1.1. Define following terms

(a) Current

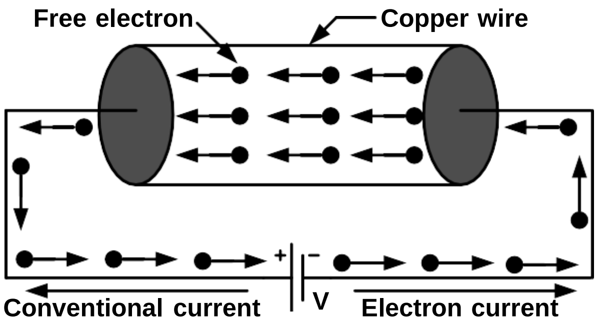


Figure 1.1Concept of electric current

- Flow of electron in closed circuit is called current.
- Amount of charge passing through the conductor in unit time also called current.
- Unit of current is charge/second or Ampere (A).

$$I = \frac{Q}{t}$$

Where, $I =$ Current

$Q =$ Charge

$t =$ Time

(b) Potential or Voltage

- The capacity of a charged body to do work is called potential.
- Unit of potential is joule/coulomb or Volt (V).

$$V = \frac{W}{Q}$$

Where, V = Potential or Voltage

W = Workdone

(c) Potential difference

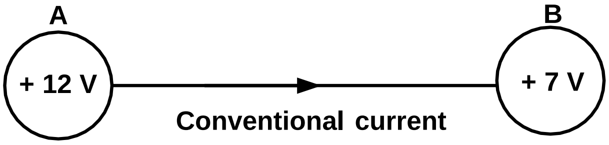


Figure 1. 1Potential differences

- The difference of electrical potential between two charged bodies is called potential difference.
- Unit of Potential Difference is Volt (V).
- If potential of body A is +12V and potential of body B is +7V then potential difference is +5V.
  i.e. (+12V) - (+7V) = +5V

(d) Electro Motive Force (emf)

- The force is required to move electron from negative terminal to positive terminal of electrical source in electrical circuit is called emf.
- Unit of emf is volt (V).
- Emf is denoted as ε.

(e) Energy

- Ability to do work is called energy.
- Unit of energy is Joule or Watt-sec or Kilowatt-hour (KWh).
- 1KWh is equal to 1 Unit.

$$W = P \times t = VIt = I^2Rt = \frac{V^2t}{R}$$

Where, W =Energy

P =Power

t =Time

(f) Power

- Energy per unit in time is called power.
- Unit of Power is Joule/Second or Watt (W).

$$P = \frac{W}{t}$$

(g) Resistance

- Property of a material that opposes the flow of electron is called resistance.
- Unit of resistance is Ohm (Ω).

$$R=\frac{V}{I}$$

Where, $R =$ Resistance

(h) Conductance

- Property of a material that allows flow of electron.
- It is reciprocal of resistance.
- Unit of conductance is (Ω^{-1}) or mho or Siemens(S).

$$G = \frac{1}{R}$$

Where, $G =$ Conductance

(i) Resistivity or Specific Resistance

- Amount of resistance offered by 1m length of wire of $1m^2$ cross-sectional area.
- Resistivity is denoted as a ρ.
- Unit of Resistivity is Ohm-meter (Ω-m).

$$R \propto \frac{l}{a}$$

$$R = \rho \frac{l}{a}$$

$$\rho = \frac{Ra}{l}$$

Where, $R =$ Resistance

$\rho =$ Resistivity

$l =$ Length of wire

$a =$ Cross section area of wire

(j) Conductivity

- Ability of a material to allow flow of electron of a given material for 1 m length & 1 m^2cross-sectional area is called conductivity. Unit of conductivity is$\Omega^{-1}m^{-1}$ or Siemens m^{-1}.

$$\sigma = \frac{1}{\rho}$$

Where, $\sigma =$ Conductivity

1.2. Explain types of electrical energysource

- Electrical source is an element which supplies energy to networks. There are two types of electrical sources.

(a) Independent sources

Independent voltage source

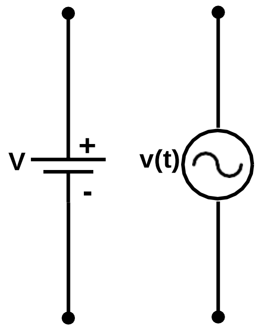


Figure 1. 2Independent voltage source

- It is a two terminal element that provide a specific voltage across its terminal.
- The value of this voltage at any instant is independent of value or direction of the current that flow through it.

Independent current source

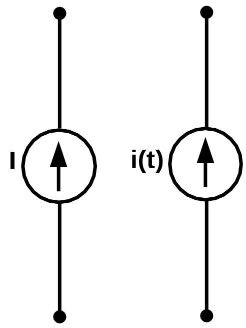


Figure 1. 3Independent current source

- It is two-terminal elements that provide a specific current across its terminal.
- The value and direction of this current at any instant is independent of value or direction of the voltage that appears across the terminal of source

(b) Dependent sources

Voltage controlled voltage source (VCVS)

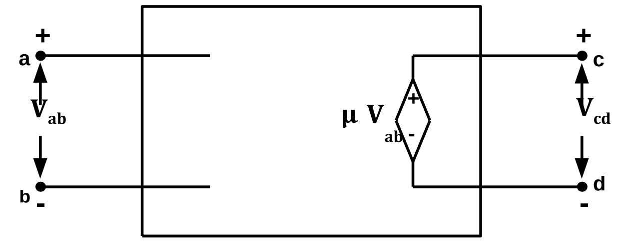


Figure 1.5VCVS

- Voltage controlled voltage source is four terminal network components that established a voltage V_{cd} between two-point c and d.

  $V_{cd} = \mu V_{ab}$
- The voltage V_{cd} depends upon the control voltage V_{ab} and μ is constant so it is dimensionless.
- μ is known as a voltage gain.

Voltage controlled current source (VCCS)

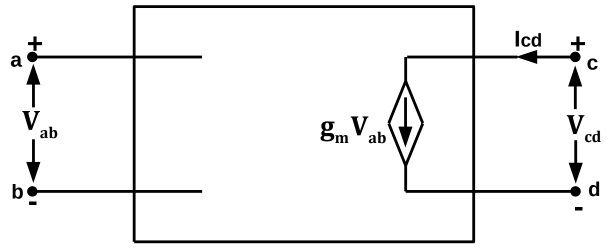


Figure 1.6VCCS

- Voltage controlled current source is four terminal network components that established a current i_{cd} in the branch of circuit.

  $i_{cd} = g_m V_{ab}$
- i_{cd} depends only on the control voltage V_{ab} and constant g_m ,is called trans conductance or mutual conductance.
- Unit of transconductance is Ampere/Volt or Siemens(S).

Current controlled voltage source (CCVS)

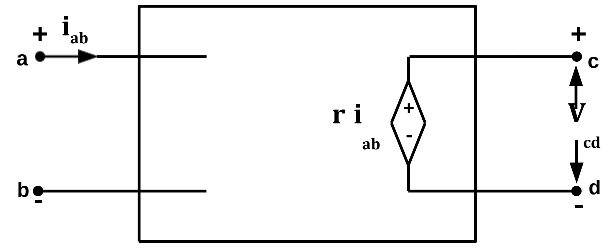


Figure 1.7CCVS

- Current controlled voltage source is four terminal network components that established a voltage V_{cd} between two-point c and d.

  $V_{cd} = r i_{ab}$
- V_{cd} depends on only on the control current i_{ab} and constant r and r is called trans resistance or mutual resistance.
- Unit of transresistance is Volt/Ampere or Ohm (Ω).

Current controlled current source (CCCS)

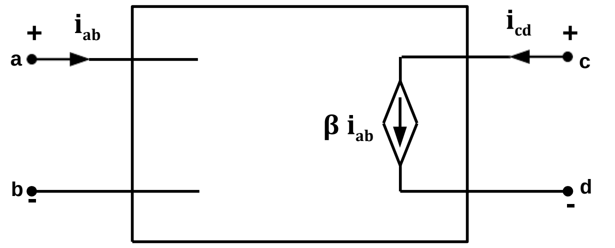


Figure 1.8CCCS

- Current controlled current source is four terminal network components that established a current I_{cd} in the branch of circuit.

  $i_{cd} = \beta i_{ab}$
- i_{cd} depends on only on the control current i_{ab} and constantβ and β is called current gain. Current gain is constant.
- Current gain is dimensionless.

1.3. Explain source conversion

- A voltage source with a series resistor can be converted into an equivalent current source with a parallel resistor. Conversely, a current source with a parallel resistor can be converted into a voltage source with a series resistor.
- Open circuit voltages in both the circuits are equal and short circuit currents in both the circuit are equal.Source transformation can be applied to dependent source as well.

