Made by: Eng/Alaa Nassar & Eng/General

Discrete Mathematics – Assignment 1

Instructor: Dr. Ahmed Hagag

Answer - Assignment
One

Answer the following questions:

1) Which of these sentences are propositions? What are the truth values of those that are propositions?

a) Boston is the capital of Massachusetts.
b) Miami is the capital of Florida.
c) $2 + 3 = 5$.
d) $5 + 7 = 10$.
e) $x + 2 = 11$.
f) Answer this question.

a) This is a true proposition.
b) This is a false proposition (Tallahassee is the capital
c) This is a true proposition.
d) This is a false proposition.
e) This is not a proposition (it contains a variable).
f) This is not a proposition, since it does not assert anything.

2) Let p and q be the propositions

p : It is below freezing.

q : It is snowing.

Write these propositions using p and q and logical connectives (including negations).

a) It is below freezing and snowing.
b) It is below freezing but not snowing.
c) It is not below freezing and it is not snowing.
d) It is either snowing or below freezing (or both).

a) Here we have the conjunction $p \wedge q$.
b) Here we have a conjunction of p with the negation of q, namely $p \wedge \neg q$.
c) Again this is a conjunction: $\neg p \wedge \neg q$.
d) Here we have a disjunction, $p \vee q$.

3) How many rows appear in a truth table for each of these compound propositions?

a) $p \to \neg p$
b) $(p \vee \neg r) \wedge (q \vee \neg s)$

A truth table will need 2^n rows if there are n variables.

a) $2^1 = 2$ b) $2^4 = 16$

4) Construct a truth table for each of these compound propositions.

a) $(p \vee q) \rightarrow (p \oplus q)$ **b)** $(p \vee \neg q) \rightarrow q$

c) $(p \rightarrow q) \vee (\neg p \rightarrow r)$

p	q	$p \vee q$	$p \oplus q$	$(p \vee q) \rightarrow (p \oplus q)$
T	T	T	F	F
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

p	q	$\neg q$	$p \vee \neg q$	$(p \vee \neg q) \rightarrow q$
T	T	F	T	T
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

p	q	r	$p \rightarrow q$	$\neg p$	$\neg p \rightarrow r$	$(p \rightarrow q) \vee (\neg p \rightarrow r)$
T	T	T	T	F	T	T
T	T	F	T	F	T	T
T	F	T	F	F	T	T
T	F	F	F	F	T	T
F	T	T	T	T	T	T
F	T	F	T	T	F	T
F	F	T	T	T	T	T
F	F	F	T	T	F	T

5) Find the bitwise *OR*, bitwise *AND*, and bitwise *XOR* of each of these pairs of bit strings.

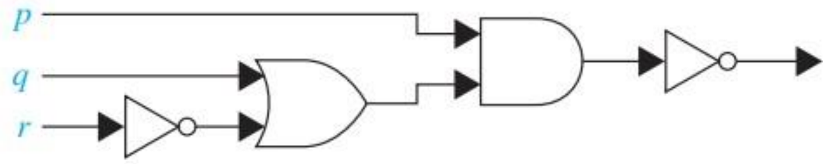
a) 101 1110, 010 0001

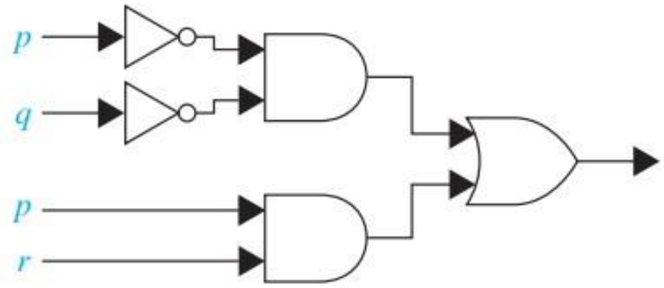
b) 1111 0000, 1010 1010

a) bitwise OR = 111 1111; bitwise AND = 000 0000; bitwise XOR = 111 1111

b) bitwise OR = 1111 1010; bitwise AND = 1010 0000; bitwise XOR = 0101 1010

6) Find the output of each of these combinatorial circuits.

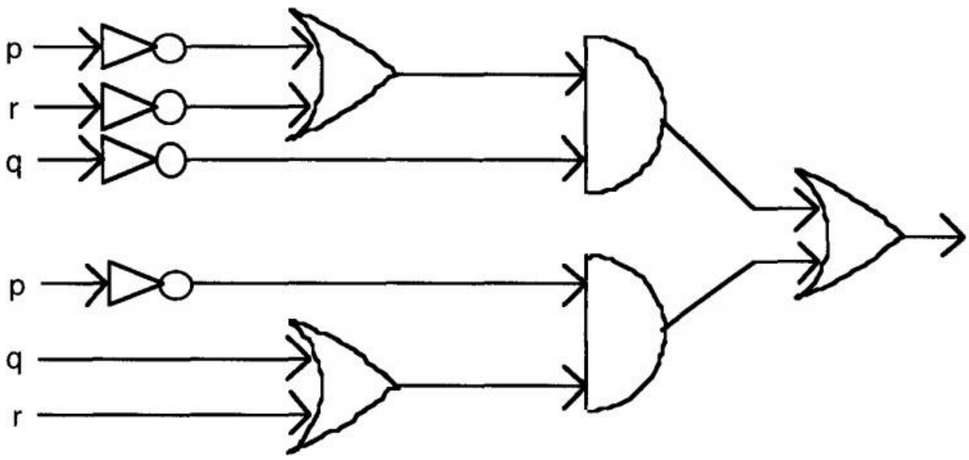
a) 

b) 

a) The output of the OR gate is $q \vee \neg r$.
Therefore the output of the AND gate is $p \wedge (q \vee \neg r)$.
Therefore the output of this circuit is $\neg(p \wedge (q \vee \neg r))$.

b) The output of the top AND gate is $(\neg p) \wedge (\neg q)$.
The output of the bottom AND gate is $p \wedge r$. Therefore the output of this circuit is $((\neg p) \wedge (\neg q)) \vee (p \wedge r)$.

7) Construct a combinatorial circuit using inverters, OR gates, and AND gates that produces the output $((\neg p \vee \neg r) \wedge \neg q) \vee (\neg p \wedge (q \vee r))$ from input bits p, q, and r.



Discrete Mathematics **Quiz (02) – V#1**

Dr. Ahmed Hagag

Answer

Question 1 **What is the power set of:** **[4 points]**

1) $\{a, \{b\}\} = \{\emptyset,\ \{a\}, \{\{b\}\}, \{a, \{b\}\}\}$

2) $\{\{\ \}\} = \{\emptyset, \{\emptyset\}\}\ \ or\ \ \{\{\ \}, \{\{\ \}\}\}$

Question 2 **Find these values:** **[3 points]**

a) $\lfloor -0.7 \rfloor = -1$

b) $\lfloor \lceil -0.7 \rceil \rfloor = 0$

c) $\lceil -2.7 \rceil = -2$

d) $\lfloor 2 + 0.1 \rfloor = 2$

Question 3 **What is the cm$(24, 36)$?** **[3 points]**

What is the lcm$(24, 36)$?

$\sqrt{24}$ are $2, 3$

$\sqrt{36}$ are $2, 3, 5$

$$\left(\begin{array}{c|c} 24 & 2 \\ 12 & 2 \\ 6 & 2 \\ 3 & 3 \\ 1 & \end{array}\right) = 2^3 \cdot 3$$

$$\left(\begin{array}{c|c} 36 & 2 \\ 18 & 2 \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array}\right) = 2^2 \cdot 3^2$$

$$\text{lcm}(24,36) = 2^3 \cdot 3^2 = 72$$

Question 4 **Is R reflexive, symmetric, and/or antisymmetric?** **[9 points]**

Where the relation R on a set is represented by the following matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

Not reflexive, **Not** symmetric, **antisymmetric**

Discrete Mathematics **Quiz (02) – V#2**

Dr. Ahmed Hagag

Answer

Question 1 **What is the power set of:** **[4 points]**

1) $\{a, \{a, b\}\} = \{\emptyset, \ \{a\}, \{\{a, b\}\}, \{a, \{a, b\}\}\}$

2) $\{\emptyset\} = \{\emptyset, \{\emptyset\}\} \quad or \quad \{\{\ \}, \{\{\ \}\}\}$

Question 2 **Find these values:** **[4 points]**

a) $\lceil -0.1 \rceil = 0$

b) $\lfloor \lceil -0.3 \rceil \rfloor = 0$

c) $\lfloor -5.2 \rfloor = -6$

d) $\lceil 1 - 0.2 \rceil = 1$

Question 3 **What is the $\mathbf{gcd(24, 36)}$?** **[6 points]**

$\sqrt{24}$ are $2, 3$

$\sqrt{36}$ are $2, 3, 5$

$$\left(\begin{array}{c|c} 24 & 2 \\ 12 & 2 \\ 6 & 2 \\ 3 & 3 \\ 1 & \end{array}\right) = 2^3 \cdot 3$$

$$\left(\begin{array}{c|c} 36 & 2 \\ 18 & 2 \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array}\right) = 2^2 \cdot 3^2$$

$$\gcd(24,36) = 2^2 \cdot 3 = 12$$

Question 4 **Is R reflexive, symmetric, and/or antisymmetric?** **[6 points]**

Where the relation R on a set is represented by the following matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

reflexive, **Not** symmetric, **Not** antisymmetric

Discrete Mathematics – Assignment 2

Instructor: Dr. Ahmed Hagag

Answer - Assignment
Two

Answer the following questions:

1) Use truth tables to verify the absorption laws.

a) $p \vee (p \wedge q) \equiv p$ **b)** $p \wedge (p \vee q) \equiv p$

p	q	$p \wedge q$	$p \vee (p \wedge q)$	$p \vee q$	$p \wedge (p \vee q)$
T	T	T	T	T	T
T	F	F	T	T	T
F	T	F	F	T	F
F	F	F	F	F	F

2) Show that each of these conditional statements is a tautology by using truth tables.

a) $(p \wedge q) \to p$ **b)** $p \to (p \vee q)$
c) $\neg p \to (p \to q)$ **d)** $(p \wedge q) \to (p \to q)$

p	q	$p \wedge q$	$(p \wedge q) \to p$	$p \vee q$	$p \to (p \vee q)$
T	T	T	T	T	T
T	F	F	T	T	T
F	T	F	T	T	T
F	F	F	T	F	T

p	q	$\neg p$	$p \to q$	$\neg p \to (p \to q)$	$p \wedge q$	$(p \wedge q) \to (p \to q)$
T	T	F	T	T	T	T
T	F	F	F	T	F	T
F	T	T	T	T	F	T
F	F	T	T	T	F	T

3) Let $P(x)$ be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students. Express each of these quantifications in English.

a) $\exists x P(x)$ **b)** $\forall x P(x)$
c) $\exists x \neg P(x)$ **d)** $\forall x \neg P(x)$

a) There is a student who spends more than five hours every weekday in class.

b) Every student spends more than five hours every weekday in class.

c) There is a student who does not spend more than five hours every weekday in class.

d) No student spends more than five hours every weekday in class.
(Or, equivalently, every student spends less than or equal to five hours every weekday in class.)

4) Translate these statements into English, where $C(x)$ is "x is a comedian" and $F(x)$ is "x is funny" and the domain consists of all people.

a) $\forall x(C(x) \to F(x))$ **b)** $\forall x(C(x) \wedge F(x))$

a) This statement is that for every x, if x is a comedian, then x is funny. In English, this is most simply stated, "Every comedian is funny."

b) This statement is that for every x in the domain (universe of discourse), x is a comedian *and* x is funny. In English, this is most simply stated, "Every person is a funny comedian." Note that this is not the sort of thing one wants to say. It really makes no sense and doesn't say anything about the existence of boring comedians; it's surely false, because there exist lots of x for which $C(x)$ is false. This illustrates the fact that you rarely want to use conjunctions with universal quantifiers.

5) Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people.

a) Someone in your class can speak Hindi.
b) Everyone in your class is friendly.

In order to do the translation the second way, we let $C(x)$ be the propositional function "x is in your class." Note that for the second way, we always want to use conditional statements with universal quantifiers and conjunctions with existential quantifiers.

a) Let $H(x)$ be "x can speak Hindi." Then we have $\exists x\, H(x)$ the first way, or $\exists x(C(x) \wedge H(x))$ the second way.

b) Let $F(x)$ be "x is friendly." Then we have $\forall x\, F(x)$ the first way, or $\forall x(C(x) \rightarrow F(x))$ the second way.

6) Use a direct proof to show that the sum of two odd integers is even.

We must show that whenever we have two odd integers, their sum is even. Suppose that a and b are two odd integers. Then there exist integers s and t such that $a = 2s + 1$ and $b = 2t + 1$. Adding, we obtain $a + b = (2s + 1) + (2t + 1) = 2(s + t + 1)$. Since this represents $a + b$ as 2 times the integer $s + t + 1$, we conclude that $a + b$ is even, as desired.

7) Show that if n is an integer and $n^3 + 5$ is odd, then n is even using

a) a proof by contraposition.
b) a proof by contradiction.

a) We must prove the contrapositive: If n is odd, then $n^3 + 5$ is even. Assume that n is odd. Then we can write $n = 2k+1$ for some integer k. Then $n^3+5 = (2k+1)^3+5 = 8k^3+12k^2+6k+6 = 2(4k^3+6k^2+3k+3)$. Thus $n^3 + 5$ is two times some integer, so it is even.

b) Suppose that $n^3 + 5$ is odd and that n is odd. Since n is odd, and the product of odd numbers is odd, in two steps we see that n^3 is odd. But then subtracting we conclude that 5, being the difference of the two odd numbers $n^3 + 5$ and n^3, is even. This is not true. Therefore our supposition was wrong, and the proof by contradiction is complete.

