
بسم الله الرحمن الرحيم

EELU

الجامعة المصرية للتعلم الإلكترونى

Egyptian E-Learning University

MATH - 1

BY

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CHAPTER 1

Functions and Their Graphs

Lecture 2

CLASSIFICATION OF FUNCTIONS

Aims and Objectives:

(1) GRAPH THE ABSOLUTE FUNCTION.

(2) INTRODUCE THE NOTION OF ODD AND EVEN FUNCTIONS.

(3) APPLY OPERATIONS ON FUNCTIONS.

(4) DEFINE THE DISTANCE BETWEEN TWO POINTS.

(5) DEVELOP EQUATION OF A CIRCLE.

(6) SOLVE EXAMPLES USING CIRCLE EQUATION.

(7) DEFINE THE GENERAL EQUATION OF THE CIRCLE.

(8) UNDERSTAND PROPERTIES OF PARABOLAS.

(9) USE DIFFERENT FORMS OF PARABOLAS.

(10) DEFINE THE GENERAL EQUATION OF A PARABOLA.

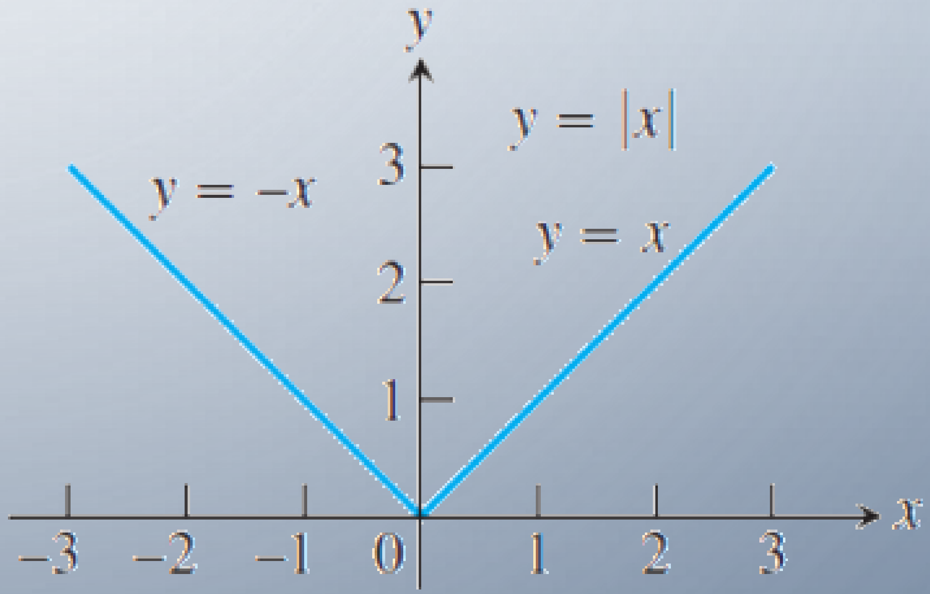
(11) SOLVE EXAMPLES ON THE GENERAL FORM OF A PARABOLA

CLASSIFICATION OF FUNCTIONS

I. Piecewise-Defined Functions :

A piecewise-defined function is a function defined by different formulas in different parts of its domain.

$$|\mathbf{x}| = \begin{cases} \mathbf{x} & \text{if } \mathbf{x} \geq 0 \\ -\mathbf{x} & \text{if } \mathbf{x} < 0 \end{cases}$$



$$Domain = \mathbb{R}, \qquad Range = [0, \infty)$$

EXAMPLE:

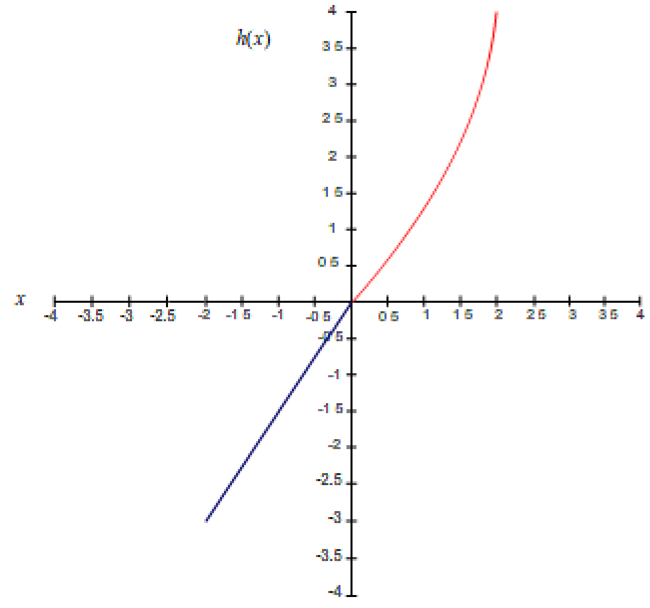
Graph the function

$$h(x) = \begin{cases} x^2 & if\ \ x \geq 0 \\ 2x+1 & if\ x < 0 \end{cases}$$

over the interval $-2 \leq x \leq 2$

SOLUTION:

x	$h(x)$
-2	-3
-1	-1
-.25	.5
0	0
.25	.625
1	1
2	4



EXAMPLE:

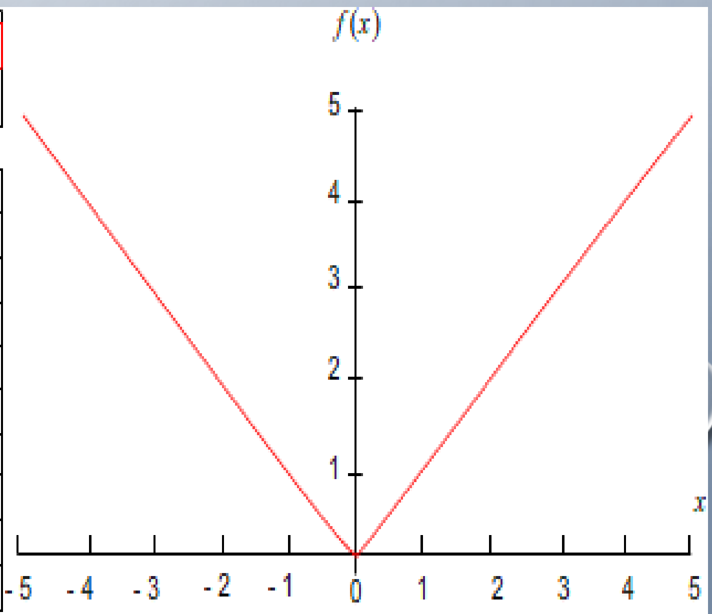
Graph the function

$$|x| = \begin{cases} x & if\ \ x \geq 0 \\ -x & if\ x < 0 \end{cases}$$

over the interval $-5 \leq x \leq 5$

SOLUTION:

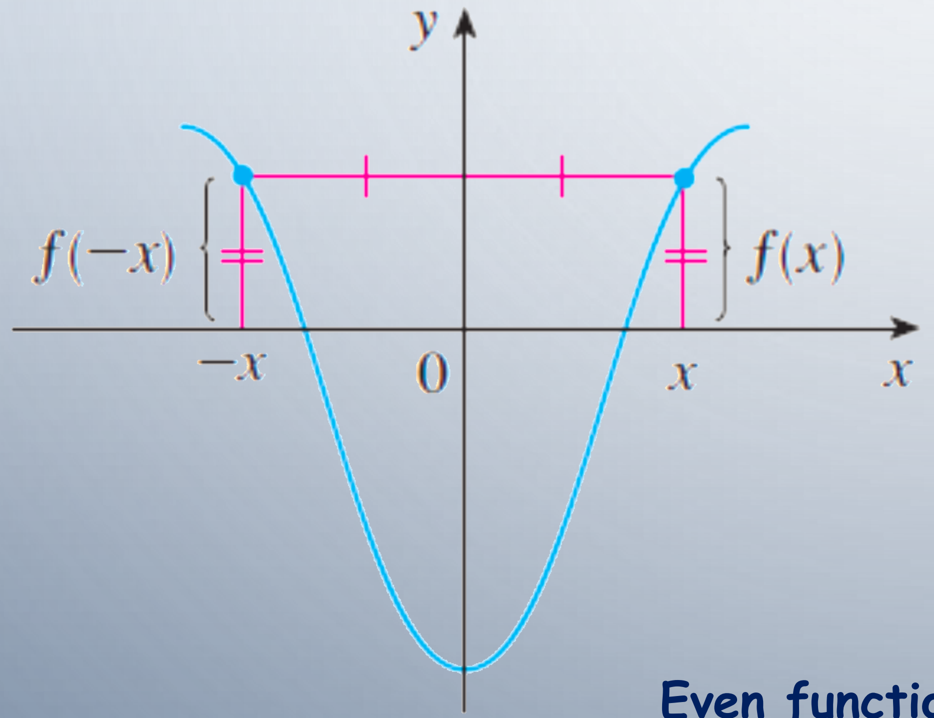
x	$|x| = \begin{cases} x & if\ x \geq 0 \\ -x & if\ x < 0 \end{cases}$
- 5.0	5
- 4.0	4
- 3.0	3
- 2.0	2
- 1.0	1
0	0
1.0	1
2.0	2
3.0	3
4.0	4
5.0	5