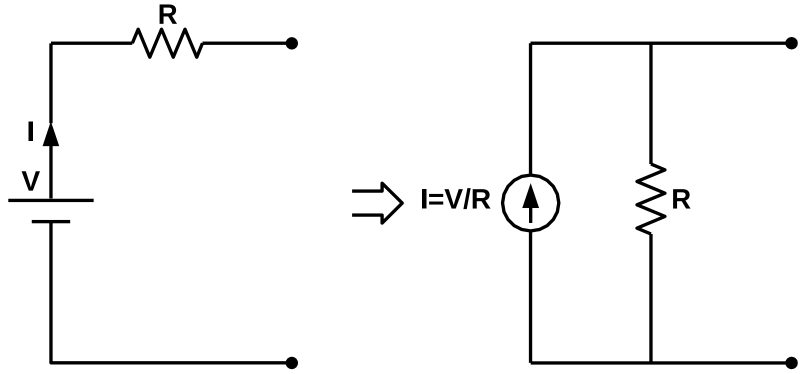
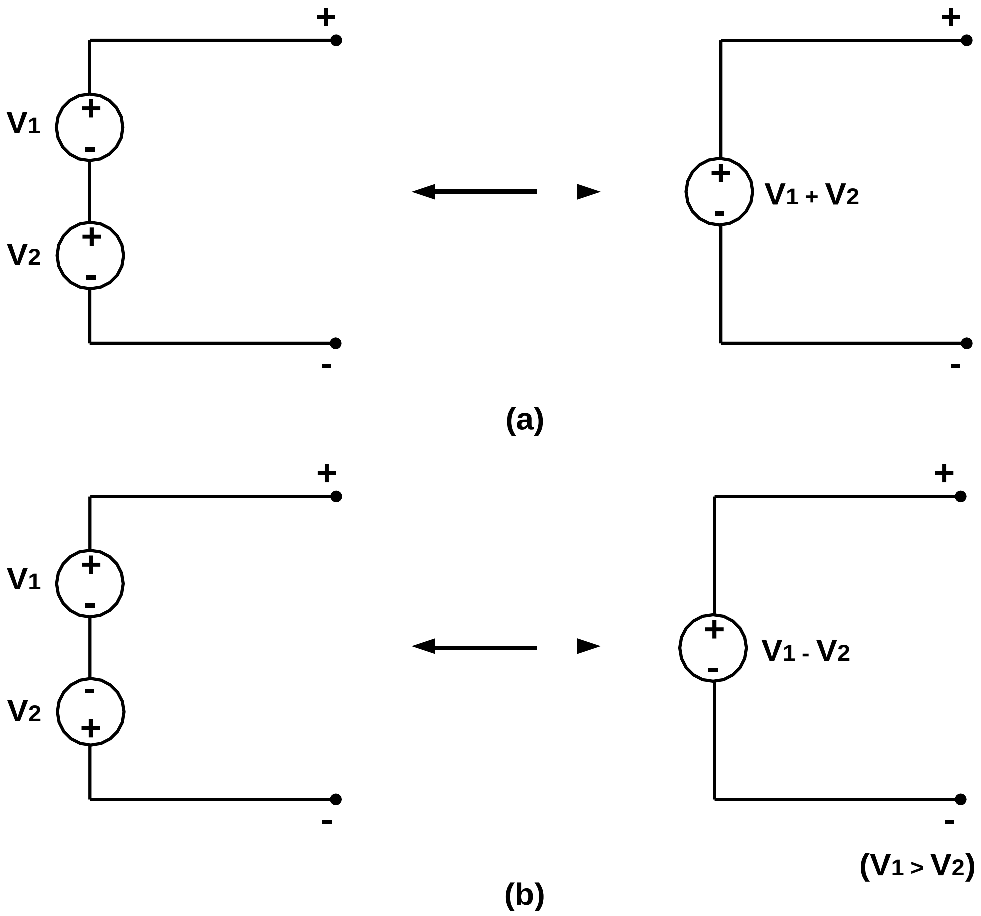
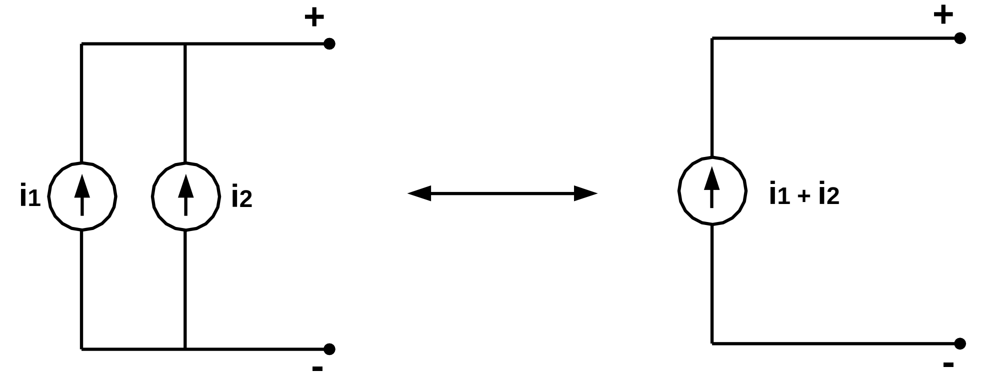


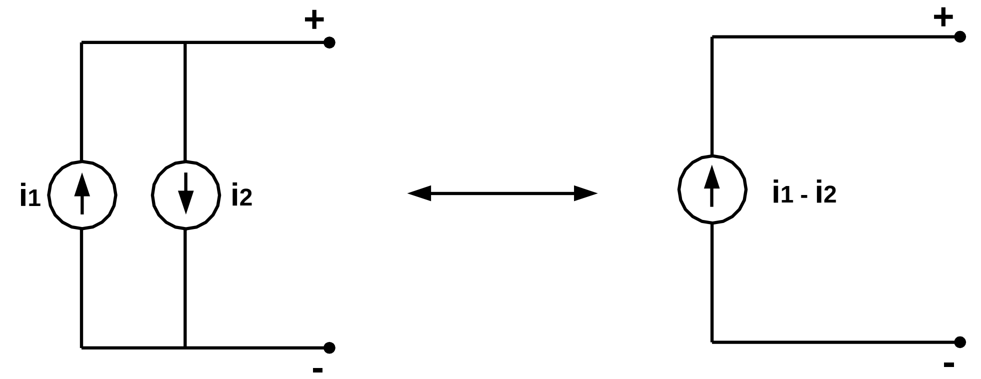
Figure 1. 9Source conversion

Network simplification techniques



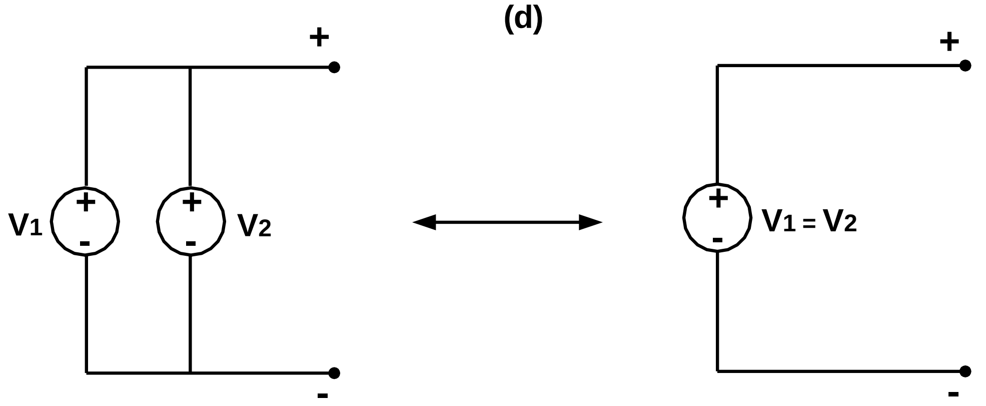


(c)

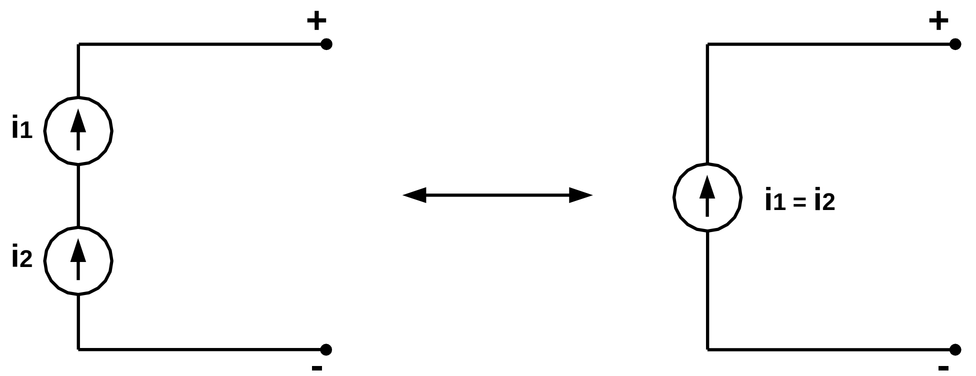


($i_1 > i_2$)

(d)



(e)



(f)

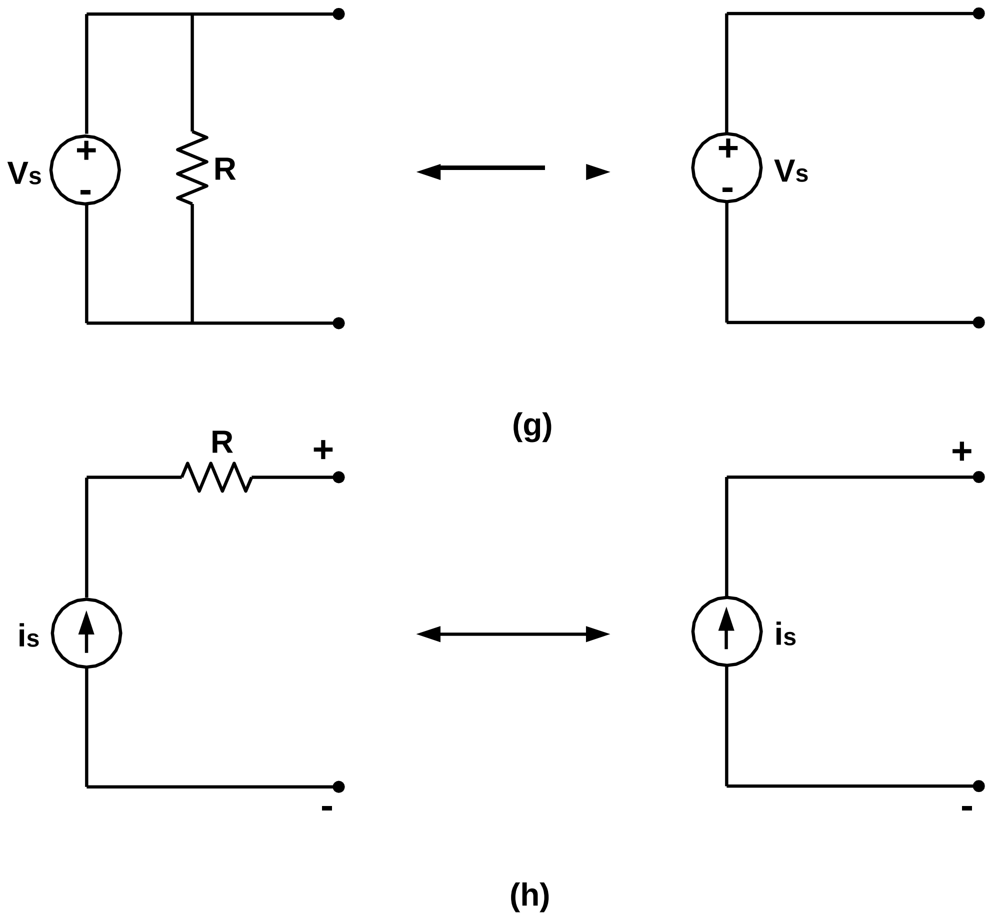


Figure 1.10Rules under which source may be combined and separated

1.4. Explain ideal electrical circuit element.

- There are major three electrical circuit elements which are discussed below.

(a) Resistor

- Resistor is element which opposes the flow of current.

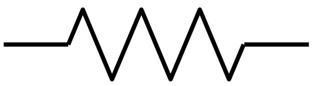


Figure 1.11Resistor

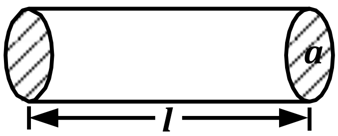


Figure 1.12Conductor

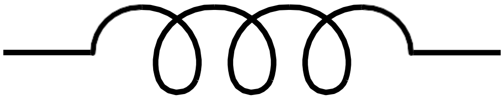
- Resistance is property of material which opposes the flow current. It is measured in Ohms (Ω).
- Value of resistance of conductor is
  - ✓ Proportional to its length.
  - ✓ Inversely proportional to the area of cross section.
  - ✓ Depends on nature of material.
  - ✓ Depends on temperature of conductor.

$$R \propto \frac{l}{a}$$

$$R = \frac{\rho l}{a}$$

(b) Inductor

- An inductor is element which store energy in form of magnetic field.
- The property of the coil of inducing emf due to the changing flux linked with it is known as inductance of the coil.
- Inductance is denoted by L and it is measured in Henry (H).



