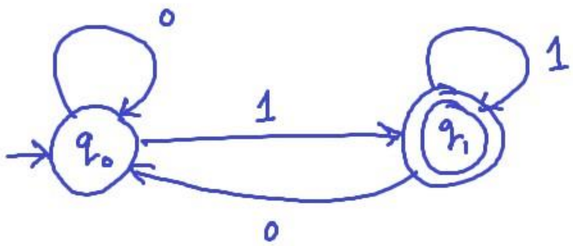
   a. Any Language c. Context-Free Language

   b. Regular Language d. Context-Sensitive Language

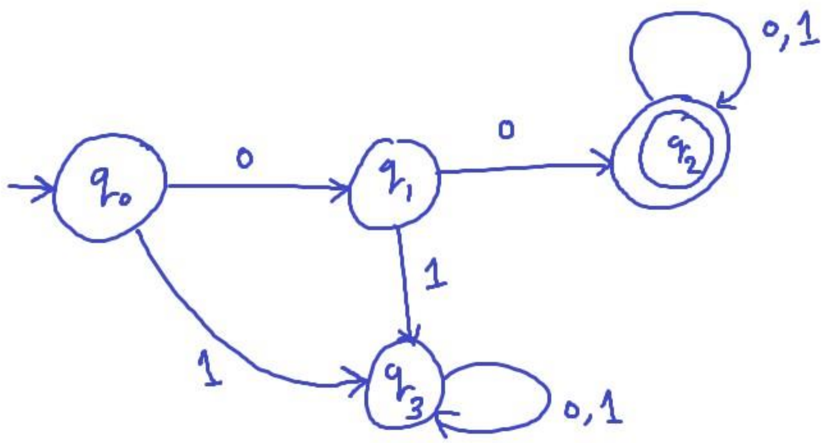
16. The simplest form of the regular expression $\boldsymbol{\lambda} + \mathbf{ab} + \mathbf{ab(ab)^*}$ is:
    a. $\mathbf{ab(ab)^*}$
    b. $\lambda + \mathbf{(ab)^*}$
    c. $\boxed{\mathbf{(ab)^*}}$
    d. $\mathbf{ab + (ab)^*}$
17. Choose a regular expression to describe the language $\{\lambda$, a, aa, aaa, aaaa, aaaaa …$\}$
    a. $\boxed{\mathbf{a^*}}$ b. $\mathbf{aa^*}$ c. $\mathbf{(aa)^*}$ d. $\mathbf{aaa^*}$
18. Choose the correct language described by the regular expression $\mathbf{a^*(a + b)}$
    a. $\{\lambda$, a, b, aa, ab,, aaa, aab, …$\}$
    b. $\boxed{\{a, b, aa, ab, aaa, aab, \ldots\}}$
    c. {a, b, aa, ba, bb, ab, aaa, baa, …}
    d. $\{\lambda$, a, b, aa, ba,, aaa, baa, …$\}$
19. The Pumping lemma for regular languages is used to prove that a given infinite language is NOT regular.
    a. $\boxed{\text{True}}$ b. False
20. Let G = < {A,S} , {a, b } , P , S > , where P consists is $S \longrightarrow bAb$, $A \longrightarrow aA|\lambda$, then the language accepted by this grammar is $L(G) = \{ba^nb : n \geq 0\}$.
    a. $\boxed{\text{True}}$ b. False

Question 2: **(10 Marks)**

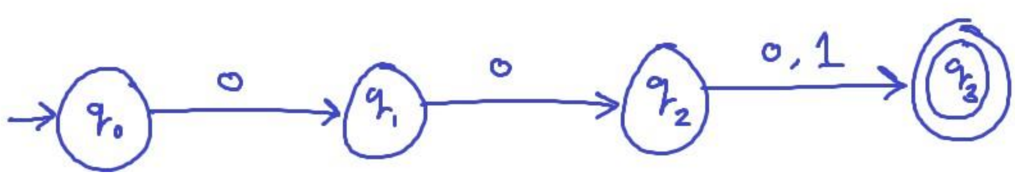
1) Construct the following machines over the alphabet {0, 1}:
   1. DFA that accepts the language of all strings ending 1.



   2. DFA that accepts the language L ={ $00w: w \in \{0,1\}^*$}



   3. NFA that accepts the language L = {001, 000}.

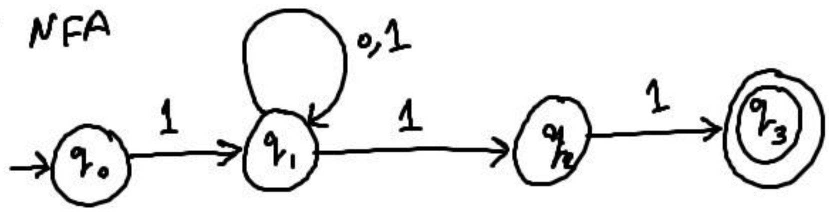


2) Show that the language $\{1w11 : w \in \{0,1\}^*\}$ is regular.

we represent it by R.E.

$$1\,(1+0)^*\,11$$

or NFA



3) Construct grammar for the set $L = \{\, c\, a^n\, b^n \;:\; n \geq 0\}$

$G = \langle \{a,b,c\}, \{S', A\}, S', P \rangle$, where P:

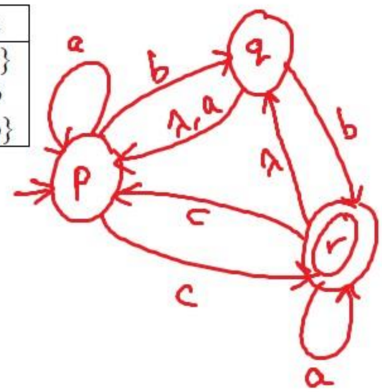
$S' \longrightarrow cA$

$A \longrightarrow aAb \mid \lambda$

Question 3:

1) Consider the NFA given by the following table: **(4 Marks)**

	λ	a	b	c
Start p	ϕ	{p}	{q}	{r}
q	{p}	{p}	{r}	ϕ
Final r	{q}	{r}	ϕ	{p}

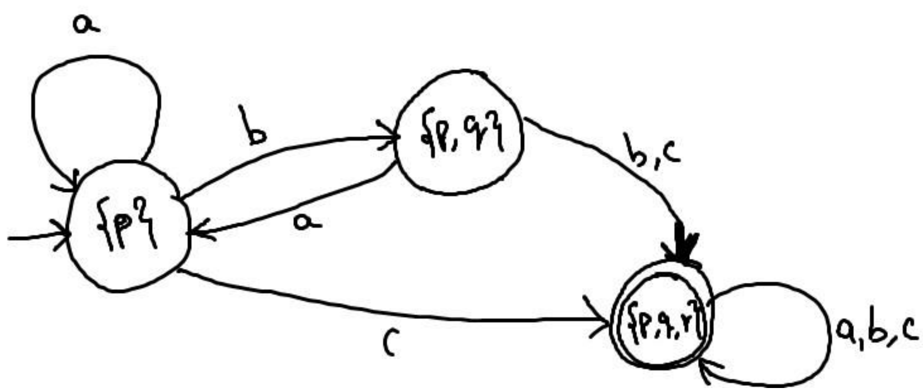


1. Find the lambda closure for all states.

state	lambda closure
p	{p}
q	{p,q}
r	{p,q,r}

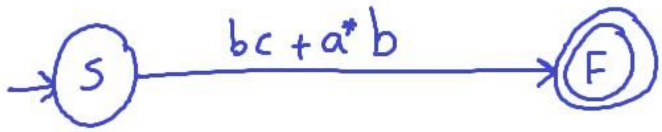
2. Convert the machine into DFA and draw the graph for the resulting machine.

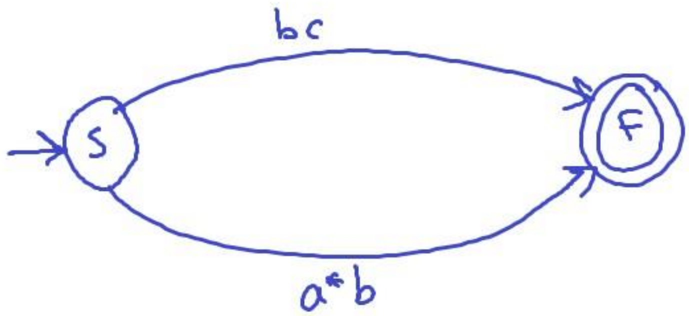
		a	b	c
Start	{p}	{p}	{p,q}	{p,q,r}
	{q}	{p}	{p,q,r}	{p,q,r}
Final	{r}	{p,q,r}	{p,q,r}	{p,q,r}
~~Start~~	{p,q}	{p}	{p,q,r}	{p,q,r}
Final	{p,q,r}	{p,q,r}	{p,q,r}	{p,q,r}

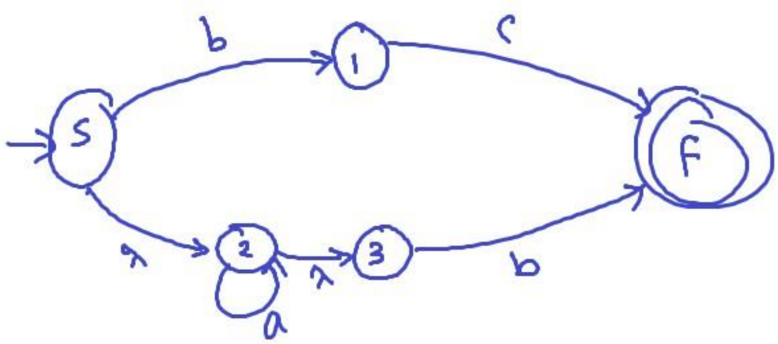


2) Construct NFA for the following regular expression using RE to FA algorithm: **(4 Marks)**

bc + a*b







3) Write regular expression to describe the language of all strings with exactly one *a* over the alphabet {a, b, c}. **(2 Marks)**

$$(b+c)^* a (b+c)^*$$

Question 4:

1) Show that the language $\{a^n b a^n : n \geq 0\}$ is NOT regular. **(3 Marks)**

By using Pumping lemma:

(1) We assume that L is regular, and we can represent it by FA with # of state $= m$.