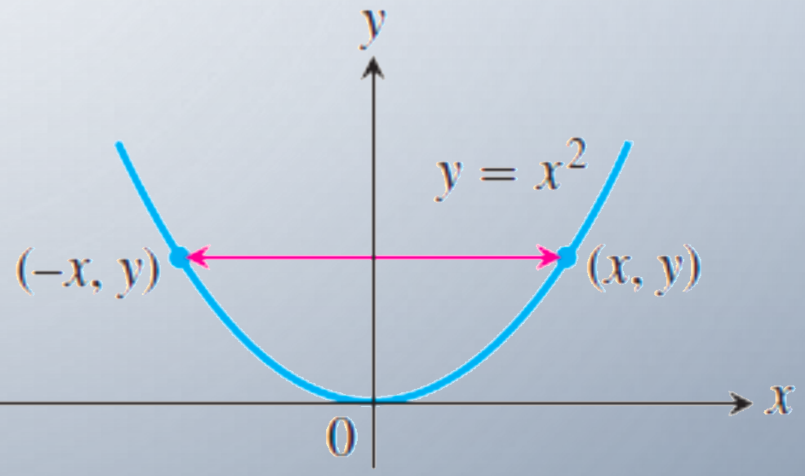


CLASSIFICATION OF FUNCTIONS

II. Even and Odd functions - Symmetry :

(i) If $f(-x) = f(x)$, then f is an even function and its graph is symmetric about the y-axis.

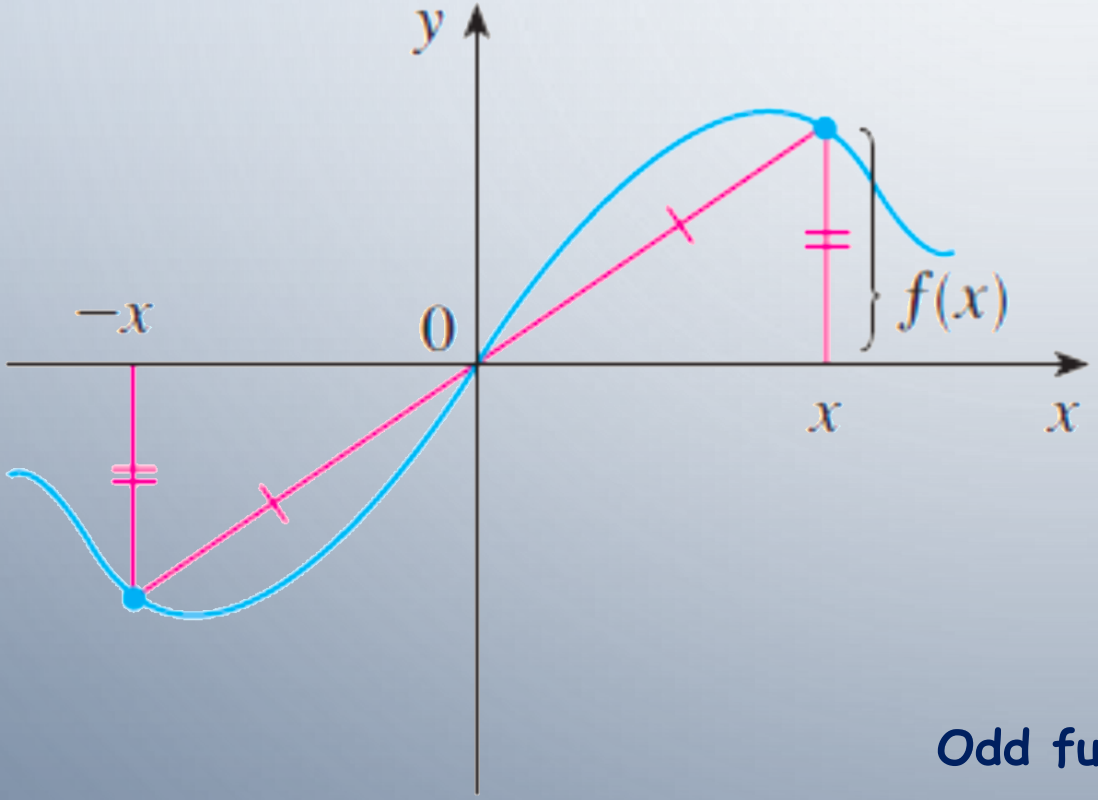


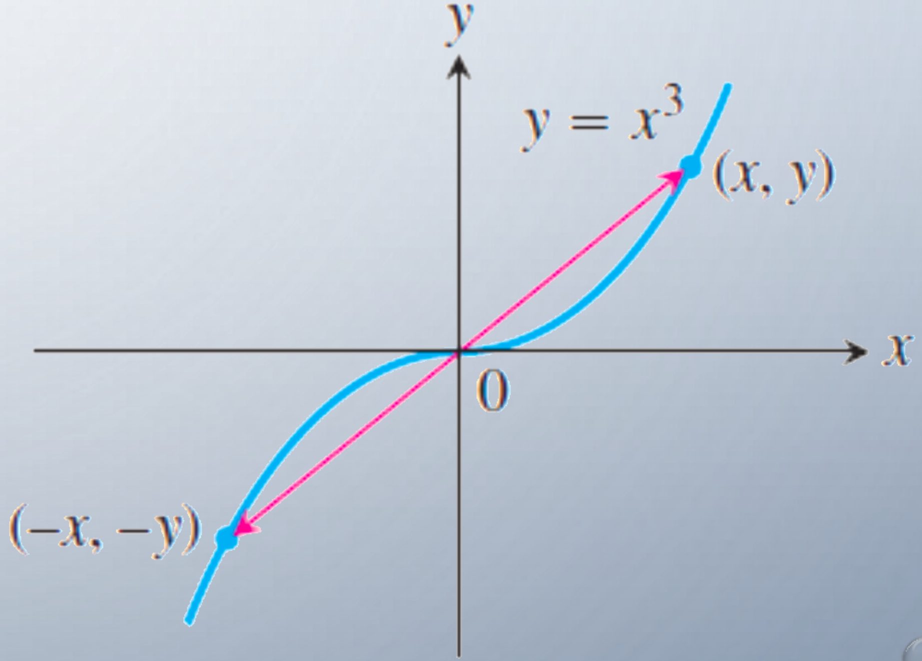


Even functions

CLASSIFICATION OF FUNCTIONS

(ii) If $f(-x) = -f(x)$, then f is an odd function and its graph is <u>symmetric about the origin.</u>





Odd functions

CLASSIFICATION OF FUNCTIONS

III. Increasing and Decreasing Functions :

(i) A function f is called increasing on an interval I if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ in I.