Discrete Mathematics – Assignment 3

Instructor: Dr. Ahmed Hagag

Answer - Assignment
Three

Answer the following questions:

1) Use the truth table to show that the hypotheses "Aly works hard," "If Aly works hard, then he is a good boy," and "If Aly is a good boy, then he will get the job" imply the conclusion "Aly willget the job."

P : Aly works hard

q : is a good boy

r : he will get the job

$$P$$

$$p \to q$$

$$q \to r$$

$$\therefore r$$

			P_1	P_2	P_3	$conclusion$	
p	q	r	p	$(p \to q)$	$(q \to r)$	r	
T	T	T	T	T	T	T	T
T	T	F	T	T	F	F	
T	F	T	T	F	T	T	
T	F	F	T	F	T	F	
F	T	T	F	T	T	T	
F	T	F	F	T	F	F	
F	F	T	F	T	T	T	
F	F	F	F	T	T	F	

2) Let f and g be the functions from the set of integers to the set of integers defined by $f(x) = 2x + 3$ and $g(x) = 3x + 2$. What is the composition of f and g? What is the composition of g and f ?

$$g(f(x)) = 3(2x+3) + 2 = 6x + 11$$

$$f(g(x)) = 2(3x+2) + 3 = 6x + 7$$

3) How can we produce the terms of the sequence $5, 25, 125, 625, \dots$?

Geometric Sequence → $a = 5, \quad r = 5$

$$\{ar^n\}_{n=0}^{\infty} \quad \rightarrow \quad \{5 * 5^n\}_{n=0}^{\infty}$$

4) Find the following summations:

$$1)\ \sum_{n=1}^{20} 3 * 4^n$$

$$\sum_{n=0}^{19} 12 * 4^n \qquad a = 12, \qquad r = 4 \qquad = \frac{12(4)^{20} - 12}{4 - 1} = 4398046511100$$

$$2)\ \sum_{k=40}^{100} k = \sum_{k=1}^{100} k - \sum_{k=1}^{39} k\ , \qquad \sum_{k=40}^{100} k = \frac{100(101)}{2} - \frac{39(40)}{2} = 5050 - 780 = 427$$

5) Determine the integer 1001 is prime or not?

The prime numbers $\leq \sqrt{1001}$ are $2,\ 3,\ 5,\ 7,\ \overline{11,13,17,19},\ 23\ \dots$

$7 \mid 1001$

Then 1001 is a composite integer (NOT prime)

6) What are the $\operatorname{lcm}(150, 400)$ and $\gcd(150, 400)$?

$\sqrt{150}$ are $2, 3, 5, 7, 11$

$\sqrt{400}$ are $2, 3, 5, 7, 11, 13, 17, 19$

$$\begin{array}{r|l} 150 & 2 \\ 75 & 3 \\ 25 & 5 \\ 5 & 5 \\ 1 & \end{array} = 2 \cdot 3 \cdot 5^2 \qquad \begin{array}{r|l} 400 & 2 \\ 200 & 2 \\ 100 & 2 \\ 50 & 2 \\ 25 & 5 \\ 5 & 5 \\ 1 & \end{array} = 2^4 \cdot 5^2$$

$\gcd(150, 400) = 2^1 \cdot 3^0 \cdot 5^2 = 50$

$\operatorname{lcm}(150, 400) = 2^4 \cdot 3^1 \cdot 5^2 = 1200$

7) List three integers that are *congruent* to 3 *modulo* 5

$a \equiv b(\boldsymbol{mod}\ \ m) \iff$ there is an integer $\boldsymbol{k}$ such that $\boldsymbol{a} = \boldsymbol{b} + \boldsymbol{km}$

$a = 3 + k * 5, \qquad k\ is\ integer$

- $k = 1 \rightarrow \quad a = 8$
- $k = 2 \rightarrow \quad a = 13$
- $k = 3 \rightarrow \quad a = 18$

Note: you can use other valuse for k

8) Use mathematical induction to show that

$$\sum_{i=1}^{n} i * i! = (n+1)! - 1$$

For all positive integers n.

Let $P(n)$ be the proposition that

$1 * 1! + 2 * 2! + 3 * 3! + \cdot \cdot \cdot + n * n! = (n+1)! - 1$

For all positive integers n.

1) Basis Step:

If $\boldsymbol{n} = \mathbf{1}$ we have $P(1) = 1 * 1! = 1 = (1+1)! - 1 = 1$, so $P(1)$ is **true**.

This completes the basis step.

2) Inductive Step:

We first **Assume** that (Induction Hypothesis) $P(k)$ is true for the positive integer k, i. e. : $P(k)$

$$1 * 1! + 2 * 2! + 3 * 3! + \ldots + k * k! = (k+1)! - 1$$

We **add** $(\boldsymbol{k}+\mathbf{1}) * (\boldsymbol{k}+\mathbf{1})!$ to both sides of the equation in $P(k)$, we obtain

$$1 * 1! + 2 * 2! + \ldots + k * k! + (k+1) * (k+1)! = (k+1)! - 1 + (k+1) * (k+1)!$$
$$= (k+1)! - 1 + (k+1) * (k+1)!$$
$$= (k+1)!\,(1 + (k+1)) - 1$$
$$= (k+1)!\,((k+2)) - 1$$
$$= (k+2)(k+1)! - 1$$
$$= (k+2)! - 1$$

- **This equation shows that $\boldsymbol{P(k+1)}$ is true under the assumption that $\boldsymbol{P(k)}$ is true.**
- **This completes the inductive step.**

So, by mathematical induction we know that $P(n)$ is true for all positive integers n.

Discrete Mathematics – Assignment 4

Instructor: Dr. Ahmed Hagag

Answer - Assignment
Four

Answer the following questions:

Question 1:

List all ordered pairs (a, b) in the relation R from $A = \{0, 1, 2, 3, 4\}$ to $B = \{0, 1, 2, 3\}$, where

a) $a = b$

$$R = \{(0,0), (1,1), (2,2), (3,3)\}$$

b) $a > b$

$$R = \{(1,0), (2,0), (2,1), (3,0), (3,1), (3,2), (4,0), (4,1), (4,2), (4,3)\}$$

c) $a \mid b$

$$R = \{(1,0), (1,1), (1,2), (1,3), (2,0), (2,2), (3,0), (3,3), (4,0)\}$$

Question 2:

List all ordered pairs (a, b) in the relation $\{(a, b) \mid a \text{ divides } b\}$ on the set $\{1, 2, 3, 4, 5, 6\}$.

$$R = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,2), (2,4), (2,6), (3,3), (3,6), (4,4), (5,5), (6,6)\}$$

Question 3:

For each of these relations on the set $\{1, 2, 3, 4\}$ decide whether it is *reflexive*, whether it is *symmetric*, whether it is *antisymmetric*, and whether it is *transitive*.

a) $\{(2,2), (2,3), (2,4), (3,2), (3,3), (3,4)\}$

b) $\{(1,1), (1,2), (2,1), (2,2), (3,3), (4,4)\}$

c) $\{(2,4), (4,2)\}$

d) $\{(1,1), (2,2), (3,3), (4,4)\}$

	reflexive	**symmetric**	**antisymmetric**	**transitive**
a)	No	No	No	Yes
b)	Yes	Yes	No	Yes
c)	No	Yes	No	No
d)	Yes	Yes	Yes	Yes

Question 4:

Let R be the relation $\{(1, 2), (1, 3), (2, 3), (2, 4), (3, 1)\}$, and let S be the relation $\{(2, 1), (3, 1), (3, 2), (4, 2)\}$. Find $S \circ R$.

$$S \circ R = \{(1,1), (1,2), (2,1), (2,2)\}$$

Question 5:

List the ordered pairs in the relations on $\{1, 2, 3\}$ corresponding to the following matrices.

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$R = \{(1,1), (1,2), (2,1), (2,3), (3,2), (3,3)\}$$

Discrete Mathematics **Final Exam**

Dr. Ahmed Hagag

Question 1 **[10 points]**

For each of these relations on the set $\{\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}\}$, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.

a) $\{(2,4), (4,2)\}$

b) $\{(1,2), (1,3)\}$

c) $\{(2,4), (4,1), (2,1)\}$

d) $\{(1,2), (2,3), (3,4)\}$

e) $\{(1,1), (2,2), (3,3), (4,4)\}$

f) $\{(1,3), (1,4), (2,3), (2,4), (3,1), (3,4)\}$

g) $\{(1,1), (2,2), (2,1), (3,3), (4,4)\}$

h) $\{(3,3), (4,4)\}$

i) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

j) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

Please check (✓) or (×) in the following Table:

	reflexive	symmetric	antisymmetric	transitive
a)	✗	✓	✗	✗
b)	✗	✗	✓	✓
c)	✗	✗	✓	✓
d)	✗	✗	✓	✗
e)	✓	✓	✓	✓
f)	✗	✗	✗	✗
g)	✓	✗	✓	✓
h)	✗	✓	✓	✓
i)	✗	✓	✗	✓
j)	✓	✗	✓	✓

<u>Question 2</u> **[8 points]**

a) Construct the *truth table* of the compound proposition $(\neg \boldsymbol{q} \to \boldsymbol{p}) \oplus (\boldsymbol{r}\ \mathrm{A}\ (\boldsymbol{p} \leftrightarrow \boldsymbol{q}))$.

p	q	r	$\neg q$	$(\neg q \to p)$	$(p \leftrightarrow q)$	$(r\ \mathrm{A}\ (p \leftrightarrow q))$	$(\neg q \to p) \oplus (r\ \mathrm{A}\ (p \leftrightarrow q))$
T	T	T	F	T	T	T	**F**
T	T	F	F	T	T	F	**T**
T	F	T	T	T	F	F	**T**
T	F	F	T	T	F	F	**T**
F	T	T	F	T	F	F	**T**
F	T	F	F	T	F	F	**T**
F	F	T	T	F	T	T	**T**
F	F	F	T	F	T	F	**F**

b) Determine whether $\neg(\boldsymbol{p} \vee \boldsymbol{q})\ \mathrm{A}\ \boldsymbol{r}$ **and** $\boldsymbol{q}\ \mathrm{A}\ (\boldsymbol{p} \to \boldsymbol{r})$ are *logically equivalent* or not.

p	q	r	$(p \vee q)$	$\neg(p \vee q)$	$\neg(p \vee q)\ \mathrm{A}\ r$	$(p \to r)$	$q\ \mathrm{A}\ (p \to r)$
T	T	T	T	F	F	T	T
T	T	F	T	F	F	F	F
T	F	T	T	F	F	T	F
T	F	F	T	F	F	F	F
F	T	T	T	F	F	T	T
F	T	F	T	F	F	T	T
F	F	T	F	T	T	T	F
F	F	F	F	T	F	T	F

Not Logically Equivalent

c) Determine whether $\neg \boldsymbol{p} \vee \boldsymbol{p}$ is a *tautology* or not.

p	$\neg p$	$\neg p \vee p$
T	F	T
F	T	T

is a *tautology*

Question 3 **[4 points]**

Let $P(x)$ be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students. Express each of these quantifications in English.

a) $\exists x\ P(x)$

There exist at least one student spends more than five hours every weekday in class

b) $\forall x\ P(x)$

All students spend more than five hours every weekday in class

c) $\exists x\ \neg P(x)$

There exist at least one student spends less than or equal five hours every weekday in class

d) $\forall x\ \neg P(x)$

All students spend less than or equal five hours every weekday in class

Question 4 **[10 points]**

a) Find the **power** sets of the following sets:

1) $\{a, \{b\}, c\}$

$$= \{\emptyset, \{\boldsymbol{a}\}, \{\{\boldsymbol{b}\}\}, \{\boldsymbol{c}\}, \{\boldsymbol{a}, \{\boldsymbol{b}\}\}, \{\boldsymbol{a}, \boldsymbol{c}\}, \{\{\boldsymbol{b}\}, \boldsymbol{c}\}, \{\boldsymbol{a}, \{\boldsymbol{b}\}, \boldsymbol{c}\}\}$$

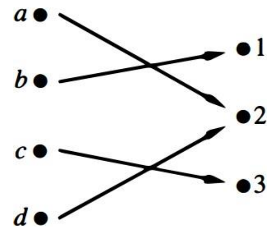
2) $\{\{\ \}\}$

$$= \{\emptyset, \{\{\ \}\}\}$$

3) $\{\emptyset, a, b\}$

$$= \{\emptyset, \{\emptyset\}, \{\boldsymbol{a}\}, \{\boldsymbol{b}\}, \{\emptyset, \boldsymbol{a}\}, \{\emptyset, \boldsymbol{b}\}, \{\boldsymbol{a}, \boldsymbol{b}\}, \{\emptyset, \boldsymbol{a}, \boldsymbol{b}\}\}$$

b) Is the following function *bijection* or not? Why?