1.13Inductor

- Value of inductance of coil is
  - ✓ Directly proportional to the square of number of turns.
  - ✓ Directly proportional to the area of cross section.
  - ✓ Inversely proportional to the length.
  - ✓ Depends on absolute permeability of magnetic material.

$$\Phi = \frac{F}{S} = \frac{NI}{S} = \frac{NI}{\frac{l}{\mu_0 \mu_r A}} = \frac{NI\mu_0\mu_r A}{l}$$

$$Now, L = \frac{N\Phi}{I} = \frac{N\left(\frac{NI\mu_0\mu_r A}{l}\right)}{I} = \frac{N^2\mu_0\mu_r A}{l}$$

$Where, L$ = Inductance of coil

N = Number of turns of coil

Φ = Flux link in coil

F = Magneto motive force(MMF)

I = Current in the coil

l = Mean length of coil

μ_0 = Permiability of free space

μ_r = Relative permiability of magnetic material

A = Cross sectional area of magnetic material

(c) Capacitor

- Capacitor is an element which stored energy in form of charge.
- Capacitance is the capacity of capacitor to store electric charge.
- It is denoted by C and measured in Farad (F).

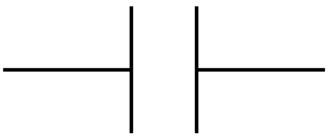


Figure 1.14Capacitor

- Value of capacitance is
  - ✓ Directly proportional to the area of plate.
  - ✓ Inversely proportional to distance between two plates.
  - ✓ Depends on absolute permittivity of medium between the plates.

$$C \propto \frac{A}{d}$$

$$C = \frac{\varepsilon A}{d}$$

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

Where, C=Capacitance of capacitor

A =Cross sectional area of plates

d =Distance between two plates

ε = Abolute Permittivity

ε_0 = Permittivity of free space

ε_r = Relative permittivity of dielectric material

1.5. Explain Ohm's law and its limitations.

- Current flowing through the conductor is directly proportional to the potential difference applied to the conductor, provided that no change in temperature.

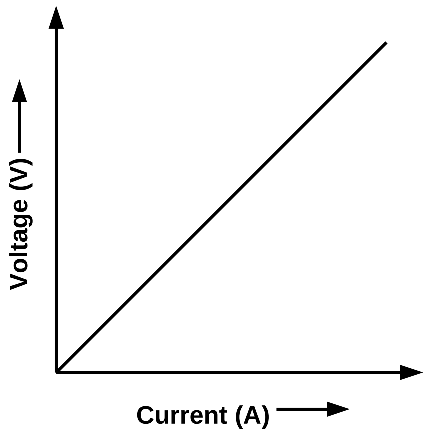


Figure 1.15Change in current w.r.t change in voltage for conducting material

$$V \propto I$$

$$\therefore V = IR$$

- Where R is constant which is called resistance of the conductor.

$$\therefore R = \frac{V}{I}$$

- Limitations of Ohm's Law:
  - ✓ It cannot be applied to non-linear device e.g. Diode, Zener diode etc.
  - ✓ It cannot be applied to non-metallic conductor e.g. Graphite, Conducting polymers
  - ✓ It can only be applied in the constant temperature condition.

1.6. State and explain the Kirchhoff's current and voltage laws

(a) Kirchhoff's current law (KCL)

- **Statement**:
  "Algebraic sum of all current meeting at a junction is zero"
- Let, Suppose

- ✓ Branches are meeting at a junction 'J'
- ✓ Incoming current are denoted with (+ve) sign
- ✓ Outgoing currents are denoted with (-ve) sign

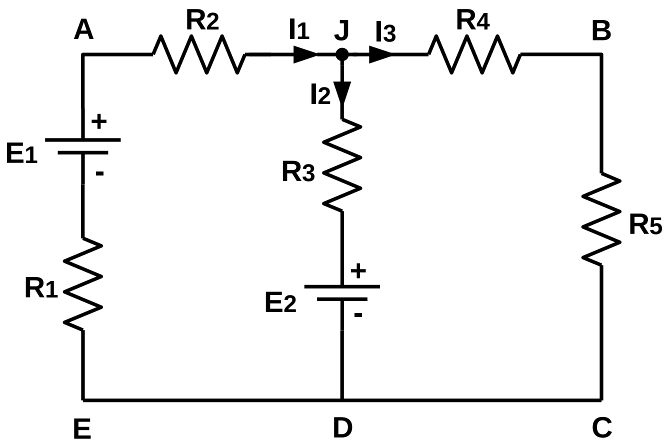


Figure 1.16Kirchhoff's law diagram

- Then,

$$\sum I = 0$$

$$(+I_1) + (-I_2) + (-I_3) = 0$$

$$I_1 - I_2 - I_3 = 0$$

$$I_1 = I_2 + I_3$$

$\therefore$ *Incoming current = Outgoing current*

(b) Kirchhoff's voltage law (KVL)

- **Statement**:
  "Algebraic sum of all voltage drops and all emf sources in any closed path is zero"
- Let, Suppose
  - ✓ Loop current in clockwise or anticlockwise direction
  - ✓ Circuit current and loop current are in same direction than voltage drop is denoted by (-ve) sign.
  - ✓ Circuit current and loop current are in opposite direction than voltage drop is denoted by (+ve) sign.
  - ✓ Loop current move through (+ve) to (-ve) terminal of source than direction of emf is (-ve).
  - ✓ If Loop current move through (-ve) to (+ve) terminal of source than direction of emf is (+ve).

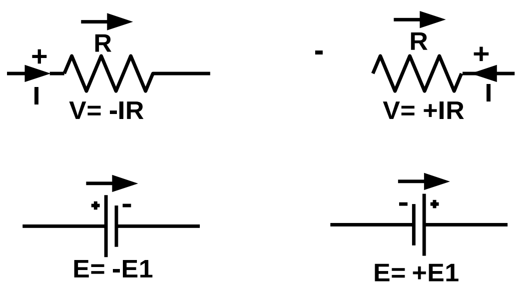


Figure 1.17Sign convention for Kirchhoff's voltage law

$$\therefore \sum IR + \sum E = 0$$

KVL to loop AJDEA

$$-I_1R_2 - I_2R_3 - E_2 - I_1R_1 + E_1 = 0$$

KVL to loop JBCDJ

$$-I_3R_4 - I_3R_5 + E_2 + I_2R_3 = 0$$

1.7. Explain series and parallel combination of resistor

Series combination of resistor

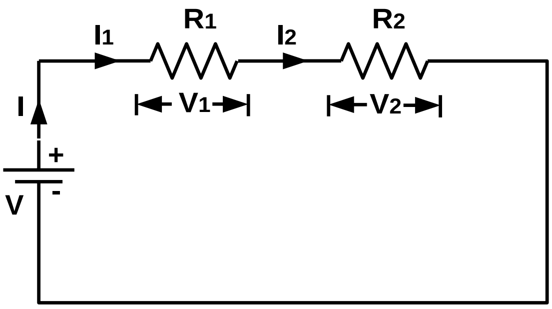


Figure 1.18Series combination of resistors

Here, $I_1 = I_2 = I$

As per KVL,

$$V = V_1 + V_2$$

$$V = IR_1 + IR_2$$

$$V = I(R_1 + R_2)$$

$$\frac{V}{I} = (R_1 + R_2)$$

$$R_{eq} = R_1 + R_2$$

For n resistor are connected in series

$$R_{eq} = R_1 + R_2 + R_3 + \dots\dots + R_n$$

- Value of equivalent resistance of series circuit is bigger than the biggest value of individual resistance of circuit.

Parallel combination of resistor

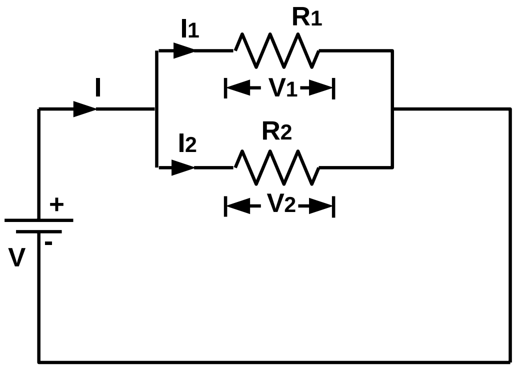


Figure 1.19Parallel combinations of resistors

Here, $V_1 = V_2 = V$

As per KCL,

$$I = I_1 + I_2$$

$$I = \frac{V}{R_1} + \frac{V}{R_2}$$

$$I = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{I}{V} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{1}{R_{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

For n resistor are connected in Parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots\dots\dots + \frac{1}{R_n}$$

- Value of equivalent resistance of parallel circuit is smaller than the smallest value of individual resistance of circuit.

1.8. Explain Voltage divider law and current divider Law.

Voltage Divider Law

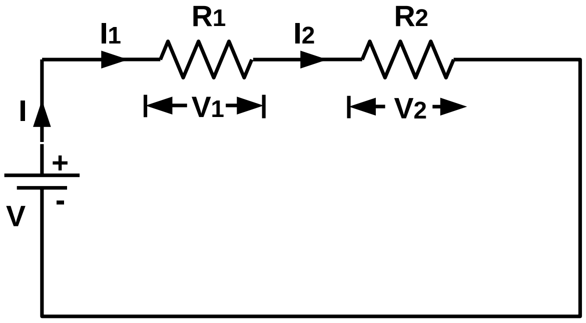


Figure 1.20Voltage divider circuit

$$Here, I_1 = I_2 = I$$

As per KVL,

$$V = V_1 + V_2$$

$$V = I_1R_1 + I_2R_2$$

$$V = IR_1 + IR_2$$

$$V = I(R_1 + R_2)$$

$$I = I_1 = I_2 = \frac{V}{(R_1 + R_2)}$$

$$Now, V_1 = I_1R_1$$

$$V_1 = \frac{V}{R_1 + R_2} R_1$$

$$V_1 = V\left(\frac{R_1}{R_1 + R_2}\right)$$

$$Now, V_2 = I_2R_2$$

$$V_2 = \frac{V}{R_1 + R_2} R_2$$

$$V_2 = V\left(\frac{R_2}{R_1 + R_2}\right)$$

Current Divider Law

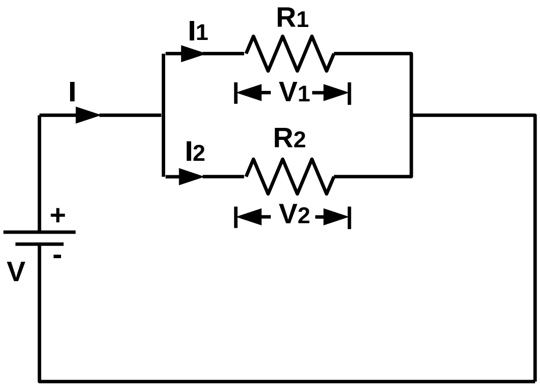


Figure 1.21Current divider circuit

$$Here, V_1 = V_2 = V$$

As per KCL,

$$I = I_1 + I_2$$

$$I = \frac{V_1}{R_1} + \frac{V_2}{R_2}$$

$$I = \frac{V}{R_1} + \frac{V}{R_2}$$

$$I = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{I}{V} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$V = V_1 = V_2 = I\left(\frac{R_1R_2}{R_1 + R_2}\right)$$