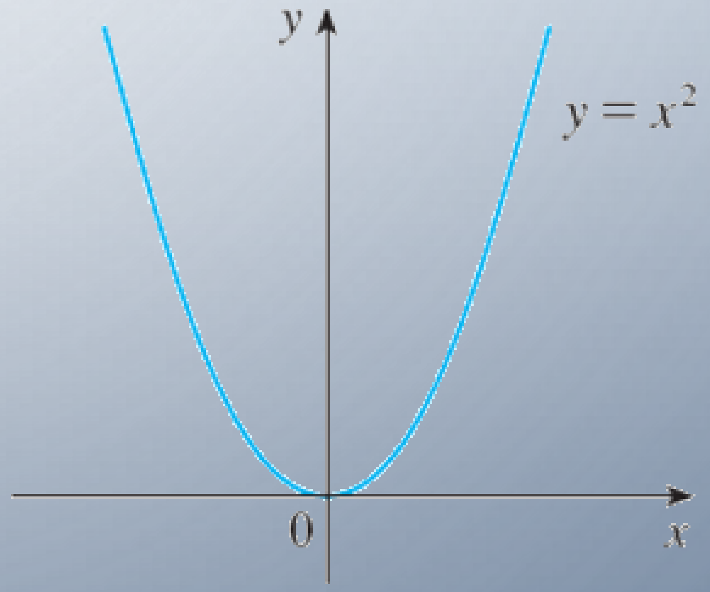
(ii) A function f is called decreasing on an interval I if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ in I.

EXAMPLE

Check the function $f(x) = x^2$ increasing or decreasing?

SOLUTION

From the opposite figure, the function $f(x) = x^2$ is decreasing on $(-\infty, 0]$ and increasing on $[0, \infty)$.



CLASSIFICATION OF FUNCTIONS

IV. Algebraic and Transcendental Functions :

A function f is called an algebraic function if its value is obtained from x using the five algebraic operations (addition, subtraction, multiplication, division, and taking roots). Functions that are not algebraic are called transcendental.

EXAMPLES OF ALGEBRAIC FUNCTIONS

(1) Polynomials :

A function f is called a polynomial if

$$f(x) = \sum_{i=0}^{n} a_i x^i = a_0 + a_1 x + a_2 x^2 + \cdots + a_{n-1} x^{n-1} + a_n x^n$$

where n is a nonnegative integer and $a_0, a_1, a_2, \dots, a_{n-1}, a^n$ are constants.

Remark : The domain of polynomial function is $\mathbb{R}$ or $(-\infty, \infty)$.

(2) Rational Functions :

A rational function f is in the form:

$$f(x) = \frac{P(x)}{Q(x)}, Q(x) \neq 0$$

where P and Q are polynomials.

Remark : The domain of the rational function is $\mathbb{R} - \{Zeros\ of\ Q(x)\}$

Remark : Functions in the form $f(x) = \sqrt[n]{g(x)}$

where g(x) is a polynomial function , are also algebraic ?

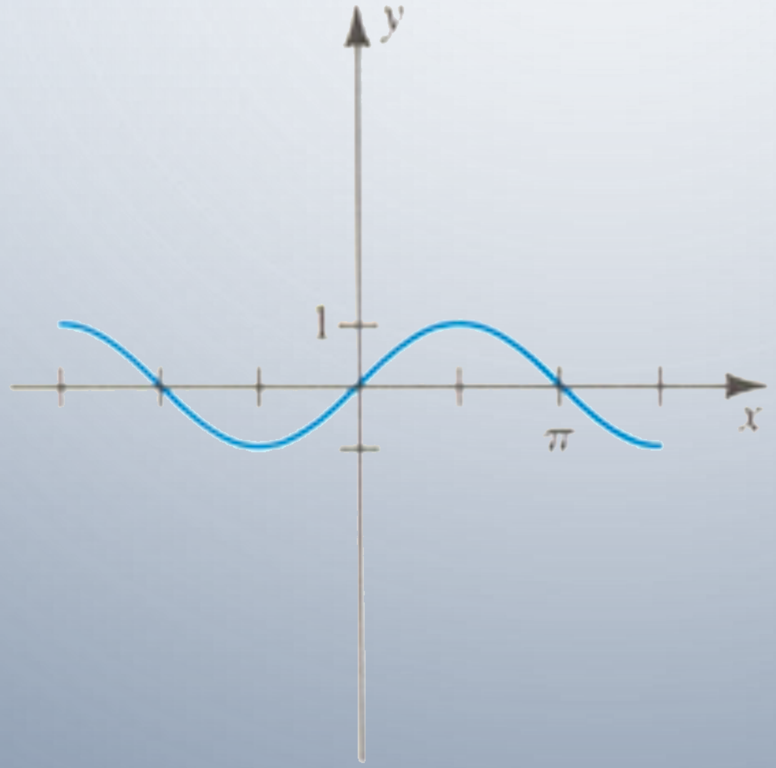
EXAMPLES OF TRANSCENDENTAL FUNCTIONS :

(1) Trigonometric functions : y = sin x , y = cos x,…

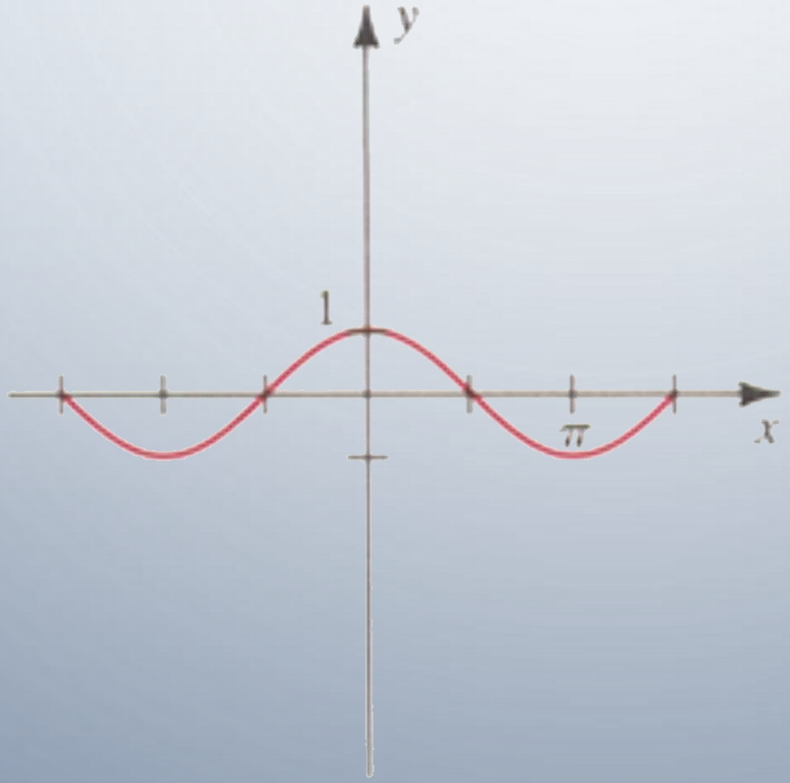
(2) Exponential functions : $y = e^x$, $y = 2^x$,…

(3) Logarithmic functions : y = ln x , $y = \log_{10} x$,…

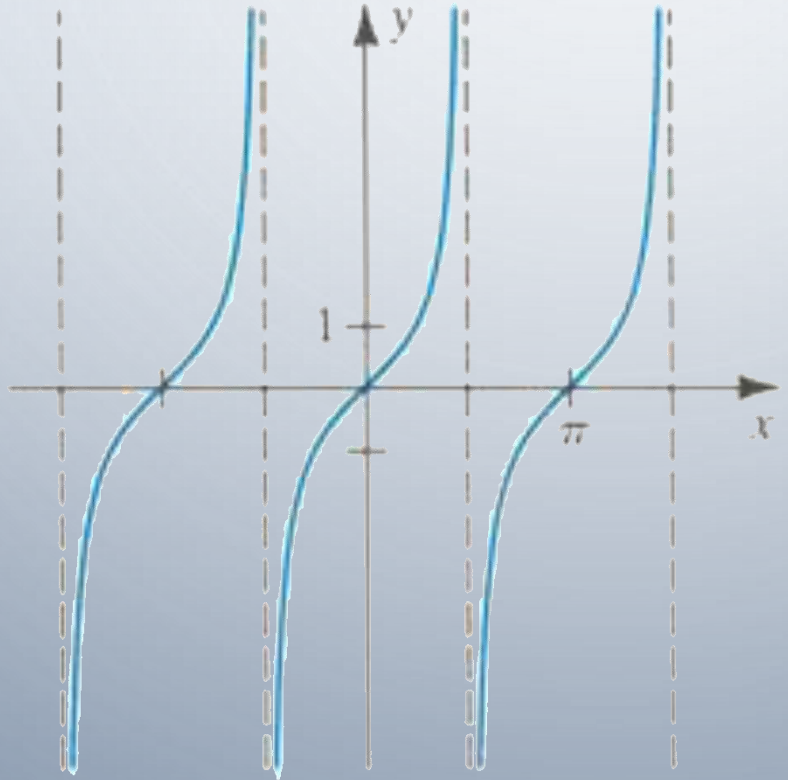
TRIGONOMETRIC FUNCTIONS

Function f	***Graph***	***Domain and Range***
$f(x) = \sin x$		***Domain*** $= \mathbb{R}$ ***or*** $(-\infty, \infty)$ ***Range*** $= [-1, 1]$ ***Period*** $= 2\pi$ ***Odd function ;*** $\sin(-x) = -\sin x$