Not bijection

Because it is not one-to-one

c) Let f and g be the functions from the set of integers to the set of integers defined by $f(x) = 2x + 3$ and $g(x) = 3x + 2$. What is the composition of f and g? What is the composition of g and f ?

$$g(f(x)) = 3(2x+3) + 2 = 6x + 11$$

$$f(g(x)) = 2(3x+2) + 3 = 6x + 7$$

d) Find the following summations:

1) $\sum_{n=1}^{20} 3 * 4^n$

$$\sum_{n=0}^{19} 12 * 4^n$$

$$a = 12, \qquad r = 4$$

$$= \frac{12(4)^{20} - 12}{4 - 1} = 4398046511100$$

Some Useful Summation Formulae.	
Sum	***Closed Form***
$\sum_{k=0}^{n} ar^k \ (r \neq 0)$	$\frac{ar^{n+1} - a}{r - 1}, r \neq 1$
$\sum_{k=1}^{n} k$	$\frac{n(n+1)}{2}$
$\sum_{k=1}^{n} k^2$	$\frac{n(n+1)(2n+1)}{6}$
$\sum_{k=1}^{n} k^3$	$\frac{n^2(n+1)^2}{4}$

2) $\sum_{k=40}^{100} k$

$$\sum_{k=40}^{100} k = \sum_{k=1}^{100} k - \sum_{k=1}^{39} k$$

$$\sum_{k=40}^{100} k = \frac{100(101)}{2} - \frac{39(40)}{2} = 5050 - 780 = 4270$$

e) How can we produce the terms of the following sequences:

1) $-2, 2, 6, 10, 14, \ldots$

Arithmetic Sequence → $a = -2, \quad d = 4$

$$\{a + nd\}_{n=0}^{\infty} \quad \rightarrow \quad \{-2 + 4n\}_{n=0}^{\infty}$$

2) $1, 5, 25, 125, 625, \ldots$

Geometric Sequence → $a = 1, \quad r = 5$

$$\{ar^n\}_{n=0}^{\infty} \quad \rightarrow \quad \{5^n\}_{n=0}^{\infty}$$

Question 5 **[4 points]**

a) What is the composite of the relations R and S, where R is the relation from $\{1, 2, 3\}$ to $\{1, 2, 3, 4\}$ with $R = \{(1,1), (1,4), (2,3), (3,1), (3,4)\}$ and S is the relation from $\{1, 2, 3, 4\}$ to $\{0, 1, 2\}$ with $S = \{(1,0), (2,0), (3,1), (3,2), (4,1)\}$?

$$S \circ R = \{(1\ ,\ 0),\ (\ 1\ ,1),\ (2,\ 1),\ (2,\ 2),\ (3\quad 0)\quad (3\quad 1)\}$$

b) Let $R = \{(1,1), (2,1), (3,2), (4,3)\}$. Find the powers $R^n, n = 2, 3, 4$

$$R^2 = R \circ R = \{(1,1), (2,1), (3,1), (4,2)\}$$

$$R^3 = R^2 \circ R = \{(1,1), (2,1), (3,1), (4,1)\}$$

$$R^4 = R^3 \circ R = \{(1,1), (2,1), (3,1), (4,1)\}$$

Question 6 **[6 points]**

a) List *three* integers that are *congruent* to 3 *modulo* 5.

$a \equiv b(\boldsymbol{mod}\ m) \quad \Leftrightarrow \quad$ there is an integer $\boldsymbol{k}$ such that $\boldsymbol{a} = \boldsymbol{b} + \boldsymbol{km}$

$$a = 3 + k * 5, \qquad k\ is\ integer$$

- $k = 1 \rightarrow a = 8$
- $k = 2 \rightarrow a = 13$
- $k = 3 \rightarrow a = 18$

b) Determine the integer 122 is *prime* or not?

The prime numbers $\leq \sqrt{122}$ are $2, 3, 5, 7$, and 11

$2|122$ |

So, 122 is not a prime integer. 122 is a composite integer.

c) Find the prime factorization of 122?

122 2
(61 |61)
1

$$122 = 2 \cdot 61$$

d) What are the **lcm**(122, 200) and **gcd**(122, 200)?

200 2
100 2
50 2
25 | 5
5 5
1

$= 2^3 \cdot 5^2 \cdot 61^0$

122 2
(61 |61) $= 2^1 \cdot 5^0 \cdot 61^1$
1

$\gcd(122, 200) = 2^1 \cdot 5^0 \cdot 61^0 = 2$

$\mathrm{lcm}(122, 200) = 2^3 \cdot 5^2 \cdot 61^1 = 12200$

Question 7 **[8 points]**

a) Use mathematical induction to show that

$$\sum_{i=1}^{n} i = 1 + 2 \ + 3 \ \cdot \ \cdot \ \cdot \ + n \ = \ \frac{n(n+1)}{2}$$

For all positive integers n.

Let $P(n)$ be the proposition that

$$1 + 2 \ + 3 \ \cdot \ \cdot \ \cdot \ + n \ = \frac{n(n+1)}{2}$$

1) Basis Step:

If $\boldsymbol{n} \ = \ \mathbf{1}$ we have $P(1) = 1 = \frac{(1)(2)}{2} = 1$, so $P(1)$ is **true**.

2) Inductive Step:

We first **Assume** that (Induction Hypothesis) $P(k)$ is true for the positive integer k, i.e.: $P(k)$

"$1 + 2 \ + 3 \ \cdot \ \cdot \ \cdot \ + \ k = \dfrac{k(k+1)}{2}$ ".

We **add** $(\boldsymbol{k} + \mathbf{1})$ to both sides of the equation in $P(k)$, we obtain

$$1 + 2 \ + 3 \ \cdot \ \cdot \ \cdot \ + \ k + (k+1) = \frac{k(k+1)}{2} + (k+1)$$

$$= \frac{k(k+1) + 2(k+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2}$$

- This equation show that $P(k+1)$ is true under the assumption that $P(k)$ is true.
- This completes the inductive step.
- So by mathematical induction we know that $P(n)$ is true for all positive integers n.

That is, we proven that

$1 + 2 \ + 3 \ \cdot \ \cdot \ \cdot \ + n \ = \frac{n(n+1)}{2}$ for all positive integers n.

b) How many bit strings of length *five* either start with a **00** bit or end with the two bits **0** ?

start with a 00 bit	$2^3 = 8$
end with the two bits 0	$2^3 = 8$
intersect	$2^1 = 2$

$$8 + 8 - 2 = 14$$

Or

start with a 00 bit	$2^3 = 8$
end with the bits 0	$2^4 = 16$
intersect	$2^2 = 4$

$$8 + 16 - 4 = 20$$

c) What is the *minimum* number of students required in a discrete mathematics class to be surethat at least **two** will receive the same mark, if the mark is start from 0 to 100 points?

There are 101 possible scores on the final. The pigeonhole principle shows that among any **102** students there must be at least 2 students with the same score.

---End of the Exam---

Discrete Mathematics **Final Exam**

Dr. Ahmed Hagag

Question 1 **[8 points]**

Part 1: (3 points)

Determine whether $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$ are logically equivalent or not.

p	q	r	$(q \rightarrow r)$	$\neg p$	$\neg p \rightarrow (q \rightarrow r)$	$(p \vee r)$	$q \rightarrow (p \vee r)$
T	T	T	T	F	T	T	T
T	T	F	F	F	T	T	T
T	F	T	T	F	T	T	T
T	F	F	T	F	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	T	F	F	F
F	F	T	T	T	T	T	T
F	F	F	T	T	T	F	T

Logically Equivalent

Part 2: (2 points)

Let $P(x)$ be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students. Express each of these quantifications in English.

a) $\exists x \, \neg P(x)$

There exist at least one student spends less than or equal five hours every weekday in class

b) $\forall x \, \neg P(x)$

All students spend less than or equal five hours every weekday in class

Part 3: (3 points)

Use the truth table to show that the hypotheses "Aly works hard," "If Aly works hard, then he is a good boy," and "If Aly is a good boy, then he will get the job" imply the conclusion "Aly will get the job."

P : Aly works hard

q : is a good boy

r : he will get the job

P

$p \to q$

$q \to r$

$\therefore r$

			P_1	P_2	P_3	$conclusion$	
p	q	r	p	$(p \to q)$	$(q \to r)$	r	
T	T	T	T	T	T	T	T
T	T	F	T	T	F	F	
T	F	T	T	F	T	T	
T	F	F	T	F	T	F	
F	T	T	F	T	T	T	
F	T	F	F	T	F	F	
F	F	T	F	T	T	T	
F	F	F	F	T	T	F	

Question 2 **[10 points]**

Part 1: (2 points)

How many relations are there on a set with 5 elements?

$$2^{5^2} = 33554432$$

Part 2: (2 points)

Let f and g be the functions from the set of integers to the set of integers defined by $f(x) = 2x + 3$ and $g(x) = 3x + 2$. What is the composition of f and g? What is the composition of g and f ?

$$g(f(x)) = 3(2x + 3) + 2 = 6x + 11$$

$$f(g(x)) = 2(3x + 2) + 3 = 6x + 7$$

Part 3: (2 points)

How can we produce the terms of the sequence 5, 25, 125, 625, ... ?

Geometric Sequence → $a = 5, \quad r = 5$

$$\{ar^n\}_{n=0}^{\infty} \quad \rightarrow \quad \{5 * 5^n\}_{n=0}^{\infty}$$

Part 4: (4 points)

Find the following summations:

1) $\sum_{n=1}^{20} 3 * 4^n$

$$\sum_{n=0}^{19} 12 * 4^n \qquad a = 12, \qquad r = 4 \qquad = \frac{12(4)^{20} - 12}{4 - 1} = 4398046511100$$

2) $\sum_{k=40}^{100} k = \sum_{k=1}^{100} k - \sum_{k=1}^{39} k$, $\quad \sum_{k=40}^{100} k = \frac{100(101)}{2} - \frac{39(40)}{2} = 5050 - 780 = 4270$

Question 3 **[7 points]**

Part 1: (1 points)

Determine the integer 1001is prime or not?

The prime numbers $\leq \sqrt{1001}$ are $2,\ 3,\ 5,\ 7,\ 11,13,17,19,\ 23\ \ldots$

$$7 \mid 1001$$

Then 1001 is a composite integer (NOT prime)

Part 2: (4 points)

What are the $\operatorname{lcm}(120, 500)$ and $\gcd(120,\ 500)$?

$\sqrt{120}$ are $2,\ 3,5,7$

$\sqrt{500}$ are $2,\ 3,\ 5,7,11,13,17,19$

$$\left(\begin{array}{r|l} 120 & 2 \\ 60 & 2 \\ 30 & 2 \\ 15 & 3 \\ 5 & 5 \\ 1 & \end{array}\right) = 2^3 \cdot 3 \cdot 5 \qquad \left(\begin{array}{r|l} 500 & 2 \\ 250 & 2 \\ 125 & 5 \\ 25 & 5 \\ 5 & 5 \\ 1 & \end{array}\right) = 2^2 \cdot 5^3$$

$$\gcd(120,\ 500) = 2^2 \cdot 3^0 \cdot 5 = 20$$

$$\operatorname{lcm}(120,\ 500) = 2^3 \cdot 3^1 \cdot 5^3 = 3000$$

Part 3: (2 points)

List three integers that are *congruent* to 3 *modulo* 5

$$a \equiv b(\boldsymbol{mod}\ \ m) \quad \Leftrightarrow \quad \text{there is an integer } \boldsymbol{k} \text{ such that } \boldsymbol{a = b + km}$$

$$a = 3 + k * 5, \qquad k \text{ is integer}$$

- $k = 1 \ \rightarrow \ a = 8$
- $k = 2 \ \rightarrow \ a = 13$
- $k = 3 \ \rightarrow \ a = 18$

Question 4 **[14 points]**

Part 1: (10 points)

For each of these relations on the set {1, 2, 3, 4}, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.

a) {(2 , 4), (4, 2)}

b) {(1 , 2), (1, 3)}

c) {(2 , 4), (4 , 1), (2, 1)}

d) {(1 , 2), (2, 3), (3 , 4)}

e) {(1 , 1), (2, 2), (3 , 3), (4, 4)}

f) {(1 , 3), (1 , 4), (2, 3), (2, 4), (3 , 1), (3 , 4)}

g) {(1 , 1), (2, 2), (2, 1), (3 , 3), (4 , 4)}

h) {(3 , 3), (4 , 4)}

i) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

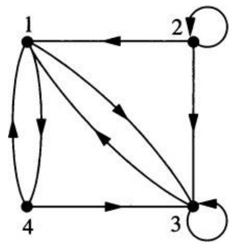
j) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

Please check (✓) or (✗) in the following Table:

	reflexive	symmetric	antisymmetric	transitive
a)	✗	✓	✗	✗
b)	✗	✗	✓	✓
c)	✗	✗	✓	✓
d)	✗	✗	✓	✗
e)	✓	✓	✓	✓
f)	✗	✗	✗	✗
g)	✓	✗	✓	✓
h)	✗	✓	✓	✓
i)	✗	✓	✗	✗
j)	✓	✗	✓	✓

Part 2: (2 points)

What are the ordered pairs in the relation R represented by the following directed graph? then find the reflexive closure of R ?



$R = \{(1, 4), (1, 3), (2,1), (2,2), (2,3), (3,1), (3,3), (4,1), (4,3)\}$

reflexive closure of $R = \{(1,1), (4,4), (1, 4), (1, 3), (2,1), (2,2), (2,3), (3,1), (3,3), (4,1), (4,3)\}$

Part 3: (2 points)

Find the matrix representing the relations R^2, where the matrix representing R is

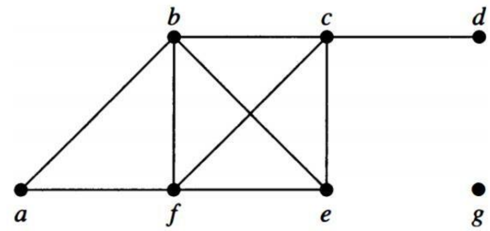
$$\mathbf{M}_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$R^2 = R \circ R = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \odot \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Question 5 **[7 points]**

Part 1: (3 points)

Find the number of vertices, the number of edges, and the degree of each vertex in the given undirected graph. Identify all isolated and pendant vertices.