$$Now, I_1 = \frac{V_1}{R_1}$$

$$I_1 = \frac{I\left(\frac{R_1R_2}{R_1 + R_2}\right)}{R_1}$$

$$I_1 = I\left(\frac{R_2}{R_1 + R_2}\right)$$

$$Now, I_2 = \frac{V_2}{R_2}$$

$$I_2 = \frac{I\left(\frac{R_1R_2}{R_1 + R_2}\right)}{R_2}$$

$$I_2 = I\left(\frac{R_1}{R_1 + R_2}\right)$$

1.9. Derive the equation of delta to star and star to delta transformation

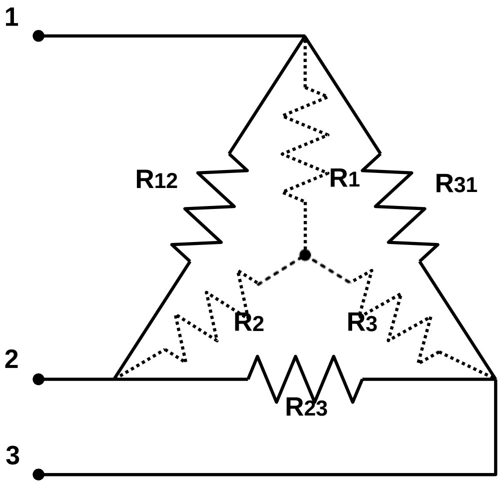


Figure 1.22Delta connected network

Resistance between terminal (1) & (2)

$$= R_{12} \parallel (R_{23} + R_{31})$$

$$= \frac{R_{12}(R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}}$$

Resistance between terminal (2) & (3)

$$= R_{23} \parallel (R_{12} + R_{31})$$

$$= \frac{R_{23}(R_{12} + R_{31})}{R_{12} + R_{23} + R_{31}}$$

Resistance between terminal (3) & (1)

$$= R_{31} \parallel (R_{12} + R_{23})$$

$$= \frac{R_{31}(R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}}$$

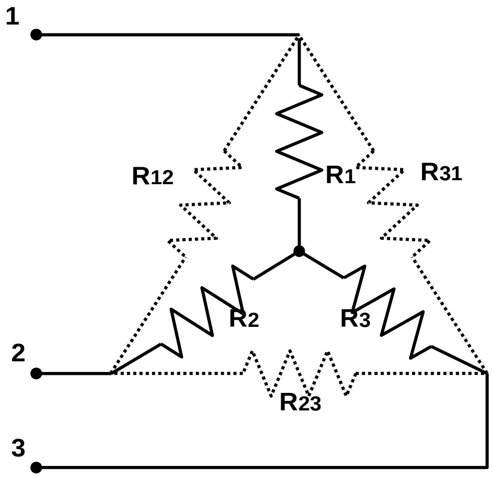


Figure 1.23Star connected network

Resistance between terminal (1) & (2)

$$= R_1 + R_2$$

Resistance between terminal (2) & (3)

$$= R_2 + R_3$$

Resistance between terminal (3) & (1)

$$= R_3 + R_1$$

Resistance between terminals (1) & (2) *in delta equal to resistance between terminals* (1) & (2) *in star*

$$R_1 + R_2 = \frac{R_{12}(R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}} \qquad (i)$$

Similarly,

$$R_2 + R_3 = \frac{R_{23}(R_{12} + R_{31})}{R_{12} + R_{23} + R_{31}} \qquad (ii)$$

$$R_3 + R_1 = \frac{R_{31}(R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}} \qquad (iii)$$

(a) Delta to star conversion

Simplify $(i) + (ii) - (iii)$ *on both the side of equations*

$$R_1 + R_2 + R_2 + R_3 - R_3 - R_1 = \frac{R_{12}(R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}} + \frac{R_{23}(R_{12} + R_{31})}{R_{12} + R_{23} + R_{31}} - \frac{R_{31}(R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}}$$

$$= \frac{(R_{12}R_{23}+R_{12}R_{31})}{R_{12}+R_{23}+R_{31}} + \frac{(R_{23}R_{12}+R_{23}R_{31})}{R_{12}+R_{23}+R_{31}} - \frac{(R_{31}R_{12}+R_{31}R_{23})}{R_{12}+R_{23}+R_{31}}$$

$$= \frac{(R_{12}R_{23}+R_{12}R_{31}+R_{23}R_{12}+R_{23}R_{31}-R_{31}R_{12}-R_{31}R_{23})}{(R_{12}+R_{23}+R_{31})}$$

$$2R_2 = \frac{2R_{12}R_{23}}{R_{12}+R_{23}+R_{31}}$$

$$R_2 = \frac{R_{12}R_{23}}{R_{12}+R_{23}+R_{31}}$$

Similarly , $R_1 = \dfrac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}}$

$$R_3 = \frac{R_{23}R_{31}}{R_{12}+R_{23}+R_{31}}$$

(b) Star to delta conversion

Simplify $(i)(ii)+(ii)(iii)+(iii)(i)$ *on both the side of equation*

$$(R_1+R_2)(R_2+R_3)+(R_2+R_3)(R_3+R_1)+(R_3+R_1)(R_1+R_2)$$

$$= \left(\frac{R_{12}(R_{23}+R_{31})}{R_{12}+R_{23}+R_{31}}\right)\left(\frac{R_{23}(R_{12}+R_{31})}{R_{12}+R_{23}+R_{31}}\right) + \left(\frac{R_{23}(R_{12}+R_{31})}{R_{12}+R_{23}+R_{31}}\right)\left(\frac{R_{31}(R_{12}+R_{23})}{R_{12}+R_{23}+R_{31}}\right) + \left(\frac{R_{31}(R_{12}+R_{23})}{R_{12}+R_{23}+R_{31}}\right)\left(\frac{R_{12}(R_{23}+R_{31})}{R_{12}+R_{23}+R_{31}}\right)$$

$$R_1R_2+R_1R_3+R_2^2+R_2R_3+R_2R_3+R_2R_1+R_3^2+R_3R_1+R_3R_1+R_3R_2+R_1^2+R_1R_2$$

$$= \left(\frac{R_{12}R_{23}+R_{12}R_{31}}{R_{12}+R_{23}+R_{31}}\right)\left(\frac{R_{23}R_{12}+R_{23}R_{31}}{R_{12}+R_{23}+R_{31}}\right) + \left(\frac{R_{23}R_{12}+R_{23}R_{31}}{R_{12}+R_{23}+R_{31}}\right)\left(\frac{R_{31}R_{12}+R_{31}R_{23}}{R_{12}+R_{23}+R_{31}}\right) + \left(\frac{R_{31}R_{12}+R_{31}R_{23}}{R_{12}+R_{23}+R_{31}}\right)\left(\frac{R_{12}R_{23}+R_{12}R_{31}}{R_{12}+R_{23}+R_{31}}\right)$$

$$3R_1R_2+3R_2R_3+3R_3R_1+R_1^2+R_2^2+R_3^2$$

$$= \left(\frac{R_{23}^2R_{12}^2+R_{12}R_{23}^2R_{31}+R_{12}^2R_{23}R_{31}+R_{12}R_{23}R_{31}^2}{(R_{12}+R_{23}+R_{31})^2}\right) + \left(\frac{R_{12}^2R_{23}R_{31}+R_{12}R_{23}^2R_{31}+R_{12}R_{23}R_{31}^2+R_{23}^2R_{31}^2}{(R_{12}+R_{23}+R_{31})^2}\right) + \left(\frac{R_{12}^2R_{23}R_{31}+R_{12}^2R_{31}^2+R_{12}R_{23}^2R_{31}+R_{12}R_{23}R_{31}^2}{(R_{12}+R_{23}+R_{31})^2}\right)$$

$$= \frac{R_{23}^2R_{12}^2+R_{12}R_{23}^2R_{31}+R_{12}^2R_{23}R_{31}+R_{12}R_{23}R_{31}^2+R_{12}^2R_{23}R_{31}+R_{12}R_{23}^2R_{31}+R_{12}R_{23}R_{31}^2+R_{23}^2R_{31}^2+R_{12}^2R_{23}R_{31}+R_{12}^2R_{31}^2+R_{12}R_{23}^2R_{31}+R_{12}R_{23}R_{31}^2}{(R_{12}+R_{23}+R_{31})^2}$$

$$= \frac{(R_{12}R_{23}^2R_{31}+R_{12}^2R_{23}R_{31}+R_{12}R_{23}R_{31}^2+R_{12}^2R_{23}R_{31}+R_{12}R_{23}^2R_{31}+R_{12}R_{23}R_{31}^2+R_{12}^2R_{23}R_{31}+R_{12}R_{23}^2R_{31}+R_{12}R_{23}R_{31}^2)+(R_{23}^2R_{12}^2+R_{23}^2R_{31}^2+R_{12}^2R_{31}^2)}{(R_{12}+R_{23}+R_{31})^2}$$

$$= \frac{R_{12}R_{23}R_{31}(R_{23}+R_{12}+R_{31}+R_{12}+R_{23}+R_{31}+R_{12}+R_{23}+R_{31})}{(R_{12}+R_{23}+R_{31})^2} + \frac{(R_{23}^2R_{12}^2+R_{23}^2R_{31}^2+R_{12}^2R_{31}^2)}{(R_{12}+R_{23}+R_{31})^2}$$

$$= \frac{R_{12}R_{23}R_{31}(3R_{12}+3R_{23}+3R_{31})}{(R_{12}+R_{23}+R_{31})^2} + \left(\frac{R_{23}^2R_{12}^2}{(R_{12}+R_{23}+R_{31})^2} + \frac{R_{23}^2R_{31}^2}{(R_{12}+R_{23}+R_{31})^2} + \frac{R_{12}^2R_{31}^2}{(R_{12}+R_{23}+R_{31})^2}\right)$$

$$= \frac{3R_{12}R_{23}R_{31}(R_{12}+R_{23}+R_{31})}{(R_{12}+R_{23}+R_{31})^2} + \left(\frac{R_{23}^2R_{12}^2}{(R_{12}+R_{23}+R_{31})^2} + \frac{R_{23}^2R_{31}^2}{(R_{12}+R_{23}+R_{31})^2} + \frac{R_{12}^2R_{31}^2}{(R_{12}+R_{23}+R_{31})^2}\right)$$