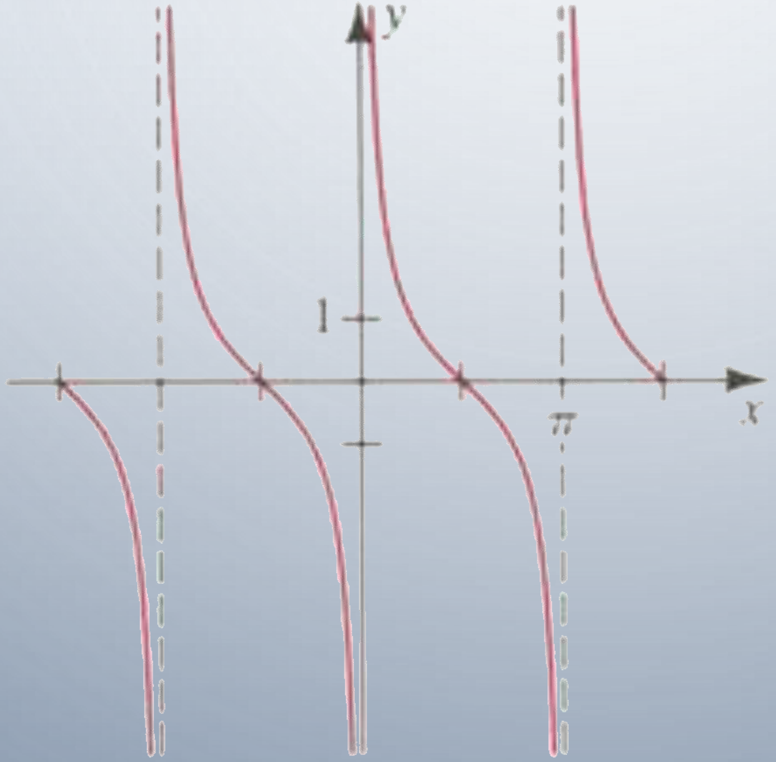
TRIGONOMETRIC FUNCTIONS

Function f	Graph	Domain and Range
$f(x) = \cos x$		$Domain = \mathbb{R}$ or $(-\infty, \infty)$ $Range = [-1, 1]$ $Period = 2\pi$ Even function ; $\cos(-x) = \cos x$

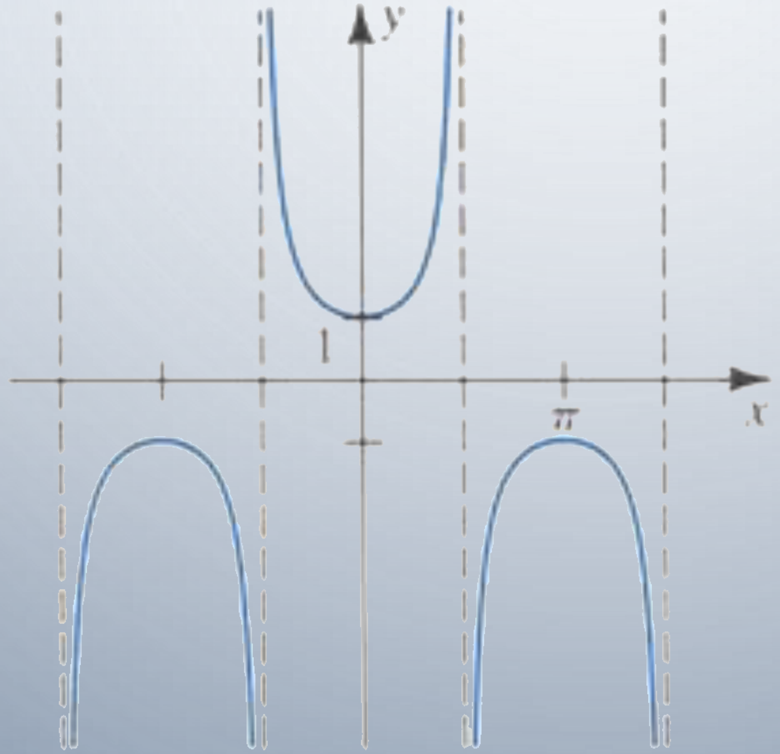
TRIGONOMETRIC FUNCTIONS

Function f	Graph	Domain and Range
$f(x) = \tan x = \dfrac{\sin x}{\cos x}$		Domain $= \mathbb{R} - \left\{ (2n+1)\dfrac{\pi}{2}, n \in \mathbb{Z} \right\}$ Range $= \mathbb{R}$ or $(-\infty, \infty)$ Period $= \pi$ Odd function ; $\tan(-x) = -\tan x$

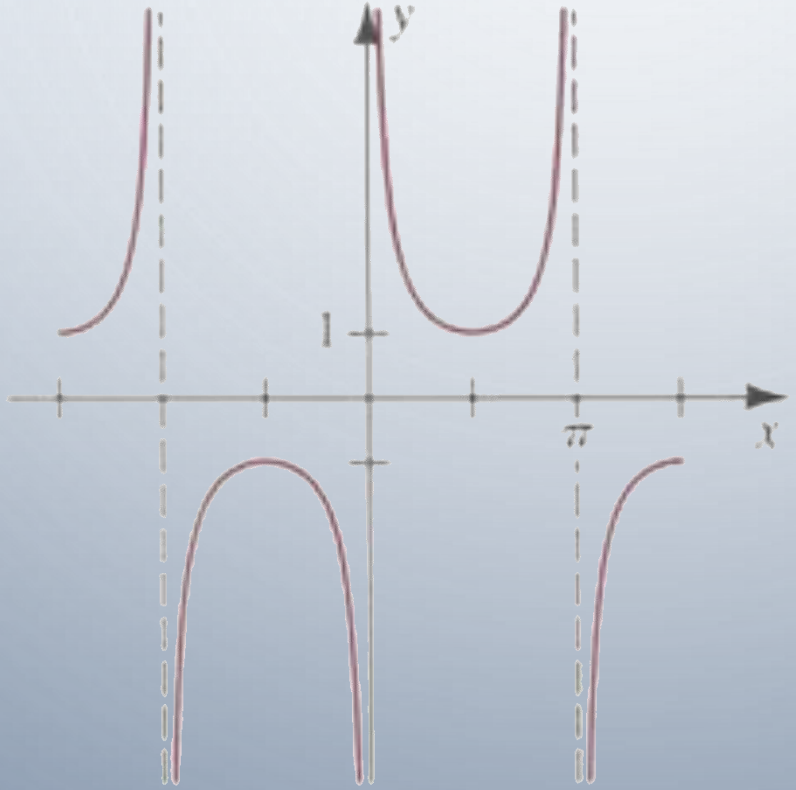
TRIGONOMETRIC FUNCTIONS

Function f	*Graph*	*Domain and Range*
$f(x) = \cot x = \dfrac{\cos x}{\sin x}$		*Domain* $= \mathbb{R} - \{ n\pi , n \in \mathbb{Z} \}$ *Range* $= \mathbb{R}$ *or* $(-\infty, \infty)$ *Period* $= \pi$ *Odd function ;* $\cot(-x) = -\cot x$

TRIGONOMETRIC FUNCTIONS

Function f	Graph	Domain and Range
$f(x) = \sec x = \dfrac{1}{\cos x}$		Domain $= \mathbb{R} - \left\{ (2n+1)\frac{\pi}{2}, n \in \mathbb{Z} \right\}$ Range $= (-\infty, -1] \cup [1, \infty)$ or $\mathbb{R} - (-1, 1)$ Period $= 2\pi$ Even function ; $\sec(-x) = \sec x$

TRIGONOMETRIC FUNCTIONS

<table>
<tr><th>Function f</th><th>Graph</th><th>Domain and Range</th></tr>
<tr><td>$f(x) = \csc x$
$= \dfrac{1}{\sin x}$</td><td>
</td><td>Domain $=$
$\mathbb{R} - \{ n\pi \,,\, n \in \mathbb{Z} \}$
Range $= (-\infty, -1] \cup [1, \infty)$
or $\mathbb{R} - (-1, 1)$
Period $= 2\pi$
Odd function ;
$\csc(-x) = -\csc x$</td></tr>
</table>

Identities of The Trigonometric Functions :

* $\cos^2\theta + \sin^2\theta = 1$,

  $1 + \tan^2\theta = \sec^2\theta$,

  $\cot^2\theta + 1 = \csc^2\theta$.