(2) we select $w \in L$, where

$w = a^n b a^n \; : \; n > m$

we write the string $w = xyz$, where $|xy| \leq m$, $|y| > 0$

$\therefore$ xy must contains a's

& y also must contains a's

$\because$ $xy^i z \in L$ for $i \geq 0$

(3) If we let $i = 4$

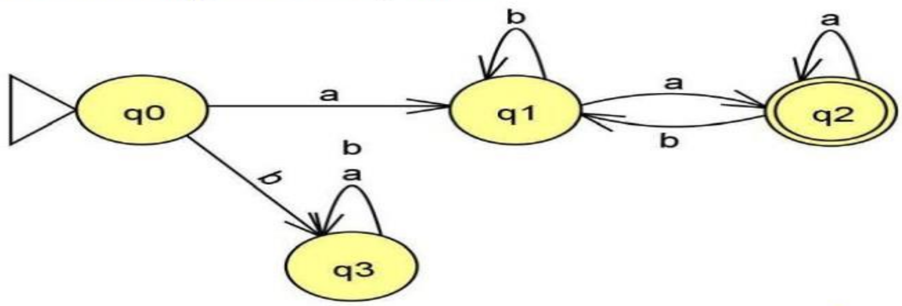
then $xy^4z \in L$, But for this case the number of a's before the $\underline{b}$ is greater than the number of a's after the $\underline{b}$.

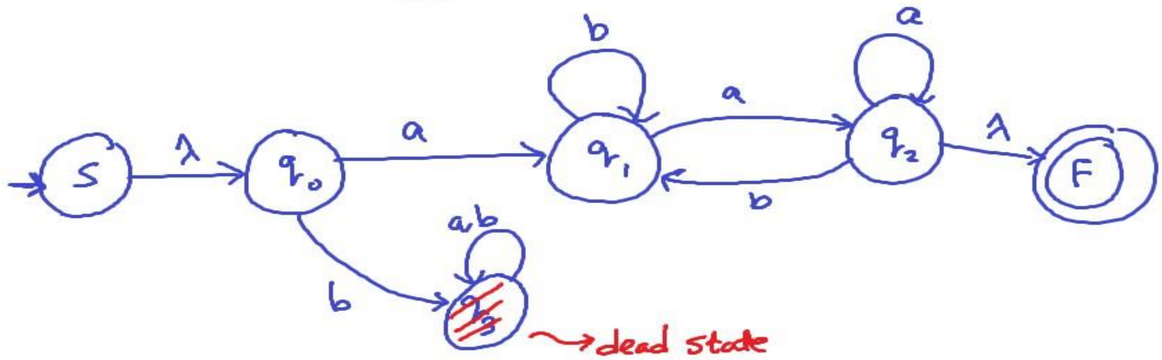
$a^n b a^n$

(4) by Contradiction

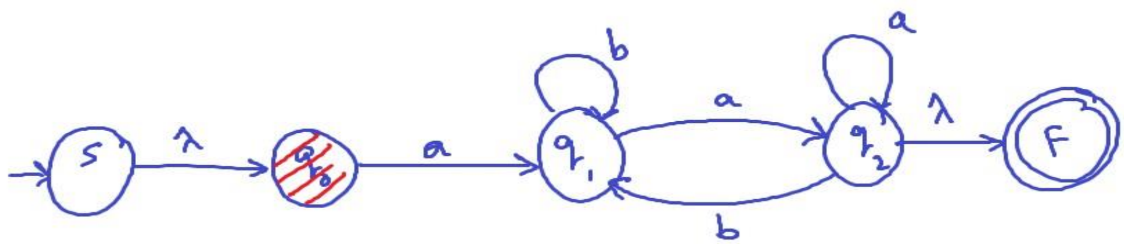
$\therefore$ L is Not regular langaye.

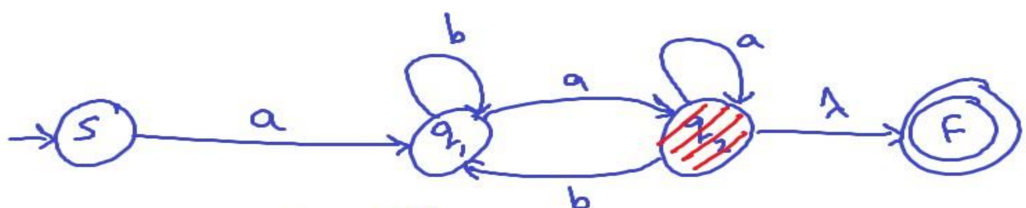
2) Step by step find a regular expression describes the language accepted by the following automaton using FA to RE algorithm: **(4 Marks)**

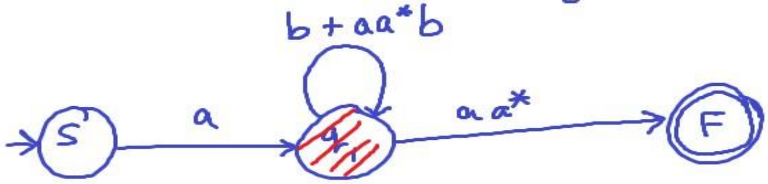


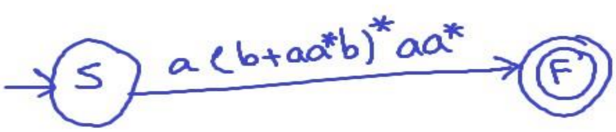


we remove the state q_3 , because it's a dead state.



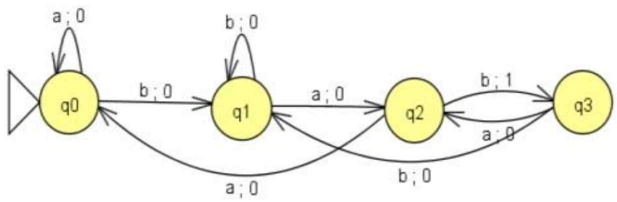






R.E. = $a(b+aa^*b)^*aa^*$

3) Consider the Mealy machine given by the following graph: **(3 Marks)**
   a. Describe the parts of the machine.
   b. Trace the output of this machine for the input *bababab*.



The Mealy machine Contains 6 parts :

1. $Q = \{q_0, q_1, q_2, q_3\}$ is the set of states.
2. $\Sigma = \{a, b\}$ is the set of alphabet.
3. $\Gamma = \{0, 1\}$ is the set of output symbols.
4. $q_0 \in Q$ is the start state.
5. δ_1 is the transition function:

$\delta_1(q_0, a) = q_0$ | $\delta_1(q_2, a) = q_0$
$\delta_1(q_0, b) = q_1$ | $\delta_1(q_2, b) = q_3$
$\delta_1(q_1, a) = q_2$ | $\delta_1(q_3, a) = q_2$
$\delta_1(q_1, b) = q_1$ | $\delta_1(q_3, b) = q_1$

6. δ_2 is the output function:

$\delta_2(q_0, a) = 0$ | $\delta_2(q_2, a) = 0$
$\delta_2(q_0, b) = 0$ | $\delta_2(q_2, b) = 1$
$\delta_2(q_1, a) = 0$ | $\delta_2(q_3, a) = 0$
$\delta_2(q_1, b) = 0$ | $\delta_2(q_3, b) = 0$

b) $w =$ bababab $\Longrightarrow$ output is [0010101]

State	q_0	q_1	q_2	q_3	q_2	q_3	q_2
input	b	a	b	a	b	a	b
output	0	0	1	0	1	0	1

Question 5:

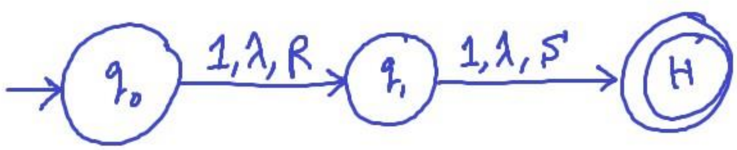
1) Show that the function $f(n) = n - 2$, $n \geq 2$ is Turing computable where the number n is represented in unary form. **(3 Marks)**

$\lambda \rightarrow 0$
$1 \rightarrow 1$
$11 \rightarrow 2$
$111 \rightarrow 3$
$\vdots$

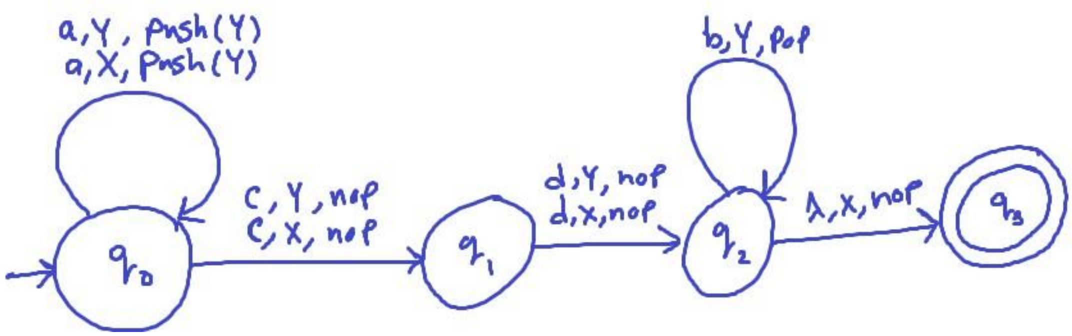
we write a turing machine that take input $(n) \rightarrow w$ & output $(n-2) \rightarrow w'$

i.e. $f(w) = w'$

$q_0 w \vdash^* H w'$



2) Construct PDA to describe the language $\{a^n\, c\, d\, b^n : n \geq 0\}$ **(4 Marks)**



3) Show that the language $\{ww^R : w \in \{0,1\}^*\}$ is context free. **(3 Marks)**

we represent this language by using a Context-free Grammar:

$G = \langle \{0,1\}, \{S\}, S, P \rangle$, where P:

$S \longrightarrow 0S0 \mid 1S1 \mid \lambda$

Faculty of Computers and Information Technology
Course Name: Automata Models
Course code: CAS 205
Instructor: Dr. Azza Taha

EELU
الجامعة المصرية للتعلم الإلكتروني
Egyptian E-Learning University

Year:2013-2014 (Fall Semester)
Final Exam. (16-1-2014)
Time allowed: 3 hrs.
Marks: 50

Answer the following questions:

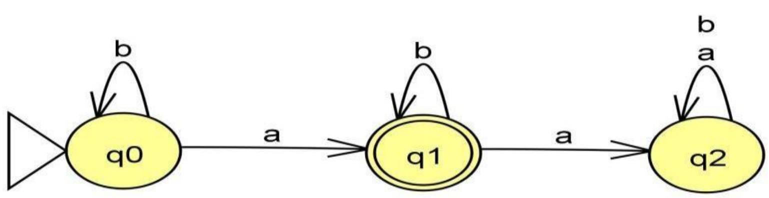
Question 1: Choose the correct answer: **(10 Marks)**