$$= 3R_3R_{12} + R_2^2 + R_3^2 + R_1^2$$

Now equation become

$$3R_1R_2+3R_2R_3+3R_3R_1+R_1^2+R_2^2+R_3^2 = 3R_3R_{12}+R_2^2+R_3^2+R_1^2$$

$$3R_1R_2+3R_2R_3+3R_3R_1 = 3R_3R_{12}$$

$$R_{12} = R_1 + R_2 + \frac{R_1R_2}{R_3}$$

Similarly

$$R_{23} = R_2 + R_3 + \frac{R_2R_3}{R_1}$$

$$R_{31} = R_3 + R_1 + \frac{R_3R_1}{R_2}$$

1.10. Explain Node analysis

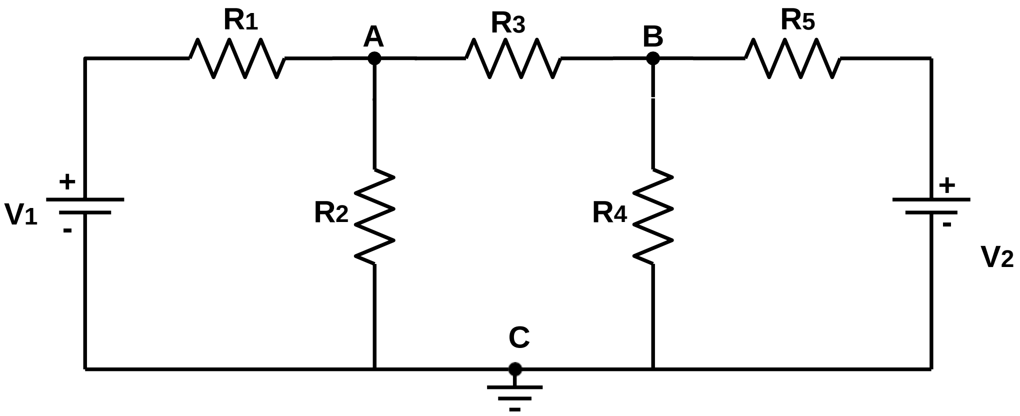


Figure 1.24Node analysis network

- **Node**: Node refers to any point on circuit where two or more circuit elements meet.
- Node analysis based on Kirchhoff's current law states that algebraic summation of currents meeting at junction is zero.
- Node C is taken as reference node in this network. If there are n nodes in any network, the number of equation to be solved will be (n-1).
- Node A,B and C are shown in given network and their voltages areV_A,V_B andV_C . Value of node V_C is zero because V_C is reference node.
- Steps to follow in node analysis:
  - ✓ Consider node in the network, assign current and voltage for each branch and node respectively.
  - ✓ Apply the KCL for each node and apply ohm's law to branch current.
  - ✓ Solve the equation for find the unknown node voltage.
  - ✓ Using these voltages, find the required branch currents.
- **Node A**

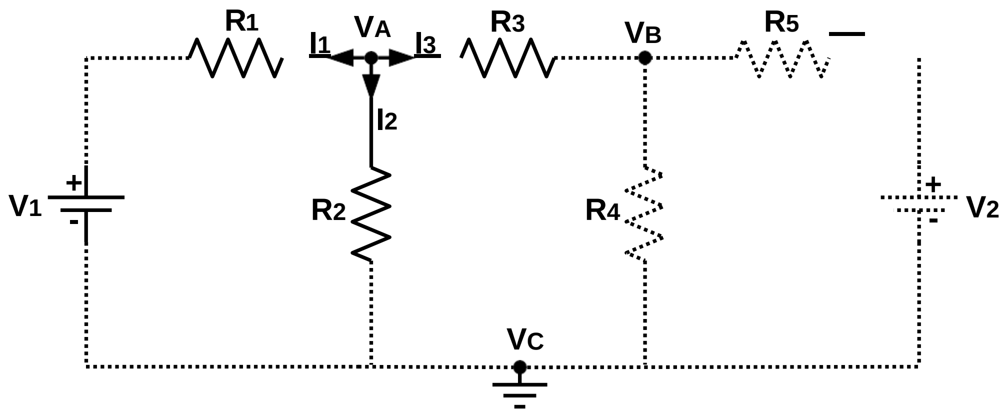


Figure 1.25Node analysis network for node A

Apply KCL at node A,

$$(-I_1)+(-I_2)+(-I_3)=0$$

$$I_1+I_2+I_3=0$$

$$\frac{V_A - V_1}{R_1}+\frac{V_A - V_C}{R_2}+\frac{V_A - V_B}{R_3}=0$$

$$V_A\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]+V_B\left[-\frac{1}{R_3}\right]=\frac{V_1}{R_1} \qquad (i)$$

- **Node B**

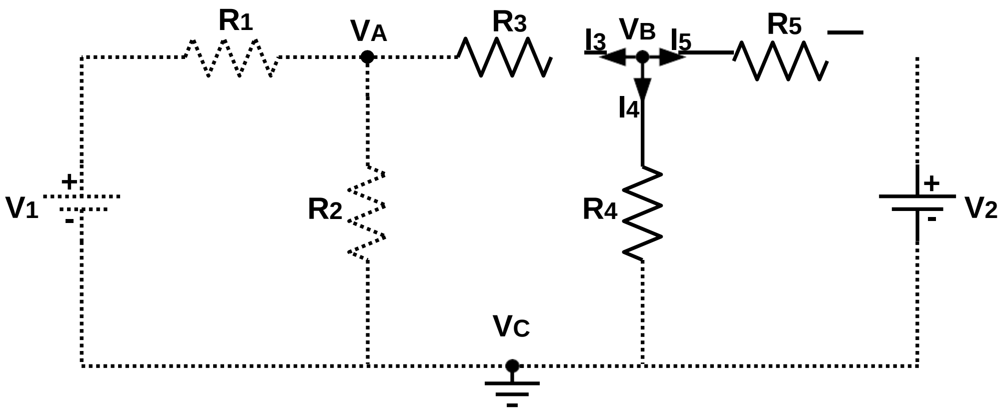


Figure 1.26Node analysis network for node B

Apply the KCL at node B,

$$(-I_3)+(-I_4)+(-I_5)=0$$

$$I_3+I_4+I_5=0$$

$$\frac{V_B-V_A}{R_3}+\frac{V_B-V_C}{R_4}+\frac{V_B-V_2}{R_5}=0$$

$$V_A\left[-\frac{1}{R_3}\right]+V_B\left[\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right]=\frac{V_2}{R_5} \qquad (ii)$$

From, equation (i) & (ii)

$$\begin{pmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & -\frac{1}{R_3} \\ -\frac{1}{R_3} & \frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5} \end{pmatrix}\begin{pmatrix} V_A \\ V_B \end{pmatrix}=\begin{pmatrix} \frac{V_1}{R_1} \\ \frac{V_2}{R_5} \end{pmatrix}$$

- One can easily find branch current of this network by solving equation (i) and (ii),if V_1 , V_2 and all resistance value are given.

1.11. Explain Mesh analysis

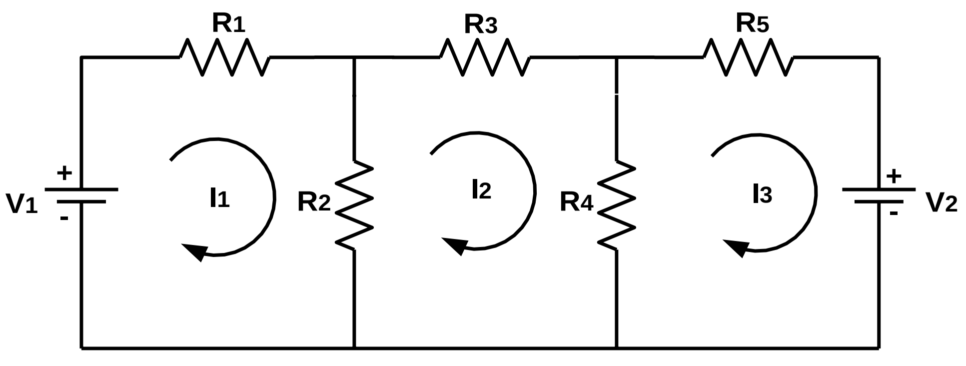


Figure 1.27Mesh analysis network

- **Mesh:** It is defined as a loop which does not contain any other loops within it.

- The current in different meshes are assigned continues path that they do not split at a junction into a branch currents.
- Basically, this analysis consists of writing mesh equation by Kirchhoff's voltage law in terms of unknown mesh current.

- Steps to be followed in mesh analysis:
  - ✓ Identify the mesh, assign a direction to it and assign an unknown current in it.
  - ✓ Assigned polarity for voltage across the branches.
  - ✓ Apply the KVL around the mesh and use ohm's law to express the branch voltage in term of unknown mesh current and resistance.
  - ✓ Solve the equations for unknown mesh current.
- **Loop 1**

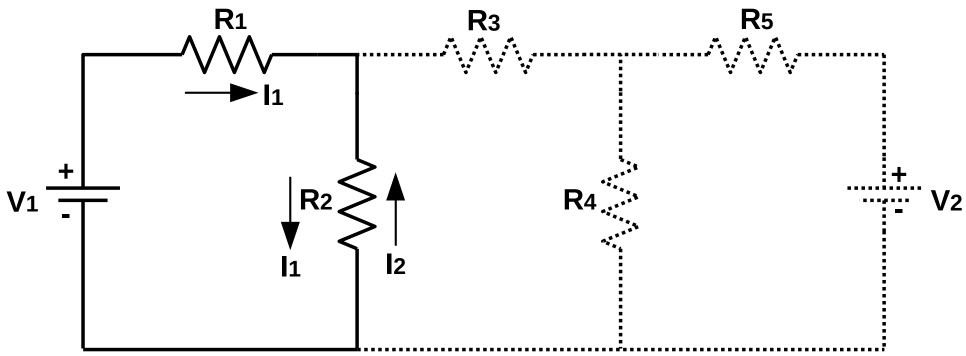


Figure 1.28Mesh analysis network for loop-1

Now apply the KVL in loop -1,

$$-I_1R_1-(I_1-I_2)R_2+V_1=0$$

$$-I_1R_1-I_1R_2+I_2R_2+V_1=0$$

$$-(R_1+R_2)I_1+R_2I_2=-V_1 \qquad \text{(i)}$$

- **Loop 2**

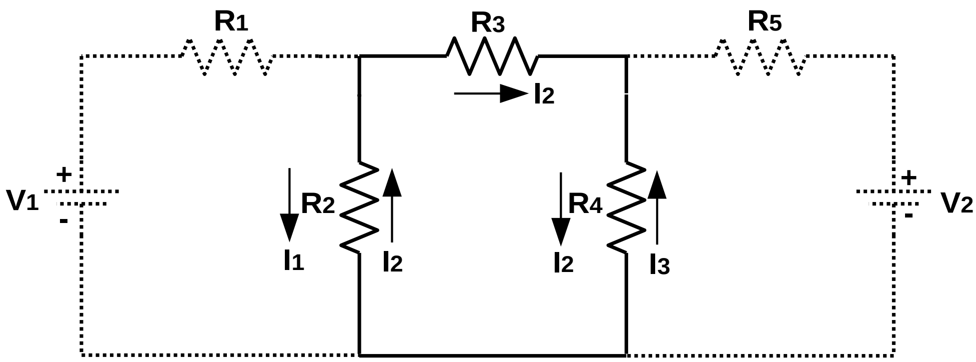


Figure 1.29Mesh analysis network for loop-2

Now Apply the KVL loop -2,

$$-I_2R_3-(I_2-I_3)R_4-(I_2-I_1)R_2=0$$

$$-I_2R_3-I_2R_4+I_3R_4-I_2R_2+I_1R_2=0$$

$$I_1R_2-I_2(R_3+R_4+R_2)+I_3R_4=0$$

$$R_2I_1-(R_3+R_4+R_2)I_2+R_4I_3=0 \qquad \text{(ii)}$$

- **Loop 3**

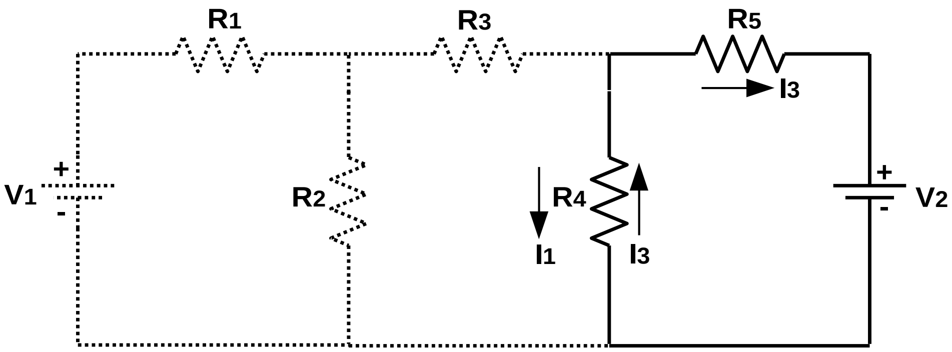


Figure 1.30Mesh analysis network for loop-3

Now Apply the KVL loop -3,

$$-I_3R_5 - V_2 - (I_3 - I_2)R_4 = 0$$

$$-I_3R_5 - V_2 - I_3R_4 + I_2R_4 = 0$$

$$I_2R_4 - I_3(R_5 + R_4) = V_2$$

$$R_4I_2 - (R_5 + R_4)I_3 = V_2 \qquad \text{(iii)}$$

From equation $(i),(ii)$ & (iii)