* **Additional Formulas :**

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B \;,$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B \;,$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \;.$$

Identities of The Trigonometric Functions :

* Double-Angle Formulas :

$$\sin 2\theta = 2\sin\theta\cos\theta \;,$$

$$\begin{aligned}\cos 2\theta &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta \;,\end{aligned}$$

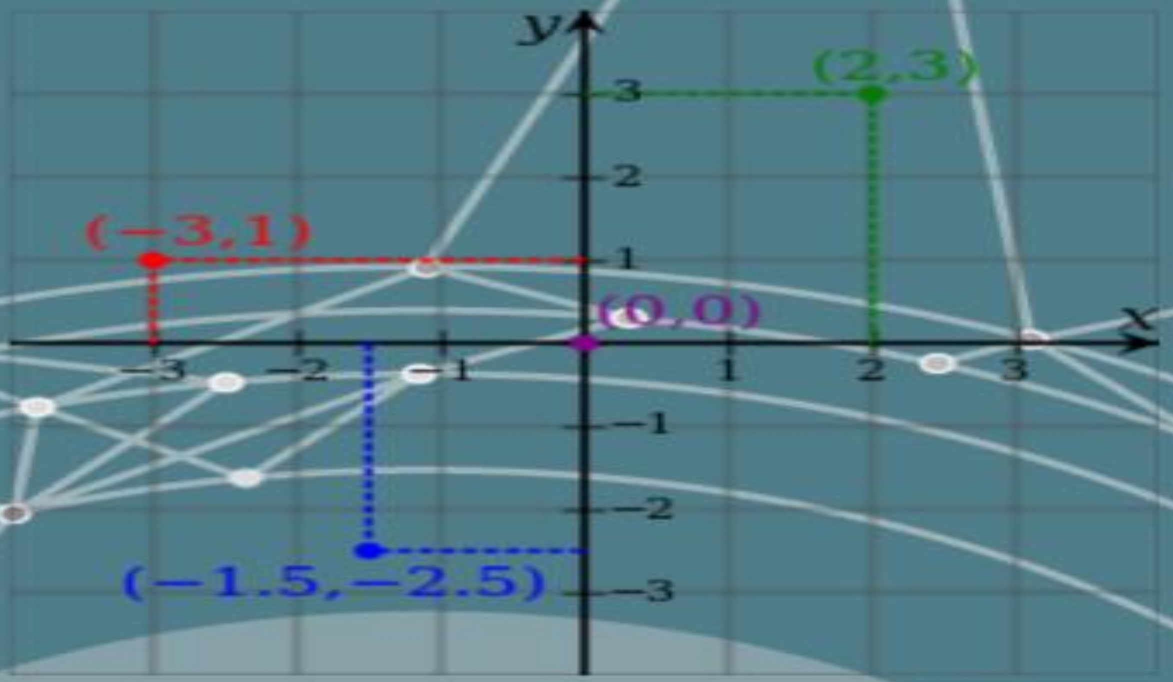
$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} \;.$$

* Half-Angle Formulas :

$$\cos^2\theta = \frac{1}{2}\left(1+\cos 2\theta\right) \;,$$

$$\sin^2\theta = \frac{1}{2}\left(1-\cos 2\theta\right).$$

A graph cannot exist without the Cartesian coordinate system, the very foundation of analytic geometry