Number of vertices = 7

Number of edges = 9

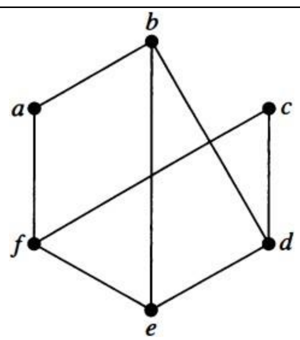
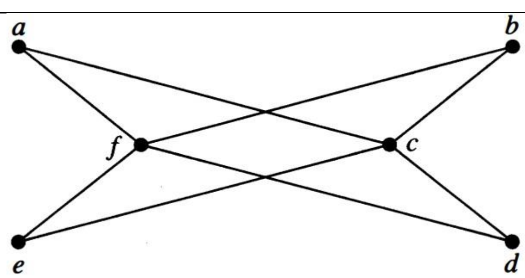
Isolated vertices = g

Pendant vertices = d

$\deg(a) = 2$ $\deg(b) = 4$ $\deg(c) = 4$ $\deg(d) = 1$ $\deg(e) = 3$ $\deg(f) = 4$ $\deg(g) = 0$

Part 2: (4 points)

Determine whether the graph is bipartite or not?

a)		NOT
b)		YES $V_1 = \{a, b, d, e\}$ $V_2 = \{f, c\}$

Question 6 **[4 points]**

Use mathematical induction to show that

$$\sum_{i=1}^{n} i = 1 + 2 + 3 \cdot \cdot \cdot + n = \frac{n(n+1)}{2}$$

Let $P(n)$ be the proposition that

$$1 + 2 + 3 \cdot \cdot \cdot + n = \frac{n(n+1)}{2}$$

1) Basis Step:

If $\boldsymbol{n} = \mathbf{1}$ we have $P(1) = 1 = \frac{(1)(2)}{2} = 1$, so $P(1)$ is **true**.

2) Inductive Step:

We first **Assume** that (Induction Hypothesis) $P(k)$ is true for the positive integer k, i.e.: $P(k)$

"$1 + 2 + 3 \cdot \cdot \cdot + k = \frac{k(k+1)}{2}$".

We **add** $(\boldsymbol{k} + \mathbf{1})$ to both sides of the equation in $P(k)$, we obtain

$$1 + 2 + 3 \cdot \cdot \cdot + k + (k+1) = \frac{k(k+1)}{2} + (k+1)$$

$$= \frac{k(k+1) + 2(k+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2}$$

- This equation show that $P(k+1)$ is true under the assumption that $P(k)$ is true.
- This completes the inductive step. So by mathematical induction we know that $P(n)$ is true for all positive integers
  n. That is, we proven that $1 + 2 + 3 \cdot \cdot \cdot + n = \frac{n(n+1)}{2}$ for all positive integers n.

Discrete Mathematics **Midterm Exam**

Dr. Ahmed Hagag

Question 1 **[1 points]**

Which of these sentences are propositions? What are the truth values of those that are propositions?

a) $x + y = 5$, where $x = 2$

Not Proposition

b) $4 - 2 = 5$

Proposition false

c) $1 + 2 = 3$

Proposition true

Question 2 **[2 points]**

Let p be the statement "Maria learns discrete mathematics" and q the statement "Maria will find a good job." Express the statement $p \to q$ as a statement in English.

If Maria learns discrete mathematics, then she will find a good job

Question 3 **[2 points]**

Construct the truth table of the compound proposition $(\neg q \vee p) \oplus (\neg p \wedge (p \to q))$.

p	q	$\neg q$	$(\neg q \vee p)$	$\neg p$	$p \to q$	$(\neg p \wedge (p \to q))$	$(\neg q \vee p) \oplus (\neg p \wedge (p \to q))$
T	T	F	T	F	T	F	**T**
T	F	T	T	F	F	F	**T**
F	T	F	F	T	T	T	**T**
F	F	T	T	T	T	T	**F**

Question 4 **[2 points]**

What are the negations of the following statements

a) $\forall x(x^2 = -2)$

$$\exists x(x^2 \neq -2)$$

b) $\neg\exists x(x > 1)$

$$\exists x(x \leq 1)$$

Question 5 **[3 points]**

What are the converse, inverse, and contrapositive of the conditional statement **"**If you study well, then you will find a good job**"**

Converse (If found a good job, then you studied well)

Inverse (If you not study well, then you will not find a good job)

Contrapositive (If you not found a good job, then you did not studied well)

Question 6 **[3 points]**

Determine whether $\neg p \rightarrow (r \wedge q)$ and $q \wedge (p \leftrightarrow r)$ are logically equivalent or not.

p	q	r	$\neg p$	$(r \wedge q)$	$\neg p \rightarrow (r \wedge q)$	$(p \leftrightarrow r)$	$q \wedge (p \leftrightarrow r)$
T	T	T	F	T	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	F	T	T	F
T	F	F	F	F	T	F	F
F	T	T	T	T	T	F	F
F	T	F	T	F	F	T	T
F	F	T	T	F	F	F	F
F	F	F	T	F	F	T	F

Not Logically Equivalent

Question 7 **[3 points]**

Let p and q be the propositions

p **:** It is below freezing.

q **:** It is snowing.

Write these propositions using p and q and logical connectives.

a) It is either snowing or below freezing (or both).

$$(\boldsymbol{p} \vee \boldsymbol{q})$$

b) If it is below freezing, it is also snowing.

$$(\boldsymbol{p} \rightarrow \boldsymbol{q})$$

c) That it is below freezing is necessary sufficient for it to be snowing.

$$(\boldsymbol{p} \leftrightarrow \boldsymbol{q})$$

Question 8 **[2 points]**

Translate these statements into English, where $P(x)$ is "x has taken a course in programming" and the domain consists of the students in your class.

a) $\forall x P(x)$

Every student in your class has taken a course in programming

b) $\exists x \neg P(x)$

There is at least one student in your class who has not taken a course in programming

Question 9 **[2 points]**

Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction.

Let p the proposition "$\sqrt{2}$ is irrational".

1) To start a proof by contradiction, we suppose that $\neg p$ is true, then $\sqrt{2}$ is rational.
2) $\sqrt{2} = \frac{a}{b}$, where a and b are intergers without common factor and $b \neq 0$
3) $2 = \frac{a^2}{b^2}$
4) $a^2 = 2b^2$, then a is even, and you can write $a = 2m$, where m is integer.
5) $(2m)^2 = 2b^2$, then $4m^2 = 2b^2$.
6) $b^2 = 2m^2$, then b is also even.
7) Because a and b are even, the a and b have a common factor 2.
8) Therefore, $\sqrt{2} = \frac{a}{b}$ is not rational, then $\neg p$ is false.
9) Therefore, p is true and $\sqrt{2}$ is irrational.

---End of the Exam---

Discrete Mathematics **Midterm Exam**

Dr. Ahmed Hagag

Question 1 **[8 points]**

Part A: [1 points]

Which of these sentences are propositions? What are the truth values of those that are propositions?

a) $2x + 1 = 9$, where $x = 4$

Proposition true

b) $1 + 7 = 7$

Proposition false

c) What time is it?

Not Proposition

d) $x + y = 2$, where $x = 2$

Not Proposition

Part B: [3 points]

What are the converse, inverse, and contrapositive of the conditional statement "If you study well, then you will succeed"

Converse (If you succeed, then you studied well)

Inverse (If you not study well, then you will not succeed)

Contrapositive (If you not succeed, then you did not studied well)

Part C: [1 points]

Determine whether $\neg \boldsymbol{p} \lor \boldsymbol{p}$ is a *tautology* or not.

p	$\neg p$	$\neg p \lor p$
T	F	T
F	T	T

is a *tautology*

Part D: [3 points]

Determine whether $\neg(\boldsymbol{p} \lor \boldsymbol{q}) \leftrightarrow \boldsymbol{r}$ **and** $\boldsymbol{q}$ A $(\boldsymbol{p} \rightarrow \boldsymbol{r})$ are *logically equivalent* or not.

p	q	r	$(p \lor q)$	$\neg(p \lor q)$	$\neg(p \lor q) \leftrightarrow r$	$(p \rightarrow r)$	q A $(p \rightarrow r)$
T	T	T	T	F	F	T	T
T	T	F	T	F	T	F	F
T	F	T	T	F	F	T	F
T	F	F	T	F	T	F	F
F	T	T	T	F	F	T	T
F	T	F	T	F	T	T	T
F	F	T	F	T	T	T	F
F	F	F	F	T	F	T	F

Not Logically Equivalent

Question 2 **[10 points]**

Part A: [2 points]

Let $P(x)$ be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students. Express each of these quantifications in English.

a) $\exists x \; P(x)$

There exist at least one student spends more than five hours every weekday in class

b) $\forall x \; \neg P(x)$

All students spend less than or equal five hours every weekday in class

Part B: [3 points]

Let p and q be the propositions

p **:** You drive over 90 kilometer per hour.

q **:** You get a speeding ticket.

Write these propositions using p and q and logical connectives.

a) You will get a speeding ticket if you drive over 90 kilometer per hour.

$$p \rightarrow q$$

b) You drive over 90 kilometer per hour, but you do not get a speeding ticket.

$$p \text{ A } \neg q$$

c) Driving over 90 kilometer per hour is sufficient for getting a speeding ticket.

$$p \rightarrow q$$

Part C: [3 points]

Use rules of inference (see Table 1) to show that the hypotheses " George studies well," " If George study well then he will succeed," and " George will graduate" imply the conclusion " George will succeed and graduate."

P : George studies well

q : George will succeed

r : George will graduate

TABLE 1 Rules of Inference.

Rule of Inference	Tautology	Name
p $p \to q$ $\therefore q$	$[p \land (p \to q)] \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$[\neg q \land (p \to q)] \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$[(p \to q) \land (q \to r)] \to (p \to r)$	Hypothetical syllogism
$p \lor q$ $\neg p$ $\therefore q$	$[(p \lor q) \land \neg p] \to q$	Disjunctive syllogism

p

$p \to q$

r

$\therefore q$ A r

(1) Using Modus ponens

p

$p \to q$

--_-____

$\therefore q$

(2) Using Conjunction

q

r

--_-____

$\therefore q$ A r

Part D: [2 points]

Prove by contraposition that if n is an integer and $3n+2$ is odd, then n is odd.

$$P : 3n \ + \ 2 \text{ is odd}$$

$$q : n \text{ is odd}$$

$$p \to q$$

$$\text{contraposition } \ \neg q \to \neg p$$

1. we assume that(n is even)

   $n = 2m,$ where m is integer.

2. $(3n+2) = (3(2m)+2)$

   $(3n+2) = (6m+2) = 2(3m+1) = \text{even}$

   $\therefore (3n+2)$is even

3. $\therefore \neg q \to \neg p$ is true, then $p \to q$ is also true.

Question 3 **[2 points]**

Find the power sets of the following sets:

a) $\{a, \{b\}, c\}$

$$= \{\emptyset, \{\boldsymbol{a}\}, \{\{\boldsymbol{b}\}\}, \{\boldsymbol{c}\}, \{\boldsymbol{a}, \{\boldsymbol{b}\}\}, \{\boldsymbol{a}, \boldsymbol{c}\}, \{\{\boldsymbol{b}\}, \boldsymbol{c}\}, \{\boldsymbol{a}, \{\boldsymbol{b}\}, \boldsymbol{c}\}\}$$

b) {{ }}

$$= \{\emptyset, \{\{\ \}\}\}$$

Discrete Mathematics **Midterm Exam**

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Question 1 **[10 points]**

Part A: [2 points]

What are the negations of the following statements

a) $\forall x(x^2 = -2)$

$$\exists x(x^2 \neq -2)$$

b) $\neg\exists x(x > 1)$

$$\exists x(x \leq 1)$$

Part B: [3 points]

What are the converse, inverse, and contrapositive of the conditional statement "You will find a goodjob, if you study well"

Converse (If found a good job, then you studied well)

Inverse (If you not study well, then you will not find a good job)

Contrapositive (If you not found a good job, then you did not studied well)

Part C: [2 points]

Construct the truth table of the compound proposition $(\neg q \vee p) \oplus (\neg p \wedge (p \rightarrow q))$.

p	q	$\neg q$	$(\neg q \vee p)$	$\neg p$	$p \rightarrow q$	$(\neg p \wedge (p \rightarrow q))$	$(\neg q \vee p) \oplus (\neg p \wedge (p \rightarrow q))$
T	T	F	T	F	T	F	**T**
T	F	T	T	F	F	F	**T**
F	T	F	F	T	T	T	**T**
F	F	T	T	T	T	T	**F**

Part D: [3 points]

Determine whether $\neg p \rightarrow (r \wedge q)$ and $q \wedge (p \leftrightarrow r)$ are *logically equivalent* or not.

p	q	r	$\neg p$	$(r \wedge q)$	$\neg p \rightarrow (r \wedge q)$	$(p \leftrightarrow r)$	$q \wedge (p \leftrightarrow r)$
T	T	T	F	T	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	F	T	T	F
T	F	F	F	F	T	F	F
F	T	T	T	T	T	F	F
F	T	F	T	F	F	T	T
F	F	T	T	F	F	F	F
F	F	F	T	F	F	T	F

Not Logically Equivalent

Question 2 **[10 points]**

Part A: [2 points]

Let $P(x)$ be the statement "x has taken a course in programming," where the domain for x consists of all students in your class. Express each of these quantifications in English.

a) $\forall x\ P(x)$

Every student in your class has taken a course in programming

b) $\exists x\ \neg P(x)$

There is at least one student in your class who has not taken a course in programming

Part B: [2 points]

Let p and q be the propositions

Determine the truth value of each of the following statements if the domain consists of the integers $-1, 0, 1,$ and 2.

c) $\forall x(x^2 \geq x)$

True

d) $\exists x(2x > x)$

True

Part C: [4 points]

Use rules of inference (see Table 1) to show that the hypotheses " Maria is a student in this class or she is graduated," "Maria is not a student in this class or she has taken a DM course," and " Maria has not yet graduated" imply the conclusion " Maria has taken a DM course."