1. If A= {1, 2, 3, 4, 5} and B = {5, 6, 7}, then | A - B| equals
   a. 4 b. 5 c. 7 d. 3
2. Select a bijective function:
   a. f: Z → Z: f (x) = $2\,x$ c. I: R →R: I(x) = x
   b. g: R →R: g(x) = x^2 d. h: Z →{0, 1} : h(x) =0 if x is even and $h(x) = 1$ otherwise
3. If the function f is bijective, then f is invertible.
   a. True b. False
4. If |A| = 3 and |B| = 2, then |A x B| equals
   a. 6 b. 9 c. 4 d. 8
5. If a = c and b = d then (a, b) = (c, d)
   a. True b. False
6. Let A = {a, b, c}, if w is a string over A and |www|= 36, what is the length of w?
   a. 6 b. 12 c. 18 d. 36
7. If L = {aba, ba}, which of the following strings is NOT in L^*:
   a. aa b. bb c. ab d. All of the previous
8. Find L in L.{λ, a} = {λ, a, b, ab, ba, aba}
   a. L = {a, b} c. L = {λ, a, b}
   b. L = {λ, b, ab} d. L = {b, ab}
9. Let G = < {D,S} , {0,1,2,…,9} , P , S > , where P is:
   S → D 0 | D2 | D4 | D6 | D8
   D → λ |D0|D 1|D2|D3|D4|D5|D7| D8| D9
   Which of the following strings is <u>NOT</u> in L(G):
   a. 13 b. 22 c. 24 d. 28
10. Choose a *correct* grammar generating the language { $a, aaa, aaaaa, \ldots, a^{2n+1}, \ldots$ }
    a. S →a | aa S
    b. S →λ | a | aa S
    c. S → a | a S
    d. S → λ | a S
11. The language L = { $a^{n^2}: 1 \leq n \leq 4$} is regular.
    a. True b. False
12. Select a regular language:
    a. $\{1^n: n \geq 0, n \; \mathrm{G} \; 1\}$
    b. $\{a^n\, b^p: n \; \mathrm{G} \; p\}$
    c. $\{a^n\, b^m: n \geq m\}$
    d. $\{\, a^n\, b^n: n \geq 0\}$

13. Regular expressions are algebraic notations used to describe regular languages.
    a. True b. False
14. Choose a regular expression to describe the language {λ, a, b, ca, bc, cca, bcc, …, $c^n a, bc^n$, …}
    a. λ + **bc*a** c. λ + ac^* + **b**c^*
    b. λ+ **c*a** +**b**c^* d. λ + **ac*** + **c**b^*

15. Choose the correct language described by the regular expression ***a*(a + b)***
    a. {λ, a, b, aa, ab,, aaa, aab, …}
    b. {a, b, aa, ab, aaa, aab, …}
    c. {a, b, aa, ba, bb, ab, aaa, baa, …}
    d. { λ, a, b, aa, ba,, aaa, baa, …}

16. Let G = < { S} , {a } , P , S > , where P is: S → λ| aS , then G is a regular grammar.
    a. True b. False
17. The language L = { $a^n b^n c^n: n \geq 0$ } is context-free.
    a. True b. False
18. The concatenation of two context-free languages is context-free.
    a. True b. False
19. Two finite machines M_1 and M_2 are said to be **equivalent** if L (M_1) = L (M_2).
    a. True b. False
20. The language accepted by the following DFA is:
    a. {a} c. { $b^n a b^m: m, n \geq 0$}
    b. { $b^n a b^n: n \geq 0$} d. { $b^n a b^m a: m, n \geq 0$}



<u>Question 2:</u> **<u>(10 Marks)</u>**

1) Construct the following machines over the alphabet {0, 1}:
   1. DFA that accepts the language {00, 01}.
   2. DFA that accepts the language L ={ $01w: w \subset \{0,1\}^*$}
   3. NFA that accepts the language L = {$u00v: u, v \subset \{0,1\}^*$}
   4. NFA without λ-transition and with a single final state that accepts the language: $\{0\} \cup \{1^n: n \geq 1\}$
   5. A Mealy machine that produces an output 1 for any input.

<u>Question 3:</u>

1) Why the NFA is less efficient in recognizing strings than DFA? **(2 Marks)**
2) Consider the NFA given by the following table: **(4 Marks)**

	λ	a	b
Start 0	{1, 2}	{1}	ϕ
1	ϕ	ϕ	{2}
Final 2	ϕ	{2}	ϕ

1. Find the lambda closure for all states.
2. Convert the machine into DFA and draw the graph for the resulting machine.

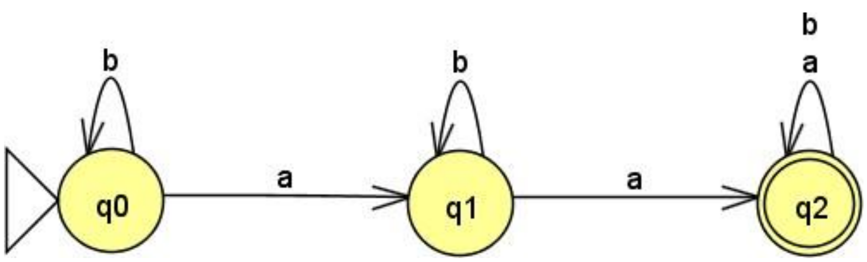
3) Construct NFA for the following regular expression using RE to FA algorithm: **(4 Marks)**

a*b c* + ac

Question 4:

1) Using Pumping lemma for regular languages, show that the language $\{a^n bc^n : n \geq 0\}$ is Not regular. **(3 Marks)**

2) Step by step find a regular expression describes the language accepted by the following automaton using FA to RE algorithm: **(4 Marks)**



3) Construct grammars for the following languages: **(3 Marks)**

a. L = $\{ ba^n b : n \geq 0 \}$

b. M = $\{a^n \ b \ c^n : n \geq 0 \}$

Question 5:

1) Show that the function f(n) = n +1 , n ≥ 0 is *Turing Computable* where the number n is represented in binary form. **(3 Marks)**

2) Construct PDA to describe the language $\{a^{n+2} \ c \ b^n : n \geq 0\}$ **(4 Marks)**

3) Write regular expression to describe each of the following languages: **(3 Marks)**

a. L = $\{a^n b^m, n, m \geq 1\}$

b. M = $\{ a^n b^m, n \geq 2, m \geq 3\}$

c. The set of all strings over the alphabet {a, b} that begin and end with the same letter.

Good Luck

Faculty of Computers and Information Technology
Course Name: Automata Models
Course code: CAS 205
Instructor: Dr. Azza Taha

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   a. L = {a, b} c. L = {λ, a, b}
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10. Choose a *correct* grammar generating the language { $a, aaa, aaaaa, \ldots, a^{2n+1}, \ldots$ }
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    b. λ+ ***c*a*** +***bc****
    c. λ + ac^* + $\boldsymbol{bc}^*$
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15. Choose the correct language described by the regular expression ***a*(a + b)***
    a. {λ, a, b, aa, ab,, aaa, aab, …}
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    c. {a, b, aa, ba, bb, ab, aaa, baa, …}
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16. Let G = < { S} , {a } , P , S > , where P is: S → λ| aS , then G is a regular grammar.
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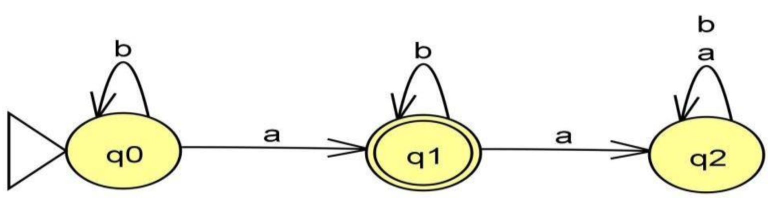
    b. { $b^n a b^n : n \geq 0$}
    c. { $b^n a b^m : m, n \geq 0$}
    d. { $b^n a b^m a : m, n \geq 0$}



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   5. A Mealy machine that produces an output 1 for any input.

Question 3:

1) Why the NFA is less efficient in recognizing strings than DFA? **(2 Marks)**
2) Consider the NFA given by the following table: **(4 Marks)**

	λ	a	b
Start 0	{1, 2}	{1}	ϕ
1	ϕ	ϕ	{2}
Final 2	ϕ	{2}	ϕ

1. Find the lambda closure for all states.
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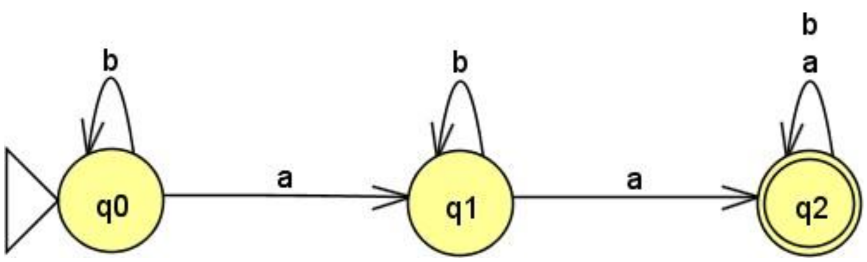
3) Construct NFA for the following regular expression using RE to FA algorithm: **(4 Marks)**

a*b c* + ac

Question 4:

1) Using Pumping lemma for regular languages, show that the language $\{a^n bc^n : n \geq 0\}$ is Not regular. **(3 Marks)**

2) Step by step find a regular expression describes the language accepted by the following automaton using FA to RE algorithm: **(4 Marks)**



3) Construct grammars for the following languages: **(3 Marks)**

a. L = { $ba^n b : n \geq 0$ }

b. M = $\{a^n\ b\ c^n : n\ \geq 0\}$

Question 5:

1) Show that the function f(n) = n +1 , n ≥ 0 is *Turing Computable* where the number n is represented in binary form. **(3 Marks)**

2) Construct PDA to describe the language $\{a^{n+2}\ c\ b^n : n\ \geq 0\}$ **(4 Marks)**

3) Write regular expression to describe each of the following languages: **(3 Marks)**

a. L = $\{a^n b^m, n, m\ \geq 1\}$

b. M = $\{\ a^n b^m, n\ \geq 2, m\ \geq 3\}$

c. The set of all strings over the alphabet {a, b} that begin and end with the same letter.