$$\begin{pmatrix} -(R_1+R_2) & R_2 & 0 \\ R_2 & (R_3+R_4+R_2) & R_4 \\ 0 & R_4 & -(R_5+R_4) \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} -V_1 \\ 0 \\ V_2 \end{pmatrix}$$

$$\Delta = \begin{vmatrix} -(R_1+R_2) & R_2 & 0 \\ R_2 & (R_3+R_4+R_2) & R_4 \\ 0 & R_4 & -(R_5+R_4) \end{vmatrix}$$

$$\Delta_1 = \begin{vmatrix} -V_1 & R_2 & 0 \\ 0 & (R_3+R_4+R_2) & R_4 \\ V_2 & R_4 & -(R_5+R_4) \end{vmatrix}$$

$$\Delta_2 = \begin{vmatrix} -(R_1+R_2) & -V_1 & 0 \\ R_2 & 0 & R_4 \\ 0 & V_2 & -(R_5+R_4) \end{vmatrix}$$

$$\Delta_3 = \begin{vmatrix} -(R_1+R_2) & R_2 & -V_1 \\ R_2 & (R_3+R_4+R_2) & 0 \\ 0 & R_4 & V_2 \end{vmatrix}$$

Now,

$$I_1 = \frac{\Delta_1}{\Delta},\ I_2 = \frac{\Delta_2}{\Delta},\ I_3 = \frac{\Delta_3}{\Delta}$$

1.12. Explain Superposition theorem

- The superposition theorem states that in any linear network containing two or more sources, the current in any element is equal to the algebraic sum of the current caused by individual sources acting alone, while the other sources are inoperative.

- According to the application of the superposition theorem. It may be noted that each independent source is considered at a time while all other sources are turned off or killed. To kill a voltage source means the voltage source is replaced by its internal resistance whereas to kill a current source means to replace the current source by its internal resistance.
- To consider the effects of each source independently requires that sources be removed and replaced without affecting the final result. To remove a voltage source when applying this theorem, the difference in potential between the terminals of the voltage source must be set to zero (short circuit) removing a current source requires that its terminals be opened (open circuit).
- Any internal resistance or conductance associated with the displaced sources is not eliminated but must still be considered.
- The total current through any portion of the network is equal to the algebraic sum of the currents produced independently by each source.
- That is, for a two-source network, if the current produced by one source is in one direction, while that produced by the other is in the opposite direction through the same resistor, the resulting current is the difference of the two and has the direction of the larger.
- If the individual currents are in the same direction, the resulting current is the sum of two in the direction of either current. This rule holds true for the voltage across a portion of a network as determined by polarities, and it can be extended to networks with any number of sources.
- The superposition principle is not applicable to power effects since the power loss in a resistor varies as the square (nonlinear) of the current or voltage.
- Steps to be followed to apply the superposition theorem:
    - ✓ Select any one energy source.
    - ✓ Replace all the other energy sources by their internal series resistances for voltage sources. Their internal shunt resistances for current sources.
    - ✓ With only one energy source calculate the voltage drops or branch currents paying attention to the voltage polarities and current directions.
    - ✓ Repeat steps 1, 2 and 3 for each source individually.
    - ✓ Add algebraically the voltage drops or branch currents obtained due to the individual source to obtain the combined effect of all the sources.
- **Example network:**

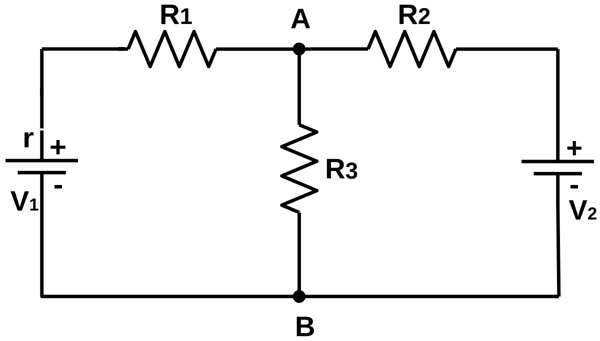


Figure 1.31Superposition theorem network

Step-1

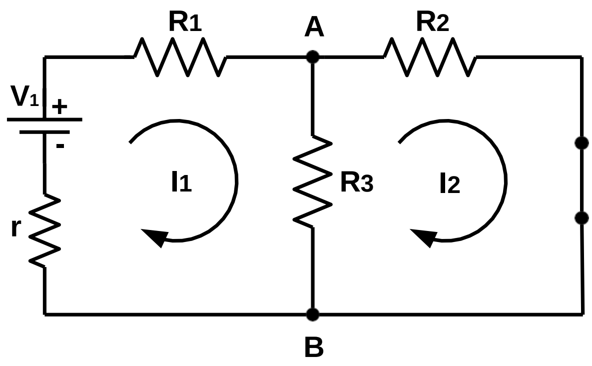


Figure 1.32Superposition theorem network for step-1

$Now\ apply\ Mesh\ analysis\ in\ loop-1,$

$-I_1R_1 - I_1R_3 + I_2R_3 - I_1r + V_1 = 0$

$Now\ apply\ Mesh\ analysis\ in\ loop-2,$

$-I_2R_2 - I_2R_3 + I_1R_3 = 0$

$Now, current\ flow\ from\ R_3\ branch\ is\ a\lg ebric\ sum\ of\ I_1\ and\ I_2$

Step-2

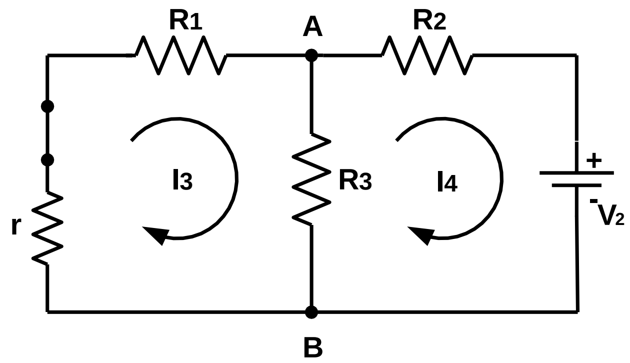


Figure 1.33Superposition theorem network for step-2

$Now\ apply\ Mesh\ analysis\ in\ loop-1,$

$-I_3R_1 - I_3R_3 + I_4R_3 - I_3r = 0$

$Now\ apply\ Mesh\ analysis\ in\ loop-2,$

$-I_4R_2 - V_2 - I_4R_3 + I_3R_3 = 0$

$Now, current\ flow\ from\ R_3\ branch\ is\ a\lg ebric\ sum\ of\ I_3\ and\ I_4$

$Finally,\ current\ flow\ from\ R_3\ is\ a\lg ebric\ sum\ of\ step-1\ and\ step-2$

1.13. Explain Thevenin's theorem

- Thevenin theorem is an analytical method used to change a complex circuit into a simple equivalent circuit consisting of a single resistance in series with a source voltage.
- Thevenin's can calculate the currents and voltages at any point in a circuit.
- Thevenin's Theorem states that "Any linear circuit containing several voltages and resistances can be replaced by just one single voltage in series with a single resistance

Connected across the load".

- In other words, it is possible to simplify any electrical circuit, no matter how complex, to an equivalent two-terminal circuit with just a single constant voltage source in series with a resistance (or impedance) connected to a load as shown below.
- Thevenin's Theorem is especially useful in the circuit analysis of power or battery systems and other interconnected resistive circuits where it will have an effect on the adjoining part of the circuit.
- **Thevenin's equivalent circuit**

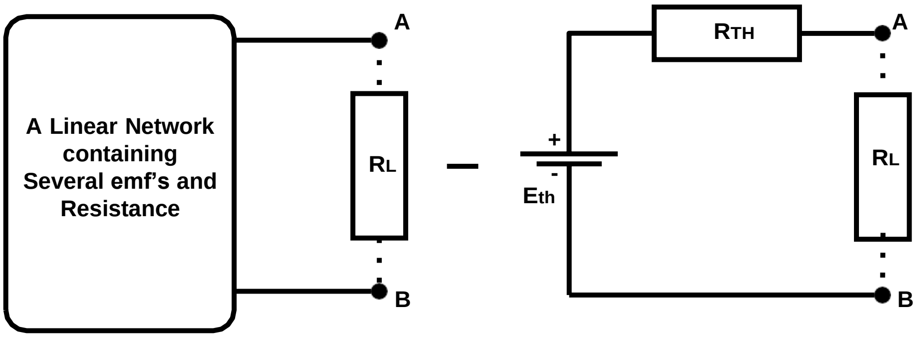


Figure 1.34Thevenin's equivalent circuit

- As far as the load resistor R_L is concerned, any complex "one-port" network consisting of multiple resistive circuit elements and energy sources can be replaced by one single equivalent resistance R_{th} and one single equivalent voltage E_{th}.
- R_{th} is the thevenin resistance value looking back into the circuit and E_{th} is the Thevenin's voltage (open circuit voltage) at the terminals.
- Steps to be followed to apply the Thevenin's theorem:
  - ✓ Remove the load resistor R_{th} or component concerned.
  - ✓ Find R_{th} by shorting all voltage sources or by open circuiting all the current sources.
  - ✓ Find E_{th} by the usual circuit analysis methods.
  - ✓ Find the current flowing through the load resistor R_{th}.
- **Example network:**