P : Maria is a student in this class

q : Maria is graduated

r : Maria has taken a DM course

$p \vee q$

$\neg p \vee r$

$\neg q$

$\therefore r$

TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \rightarrow q$ $\therefore q$	$[p \wedge (p \rightarrow q)] \rightarrow q$	Modus ponens
$\neg q$ $p \rightarrow q$ $\therefore \neg p$	$[\neg q \wedge (p \rightarrow q)] \rightarrow \neg p$	Modus tollens
$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	$[(p \vee q) \wedge \neg p] \rightarrow q$	Disjunctive syllogism

(1) Using Resolution

$p \vee q$

$\neg p \vee r$

--_-___

$\therefore q \vee r$

(2) Using Disjunctive syllogism

$q \vee r$

$\neg q$

--_-___

$\therefore r$

Part D: [2 points]

Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction.

Let p the proposition "$\sqrt{2}$ is irrational".

1) To start a proof by contradiction, we suppose that $\neg p$ is true, then $\sqrt{2}$ is rational.
2) $\sqrt{2} = \frac{a}{b}$, where a and b are intergers without common factor and $b \neq 0$
3) $2 = \frac{a^2}{b^2}$
4) $a^2 = 2b^2$, then a is even, and you can write $a = 2m$, where m is integer.
5) $(2m)^2 = 2b^2$, then $4m^2 = 2b^2$.
6) $b^2 = 2m^2$, then b is also even.
7) Because a and b are even, the a and b have a common factor 2.
8) Therefore, $\sqrt{2} = \frac{a}{b}$ is not rational, then $\neg p$ is false.
9) Therefore, p is true and $\sqrt{2}$ is irrational.

Choose the Correct Answer

1. **The statement " $x + 2 = 7$, for $x = 3$ " is ___________.**

   A. Proposition False

   B. Proposition True

   C. Not a Proposition

   D. Proposition both True and False

2. **If p is false and q is true, then $(p \vee \neg q) \rightarrow (p \wedge q)$ is __________.**

   A. False

   B. True

   C. Neither true nor false

   D. Both true and false

3. **If p is true and q is false, then $p \rightarrow q$ is___________.**

   A. False

   B. True

   C. Neither true nor false

   D. Both true and false

4. **A compound proposition $(p \wedge q) \rightarrow p$ is**_________.

   A. Tautology

   B. Contradiction

   C. Contingency

   D. Equivalent

5. **Let p is a proposition, then $p \vee \neg p$ is logical equivalent to ______ _.**

   A. True

   B. False

   C. p

   D. q

6. **Express the statement "Every student in your class has taken a course in computer". Where $P(x)$ is "x has taken a course in computer". The domain of x is the set of the students in your class.**

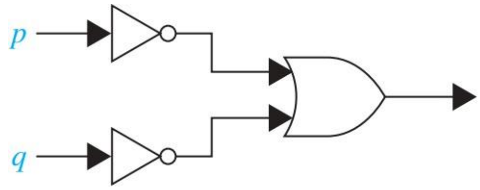
   A. $\forall x P(x)$

   B. $\exists x P(x)$

   C. $\forall x \neg P(x)$

   D. $P(x)$

7. **Find the output of the following combinatorial circuit.**



A. $\neg(p \land q)$

B. $\neg p \land \neg q$

C. $\neg(p \lor q)$

D. $p \land q$

8. **Let p is a proposition and T stands for True, then $p \lor$ T is logical equivalent to ______.**

A. True

B. False

C. p

D. q

9. **If** A **and** B **are sets, then** A **and** B **are equal if and only if __________.**

A. $\forall x(x \in A \leftrightarrow x \in B)$.

B. $\forall x(x \in A \rightarrow x \in B)$

C. $\forall x(x \in A)$

D. $\exists x(x \in A)$

10. **The power set of the set $S = \{a, \{b, c\}\}$ is___________.**

A. $\{\emptyset, \{a\}, \{\{b, c\}\}, \{a, \{b, c\}\}\}$

B. $\{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

C. $\{\emptyset, a, \{b, c\}, \{a, b, c\}\}$

D. $\emptyset$

11. **The union of the sets A and B, $A \cup B$ is equal to_________.**

A. $\{x | x \in A \lor x \in B\}$

B. $\{x | x \in A \land x \in B\}$

C. $\{x | x \in A \lor x \notin B\}$

D. $\{x\}$

12. **Let f be the function from $\{a, b, c\}$ to $\{1, 2, 3\}$ such that $f(a) = 2$, $f(b) = 3$, and $f(c) = 1$. The function f is _________.**

A. One-to-one correspondence

B. One-to-one ONLY

C. Onto ONLY

D. Not bijection

13. **The following double sums $\sum_{i=0} \sum_{j=0}^{3} j$ is equal __________.**

A. 18

B. $6i$

C. $6j$

D. 12

14. $\lceil -5.2 \rceil$ =________.

A. -5
B. 5
C. 6
D. -6

15. Binary search can be used when the list has terms occurring in ________.

A. Order of increasing size
B. Unordered
C. Even numbers
D. Odd numbers

16. The bubble sort comparing adjacent elements, interchanging them if they are in ______.

A. The wrong order
B. The right order
C. 1st location
D. 2nd location

17. If p is true and q is false, then $p \oplus q$ is ________.

A. Fals
eB.
True
C. Neither true nor false
D. Both true and false

18. Let p and q be the propositions: p: The automated reply can be sent. q: The file system is full. Express the following statement using p and q and logical connectives.
"The automated reply cannot be sent when the file system is full."

A. $p \rightarrow q$
B. $q \rightarrow \neg p$
C. $q \rightarrow p$
D. $p \rightarrow \neg q$

19. The set S of odd positive integers less than 7 can be expressed by using set builder notation as _____.

A. $S = \{x \in R^+ | x \text{ is odd and } x < 7\}$
B. $S = \{x \in Z^+ | x \text{ is odd and } x < 7\}$
C. $S = \{x | x \text{ is odd and } x < 7\}$
D. $S = \emptyset$

20. If A and B are sets, then A is said to be a subset of B if and only if ________.

A. $\forall x(x \in A \leftrightarrow x \in B)$.B.
$\forall x(x \in A \rightarrow x \in B)$
C. $\forall x(x \in A)$
D. $\exists x(x \in A)$

21. The difference of the sets A and B (i.e., $A - B$) is equal to________.

A. $\{x | x \in A \vee x \in B\}$
B. $\{x | x \in A \wedge x \notin B\}$
C. $\{x | x \in A \vee x \notin B\}$
D. $\{x\}$

22. The matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 5 \\ 2 & 5 & -1 \end{bmatrix}$**, then the transpose of A = (A^t) = ________.**

A. $\begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 5 \\ 2 & 5 & -1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & 5 \\ 2 & 5 & -1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 5 \\ 2 & 5 & -1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 5 \\ 2 & 0 & -1 \end{bmatrix}$

23. The meet of the two metrices $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ **and** $B = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ **is________.**

A. $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$

24. An algorithm should produce the correct output values for each set of input values. This means_________ _.

A. Finiteness

B. Definitenes

sC.

Correctness

D. Generality

25. $(\mathbf{10011} \vee \mathbf{01010}) \oplus \mathbf{11111}$ **is __________ _.**

A. 11111

B. 00000

C. 00100

D. 11011

26. Let p **is a proposition and F stands for False, then** $p \vee$ **F is logical equivalent to __ ___ _.**

A. True

B. Fals

eC. p

D. q

27. Let p **is a proposition and T stands for True, then** $p \wedge$ **T is logical equivalent to __ ___ _.**

A. True

B. Fals

eC. p

D. q

28. Let p and q are two propositions, then $p \to q$ is logical equivalent to_____ __.

A. $p \vee q$

B. $p \wedge q$

C. $\neg p \vee q$

D. $q \to p$

29. The set $\mathbf{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$ is the set of ___________.

A. Real Numbers

B. Natural Numbers

C. Integers Numbers

D. Complex Numbers

30. If $2^S = \{\emptyset\}$, then $S =$ ___________.

A. $\{\{\emptyset\}\}$

B. $\{\emptyset\}$

C. $\emptyset$

D. $\{\{\ \}\}$

31. Let U be the universal set. The complement of the set A is equal to__________.

A. $\{x \notin U | x \notin A\}$

B. $\{x | x \in A\}$

C. $\{x \in U | x \notin A\}$

D. $\{x\}$

32. The composition $f \circ g$ cannot be defined unless the range of g is a subset of the________.

A. Domain of g

B. Co-Domain of f

C. Domain of f

D. Co-Domain of g

33. $(01111 \wedge 10101) \vee 01000$ is ___________.

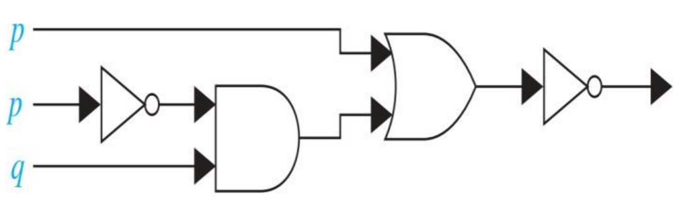
A. 10101

B. 01001

C. 01111

D. 01101

34. Find the output of the following combinatorial circuit.



A. $\neg p \vee \neg q$

B. $\neg(p \vee p \wedge q)$

C. $(\neg p \wedge q) \vee p$

D. $\neg p \wedge (p \vee \neg q)$

35. Express the statement "Every student in FCAI has an email". Where $P(x)$ is "x in FCAI", $F(x)$ is "x has an email". The domain of x is the set of all students in Egypt.

A. $\forall x F(x)$

B. $\forall x P(x)$

C. $\forall x(P(x) \wedge F(x))$

D. $\forall x(P(x) \rightarrow F(x))$

36. Let $A = \{1, 2, 3, 4\}$, and $B = \{a, b, c\}$, the $|A \times B|$ is ___________elements.

A. 3

B. 4

C. 1

6D.

12

37. The general term a_n of the sequence $15,\ 8,\ 1,\ -6,\ -13,\ -20, \ldots$ is ___________.

A. $\{15 - n\}$

B. $\{15 + 7n\}$

C. $\{7 - 15n\}$

D. $\{15 - 7n\}$

38. $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & 0 \\ 1 & 3 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & -1 \\ 1 & 1 \end{bmatrix} =$ _________.

A. $\begin{bmatrix} 9 & 2 \\ 3 & 1 \\ 9 & -2 \end{bmatrix}$

B. $\begin{bmatrix} 9 & 0 \\ 0 & 0 \\ 9 & -2 \end{bmatrix}$

C. $\begin{bmatrix} 9 & 1 \\ 0 & 1 \\ 9 & -2 \end{bmatrix}$

D. $\begin{bmatrix} 9 & 2 \\ 0 & 0 \\ 9 & -2 \end{bmatrix}$

39. $\lfloor 3.5 \rfloor =$__________.

A. -4

B. 4

C. -

3D. 3

40. A(an)_________is a sequence of the form $a + ar + ar^2, \ldots, ar^n, \ldots$

A. Arithmetic progression

B. Floor function

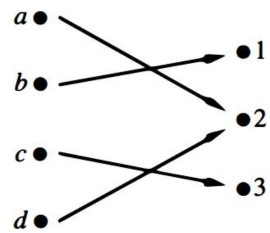
C. Fibonacci

D. Geometric progression

Made by: Eng/Alaa Nassar & Eng/General

Answer

<u>Question 1</u> Is the following function *bijection* or not? Why? **[1 points]**



Not bijection

Because it is not one-to-one

<u>Question 2</u> **Find these values:** **[2 points]**

a) $\lfloor -0.7 \rfloor = -1$

b) $\lfloor \lceil -0.7 \rceil \rfloor = 0$

c) $\lceil -2.7 \rceil = -2$

d) $\lfloor 2 + 0.1 \rfloor = 2$

<u>Question 3</u> **How can we produce the terms of the following sequences:** **[2 points]**

$-2, 2, 6, 10, 14, \ldots$

Arithmetic Sequence → $a = -2, \quad d = 4$

$$\{a + nd\}_{n=0}^{\infty} \quad \rightarrow \quad \{-2 + 4n\}_{n=0}^{\infty}$$

<u>Question 4</u> **What is the $\mathrm{lcm}(24, 36)$?** **[2 points]**

What is the $\mathrm{lcm}(24, 36)$?