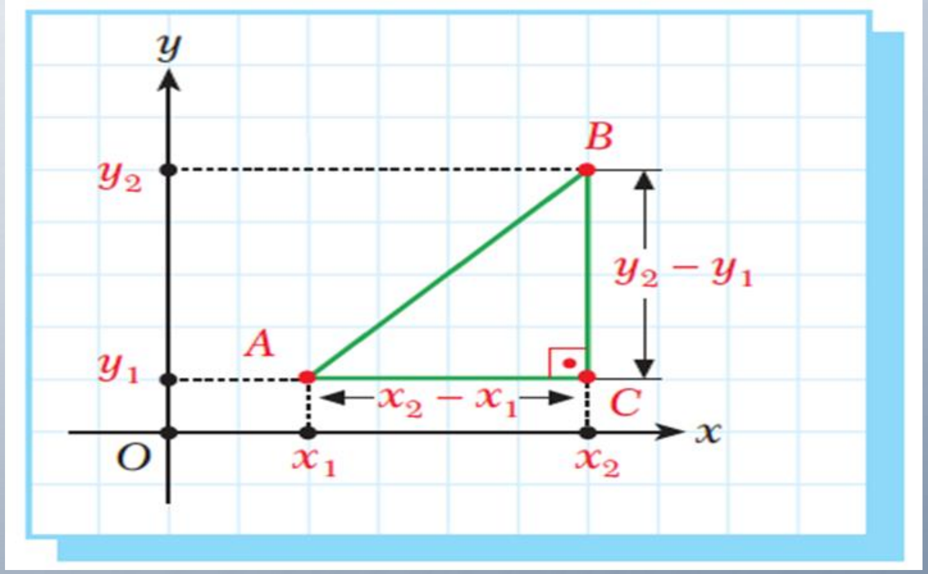
DISTANCE BETWEEN TWO POINTS

If we have the two points $A(x_1,y_1)$ and $B(x_2,y_2)$

R.T.P the distance between the two points A and B is

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

PROOF



In the opposite figure

$\because \Delta ABC$ is a right-angled triangle

$\therefore AC = x_2 - x_1$ *and* $BC = y_2 - y_1$

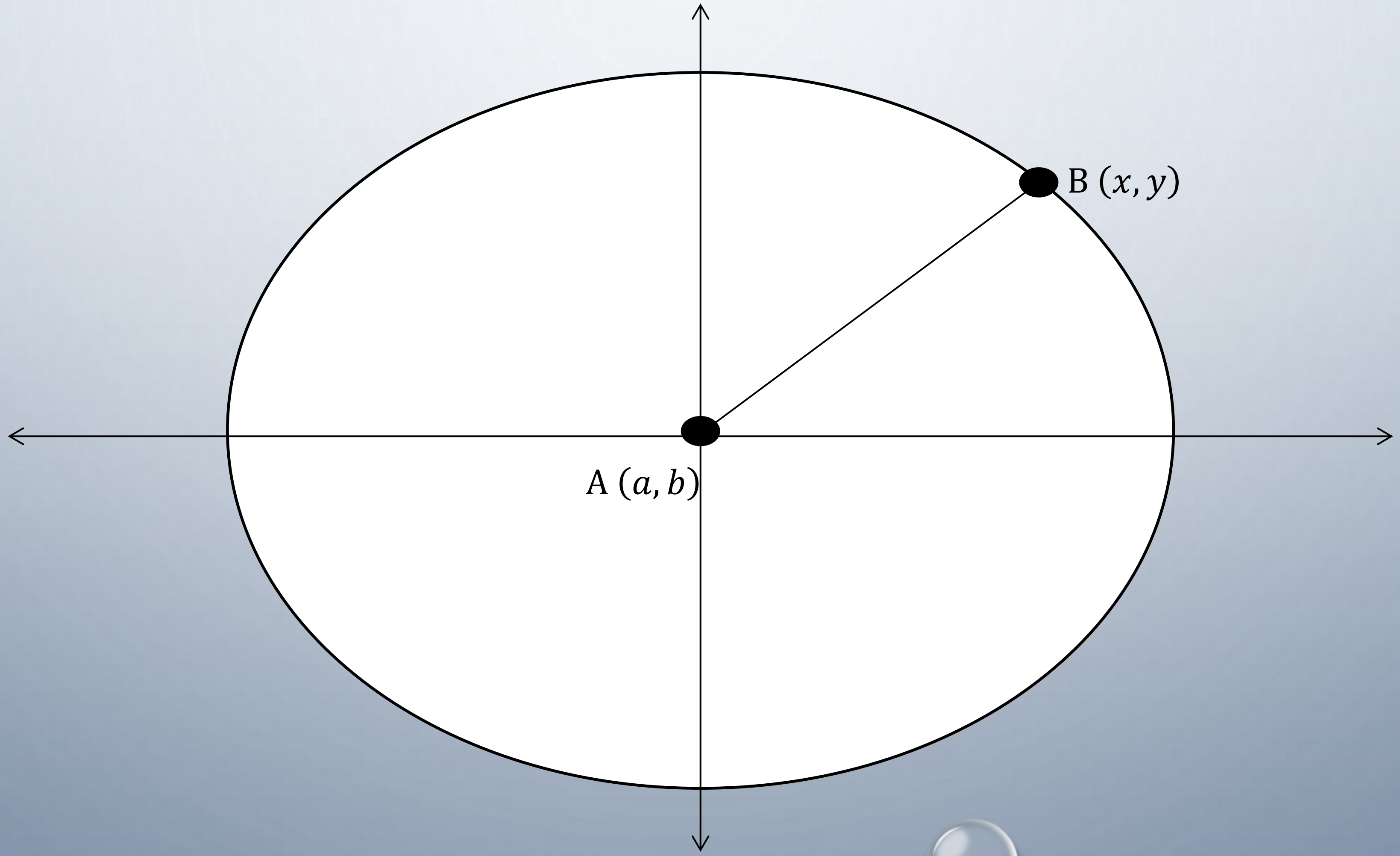
By the Pythagorean theorem

$$AB^2 = AC^2 + BC^2$$

$$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

Then $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

EQUATION OF A CIRCLE



- **Using distance formula to find the distance between points A and B, we have**

$$d^2 = (x-a)^2 + (y-b)^2 \quad (1)$$

- **Since the distance between points A and B is the radius of the circle, we will let d = r. Thus,**

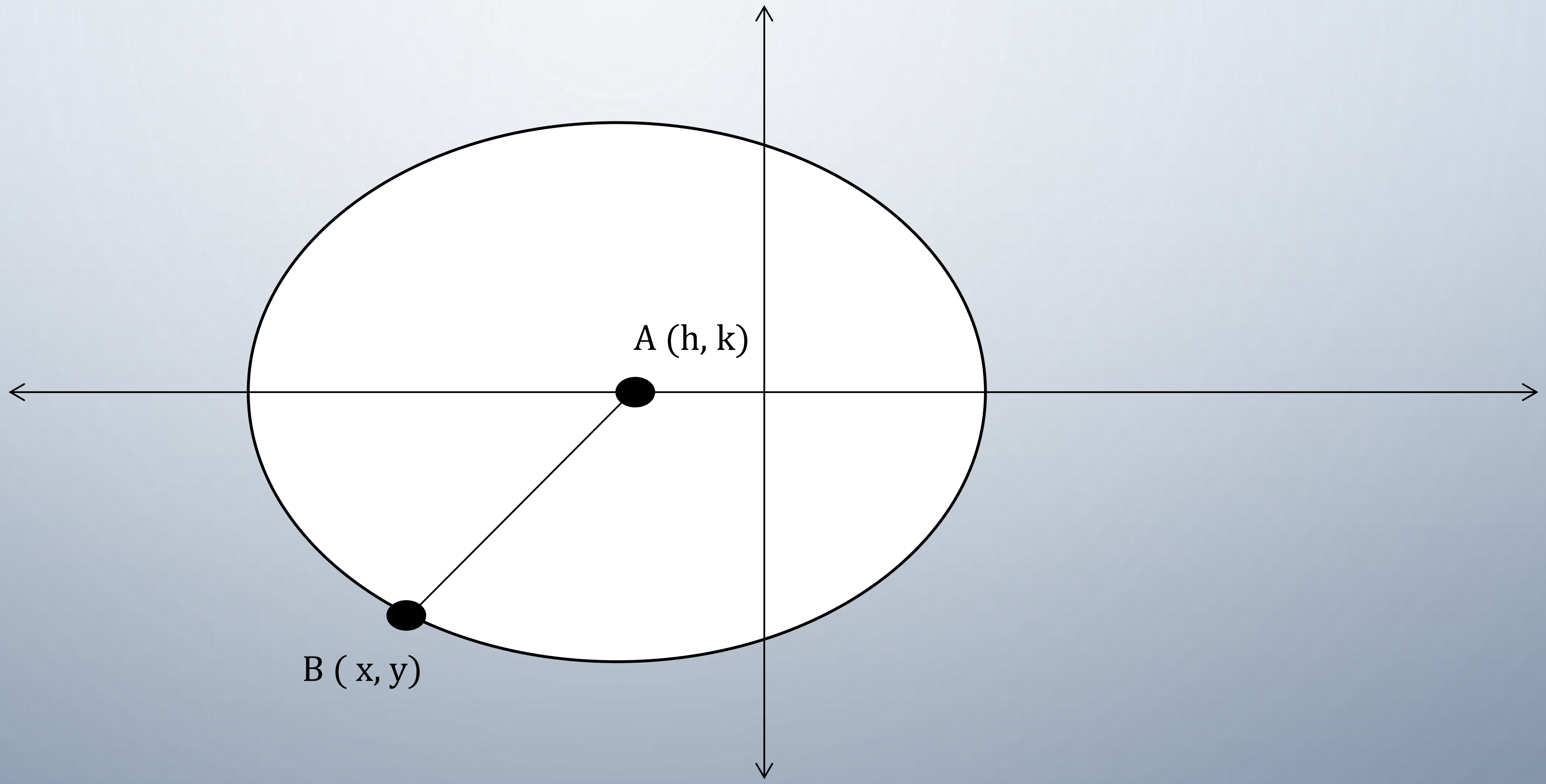
$$r^2 = (x-a)^2 + (y-b)^2 \quad (2)$$

- **Point A is in the origin, therefore $a = b = 0$, then**

$$r^2 = (x-0)^2 + (y-0)^2$$

$$r^2 = x^2 + y^2 \quad (3)$$

EQUATION OF CIRCLE (origin is not the center)



- The distance between points A and B is still the radius of the circle. So,

$$r^2 = (x - h)^2 + (y - k)^2 \quad (5)$$

- Thus, (5) is the equation of the circle if the center is at (h, k).
- Expanding (5), and letting, it will lead to the general equation of the circle. Thus,

$$x^2 + y^2 + Cx + Dy + E = 0 \quad (6)$$

EXAMPLE

Sketch the graph of the equation $x^2 + y^2 + 2x - 6y + 7 = 0$ by first showing that it represents a circle and then finding its center and radius.

SOLUTION

We first group the x-terms and y-terms as follows:

$$(x^2+2x) + (y^2-6y) = -7$$

Then we complete the square within each grouping, adding the appropriate constants

(the squares of half the coefficients of x and y) to both sides of the equation:

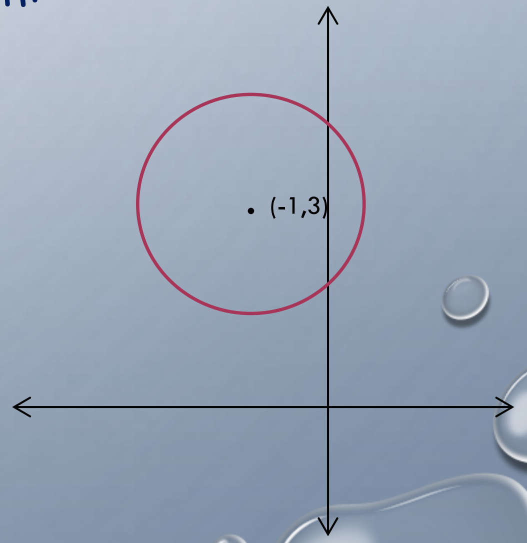
$$(x^2+2x+1) + (y^2-6y+9) = -7+1+9$$

Or $$(x+1)^2+(y-3)^2 = 3$$

Comparing this equation with the standard equation of a circle,

we see that h=-1, k=3, and $r = \sqrt{3}$, so the given equation represents

a circle with center (-1,3) and radius $\sqrt{3}$. It is sketched in the Figure:



EXAMPLE:

Find the center and radius of the circle $x^2 + y^2 + 4x - 6y = 12$

SOLUTION:

we complete the squares in the x terms and y terms

and get $(x^2 + 4x + 4) + (y^2 - 6y + 9) = 12 + 4 + 9$.

or $(x + 2)^2 + (y - 3)^2 = 25$.