Good Luck

Made by : Eng/General& Eng/Alaa Nassar

Model Answer

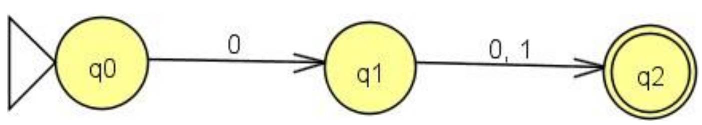
Question 1: Choose the correct answer: **(10 Marks)**

1. If A = {1, 2, 3, 4, 5} and B = {5, 6, 7}, then |A ∩ B| equals
   a. 4 b. 5 c. 1 d. 3
2. The function I: R→ R, defined as I (x) = x, is bijective.
   a. True b. False
3. If the function g is invertible, then it is onto.
   a. True b. False
4. If |A| = 5, then |A x A| equals
   a. 25 b. 10 c. 50 d. 150
5. If L = {ab, bb}, which of the following strings is NOT in L^*?
   a. aa b. ab c. abbb d. bbbb
6. If L = { λ, a, aa, aaa, …} and M = {b, bb, bbb, …}, which of the following strings is NOT in L.M?
   a. aa b. aab c. bb d. abbb
7. Find L in {λ, a, ab}.L = {b, ab, ba, aba, abb, abba}
   a. L = {b, ba} c. L = {ab, ba}
   b. L = {b, ab} d. L = {λ, b, ba}
8. Let G = < {D,S} , {0,1,2,…,9} , P , S > , where P is:
   S → D 0 | D2 | D4 | D6 | D8
   D → λ |D0|D 1|D2|D3|D4|D5|D7| D8| D9
   Which of the following strings is NOT in L(G)?
   a. 15 b. 22 c. 24 d. 28
9. Choose a *correct* grammar describing the language { $a, aaa, aaaaa, \dots, a^{2n+1}, \dots$ }
   a. S →a | aa S
   b. S →λ | a | aa S
   c. S → a | a S
   d. S → λ | a S
10. Select a regular language:
    a. $\{a^n\, b^p : n \neq p\}$
    b. $\{a^n\, b^m : n \geq m\}$
    c. $\{ a^{n^2} : 1 \leq n \leq 4\}$
    d. $\{ a^n\, b^n : n \geq 0\}$
11. If L and M are two regular languages, then
    a. $L \cup M$ is regular c. $\overline{L \cup M}$ is regular
    b. $L \cap M$ is regular d. All of the previous
12. Let G = < { S} , {a, b} , P , S > , where P is: S → λ| abS , then G is a regular grammar.
    a. True b. False

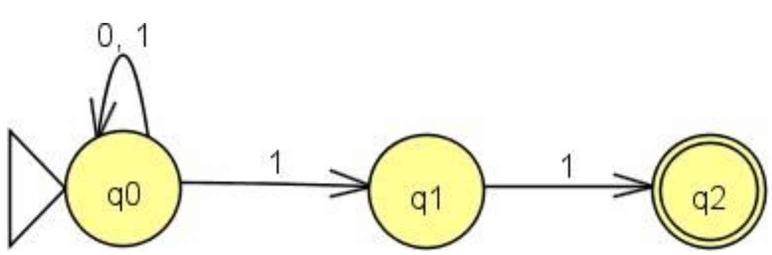
13. Let G = <{S}, {a, b}, P, S>, where P is given by: S → aSa | bSb | a | b | λ, then G is context-free grammar.
    a. True b. False
14. The language L = { $a^n b^n c^n : n \geq 0$ } is context-free.
    a. True b. False
15. If w is a string where $w = a^{m+1} b^{2m}$, then |w| is:
    a. $3m + 1$ b. $2m^2 + 1$ c. $2m^2 + 2m$ d. None of the previous
16. If L and M are two context-free languages, then
    a. $L.M$ is context-free
    b. $L \cup M$ is context-free
    c. $(L \cup M)^*$ is context-free
    d. All of the previous
17. Two finite machines M_1 and M_2 are said to be *equivalent* if L (M_1) = L (M_2).
    a. True b. False
18. Find the regular expression to describe the language {a, b, c}
    a. **abc**
    b. **a + b + c**
    c. $\boldsymbol{a} \cup \boldsymbol{b} \cup \boldsymbol{c}$
    d. $(\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c})^*$
19. Find the regular expression to describe the language $\{1w1 : w \in \{0, 1\}^*\}$
    a. **101**
    b. $\mathbf{1(0+1)^*1}$
    c. **11**
    d. $\mathbf{1(01)^*1}$
20. Find the language described by the regular expression $\mathbf{0 + 1.0^*}$
    a. {0, 1, 10, 100, 1000, …}
    b. {0, 1, 00, 10, 000, 100, …}
    c. {λ, 0, 1, 10, 100, 1000,…}
    d. {0, 1, 00, 000, 0000, …}

Question 2: **(10 Marks)**

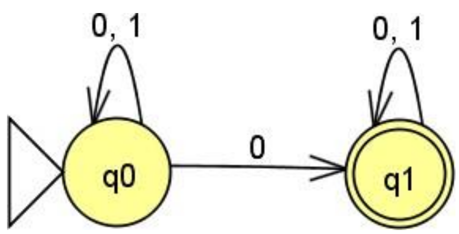
1) Construct the following finite machines, over the alphabet {0, 1}, to define the following:
   1. The language {00, 01}.
      Answer: You can either give NFA or DFA. This NFA accepts {00, 01}:



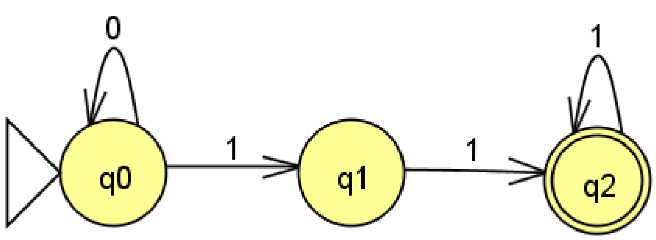
   2. The language $\{w11 : w \in \{0, 1\}^*\}$.
      Answer: NFA:



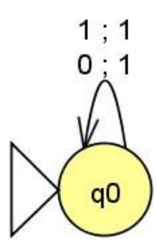
3. The language $\{u0v: u, v \in \{0,1\}^*\}$.
   Answer: NFA:



4. The language $\{0^n1^m: n \geq 0, m \geq 2\}$.
   Answer: NFA



2) Construct a Mealy machine over the alphabet {0, 1} that produces an output 1 for any input.
   Answer:



Question 3:

1) Consider the NFA given by the following table: **(5 Marks)**

	λ	a	b	c
Start p	ϕ	{p}	{q}	{r}
q	{p}	{q}	{r}	ϕ
Final r	{q}	{r}	ϕ	{p}

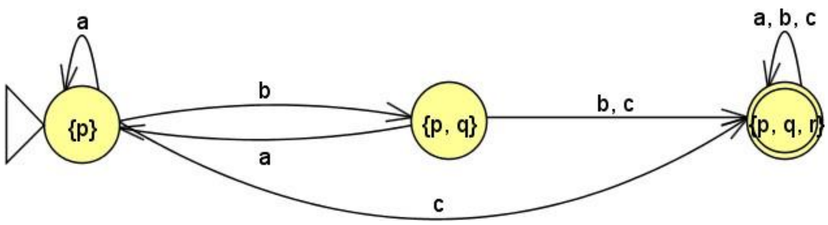
1. Find the lambda closure for all states.

Answer: (3 Marks)

	λ-Clouse
p	{p}
q	{p, q}
r	{p, q, r}

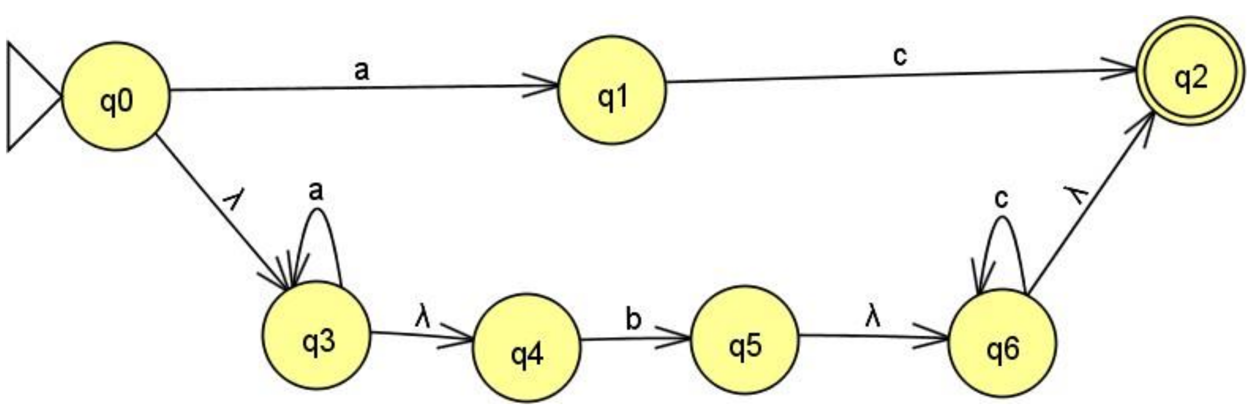
2. Convert the machine into DFA and draw the graph for the resulting machine.
Answer: (2 Marks)

	a	b	c
Start {p}	{p}	{p, q}	{p, q, r}
{p, q}	{p,q}	{p, q, r}	{p, q, r}
Final {p, q, r}	{p, q, r}	{p, q, r}	{p, q, r}



2) Construct NFA for the following regular expression using RE to FA algorithm: **(5 Marks)**
$\boldsymbol{a^*bc^* + ac}$

Answer: Apply the steps of the algorithm in lecture 8, the final machine is:

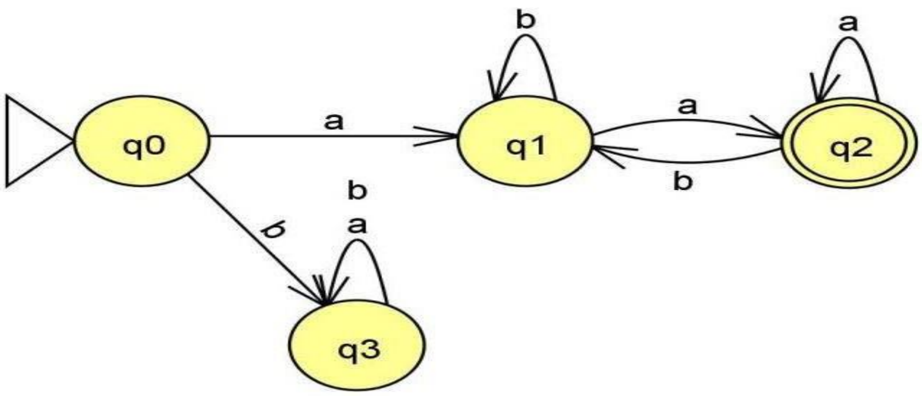


Question 4:

1) Using Pumping lemma for regular languages, show that the language $\{a^n b^{n+1} : n \geq 0\}$ is Not regular. **(3 Marks)**

   Answer: Similar to the examples in lecture 10

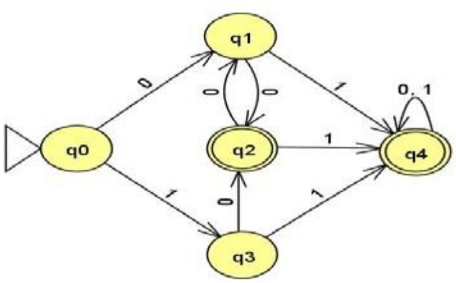
2) Step by step find a regular expression describes the language defined by the following automaton using FA to RE algorithm: **(4 Marks)**



   Answer:

   First we remove the dead state q3, then q0, q2 and finally q1. The final regular expression is: **a(b + aa*b)* aa***

3) Specify the equivalent states in the following DFA: **(3 Marks)**



Answer:	q4	q3	q2	q1
q0	x	x	x	x
q1	√	√	x	
q2	x	x		
q3	√			

Question 5:

1) Define a *Turing Computable function*, then show that the function f(n) = n +1 , n ≥ 0 is *Turing Computable* where the number n is represented in binary format. **(4 Marks)**
   Answer: lecture 11 page 45, 49-51
2) Construct PDA to describe the language $\{a^n b^{n+2} : n \geq 0\}$. **(4 Marks)**
   Answer: Similar to the examples in lecture 12
3) Construct a grammar for the language M = $\{ ba^n b : n \geq 0 \}$. **(2 Marks)**
   Answer: $S \rightarrow bAb, \quad A \rightarrow \lambda \mid aA$

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Model Answer

Question 1: Choose the correct answer: **(10 Marks)**

1. If A = {4, 5, 6, 7, 8} and B = {8, 9, 10}, then |A ∩ B| equals
   a. 8 b. 1 c. 7 d. 4
2. If A ={a, b, c} and B ={0, 1}, then |A x B| equals
   a. 6 b. 8 c. 4 d. 9
3. If A = {a, b, c} and B = {d, e}, which of the following is correct?
   a. A and B are disjoint c. B is a subset of A
   b. A is a subset of B d. | A− B | = 1
4. If A = {1, 2, 3, 4}, select a correct partition of the set A:
   a. {{1}, {3, 4}} b. {{1,2}, {2, 3, 4}} c. {{1}, {2, 3, 4}} d. {{1, 2}, {3}}
5. Consider the function f, f:{ 1, 3, 5, 7, 9} →{5, 15, 25, 35, 45} defined by f(x) = 5x, then f is:
   a. Onto c. Invertible
   b. One-to-one d. All of the Previous
6. If L = {aba, bbb}, which of the following strings is NOT in L^*?
   a. aa b. bb c. λ d. Both a. and b. are correct
7. If L = { λ, a, aa, aaa, …} and M = {b, bb, bbb, …}, which of the following strings is NOT in L.M?
   a. aa b. aab c. bb d. abbb
8. Find L in {λ, a, ab}.L = {b, ab, ba, aba, abb, abba}
   a. L = {b, ab} c. L = {ab, ba}
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9. Let G = < {D,S} , {0,1,2,…,9} , P , S > , where P is:
   S → D 0 | D2 | D4 | D6 | D8
   D → λ |D0|D 1|D2|D3|D4|D5|D7| D8| D9
   Which of the following strings is NOT in L(G)?
   a. 17 b. 26 c. 30 d. 16
10. Choose a *correct* grammar describing the language $\{ aa, aaaa, , \ldots, a^{2n}, \ldots\}$
   a. S →a | aa S c. S → aa | aa S
   b. S →λ | aa | aa S d. S → λ | a S
11. Select a regular language:
   a. $\{a^n b^p : n \neq p\}$ c. $\{ a^n b^m c^p : n, m, p \geq 0\}$
   b. $\{a^n b^n c^n : n \geq 0\}$ d. $\{ a^n b a^n : n \geq 0\}$
12. If L and M are two regular languages, then
   a. $L.M$ is regular c. $\overline{L \cup M}$ is regular
   b. (L.M)* is regular d. All of the previous

13. Which of the following grammar is regular?

a. G = <{S}, {a, b}, { S→ λ | abS}, S>
b. F = <{S}, {a, b}, {S→ λ | b | aSa}, S >
c. H = <{S, A, B}, {a, b}, {S→AB, A → aA |a, B → bB | b}, S>
d. I = <{S, A}, {a, +}, {S → S+A | A, A → a}, S>

14. Select a correct grammar describing the language {ac, abc, abbc, …}

a. S → acS | b | λ
b. S → aAc, A → bA | λ
c. S→ aAc | λ, A → bA
d. S → aSc | b

15. Choose the correct regular expression to describe the language $\{1^n 01^m : n \geq 1, m \geq 2\}$

a. $\mathbf{1^* 0\ 1^*}$
b. $\mathbf{11^* 0\ 111^*}$
c. **101**
d. $\mathbf{1^* 0\ 11^*}$

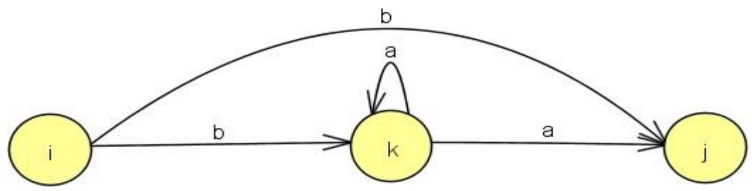
16. Choose the correct regular expression to describe the language $\{1^n : n \geq 0\ , n \neq 2\}$

a. **λ + 1 + 1111***
b. **λ + 11***
c. **1 +1111***
d. **λ + 1111***

17. Find the language described by the regular expression $\mathbf{0 + 1.0^*}$

a. {0, 1, 10, 100, 1000, …}
b. {0, 1, 00, 10, 000, 100, …}
c. {λ, 0, 1, 10, 100, 1000,…}
d. {0, 1, 00, 000, 0000, …}

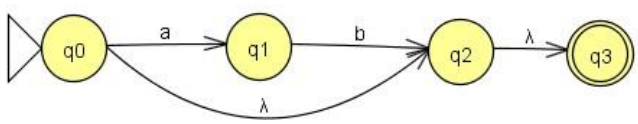
18. In removing state k from the following machine using FA to RE algorithm, the correct regular expression connecting states i and j is:



a. **ba*a**
b. **b + ba*a**
c. **ba***
d. both b and c are correct

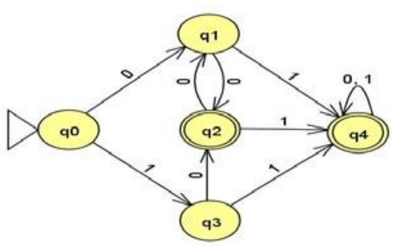
19. Select the lambda closure of state q_0 in the following automaton:

a. $\{q_2\}$
b. $\{q_0, q_2\}$
c. $\{q_0, q_2, q_3\}$
d. $\{q_2, q_3\}$



20. Select the equivalent states in the following DFA:

a. $\{q_1, q_2\}$
b. $\{q_2, q_3\}$
c. $\{q_1, q_4\}$
d. $\{q_2, q_4\}$



Question 2: (10 Marks)

1) Specify whether each of the following is correct or not, if it is not correct specify why? (5 Marks)

a. The function f: Z → Z, defined by f(x) = x^2 is bijective.

False: since this function is NOT one-to-one, (f(-1) = f(1) = 1)

b. If R and S are regular expressions, then R = S iff L(R) = L(S).

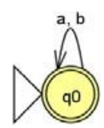
True

c. NonDeterministic PDAs are more powerful than deterministic PDAs.
True

d. NFAs are more powerful than DFAs.
False: NFAs has the same power as DFAs, they can describe the same language (regular language)
(The question is about powerful not efficiency)

e. The language $\{a^n\, b^n c^n : n \geq 0\}$ is context-free language.
False: this language is context-sensitive language.

2) Select from column B the correct representation for the languages in column A: (5 Marks)

A	B
1. $\{a^n b^{n+2} : n \geq 0\}$	a. **aaa*bbbb***
2. $\{a^n b^m a^p \,:\, n, m, p \geq 0\}$	b. S → bb | aSb
3. $\{a^n b^m : n, m \geq 0\} \cup \{ b^o a^p : o, p \geq 0\}$	c. **a*b*a***
4. $\{a^n b^m : n \geq 2, m \geq 3\}$	d. S → λ | aS | B , B →b | bB
5. $\{a^n b^m : n, m \geq 0\}$	e. 