$\sqrt{24}$ are $2, 3$

$\sqrt{36}$ are $2, 3, 5$

$$\left(\begin{array}{c|c} 24 & 2 \\ 12 & 2 \\ 6 & 2 \\ 3 & 3 \\ 1 & \end{array}\right) = 2^3 \cdot 3$$

$$\left(\begin{array}{c|c} 36 & 2 \\ 18 & 2 \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array}\right) = 2^2 \cdot 3^2$$

$$\mathrm{lcm}(24{,}36) = 2^3 \cdot 3^2 = 72$$

<u>Question 5</u> **Is R reflexive, symmetric, and/or antisymmetric?** **[3 points]**

Where the relation R on a set $\{a, b, c\}$ is represented by the following table

R	a	b	c
a	×		
b	×		×
c	×		×

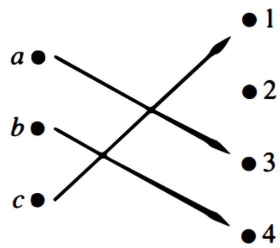
Not reflexive, **Not** symmetric, **antisymmetric**

Discrete Mathematics **Quiz (02).v2**

Dr. Ahmed Hagag

Answer

Question 1 Is the following function *bijection* or not? Why? **[1 points]**



Not bijection

Because it is onto

Question 2 **Find these values:** **[2 points]**

a) $\lceil -0.1 \rceil = 0$

b) $\lfloor \lceil -0.3 \rceil \rfloor = 0$

c) $\lfloor -5.2 \rfloor = -6$

d) $\lceil 1 - 0.2 \rceil = 1$

Question 3 **How can we produce the terms of the following sequences:** **[2 points]**

1, 5, 25, 125, 625, …

Geometric Sequence → $a = 1, \quad r = 5$

$$\{ar^n\}_{n=0}^{\infty} \rightarrow \{5^n\}_{n=0}^{\infty}$$

Question 4 **What is the $\gcd(24, 36)$?** **[2 points]**

$\sqrt{24}$ are $2, 3$ $\sqrt{36}$ are $2, 3, 5$

$$\left(\begin{array}{c|c} 24 & 2 \\ 12 & 2 \\ 6 & 2 \\ 3 & 3 \\ 1 & \end{array}\right) = 2^3 \cdot 3 \qquad \left(\begin{array}{c|c} 36 & 2 \\ 18 & 2 \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array}\right) = 2^2 \cdot 3^2$$

$$\gcd(24,36) = 2^2 \cdot 3 = 12$$

Question 5 **Is R reflexive, symmetric, and/or antisymmetric?** **[3 points]**

Where the relation R on a set is represented by the following matrix

R	a	b	c
a	×		
b		×	
c	×		×

reflexive, **Not** symmetric, **Not** antisymmetric

Year:
Course Name:
Course Code:
Instructor:

THE EGYPTIAN E-LEARNING UNIVERSITY

Date:
Time Allowed:
Mark: 20
Test Code: 01

I. Choose The Correct Answer

1- (01 1011 0110) AND (11 0001 1101) is

- (A) 00 1001 0110
- (B) 00 0011 0100
- (C) 01 0001 0100
- (D) 01 0011 0100

2- Let p and q be the propositions

p : You drive over 90 kilometer per hour.

q : You get a speeding ticket.

Write the following proposition using p and q and logical connectives.

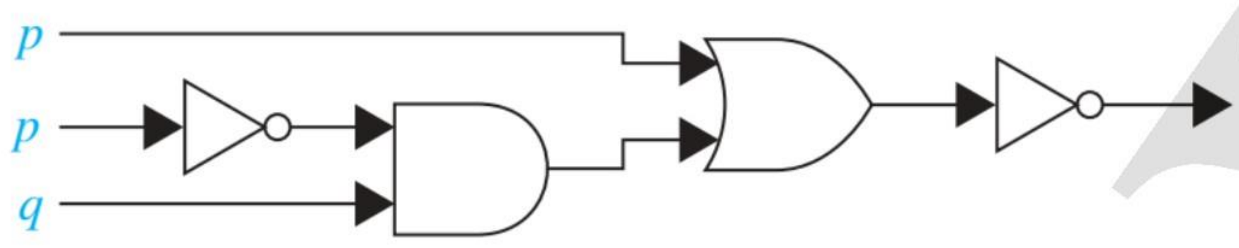
- You drive over 90 kilometer per hour, but you do not get a speeding ticket.

- (A) $p \to q$
- (B) $p \wedge \neg q$
- (C) $p \wedge q$
- (D) $p \leftrightarrow q$

3- $(01111 \wedge 10101) \vee 01000$

- (A) 01001
- (B) 01101
- (C) 11101
- (D) 11001

4- Find the output of the following combinatorial circuit



- (A) $\neg(p \vee (p \wedge q))$
- (B) $(p \vee (\neg p \wedge q))$
- (C) $\neg p \wedge (p \vee \neg q)$
- (D) $(p \vee (\neg p \vee \neg q))$

5- Express the statement "Every student in EELU has an email". Where $P(x)$ is "x in EELU", $F(x)$ is "x has an email". The domain of x is the set of all students in Egypt.

- (A) $\forall x F(x)$
- (B) $\forall x(P(x) \to F(x))$
- (C) $\forall x(P(x) \wedge F(x))$
- (D) $\exists x(P(x) \to F(x))$

6- The cardinality of the power set for $\{a, \{b, d, e\}, c\}$ is .... elements.

Ⓐ 2
Ⓑ 8
Ⓒ 16
Ⓓ 32

7- $2^S = \{\emptyset\}$, then $S = \cdots$

Ⓐ $\{\emptyset\}$
Ⓑ $\{\{\emptyset\}\}$
Ⓒ $\{\infty\}$
Ⓓ $\emptyset$

II. State True or False

1- $2 + 3 = 5$ is not a proposition.

Ⓐ [True]
Ⓑ [False]

2- If p is true and q is false, then $p \oplus q$ is true.

Ⓐ [True]
Ⓑ [False]

3- If p is false and q is true, then $(p \vee \neg q) \rightarrow (p \wedge q)$ is false.

Ⓐ [True]
Ⓑ [False]

4- $p \vee (p \wedge q) \equiv p$

Ⓐ [True]
Ⓑ [False]

5- $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is a tautology

Ⓐ [True]
Ⓑ [False]

6- The hypotheses " George studies well, " " If George study well then he will succeed," and " George will graduate" imply the conclusion " George will succeed and graduate." **is not a valid argument.**

Ⓐ [True]
Ⓑ [False]

7- $A \subset B \leftrightarrow \forall x(x \in A \rightarrow x \in B)$

Ⓐ [True]
Ⓑ [False]

8- A one-to-one correspondence is called **invertible** because we can define an inverse of this function.

Ⓐ [True]
Ⓑ [False]

9- The composition $f \circ g$ cannot be defined unless the range of f is a subset of the domain of g.

Ⓐ [True]
Ⓑ [False]

Faculty of Information Technology
GEN206 Discrete Mathematics
Dr. Ahmed Hagag

EELU
الجامعة المصرية للتعلم الإلكتروني
Egyptian E-Learning University

Fall Semester 2017/2018
Final Exam (3 Hours)
Sunday, December 24, 2017

Please answer in the examination with no extra pages

برجاء الإجابة فى نفس كراسة الأسئلة بدون أوراق خارجية

Question 1 **[10 points]**

For each of these relations on the set $\{1, 2, 3, 4\}$, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.

a) $\{(2,4),(4,2)\}$

b) $\{(1,2),(1,3)\}$

c) $\{(2,4),(4,1),(2,1)\}$

d) $\{(1,2),(2,3),(3,4)\}$

e) $\{(1,1),(2,2),(3,3),(4,4)\}$

f) $\{(1,3),(1,4),(2,3),(2,4),(3,1),(3,4)\}$

g) $\{(1,1),(2,2),(2,1),(3,3),(4,4)\}$

h) $\{(3,3),(4,4)\}$

i) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

j) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

Please check (✓) or (×) in the following Table:

	reflexive	symmetric	antisymmetric	transitive
a)				
b)				
c)				
d)				
e)				
f)				
g)				
h)				
i)				
j)				

Question 2 [8 points]

a) Construct the *truth table* of the compound proposition $(\neg q \rightarrow p) \oplus (r \land (p \leftrightarrow q))$.

b) Determine whether $\neg(p \lor q) \land r$ **and** $q \land (p \rightarrow r)$ are *logically equivalent* or not.

c) Determine whether $\neg p \lor p$ is a *tautology* or not.

Question 3 [4 points]

Let $P(x)$ be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students. Express each of these quantifications in English.

a) $\exists x\, P(x)$

---

b) $\forall x\, P(x)$

---

c) $\exists x\, \neg P(x)$

---

d) $\forall x\, \neg P(x)$

Question 4 [10 points]

a) Find the **power** sets of the following sets:

1) $\{a, \{b\}, c\}$

---

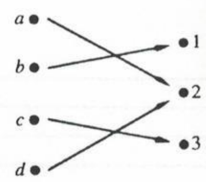
2) $\{\{\ \}\}$

---

3) $\{\emptyset, a, b\}$

---

b) Is the following function *bijection* or not? Why?



c) Let f and g be the functions from the set of integers to the set of integers defined by $f(x) = 2x + 3$ and $g(x) = 3x + 2$. What is the composition of f and g? What is the composition of g and f ?

d) Find the following summations:

1) $\sum_{n=1}^{20} 3 * 4^n$

Some Useful Summation Formulae.

Sum	*Closed Form*
$\sum_{k=0}^{n} ar^k \ (r \neq 0)$	$\frac{ar^{n+1} - a}{r - 1}, r \neq 1$
$\sum_{k=1}^{n} k$	$\frac{n(n+1)}{2}$
$\sum_{k=1}^{n} k^2$	$\frac{n(n+1)(2n+1)}{6}$
$\sum_{k=1}^{n} k^3$	$\frac{n^2(n+1)^2}{4}$

2) $\sum_{k=40}^{100} k$

c) How can we produce the terms of the following sequences:

1) $-2, 2, 6, 10, 14, \ldots$

2) $1, 5, 25, 125, 625, \ldots$

Question 5 **[4 points]**

a) What is the composite of the relations R and S, where R is the relation from $\{1, 2, 3\}$ to $\{1, 2, 3, 4\}$ with $R = \{(1,1), (1,4), (2,3), (3,1), (3,4)\}$ and S is the relation from $\{1, 2, 3, 4\}$ to $\{0, 1, 2\}$ with $S = \{(1,0), (2,0), (3,1), (3,2), (4,1)\}$?

b) Let $R = \{(1,1), (2,1), (3,2), (4,3)\}$. Find the powers $R^n, n = 2, 3, 4$

Question 6 **[6 points]**

a) List *three* integers that are *congruent* to 3 *modulo* 5.

b) Determine the integer 122 is *prime* or not?

c) Find the prime factorization of 122?

d) What are the **lcm**(122, 200) and **gcd**(122, 200)?

Question 7 [8 points]

a) Use mathematical induction to show that

$$\sum_{i=1}^{n} i = 1 + 2 \; + 3 \cdot \; \cdot \; \cdot + n = \frac{n(n+1)}{2}$$

For all positive integers n.

b) How many bit strings of length *five* either start with a **00** bit or end with the two bits **0** ?

c) What is the *minimum* number of students required in a discrete mathematics class to be sure that at least **two** will receive the same mark, if the mark is start from 0 to 100 points?

---End of the Exam---

Faculty of Information Technology
GEN206 Discrete Mathematics
Dr. Ahmed Hagag

EELU
الجامعة المصرية للتعلم الإلكتروني
Egyptian E-Learning University

Fall Semester 2018/2019
Midterm Exam (90 Minutes)
Thursday, November 08, 2018

Please answer in the examination with no extra pages برجاء الإجابة في نفس كراسة الأسئلة بدون أوراق خارجية

Question 1 **[8 points]**

Part A: [1 points]

Which of these sentences are propositions? What are the truth values of those that are propositions?

a) $2x + 1 = 9$, where $x = 4$

Proposition true

b) $1 + 7 = 7$

Proposition false

c) What time is it?

Not Proposition

d) $x + y = 2$, where $x = 2$

Not Proposition

Part B: [3 points]

What are the converse, inverse, and contrapositive of the conditional statement "If you study well, then you will succeed"

Converse (If you succeed, then you studied well)

Inverse (If you not study well, then you will not succeed)

Contrapositive (If you not succeed, then you did not studied well)

Part C: [1 points]

Determine whether ¬*p* V *p* is a *tautology* or not.

p	¬*p*	¬*p* V *p*
T	F	T
F	T	T

is a *tautology*

Part D: [3 points]

Determine whether ¬(*p* V q) ① *r* and q A (*p* → *r*) are *logically equivalent* or not.

p	*q*	*r*	(*p* V *q*)	¬(*p* V *q*)	¬(*p* V *q*) ① *r*	(*p* → *r*)	*q* A (*p* → *r*)
T	T	T	T	F	F	T	T
T	T	F	T	F	T	F	F
T	F	T	T	F	F	T	F
T	F	F	T	F	T	F	F
F	T	T	T	F	F	T	T
F	T	F	T	F	T	T	T
F	F	T	F	T	T	T	F
F	F	F	F	T	F	T	F

Not Logically Equivalent

Question 2 **[10 points]**

Part A: [2 points]

Let $P(x)$ be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students. Express each of these quantifications in English.

a) Ex $P(x)$

There exist at least one student spends more than five hours every weekday in class

b) ✯x $\neg P(x)$

All students spend less than or equal five hours every weekday in class

Part B: [3 points]

Let p and q be the propositions

p **:** You drive over 90 kilometer per hour.

q **:** You get a speeding ticket.

Write these propositions using p and q and logical connectives.

a) You will get a speeding ticket if you drive over 90 kilometer per hour.

$$p \to q$$

b) You drive over 90 kilometer per hour, but you do not get a speeding ticket.

$$p \text{ A } \neg q$$

c) Driving over 90 kilometer per hour is sufficient for getting a speeding ticket.

$$p \to q$$

Part C: [3 points]

Use rules of inference (see Table 1) to show that the hypotheses " George studies well," " If George study well then he will succeed," and " George will graduate" imply the conclusion " George will succeed and graduate."