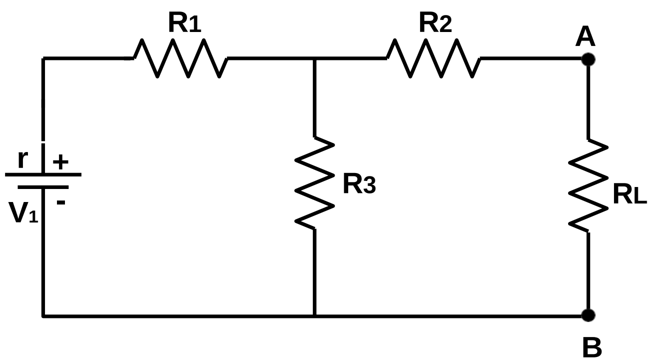


Figure 1.35Thevenin's theorem network

Step-1

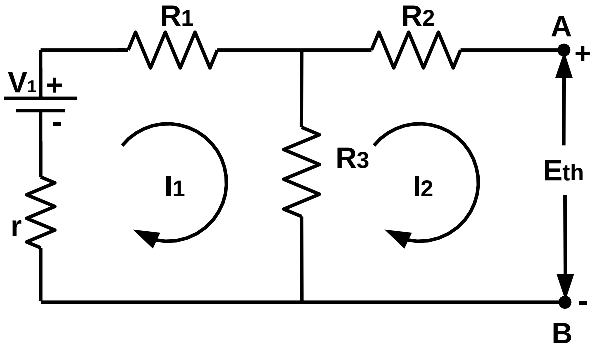


Figure 1.36Thevenin's theorem network (step-1)

Now apply Mesh analysis in loop-1,

$$-I_1R_1 - I_1R_3 + I_2R_3 - I_1r + V_1 = 0$$

Now apply Mesh analysis in loop-2,

$$-I_2R_2 - E_{th} - I_2R_3 + I_1R_3 = 0$$

Loop-2 *is open that's way* $I_2 = 0$,

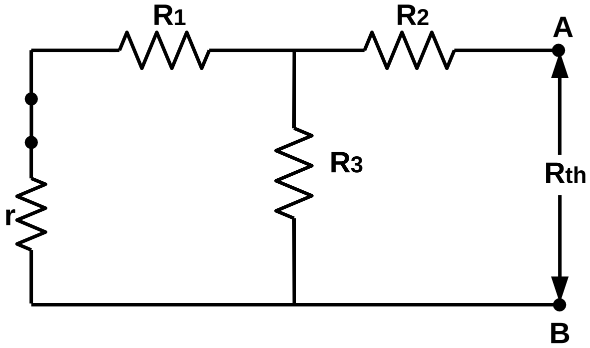
So, $E_{th} = I_1R_3$

$E_{th} =$ *Thevenin equivalent voltage*

$R_{th} =$ *Thevenin equivalent Resistance*

$R_L =$ *Load Resistance*

Step-2

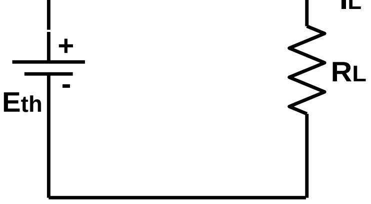


$$R_{th} = \left(\left(\left(r + R_1\right) \parallel R_3\right) + R_2\right)$$

$$R_{th} = \left(\left(\frac{(r + R_1) \times R_3}{(r + R_1) + R_3}\right) + R_2\right)$$

Figure 1.37Thevenin's theorem network (step-2)

Step-3



$$I_L = \frac{E_{th}}{R_{th} + R_L}$$

Figure 1.38Thevenin's theorem network (step-3)

1.14. Explain Norton's theorem

- Norton's theorem is an analytical method used to change a complex circuit into a simple equivalent circuit consisting of a single resistance in parallel with a current source.
- Norton's Theorem states that "Any linear circuit containing several energy sources and resistances can be replaced by a single Constant Current generator in parallel with a Single Resistor".
- As far as the load resistance, R_L is concerned this single resistance, R_N is the value of the resistance looking back into the network with all the current sources open circuited and I_N is the short circuit current at the output terminals as shown below.
- **Norton's equivalent circuit**

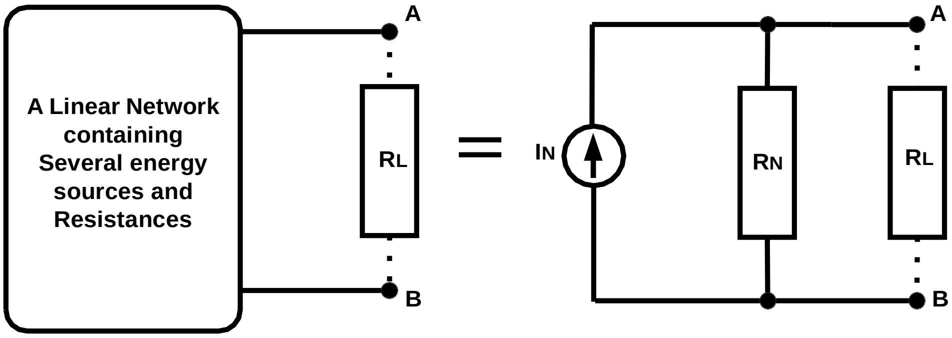


Figure 1.39Norton's theorem equivalent circuit

- The value of this "constant current" is one which would flow if the two output terminals where shorted together while the Norton's resistance would be measured looking back into the terminals.
- The basic procedure for solving a circuit using Norton's Theorem is as follows:
  - ✓ Remove the load resistor R_L or component concerned.
  - ✓ Find R_N by shorting all voltage sources or by open circuiting all the current sources.
  - ✓ Find I_N by placing a shorting link on the output terminals A and B.
  - ✓ Find the current flowing through the load resistor R_L.
- **Example network:**

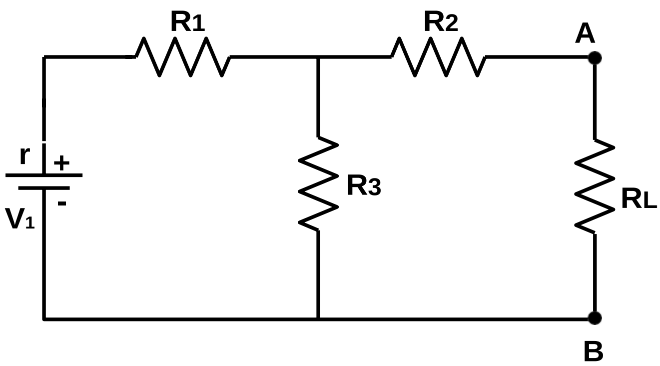


Figure 1.40Norton's theorem network

Step-1

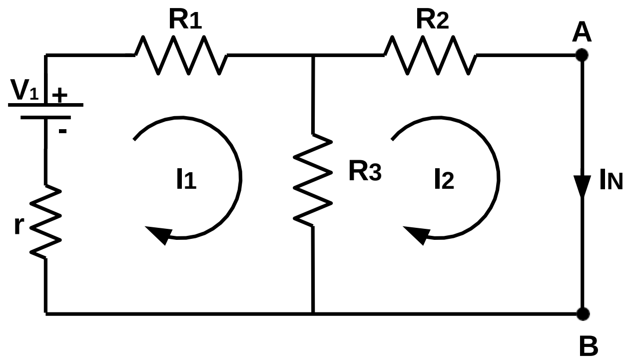


Figure 1.41 Norton's theorem network (step-1)

Now apply Mesh analysis in loop -1,

$$-I_1R_1 - I_1R_3 + I_2R_3 - I_1r + V_1 = 0$$

Now apply Mesh analysis in loop -2,

$$-I_2R_2 - I_2R_3 + I_1R_3 = 0$$

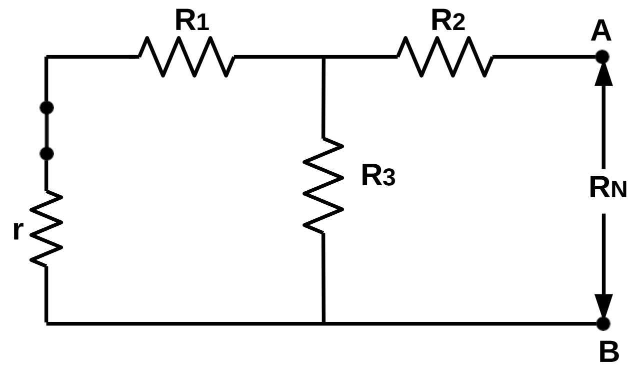
Here $I_2 = I_N$

$I_N =$ *Norton's equivalent current*

$R_N =$ *Norton's equivalent* Resistance

$R_L =$ *Load* Resistance

Step-2

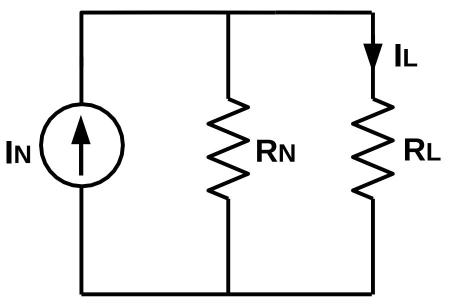


$$R_N = \left(\left(\left(r + R_1\right) \parallel R_3\right) + R_2\right)$$

$$R_N = \left(\left(\frac{\left(r + R_1\right) \times R_3}{\left(r + R_1\right) + R_3}\right) + R_2\right)$$

Figure 1.42 Norton's theorem network (step-2)

Step-3



$$I_L = I_N \frac{R_N}{R_N + R_L}$$

Figure 1.43 Norton's theorem network (step-3)

1.15. Time domain analysis of first order RC circuit

Charging of Capacitor

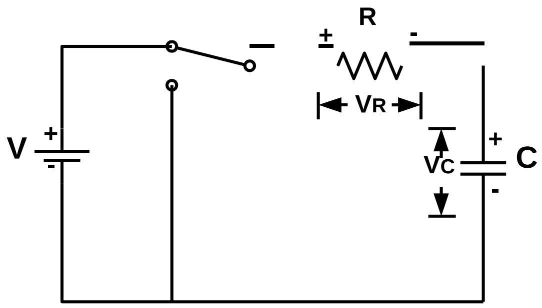


Figure 1.44Charging of capacitor

Apply KVL in circuit ,

$$V - V_R - V_c = 0$$

$$V = V_R + V_c$$

$$V = iR + V_c$$

$$V = \frac{dq}{dt} R + V_c$$

$$V = \frac{d(CV_c)}{dt} R + V_c$$

$$V = RC\frac{dV_c}{dt} + V_c$$

$$V - V_c = RC\frac{dV_c}{dt}$$

$$\int \frac{1}{V - V_c}\, dV_c = \int \frac{1}{RC} dt$$

Multiply minus sign both the side

$$\int \frac{-1}{V - V_c}\, dV_c = \int \frac{-1}{RC} dt$$

$$\log(V - V_c) = \frac{-t}{RC} + K \qquad (i)$$

When, $t = 0,\ V_c = 0$

$$\log(V) = K \qquad (ii)$$

Solve equation (i) *and* (ii)