Answer:

1. b 2. c 3. e 4. a 5. d

Question 3: (10 Marks)

1) Convert the NFA given by the following transition table into DFA: (4 Marks)

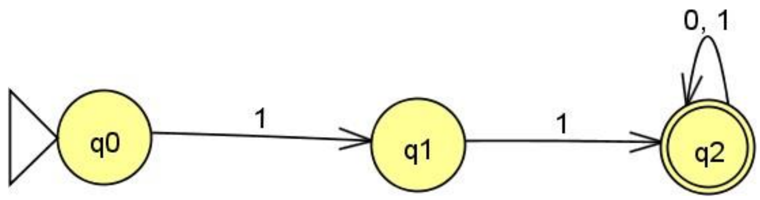
	0	1
Start p	{p, q}	{p}
q	{r}	{r}
r	{t}	ϕ
Final t	{t}	{t}

Answer: the transition table of the DFA is:

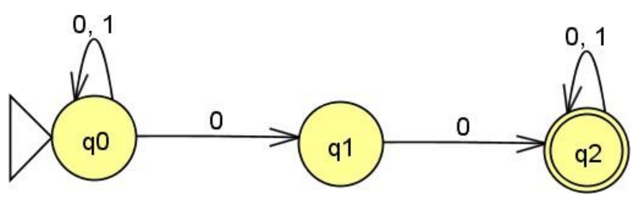
	0	1
Start {p}	{p, q}	{p}
{p, q}	{p, q, r}	{p, r}
{p, r}	{p, q, t}	{p}
{p, q, r}	{p, q, r, t}	{p, r}
Final {p, q, t}	{p, q, r, t}	{p, r, t}
Final {p, q, r, t}	{p, q, r, t}	{p, r, t}
Final {p, r, t}	{p, q, t}	{p, t}
Final {p, t}	{p, q, t}	{p, t}

2) Construct the following finite machines, over the alphabet {0, 1}, to define the following: (6 Marks)

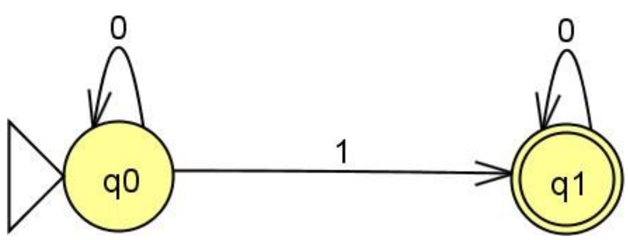
1. The language $\{11w: w \in \{0,1\}^*\}$.
NFA:



2. The language $\{u00v: u, v \in \{0,1\}^*\}$.
NFA:



3. The language with all strings with no more than one "1".
NFA:



Question 4: (10 Marks)

1) State without proof the Pumping lemma for regular languages, then use the lemma to show that the language $\{0^n10^n : n \geq 0\}$ is NOT regular.

Answer: (5 Marks)

Let L be an infinite regular language over the alphabet ∑. Suppose that L can be accepted by a DFA with m states.

Then, for any w ϵ L, such that |w| > m, there exist strings *x, y, z* in ∑*, where:

1. $|y| \geq 1$ (i.e., $y \neq \lambda$), $w = xyz$,
2. $|xy| \leq m$ and
3. $xy^tz \in L$ for all $t \geq 0$.

Assume that the language $\{0^n10^n : n \geq 0\}$ is regular. Then there is a DFA with m states that recognize this language.

Select w = 0^{m+1} 1 $0^{m+1} \in \{0^n10^n : n \geq 0\}$ and |w| > m

There are x, y, z such that:

w = xyz and y $\neq \lambda$

Let $x = 0^h$, $y = 0^k$, then $z = 0^{m+1-h-k}$ 1 0^{m+1}

By the pumping lemma, the string xy^2z must be in L

i.e. 0^{m+1+k} 1 0^{m+1} must be in L, m+1+k must equal m+1.

Hence, k must be 0, i.e., y = λ

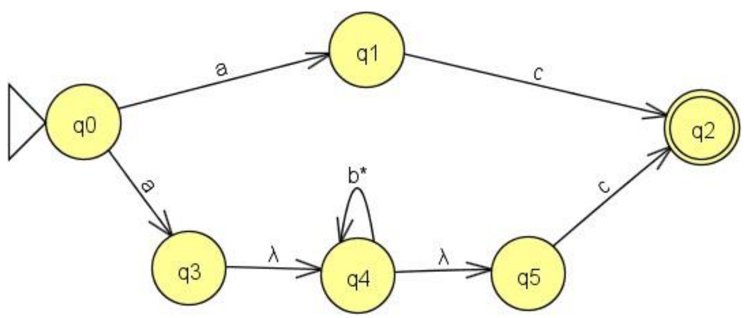
Which is a contradiction, since y cannot be λ

Therefore, the original assumption is false and the language cannot be regular.

2) Construct NFA for the following regular expression using RE to FA algorithm: (5 Marks)

$$ab^*c + ac$$

Answer: The final machine is:



Question 5: (10 Marks)

1) Define a Turing Computable function, then show that the function f: Z → Z is *Turing computable*, where:

$$f(n) = \begin{cases} n-3 & if\ n \geq 3 \\ 0 & otherwise \end{cases}$$

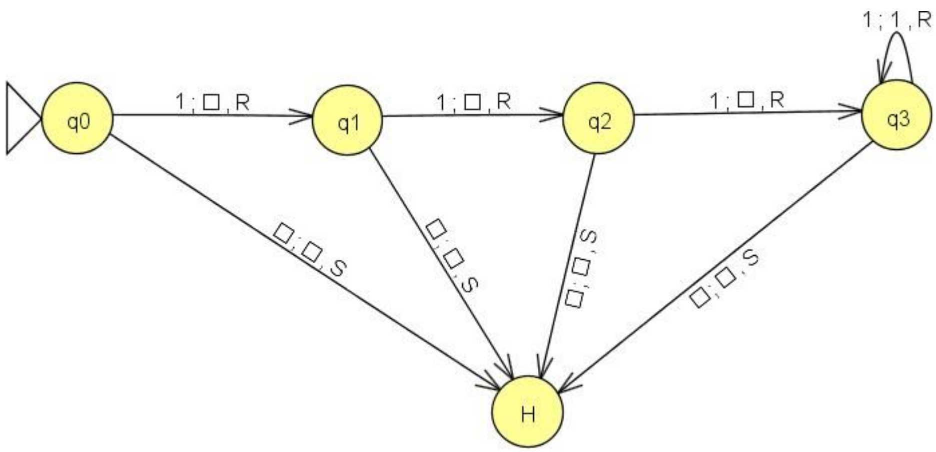
, n is represented in unary format. (5 Marks)

Answer:

A function f with domain D is said to be Turing-computable if there exists some Turing Machine such that:

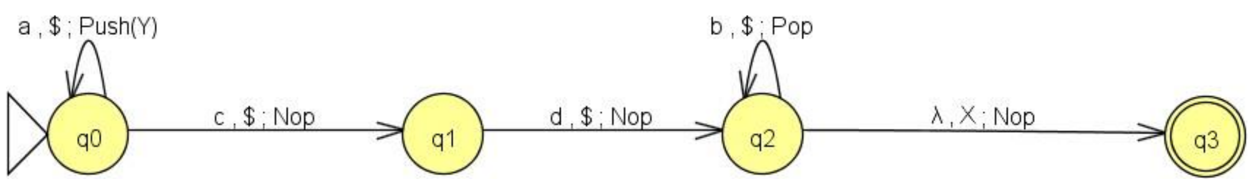
q_0w ⊢* H u for all w ∈ D, where u= f (w)

q_0 is the initial state, H is the halt state



2) Construct PDA to describe the language $\{a^n\, c\, d\, b^n\ :\ n\ \geq 0\}$. (5 Marks)

Answer:



Where the stack symbols = {X, Y}

Faculty of Computers and Information Technology
Course Name: Automata Models
Course Code: CAS 205
Instructor: Dr. AzzaTaha

EELU
الجامعة المصرية للتعلم الإلكتروني
Egyptian E-Learning University

Year: 2017-2018 (Fall Semester)
Mid-Term Exam. (30-10-2017)
Time Allowed: 1.5 hrs.
Marks: 20

Model Answer

Question 1: Choose the correct answer: **(10 Marks)**

1. If A = {1, 2, 3, 4, 5} and B = {5, 6, 7} then |A ∩ B| equals:
   a. 5 b. 1 c. 7 d. 0
2. If A = {a, b, c} then the power set of A has
   a. 6 elements b. 9 elements c. 8 elements d. 3 elements
3. If A = {a, b, c} and B = {a, b, c, d}, which of the following is correct?
   a. A is a proper subset of B. c. B is a subset of A.
   b. A and B are disjoint. d. B − A = ϕ
4. If a given function f is bijective, then f is invertible.
   a. True b. False
5. Let g: N → N, defined by g(x) = 2x, then the function g has inverse.
   a. True b. False
6. If L = $\{a, bb\}$, then $aabb \in L^*$
   a. True b. False
7. Consider the function h, h: {a, b, c} → {1, 2, 3}, such that h(a) = 1, h(b) =3 and h(c) = 3 then the *range of h* is:
   a. {3} b. {1, 2, 3} c. {a, b, c} d. {1, 3}
8. If Σ = {a, b, c}, u = aaa and v = bbc, then |uv| equals:
   a. 6 b. 7 c. 8 d. 9
9. If L = {11, 10}, which of the following strings is NOT in L^*?
   a. 010 b. 1111 c. 1110 d. 1011
10. If L = {λ, 010, 10}, which of the following strings is NOT in L^+?
    a. λ b. 1010 c. 00 d. 010
11. If L = {a, aa, aaa, …} and M = { λ, b, bb, bbb, …}, which of the following strings is NOT in L.M:
    a. aaa b. baa c. abb d. aab
12. Find L in: {λ, a, ab}. L = {b, ab, ba, aba, abb, abba}
    a. L = {b, ba} c. L = {ab, ba }
    b. L = {b, ab} d. L = {λ, b, ba}
13. Select a *correct* grammar to describe the language {00, 01}.
    a. S → 00 | 01 c. S → 0S | 1S
    b. S → λ | 00 | 01 d. S → λ | 0S | 1S
14. Select a *correct* grammar generating the language {aa, aba, abba, abbba, …}
    a. S →aAa | λ, A → bA | λ
    b. S →aAa, A →bA
    c. S →aAa, A → bA | λ
    d. S →aAa | λ, A→ bA

15. Let G = ({A,S} , {a, b } , P , S) , where P consists of $S \rightarrow 1A1$, $A \rightarrow 0A \mid \lambda$, then G is a regular grammar.
    a. True b. False

16. Let G = ({D,S} , {0,1,2,…,9} , P , S) , where P is:
    $S \rightarrow D1 \mid D3 \mid D5 \mid D7 \mid D9$
    $D \rightarrow \lambda \mid D0 \mid D1 \mid D2 \mid D3 \mid D4 \mid D5 \mid D7 \mid D8 \mid D9$
    Which of the following strings is NOT in L(G)?
    a. 19 b. 7 c. 30 d. 11

17. Let G = ({A, S}, {0, 1}, P, S), where P is: $S \rightarrow 1A1$, $A \rightarrow \lambda \mid 0A$, which of the following strings is NOT in L(G)?
    a. λ b. 11 c. 101 d. 100001