P : George studies well

q : George will succeed

r : George will graduate

TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$[p \wedge (p \to q)] \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$[\neg q \wedge (p \to q)] \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	$[(p \vee q) \wedge \neg p] \to q$	Disjunctive syllogism

p

$p \to q$

r

$\therefore q$ A r

(1) Using Modus ponens

p

$p \to q$

\- - - - - -

$\therefore q$

(2) Using Conjunction

q

r

\- - - - - -

$\therefore q$ A r

Part D: [2 points]

Prove by contraposition that if n is an integer and $3n + 2$ is odd, then n is odd.

$$P : 3n + 2 \text{ is odd}$$

$$q : n \text{ is odd}$$

$$p \rightarrow q$$

$$\text{contraposition } \neg q \rightarrow \neg p$$

1. we assume that(n is even)

   $n = 2m,$ where m is integer.

2. $(3n + 2) = (3(2m) + 2)$

   $(3n + 2) = (6m + 2) = 2(3m + 1) = \text{even}$

   $\therefore (3n + 2)$ is even

3. $\therefore \neg q \rightarrow \neg p$ is true, then $p \rightarrow q$ is also true.

Question 3 **[2 points]**

Find the power sets of the following sets:

a) $\{a, \{b\}, c\}$

$$= \{\emptyset, \{a\}, \{\{b\}\}, \{c\}, \{a, \{b\}\}, \{a, c\}, \{\{b\}, c\}, \{a, \{b\}, c\}\}$$

b) $\{\{\ \}\} = \{\emptyset, \{\{\ \}\}\}$

And $\wedge$	only T T = T
or $\vee$	at least(T) = T
Xor $\oplus$	only different = T
implies $\rightarrow$	only T F = F
if & only if $\leftrightarrow$	only same value = T

TABLE
Precedence of Logical Operators.

Operator	*Precedence*
$\neg$	1
$\wedge$ $\vee$	2 3
$\rightarrow$ $\leftrightarrow$	4 5

Tautology → T T T T T

Contradiction → F F F F F

Contingency → T T T F F F

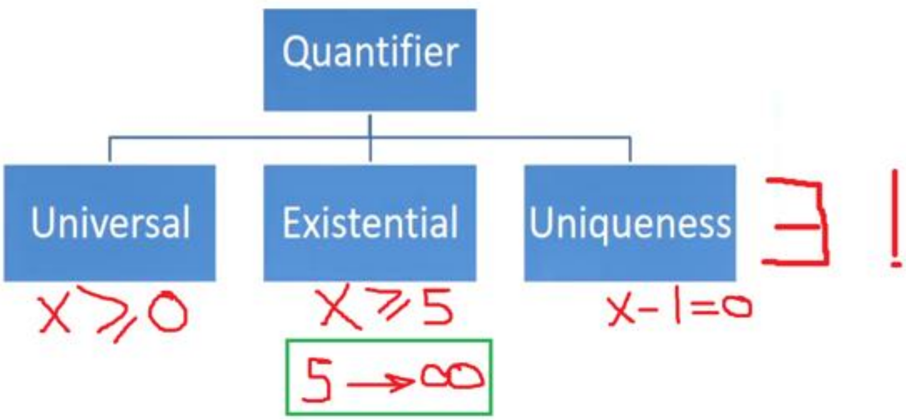
- Logical Equivalences

TABLE Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws

TABLE Logical Equivalences.

Equivalence	*Name*
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv \boxed{p}$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws



Question 1

Correct

P ---> R is

Select one:

- a. Contingency
- b. Contradiction
- c. Tautology
- d. none of the above

The correct answer is: Contingency

Question 2

Which of the following statement is a proposition?

Select one:

- a. What is the time now?
- b. Get me a glass of milkshake
- c. God bless you!
- d. The only odd prime number is 2

The correct answer is: The only odd prime number is 2

Question 3

$p \lor q$ is logically equivalent to ________

Select one:

- a. $\neg q \rightarrow \neg p$
- b. $\neg p \rightarrow \neg q$
- c. $q \rightarrow p$
- d. $\neg p \rightarrow q$

The correct answer is: $\neg p \rightarrow q$

Question 4

are these statement are equivalent : $(\neg p \lor q)$and $(\neg Q \land \neg p)$?

Select one:

- a. True
- b. False

The correct answer is: False

Question 5

The proposition $(p \oplus q) \wedge (p \leftrightarrow q)$ is:

Select one:

- a. None of the above
- b. Contingency
- c. Tautology
- d. Contradiction

The correct answer is: Contradiction

Question 1

Let p and q be the propositions

p: Ahmed bought a train ticket.

q: Ahmed is going to travel tomorrow.

Then the statement “Ahmed did not buy a train ticket, but he is going to travel tomorrow” is equivalent to which proposition?

Select one:

- a. $\neg p \vee \neg q$
- b. $\neg p \vee q$
- c. $\neg p \wedge q$
- d. $\neg p \rightarrow q$

The correct answer is: $\neg p \wedge q$

Question 2

Let P (x) denote the statement “x >7.” Which of these have truth value true?

Select one:

a. P (4)
b. P (9)
c. P (0)
d. P (6)
The correct answer is: P (9)

Question 3

p → q ≡.........................

Select one:
a. p ∨ q
b. ~ p ∨ q
c. P ∧ ~ q
d. P ∨ ~ q
The correct answer is: ~ p ∨ q

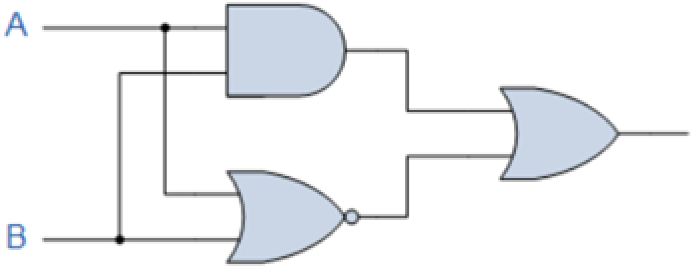
Question 4

If P then Q is called _________ statement

Select one:
a. conditional
b. disjunction
c. Conjunction
d. bi conditional
The correct answer is: conditional

Question 1

Choose the correct proposition for the following circuit



Select one:

- a. $(A \wedge B) \vee \neg(A \wedge B)$
- b. $(A \wedge B) \vee (\neg A \wedge \neg B)$
- c. $(A \wedge B) \wedge (\neg A \vee \neg B)$
- d. $(A \wedge B) \vee (A \vee B)$

The correct answer is: $(A \wedge B) \vee (\neg A \wedge \neg B)$

Question 2

let p(x) "x+1=3" and the truth value is true in domain natural numbers. What is the type of quantifier?

Select one:

- a. Both a and b
- b. Universal
- c. Existential
- d. Uniqueness

The correct answer is: Uniqueness

Question 3

p → q ≡........................

Select one:

- a. P ∨ ~ q
- b. p ∨ q
- c. ~ p ∨ q
- d. P ∧ ~ q

The correct answer is: ~ p ∨ q

Question 4

Choose the correct truth table for the following proposition

$$(p \vee \neg q) \rightarrow (p \wedge q)$$

Select one:

a.

F
T
F
T

b.

T
F
T
F

c.

T
F
T
T

d.

F
F
T
F

Feedback

The correct answer is:

T
F
T
F

Question 1

Let P: We should be honest., Q: We should be dedicated., R: We should be overconfident. Then 'We should be honest or dedicated but not overconfident.' is best represented by?

Select one:

a. P ^ ¬Q ^ R

b. P v Q ^ ¬R

c. ¬P v ¬Q v R

d. P v Q ^ R

The correct answer is: P v Q ^ ¬R

Question 2

Let P (x) denote the statement "x ≤ 4." What is the truth value of p(6) ?

Select one:

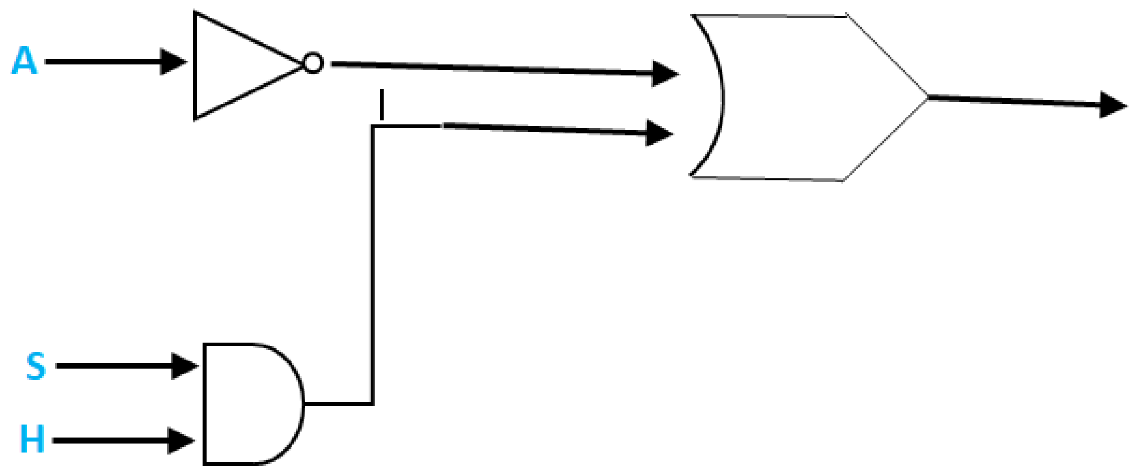
a. T

b. F

The correct answer is: F

Question 3

What is the output



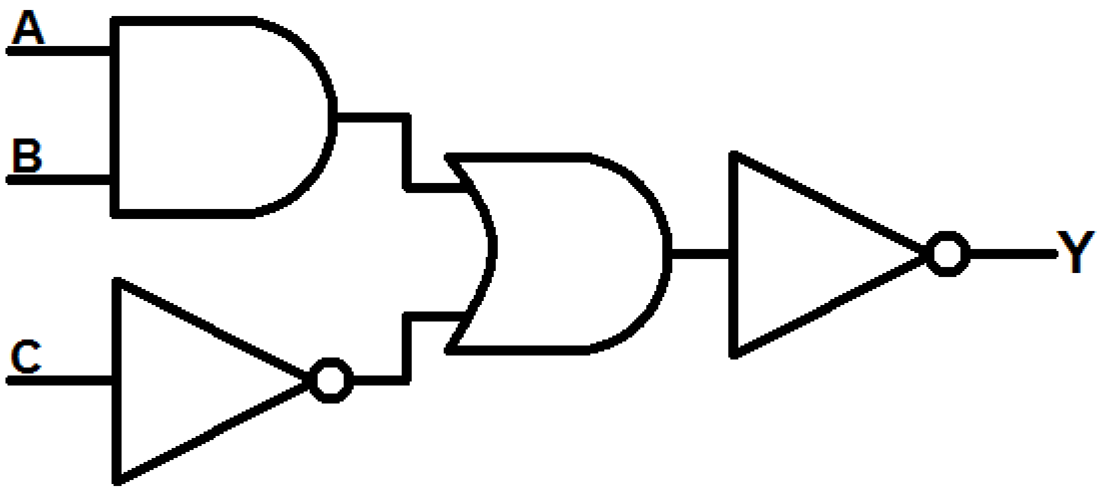
Select one:

- a. - H ^ (S V A)
- b. -A ^ (S V H)
- c. S V (A^H)
- d. -A V (S^H)

The correct answer is: -A V (S^H)

Question 4

Find the output of each of this combinational circuit:



Select one:

a. Y=~C V (A ∧ Q)

b. Y=~ (~C V (A ∧ B))

c. Y=C V (A ∧ Z)

d. Y=A ∧ ~B V C

The correct answer is: Y=~ (~C V (A ∧ B))

Question 5

If P(x) is “x spends more than five hours every weekday in class”. Then the statement “There are no students who spends more than five hours every weekday in class” is equivalent to which quantification?

Select one:

a. ∃x¬P(x)

b. ∃xP(x)

c. ∀x¬P(x)

d. ∀xP(x)

The correct answer is: ∀x¬P(x)

Question 5

If P(x) is “x spends more than five hours every weekday in class”. Then the statement “There are no students who spends more than five hours every weekday in class” is equivalent to which quantification?

Select one:

- a. $\forall x \neg P(x)$
- b. $\exists x P(x)$
- c. $\exists x \neg P(x)$
- d. $\forall x P(x)$

The correct answer is: $\forall x \neg P(x)$

5. Let $P(x)$ be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students. Express each of these quantifications in English.

a) $\exists x P(x)$ **b)** $\forall x P(x)$
c) $\exists x \neg P(x)$ **d)** $\forall x \neg P(x)$

Solution

(a) There exists a student that spends more than five hours every weekday in class.
(b) All students spend more than five hours every weekday in class.
(c) There exists a student that does not spend more than five hours every weekday in class.
(d) All students do not spend more than five hours every weekday in class.

Question 1

Let p, q, and r be the propositions

p: You have the flu.

q: You miss the final examination.

r: You pass the course.

If you have the flu then you'll not pass the course OR If you miss the final examination then you'll fail the course

Select one:

- a. $(p \to \neg r) \vee (q \to \neg r)$
- b. $(p\to\neg r)\wedge(q\to r)$
- c. $(p\to\neg r)\vee(q\to r)$
- d. $(p\leftrightarrow\neg r)\vee(q\to\neg r)$

The correct answer is: $(p \to \neg r) \vee (q \to \neg r)$

Question 2

Let P(x) be the statement "x+5≥6" choose the true statement the quantification

Select one:

- a. $\forall x \neg P(x)$
- b. None of the above
- c. $\forall x P(x)$
- d. $\exists x P(x)$

The correct answer is: $\exists x P(x)$

Question 3

which of these are proposition:

Select one:

- a. Open the door
- b. How are you
- c. Do not be late
- d. cairo is the capital of Egypt

The correct answer is: cairo is the capital of Egypt

If we have p,q. Are $p \oplus q \equiv p \leftrightarrow q$?