This is equation $(x - h)^2 + (y - k)^2 = r^2$

with $(h, k) = (2, -3)$ and $r^2 = 25$.

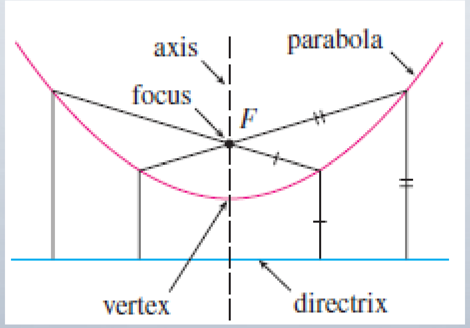
it therefore represents a circle with

Center: $C\ (-2,\ 3)$

Radius: $r = 5$.

PARABOLAS

A parabola is the set of points in a plane that are equidistant from a fixed point (called the focus) and a fixed line (called the directrix). This definition is illustrated by the Figure. Notice that the point halfway between the focus and the directrix lies on the parabola; it is called the vertex. The line through the focus perpendicular to the directrix is called the axis of the parabola.



In the 16th century Galileo showed that the path of a projectile that is shot into the air at an angle to the ground is a parabola. Since then, parabolic shapes have been used in designing automobile headlights, reflecting telescopes, and suspension bridges.

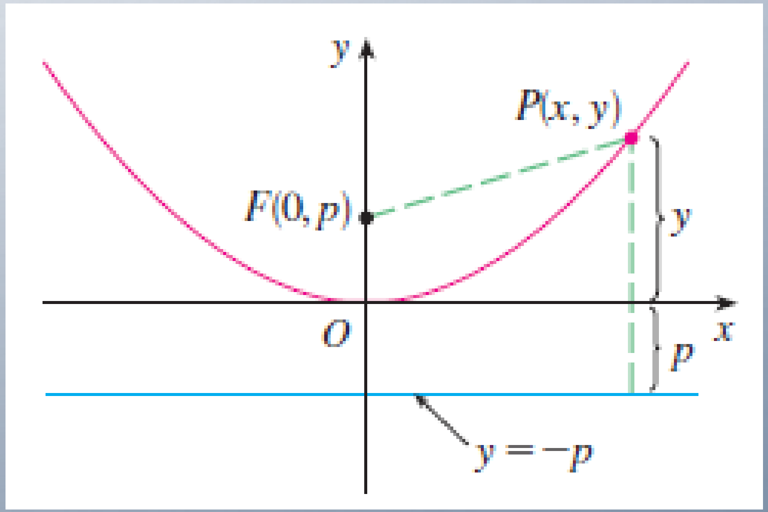
PARABOLAS

We obtain a particularly simple equation for a parabola if we place its vertex at the origin O and its directrix parallel to the x-axis as in the Figure. If the focus is the point (0,p), then the directrix has the equation $y = -p$ and the parabola has the equation

$$x^2 = 4py$$

If we write $a = 1/(4p)$, then the equation of the parabola becomes

$$y = ax^2$$



PARABOLAS

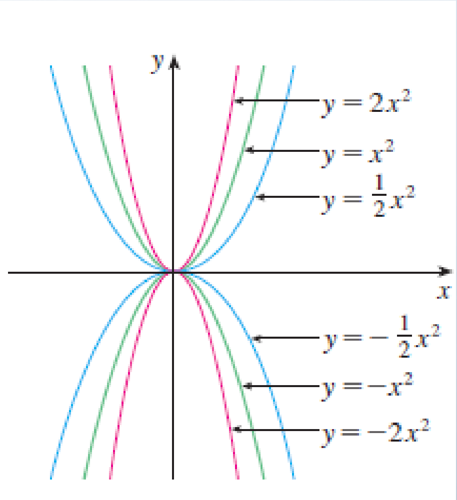
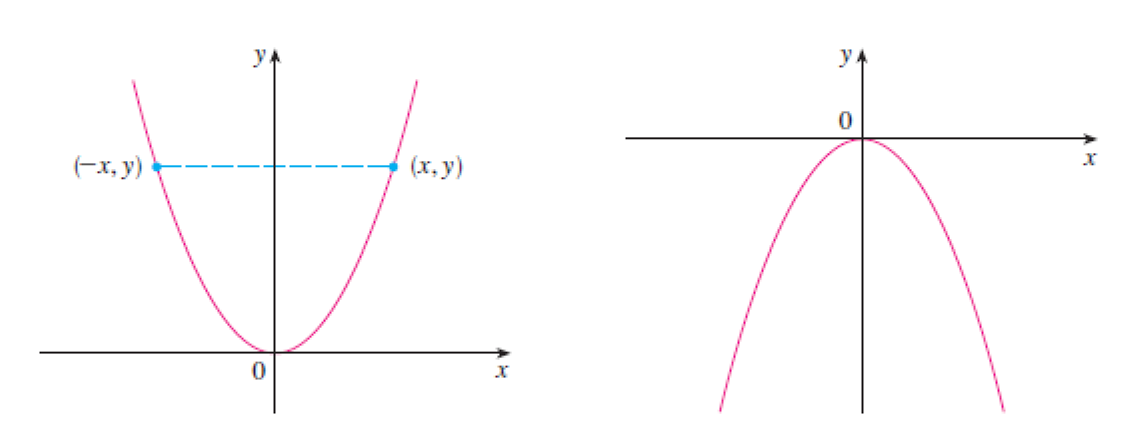


Figure shows the graphs of several parabolas with equations

$$y = ax^2$$

of the form for various values of the number a.

We see that the parabola $y = ax^2$ opens upward if $a > 0$ and downward if $a < 0$. The graph is symmetric with respect to the y-axis because its equation is unchanged when x is replaced by -x. This corresponds to the fact that the function $y = ax^2$ is an even function.



(a) $y = ax^2,\ a > 0$ (b) $y = ax^2,\ a < 0$

EXAMPLE:

Find the focus and directrix of the parabola $x^2 = 8y$

Solution:

equation ($x^2 = 8y$) is equation ($x^2 = 4py$) with

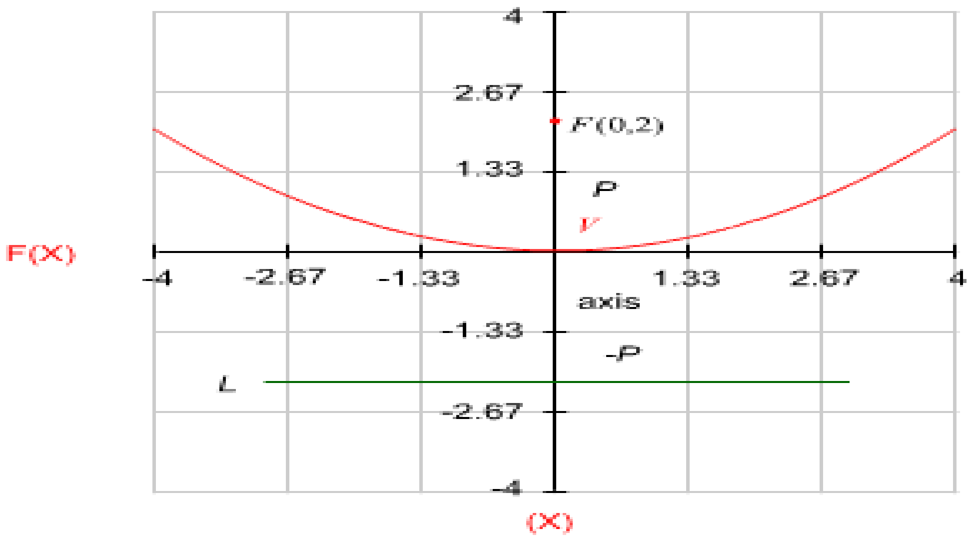
$4p = 8, \qquad p = 2$

The focus is on the y-axis $p = 2$ units from the vertex,

that is, at Focus: $F(0,2)$

The directrix $y = -p$ is the line $y = -2$:

Directrix: $y = -2$.