$$\log(V - V_c) = \frac{-t}{RC} + \log(V)$$

$$\log(V - V_c) - \log(V) = \frac{-t}{RC}$$

$$\log\left(\frac{V - V_c}{V}\right) = \frac{-t}{RC}$$

$$\left(\frac{V - V_c}{V}\right) = e^{\frac{-t}{RC}}$$

$$1 - \left(\frac{V_c}{V}\right) = e^{\frac{-t}{RC}}$$

Discharging of Capacitor

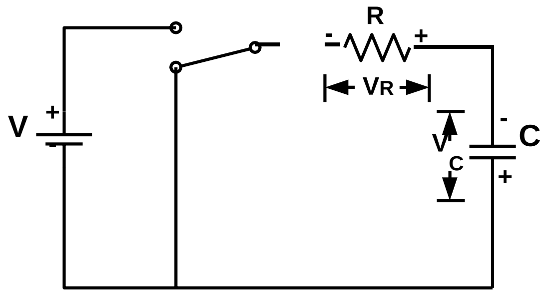


Figure 1.45Discharging of capacitor

Apply KVL in circuit ,

$$0 = V_R + V_c$$

$$0 = iR + V_c$$

$$0 = R\frac{dq}{dt} + V_c$$

$$0 = R\frac{d(CV_c)}{dt} + V_c$$

$$0 = RC\frac{dV_c}{dt} + V_c$$

$$V_c = -RC\frac{dV_c}{dt}$$

$$\int \frac{1}{V_c}\, dV_c = \int \frac{-1}{RC} dt$$

$$\log(V_c) = \frac{-t}{RC} + K \qquad (i)$$

When, $t = 0,\ V_c = V$

$$\log(V) = K \qquad (ii)$$

Solve equation (i) *and* (ii)

$$\log(V_c) = \frac{-t}{RC} + \log(V)$$

$$\log(V_c) - \log(V) = \frac{-t}{RC}$$

$$\log\left(\frac{V_c}{V}\right) = \frac{-t}{RC}$$

$$\left(\frac{V_c}{V}\right) = e^{\frac{-t}{RC}}$$

$$V_c = Ve^{\frac{-t}{RC}}$$

$$V_c = V(1 - e^{\frac{-t}{RC}})$$

Also, $i = \frac{dq}{dt}$

$$i = \frac{d(CV_c)}{dt}$$

$$i = C\frac{d}{dt}(V(1 - e^{\frac{-t}{RC}}))$$

$$i = VC\frac{d}{dt}(1 - e^{\frac{-t}{RC}})$$

$$i = VC\left(0 - \left(-\frac{1}{RC}\right)e^{\frac{-t}{RC}}\right)$$

$$i = \frac{VC}{RC}e^{\frac{-t}{RC}}$$

$$i = \frac{V}{R}e^{\frac{-t}{RC}}$$

$$i = i_m e^{\frac{-t}{RC}}$$

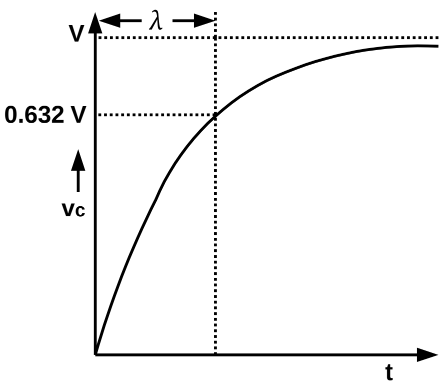


Figure 1.46Charging voltage of capacitor

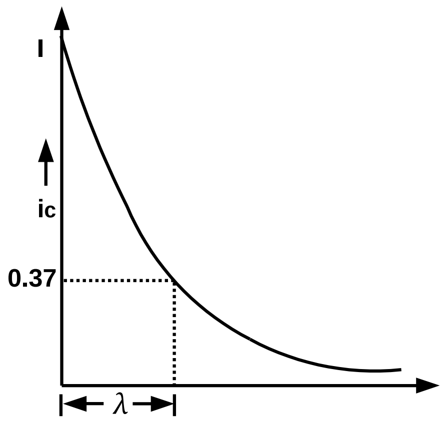


Figure 1.47Charging current of capacitor

Also,$i = \frac{dq}{dt}$

$$i = \frac{d(CV_c)}{dt}$$

$$i = C\frac{dV_c}{dt}$$

$$i = C\frac{d}{dt}(Ve^{\frac{-t}{RC}})$$

$$i = CV\frac{-1}{RC}e^{\frac{-t}{RC}}$$

$$i = -\frac{V}{R}e^{\frac{-t}{RC}}$$

$$i = -I_m e^{\frac{-t}{RC}}$$

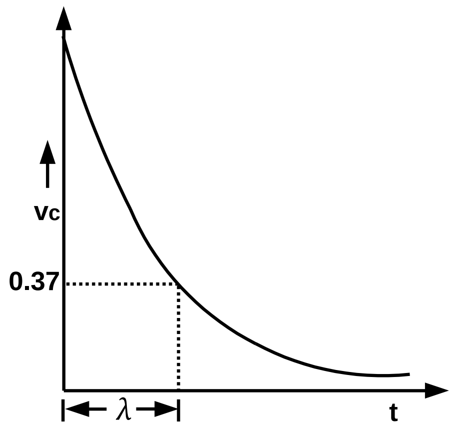


Figure 1.48Dicharging voltage of capacitor

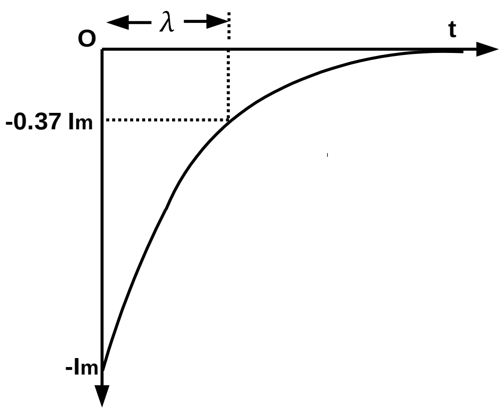


Figure 1.49Dicharging current of capacitor

1.16. Time domain analysis of first order RL circuit

Charging of Inductor

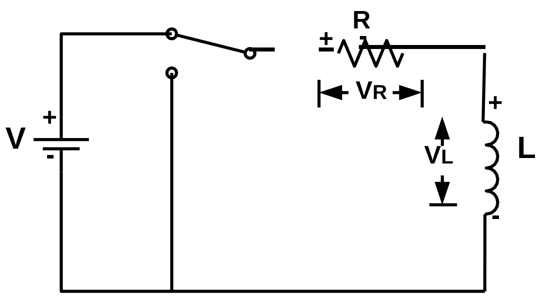


Figure 1.50Charging of inductor

From KVL,

$$V - iR - L\frac{di}{dt} = 0$$

$$\therefore V - iR = L\frac{di}{dt}$$

$$\therefore \frac{di}{V - iR} = \frac{dt}{L}$$

$$\therefore \int \frac{1}{V - iR}\, di = \frac{1}{L}\int dt$$

$$\therefore \int \frac{-R}{V - iR}\, di = \frac{-R}{L}\int dt$$

$$\therefore \log\left(V - iR\right) = \left(\frac{-R}{L}\right)t + K \qquad (i)$$

When, $t = 0,\ i = 0$

$$\log\left(V\right) = K \qquad (ii)$$

Solve equation (i) *and* (ii)

$$\therefore \log\left(V - iR\right) = \left(\frac{-R}{L}\right)t + \log\left(V\right)$$

$$\therefore \log\left(V - iR\right) - \log\left(V\right) = \left(\frac{-R}{L}\right)t$$

$$\therefore \log\left(\frac{V - iR}{V}\right) = \left(\frac{-R}{L}\right)t$$

$$\therefore \left(\frac{V - iR}{V}\right) = e^{\left(\frac{-R}{L}\right)t}$$

$$\therefore 1 - \left(\frac{R}{V}\right)i = e^{\left(\frac{-R}{L}\right)t}$$

$$\therefore i = \frac{V}{R}\left(1 - e^{\left(\frac{-R}{L}\right)t}\right)$$

$$\therefore i = I_m\left(1 - e^{\left(\frac{-R}{L}\right)t}\right)$$

$$\therefore i = I_m\left(1 - e^{-\lambda t}\right)$$

Discharging of Inductor

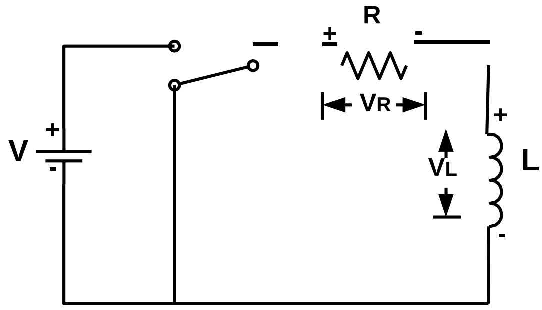


Figure 1.51Discharging of inductor

From KVL,

$$-iR - L\frac{di}{dt} = 0$$

$$\therefore -iR = L\frac{di}{dt}$$

$$\therefore \frac{di}{i} = \frac{-R}{L}di$$

$$\therefore \int \frac{1}{i}di = \frac{-R}{L}\int dt$$

$$\therefore \log\left(i\right) = \left(\frac{-R}{L}\right)t + K \qquad (i)$$

When, $t = 0,\ i = \frac{V}{R}$

$$\log\left(\frac{V}{R}\right) = K \qquad (ii)$$

Solve (i) *and* (ii)

$$\therefore \log\left(i\right) = \left(\frac{-R}{L}\right)t + \log\left(\frac{V}{R}\right)$$

$$\therefore \log\left(i\right) - \log\left(\frac{V}{R}\right) = \left(\frac{-R}{L}\right)t$$

$$\therefore \log\left(\frac{i}{V/R}\right) = \left(\frac{-R}{L}\right)t$$

$$\therefore \left(\frac{i}{V/R}\right) = e^{\left(\frac{-R}{L}\right)t}$$

$$\therefore i = \frac{V}{R}e^{\left(\frac{-R}{L}\right)t}$$

$$\therefore i = \frac{V}{R}e^{-\lambda t}$$

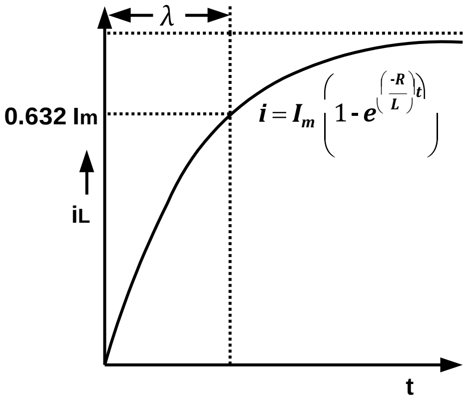


Figure 1.52Charging current of inductor

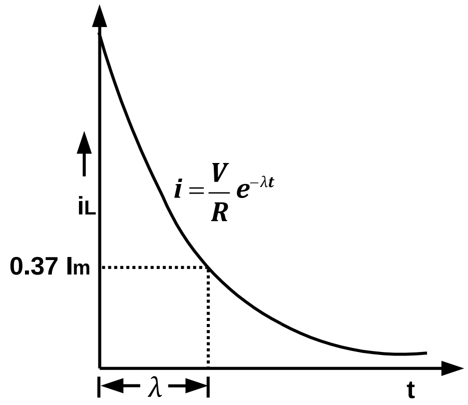


Figure 1.53Dicharging current of inductor