Select one:

- a. None
- b. False
- c. True

The correct answer is: False

Which of the following option is suitable, if A is "10110110", B is"11100000" and C is"10100000"?

Select one:

- a. C=~B
- b. C= A or B
- c. C=~A
- d. C=A and B

The correct answer is: C=A and B

Which of the following bits is the negation of the bits "010110"?

Select one:

- a. 111001
- b. 001001
- c. 111111
- d. 101001

The correct answer is: 101001

Which of the following propositions is tautology?

Select one:

- a. $(p \vee q) \rightarrow q$
- b. Both (b) & (c)
- c. $p \vee (q \rightarrow p)$
- d. $p \vee (p \rightarrow q)$

The correct answer is: $p \vee (p \rightarrow q)$

What is the truth value of quantifier $\forall x\, P(x)$, where $x > 1/x$, x in the domain of R is

Select one:

- a. None

b. True

c. False

The correct answer is: False

Find the bitwise “and” 0110 1101 , 1001 0011

Select one:

a. 1000 0000

b. 0001 0000

c. 0000 0001

d. 1000 0010

The correct answer is: 0000 0001

$(p \rightarrow q) \land (p \rightarrow r)$ is logically equivalent to_______

a. $p \rightarrow (q \lor r)$

b. $p \lor (q \land r)$

c. $p \rightarrow (q \land r)$

d. $p \land (q \lor r)$

The correct answer is: $p \rightarrow (q \land r)$

Which of the following option is true?

a. If the Sun is a planet, elephants will fly

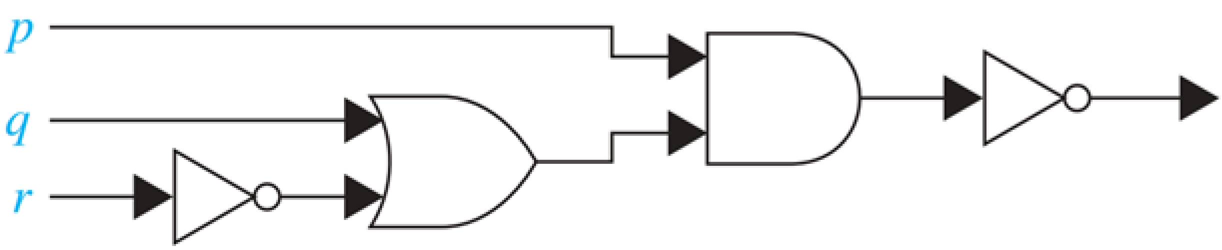
b. -2 > 3 or 3 is a negative integer

c. 1 > 3 and 3 is a positive integer

d. 3 +2 = 8 if 5-2 = 7

The correct answer is: If the Sun is a planet, elephants will fly

What is the output of the following combinatorial circuit



Select one:

- a. (p ∧ ¬r) ∨ (¬q ∧ r)
- b. (¬p ∧ (q ∨ r)) ∧ ((¬p ∨ ¬r) ∧ ¬q)
- c. ¬ (p ∧ (q ∨ ¬r))
- d. (p ∨ ¬r) ∧ (¬p ∨ (q ∨ ¬r))

The correct answer is: ¬ (p ∧ (q ∨ ¬r))

The truth table of negation of (p ^ q) is:

Select one:

a.

output
0
1
1
1

output
0
0
0
1

b.

output
1
1
1
0

c.

d. none of above

The correct answers are:

output
0
1
1
1

,

output
1
1
1
0

(i) Determine whether each of these statements is true or false.

(a) $0 \in \emptyset$

(b) $\emptyset \in \{0\}$

(c) $\{0\} \subset \emptyset$

(d) $\emptyset \subset \{0\}$

(e) $\{0\} \in \{0\}$ ✗

(f) $\{0\} \subset \{0\}$ ✗

(g) $\{\emptyset\} \subseteq \{\emptyset\}$ ✓

(ii) Determine whether these statements are true or false.

(a) $\emptyset \in \{\emptyset\}$ ✓

(b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$ ✓

(c) $\{\emptyset\} \in \{\emptyset\}$ ✗

(d) $\{\emptyset\} \in \{\{\emptyset\}\}$ ✓

(e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$ ✓

(f) $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$ ✓

(g) $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$ ✓

Determine this statement is true or false.

$\emptyset \in \{0\}$

Select one:

True

False

Determine this statement is true or false.

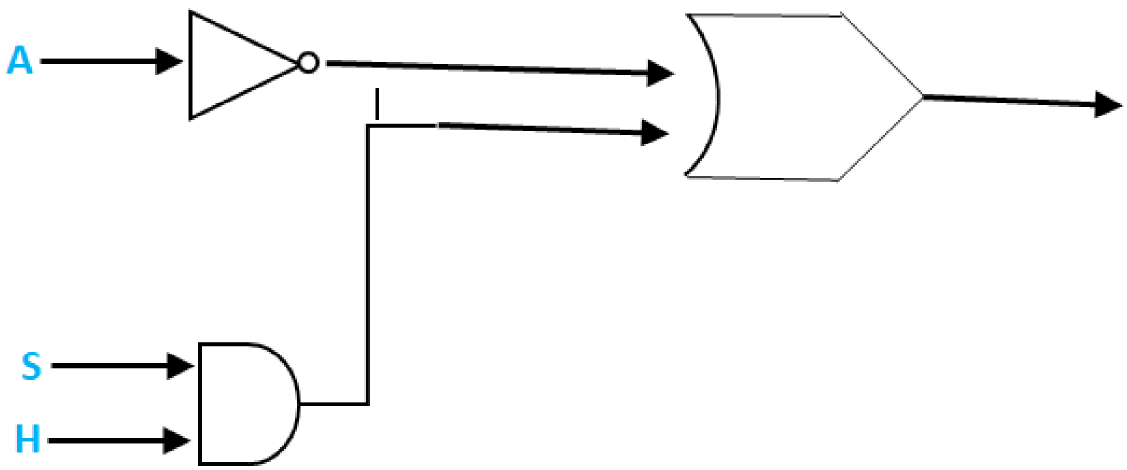
$0 \in \emptyset$

Select one:

True

False

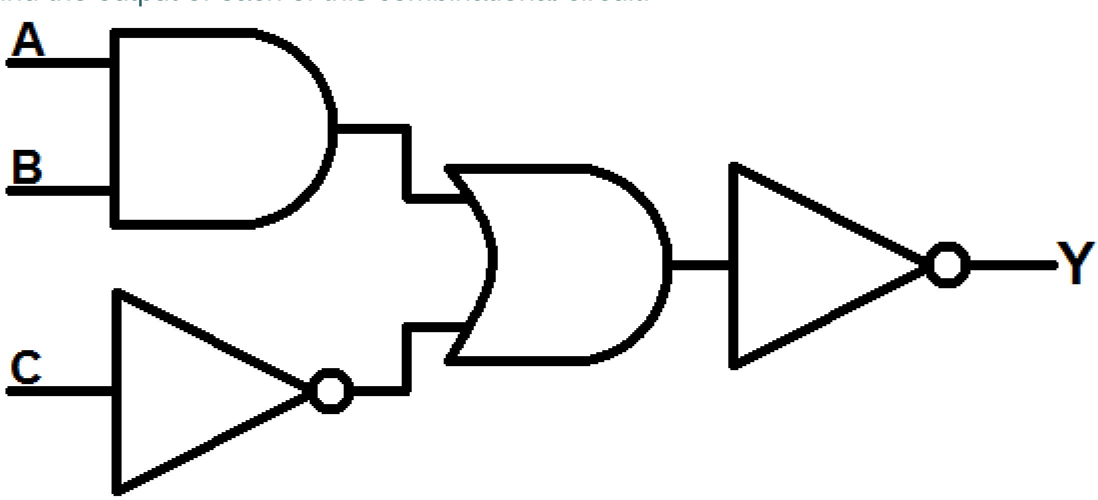
What is the output



a. - H ^ (S V A)
b. -A ^ (S V H)
c. -A V (S^H)
d. S V (A^H)

8 is odd number, is it proposition?
Select one:
a. it is proposition and his truth value is false
b. none of the above
c. it is not proposition
d. it is proposition and his truth value is true

Find the output of each of this combinational circuit:



Select one:

a. Y=C V (A ∧ Z)
b. Y=~ (~C V (A ∧ B))
c. Y=A ∧ ~B V C
d. Y=~C V (A ∧ Q)

Let P and Q are propositions

P: I am in Bangalore.

Q: I love cricket.

then q -> p (q implies p) is?

- a. I am not in Bangalore
- b. If I love cricket then I am in Bangalore
- c. I love cricket
- d. If I am in Bangalore then I love cricket

The correct answer is: If I love cricket then I am in Bangalore

Choose the correct truth table for the following proposition

$$(p \vee \neg q) \rightarrow (p \wedge q)$$

Select one:

a.

F
T
F
T

b.

T
F
T
F

T
F
T
T

c.

F
F
T
F

d.

T
F
T
F

The correct answer is:

What is the power set of the set {Ø}?
Select one:
a. P(Ø) = {Ø}
b. P(Ø) = {Ø,{Ø}}

What is the power set of the empty set?

O a. P(0) = {0,{0}}
® b. P(0) = {0}

Let P(x) be the statement $x = x^2$. If the domain consists of the integers, the truth value of P(0) is True

® True
O False

Let P (x) be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students. Express 3xP(x) quantifications in English

® a. There is a student who spends more than five hours every weekday in class.
O b. No student spends more than five hours every weekday in class
O c. There is a student who does not spend more than five hours every weekday in class
O d. Every student spends more than five hours every weekday in class.

if A and B are sets with A £ B, then A ∪ B = B

® True
O False

Let P (x) be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students. Express 3x ^P (x) quantifications in English

O a. There is a student who spends more than five hours every weekday in class.
O b. No student spends more than five hours every weekday in

class

® c. There is a student who does not spend more than five hours every weekday in class

O d. Every student spends more than five hours every weekday in class.

Let P(x) be the statement $x = x^2$. If the domain consists of the integers, the truth value of P(1) is True

® True

O False

Let P (x) be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students.
Express ∀x ¬P (x) quantifications in English

O a. Every student spends more than five hours every weekday in class.

® b. No student spends more than five hours every weekday in class

O c. There is a student who does not spend more than five hours every weekday in class

O d. There is a student who spends more than five hours every weekday in class.

are these statement are equivalent : ('pv'q)and ('Q^'P) ?

O a. True

(§) b. False

are these statement are equivalent : (¬p∨q)and (¬Q∧¬p) ?

a. True

b. False

Let P (x) be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students.
Express ∃xP(x) quantifications in English

O a. No student spends more than five hours every weekday in class

O b. Every student spends more than five hours every weekday in class.

O c. There is a student who does not spend more than five hours every weekday in class

® d. There is a student who spends more than five hours every weekday in class.

The sets {1, 3, 5} and {3, 5, 1} are equal

True

False

Let P (x) be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students.
Express ∃x ¬P (x) quantifications in English
Select one:

a. There is a student who spends more than five hours every weekday in class.

b. No student spends more than five hours every weekday in class

c. There is a student who does not spend more than five hours every weekday in class

d. Every student spends more than five hours every weekday in class.

let p(x) "x+1=3" and the truth value is true in domain natural numbers. What is the type of quantifier ?

a. Uniqueness

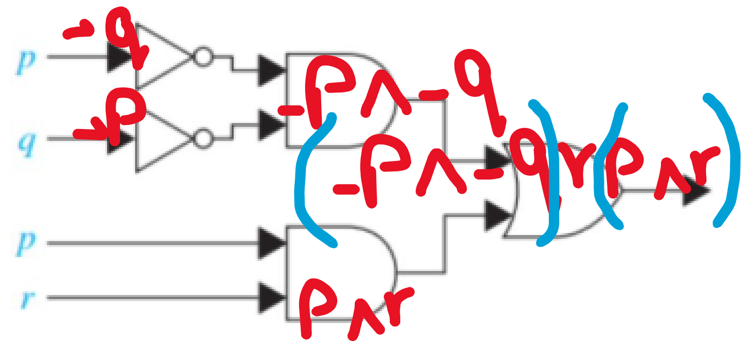
b. Both a and b
c. Existential
d. Universal

Which of the following bits is the negation of the bits “010110”?

Select one:

- a. 111001
- b. 111111
- c. 001001
- d. 101001

Choose the correct output of this combinational circuit.



Select one:

a. ((¬p) ∧ (¬q)) ∧ (p∧r)

b. ((¬p) ∧ (¬q)) ∨(¬p∧r)

c. ((p) ∧(¬q)) ∨(p∧r)

d. ((¬p) ∧ (¬q)) ∨(p∧r)

For every

The Quantifier ∀xP(x) is false, when there is an x for which P(x) is false.

Select one:

✓ True

x False

P

Let P statement "Maria takes a English course" and q Maria will find a new job . Express the statement P →q as English statement.

q

P → q

Select one:

a. Maria takes an English course but She will not find a new job

b. If Maria takes an English course, then She will find a new job

c. Maria takes an English course and she will find a new job

Choose the correct truth table for the following compound proposition

$(p \rightarrow q) \vee (\neg p \rightarrow r)$

Select one:

- [ ] a.

T
T
T
T
T
T
F
T

- [x] b.

T
T
T
T
T
T
T
T

- [ ] c.

F
T
T
F
T
T
F
T

- [ ] d.

F
F
F
T
T
F
F
F

Choose the correct output of this combinational circuit.