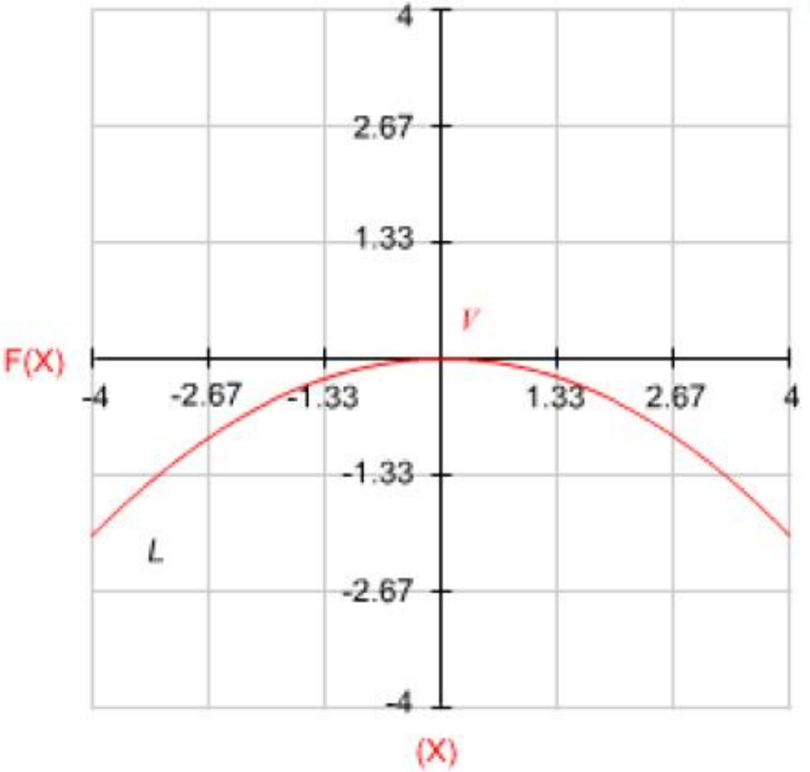
EXAMPLE:

If the axis of the parabola is the y axis and the vertex is at the origin with focus F at $(0, p)$, then the equation of the parabola is $x^2 = 4py$

If $p > 0$, then the parabola open upward.

If $p < 0$ the parabola open downward.

Solution:



Example:

Interchanging the roles of x and y yields the similar equation

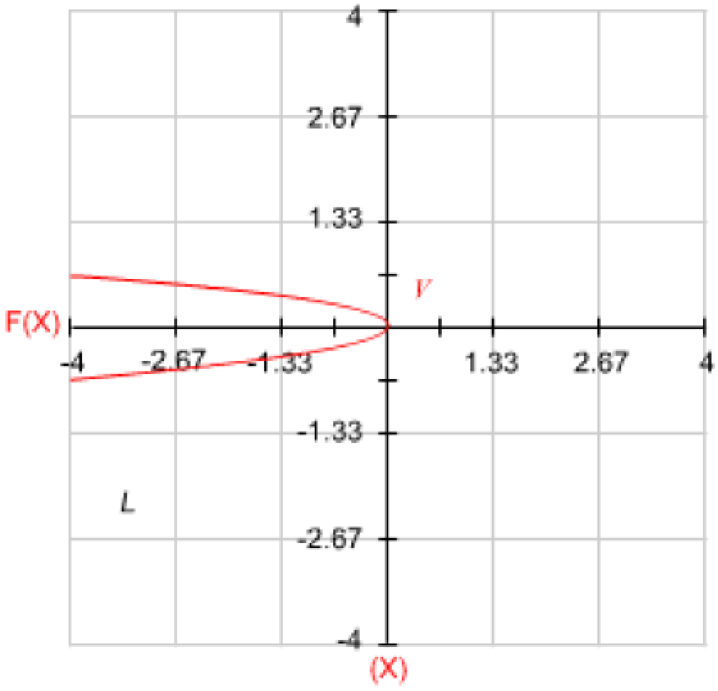
$$y^2 = 4px$$

which are the equations of the parabola with vertex at the origin and focus to F at $(p,0)$.

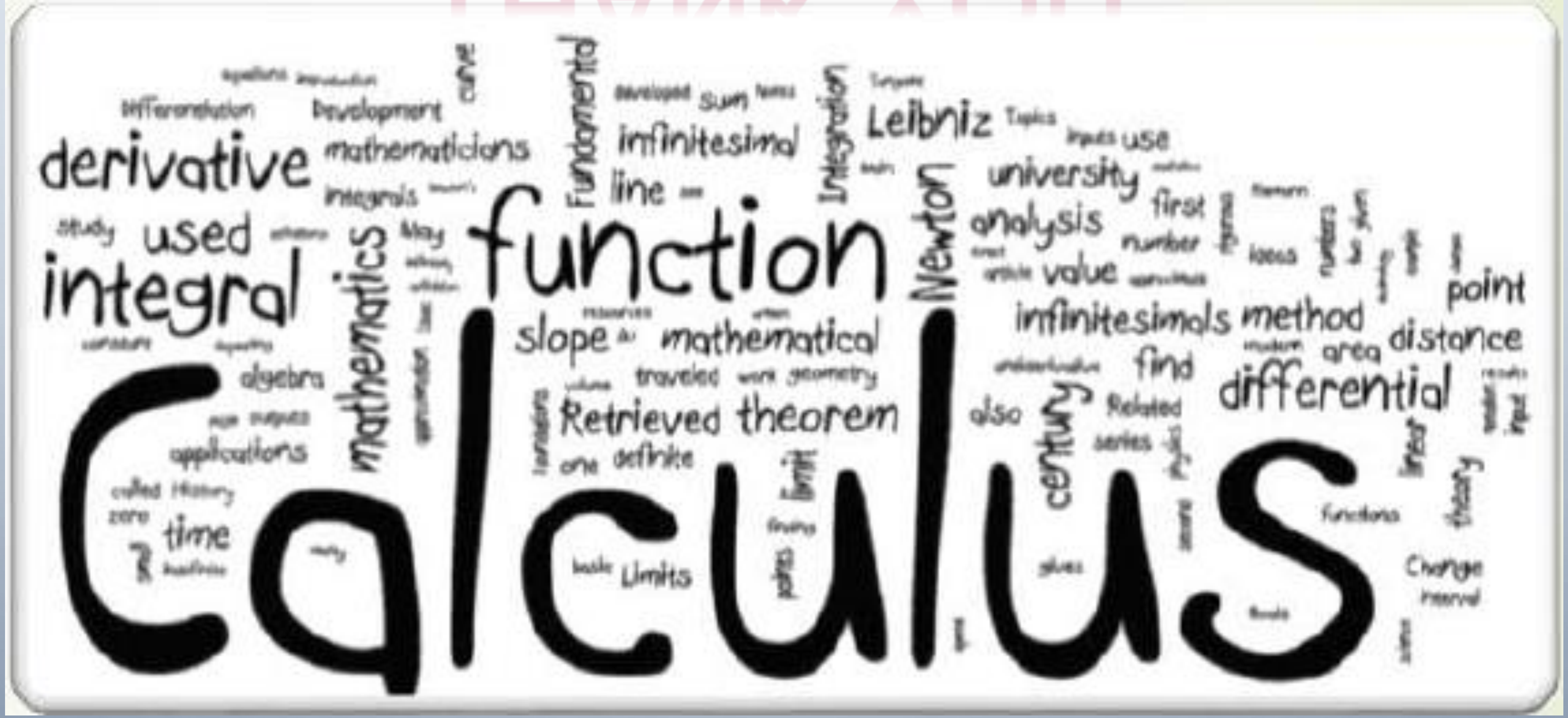
If $p > 0$, then the parabola open to the right.

and if $p < 0$ the parabola open to the left.

Solution:



THANK YOU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