18. Select a *correct* grammar to describe the language $\{aa, aaaa, \ldots, a^{2n}, \ldots\}$
    a. $S \rightarrow a \mid aS$ c. $S \rightarrow aa \mid aaS$
    b. $S \rightarrow \lambda \mid a \mid aS$ d. $S \rightarrow \lambda \mid aa \mid aaS$

19. The Pigeonhole principal states that: if m objects are placed into n holes, where … …, then some holes contain at least 2 objects.
    a. $m > n$ b. $m \geq n$ c. $m < n$ d. $m \leq n$

20. Let G = ({S, A, B}, {a, b}, P_2, S), where P is: $S \rightarrow AB$, $A \rightarrow aA \mid a$, $B \rightarrow b \mid bB$, then G is a context-free grammar.
    a. True b. False

Question 2: **(10 Marks)**

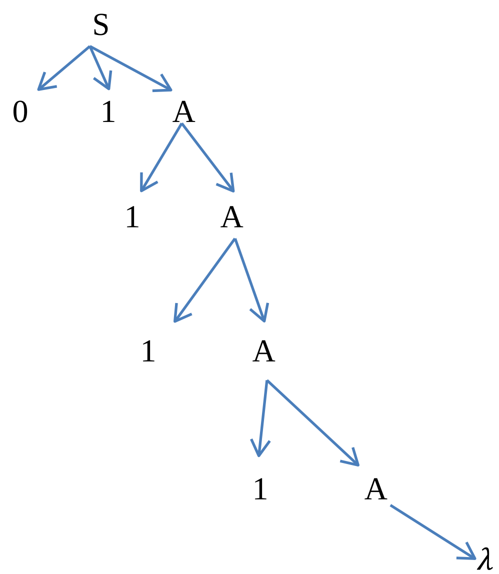
1) Construct a grammar to describe the language $\{01^n : n \geq 1\}$, and then show by either a derivation or a parse tree that the string 01111 can be derived from the constructed grammar. (4 Marks)

Answer: one possible correct grammar:

$S \rightarrow 01A$, $A \rightarrow 1A \mid \lambda$

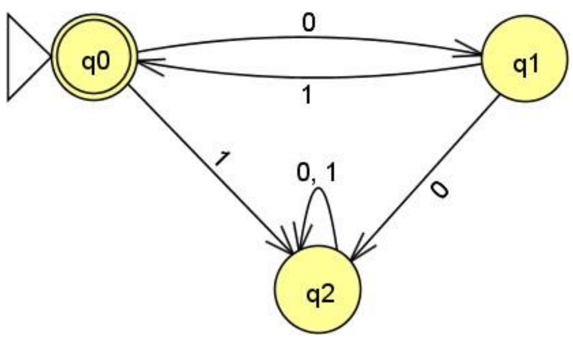
By derivation: $S \Rightarrow 01A \Rightarrow 011A \Rightarrow 0111A \Rightarrow 01111A \Rightarrow 01111\lambda \equiv 01111$

By parse tree:

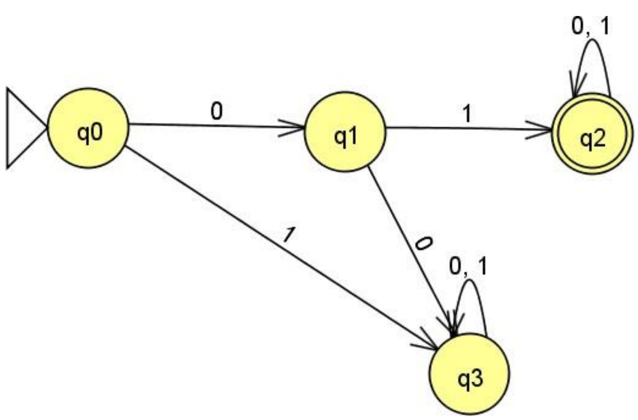


2) Show that the language {λ, 01, 0101, ... } is regular. (2 Marks)

Answer: by either a regular grammar: $S \rightarrow \lambda \mid 01\,S$ or by finite machine:



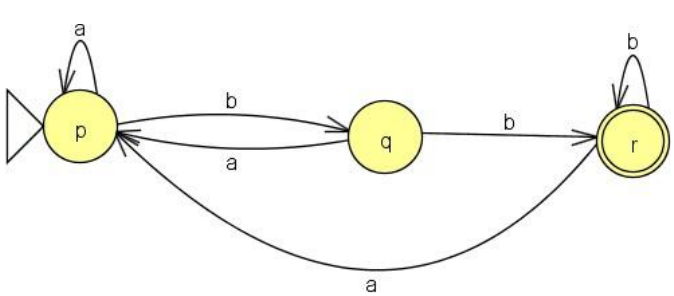
3) Design a finite state machine to describe the language $\{01w : w \in \{0,1\}^*\}$ (2 Marks)



4) Consider the finite machine given by the table:

	a	b
Start p	p	q
q	p	r
Final r	p	r

Draw the transition graph for the machine, and then describe its language. (2 Marks)



L = { ubb: u ∈ {a, b}*}

Made by : Eng/General& Eng/Alaa Nassar

Faculty of Computers and Information Technology
Course Name: Automata Models
Course Code: CAS 205
Instructor: Dr. AzzaTaha

EELU
الجامعة المصرية للتعلم الإلكتروني
Egyptian E-Learning University

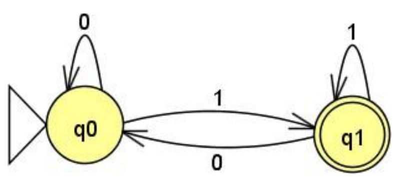
Year: 2018-2019 (Fall Semester)
Mid-Term Exam. (5-11-2018)
Time Allowed: 1.5 hrs.
Marks: 20

Model Answer

Question 1: Choose the correct answer: **(10 Marks)**

1. If A = {1, 2, 3, 4, 5} and B = {5, 6, 7} then |A − B| equals:
   a. 5 b. 4 c. 3 d. 7
2. If B = {{1}, {3, 4, 5}}, then |B| equals:
   a. 4 b. 2 c. 3 d. 1
3. If A = {3, 4, 6, 7} and B = {5, 7}, which of the following is in A x B:
   a. 7 b. (7, 7) c. (5, 6) d. {7, 7}
4. If A = {1, 2, 3, 4} and B = {1, 2, 3}, which of the following is correct?
   a. A is a subset of B. c. B is a proper subset of A.
   b. A and B are disjoint. d. A − B = ϕ
5. Consider the function f, f: {a, b, c} ⟶ {1, 2, 3}, such that f(a) = 1, f(b) = 3 and f(c) = 3 then the *domain* of f is:
   a. {3} b. {1, 2, 3} c. {a, b, c} d. {1, 3}
6. Consider the function h, h: Z ⟶{0, 1} defined by h(x) = $\begin{cases} 0 & \text{if } x \text{ is even} \\ 1 & \text{otherwise} \end{cases}$, then:
   a. h is one to one c. h is bijective
   b. h is onto d. h is invertible
7. If ∑ = {a, b, c}, w is a string over ∑ and |www| = 18, what is the length of w?
   a. 18 b. 12 c. 6 d. 3
8. If L = {aba, ba}, which of the following strings is NOT in L^+?
   a. λ b. baba c. baaba d. ba
9. If L = { λ, 011, 00}, which of the following strings is NOT in L^+?
   a. λ b. 11 c. 01100 d. 00011
10. If L = {aa, ab}, which of the following strings is NOT in L^*?
    a. aba b. aaaa c. aaab d. abaa
11. If L = {a, aa, aaa, …} and M = { λ, b, bb, bbb, …}, which of the following strings is NOT in L.M:
    a. aaa b. λ c. abb d. aab
12. If $L_1 = \{a^n b^n : n > 0\}$ and $L_2 = \{c^m : m > 0\}$, which of the following strings NOT in $L_1.L_2$?
    a. λ b. ccc c. ab d. All of the previous
13. Find L in L.{a , b} = {a, baa, b, bab}
    a. L = {a, ba} c. L = {ba, λ}
    b. L = {b, ba} d. L = { ab, ba}
14. Select *a correct* grammar generating the language {b, abc, aabcc, …}
    a. S → aAc, A → b A | λ c. S → λ |b | aSc
    b. S → b| aSc d. S → aAc | λ, A → bA

15. Select a *correct* grammar generating the language {ac, abc, abbc, abbbc, …}
   a. $S \rightarrow aAc, \quad A \rightarrow b\,A \mid \lambda$ c. $S \rightarrow ac\,S \mid a \mid \lambda$
   b. $S \rightarrow aAc, \quad A \rightarrow b\,A$ d. $S \rightarrow aAc \mid \lambda, \quad A \rightarrow b\,A$
16. Select a correct grammar to define the language {0, 01, 011, 0111, …}
   a. $S \rightarrow 0A, \quad A \rightarrow 1 \mid 1A$ c. $S \rightarrow 0A, \quad A \rightarrow \lambda \mid 1A$
   b. $S \rightarrow 0 \mid 1S$ d. $S \rightarrow 1 \mid 0S$
17. Which of the following grammar is regular?
   a. G = <{S}, {a, b}, { S→ λ | abS}, S>
   b. F = <{S}, {a, b}, {S→ λ | b | aSa}, S >
   c. H = <{S, A, B}, {a, b}, {S→AB, A → aA |a, B → bB | b}, S>
   d. I = <{S, A}, {a, +}, {S → S+A | A, A → a}, S>
18. Let G = < {D,S} , {0,1,2,…,9} , P , S>, where P is:
   S → D1 | D3 | D5 | D7 | D9 D → λ | D0 | D1 | D2 | D3 | D4 | D5 | D7 | D8| D9
   Then the string 2018 ∈ L(G)
   a. True b. False
19. If a given function f is bijective, then it is invertible.
   a. True b. False
20. Select the language defined by the following automaton:



   a. $\{1w : w \in \{0,1\}^*\}$ c. $\{1w : w \in \{1\}^*\}$
   b. $\{w1 : w \in \{0,1\}^*\}$ d. $\{w1 : w \in \{0\}^*\}$

Question 2: **(10 Marks)**

1) Construct a grammar to describe the language $\{10^n1 : n \geq 1\}$, and then show by either a derivation or a parse tree that the string 10001 can be derived from the constructed grammar.

Answer:

$S \rightarrow 1A1, \quad A \rightarrow 0 \mid 0\,A$ (2 Marks)

Derivation: $S \Rightarrow 1A\,1 \Rightarrow 10A\,1 \Rightarrow 100A1 \Rightarrow 10001$ (2 Marks)

Parse Tree: S

1 A 1

0 A

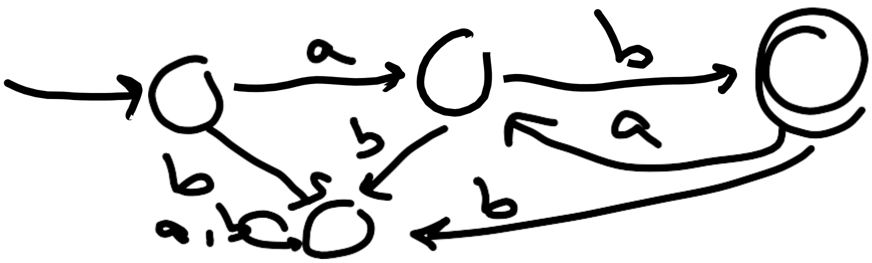
0 A

0

2) Show that the language {ab, abab, ababab, ... } is regular. (2 Marks)

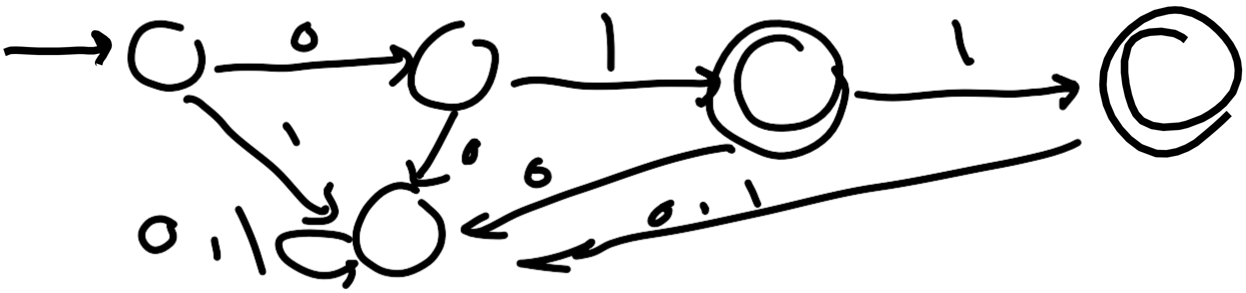
Answer: only one of the following representations is enough (regular grammar or finite machine)

1. This language can be represented by the regular grammar: S ⟶ ab | abS
2. This language can be represented by the machine:

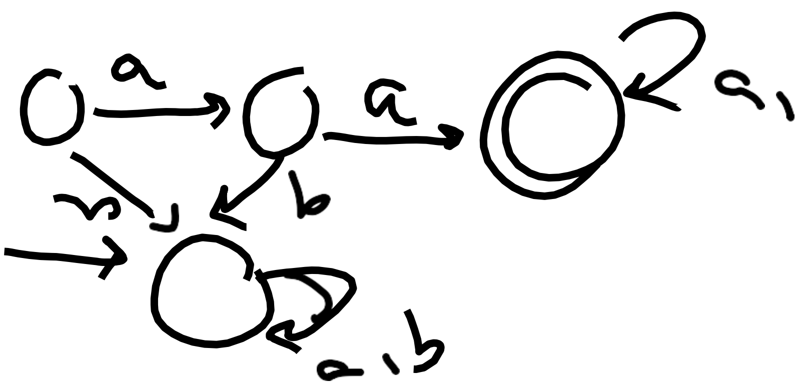


3) Design a finite state machine to describe each of the following languages: (4 Marks)

a. L = $\{01, 011\}$



b. M = $\{aaw : w \in \{a, b\}^*\}$



Question 1

Question text

Types of Finite automata include low-fidelity automata and high-fidelity automata

Select one:

True

False

Feedback

The correct answer is 'False'.

Question **2**

Question text

Consider the regular expression 0 * (10 *) which is similar to the same set as

Select one:

a. 0 + (0 + 10) *

b. (0 +1) * 10 (0 + 1) *

c. (1 * 0) * 1*

d. None of the above

Feedback

Your answer is correct.
The correct answer is: None of the above

Question **4**

Question text

The language described by the regular expression (0+1)*0(0+1)*0(0+1)* over the alphabet {0,1} is the set of

Select one:

a. All strings containing at least two 1's

b. All strings that begin and end with either 0's or 1's

c. All strings containing the substring 00

d. All strings containing at least two 0's

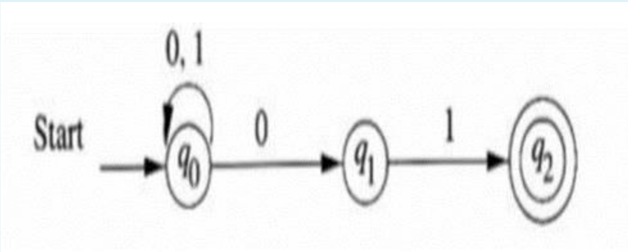
Feedback

Your answer is correct.
The correct answer is: All strings containing at least two 0's

Question **5**

Question text

Find the language of NFA



Select one:

a. L={ set of strings of 0's and 1's ending with 01}

b. L={ set of strings of 0's and 1's with starting with 01

c. L={ set of strings of 0's and 1's with odd number of 1's}

d. L={set of strings of 0's and 1's}

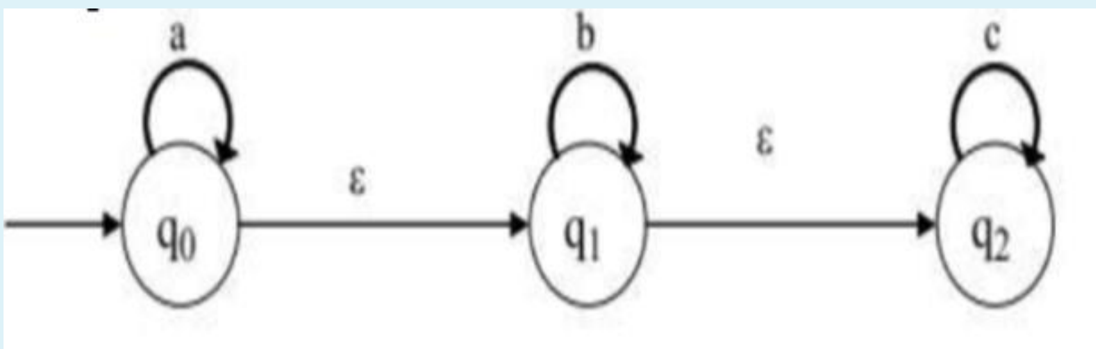
Feedback

Your answer is correct.
The correct answer is: L={ set of strings of 0's and 1's ending with 01}

Question **6**

Question text

According to given transitions,which among the following are the epsilon closures of q1 for the following NFA



Select one:

a.
{q0,q1,q2}

b. {q1,q2}

c. {q2}

d. None of them

Feedback

Your answer is correct.
The correct answers are: {q1,q2}, {q2}

Question **7**

Question text

The regular expression for { w ϵ {0,1}*: w has two consecutive 0's or three consecutive 1's}is :

Select one:

a. $(00 + 111)(0 + 1)^*$

b. $(0 + 1)^*(00 + 111)$

c. $(0 + 1)^*(000 + 11)(0 + 1)^*$

d. $(0 + 1)^*(00 + 111)(0 + 1)^*$

Feedback

Your answer is correct.

The correct answer is: $(0 + 1)^*(00 + 111)(0 + 1)^*$

Question **8**

Question text

A finite automata recognizes ....

a. Context sensitive language

b. Context free language

c. Any language

d. Regular language

Feedback

Your answer is correct.
The correct answer is: Regular language

Question **9**

Question text

what is the regular expression of this language L = {w ϵ {0, 1}*: w contains all strings that begin with 11 and end with 11}?

Select one:

a. None

b. 11(1+0)11*

c. 11(1+0)*11

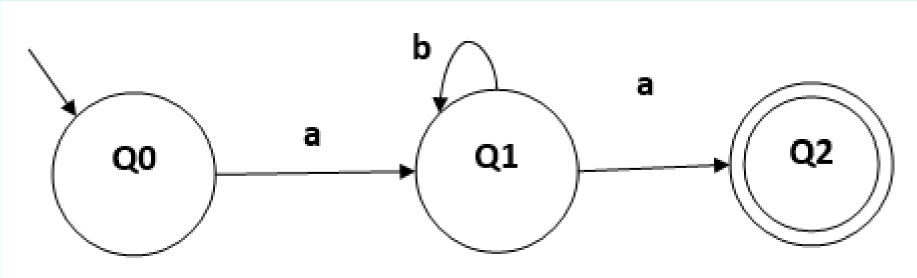
d. 11(1+0)11

Your answer is correct.
The correct answer is: 11(1+0)*11

Question **10**

Question text

What is the language identified by the following DFA ?



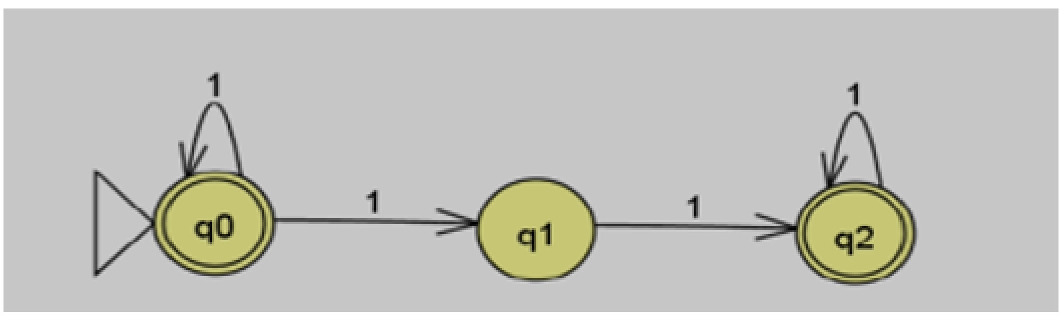
a. {aba}
b. $\{ab^n, n>=0\}$
c. $\{ab^na, n>=0\}$
d. {abba}

Your answer is correct.
The correct answer is: $\{ab^na, n>=0\}$

Question **10**

Question text

Select the language defined by the following finite automaton:



Select one:

a. $\{11, 1111, 1111, \ldots\}$

b. $\{1, 11, 111, 1111, \ldots\}$

c. $\{1^n, n \geq 0, n \neq 1\}\}$

d. $\{1^n, n \geq 0\}$

The correct answer is: $\{1^n, n \geq 0\}$