Factors of Safety and Reliability in Geotechnical Engineering

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Abstract: Simple reliability analyses, involving neither complex theory nor unfamiliar terms, can be used in routine geotechnical engineering practice. These simple reliability analyses require little effort beyond that involved in conventional geotechnical analyses. They provide a means of evaluating the combined effects of uncertainties in the parameters involved in the calculations, and they offer a useful supplement to conventional analyses. The additional parameters needed for the reliability analyses—standard deviations of the parameters—can be evaluated using the same amount of data and types of correlations that are widely used in geotechnical engineering practice. Example applications to stability and settlement problems illustrate the simplicity and practical usefulness of the method.

INTRODUCTION

The factors of safety used in conventional geotechnical practice are based on experience, which is logical. However, it is common to use the same value of factor of safety for a given type of application, such as long-term slope stability, without regard to the degree of uncertainty involved in its calculation. Through regulation or tradition, the same value of safety factor is often applied to conditions that involve widely varying degrees of uncertainty. This is not logical.

Reliability calculations provide a means of evaluating the combined effects of uncertainties, and a means of distinguishing between conditions where uncertainties are particularly high or low. In spite of the fact that it has potential value, reliability theory has not been much used in routine geotechnical practice. There are two reasons for this. First, reliability theory involves terms and concepts that are not familiar to most geotechnical engineers. Second, it is commonly perceived that using reliability theory would require more data, time, and effort than are available in most circumstances.

Christian et al. (1994), Tang et al. (1999), and others have described excellent examples of use of reliability in geotechnical engineering, and clear expositions of the underlying theories. The purpose of this paper is to show that reliability concepts can be applied in simple ways, without more data, time, or effort than are commonly available in geotechnical engineering practice. Working with the same quantity and types of data, and the same types of engineering judgments that are used in conventional analyses, it is possible to make approximate but useful evaluations of reliability.

The results of simple reliability analyses, of the type described in this paper, will be neither more nor less accurate than conventional deterministic analyses that use the same types of data, judgments, and approximations. While neither deterministic nor reliability analyses are precise, they both have value and each enhances the value of the other.

It is not advocated here that factor of safety analyses be abandoned in favor of reliability analyses. Instead, it is suggested that factor of safety and reliability be used together, as complementary measures of acceptable design. The simple types of reliability analyses described in this paper require only modest extra effort as compared to that required to evaluate factors of safety, but they will add considerable value to the results of the analyses.

EXAMPLE—RETAINING WALL STABILITY

A cantilever retaining wall is shown in Fig. 1. It will be backfilled with compacted silty sand, and the concrete footing will be cast on a layer of silty sand. The backfill will be drained to prevent buildup of water pressures behind the wall.

Factor of Safety against Sliding

The factor of safety against sliding on the sand layer beneath the footing is given by the following formula:

$$F_{ss} = \frac{W \tan \delta}{E} \tag{1}$$

in which W = weight of wall and backfill over the heel of the wall (lb/ft or kN/m); tan δ = tangent of friction angle between base of wall and sand; and E = earth pressure force on vertical plane through heel of wall (lb/ft or kN/m).

For the conditions shown in Fig. 1, the value of F_{SS} is 1.50, as shown at the top of Table 1. This is called the most likely value of factor of safety, F_{MLV}.

Uncertainty in Factor of Safety against Sliding

The terms involved in computing F_{ss} [W, tan δ, and E in (1)] all involve some degree of uncertainty. Therefore the computed value of F_{ss} also involves some uncertainty. It is useful

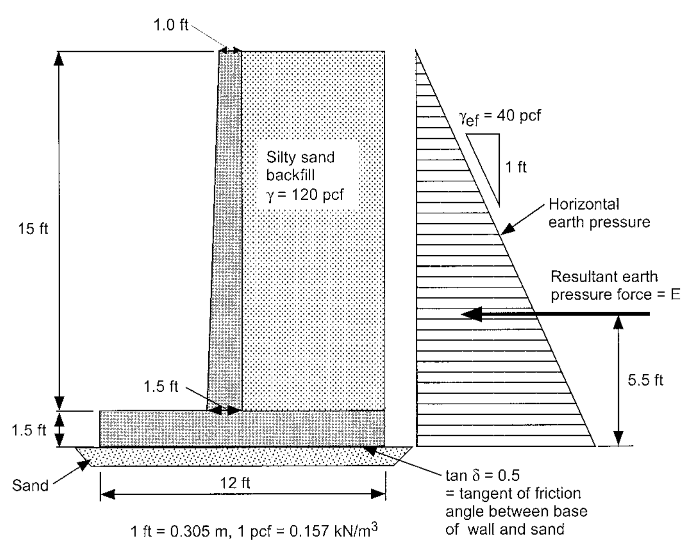

FIG. 1. Cantilever Retaining Wall with Silty Sand Backfill

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TABLE 1. Taylor Series Reliability Analysis for Retaining Wall (with All Variables Assigned Their Most Likely Values, $F_{ss} = 1.50$)

Variable (1)	Values (2)	Factors of safety (3)	ΔF (4)
Equivalent fluid unit weight, γ_{ef}			
Most likely value plus σ	45 pcf	$F^+ = 1.33$	
Most likely value minus σ	35 pcf	$F^- = 1.71$	-0.38
Tangent of δ			
Most likely value plus σ	0.55	$F^+ = 1.65$	
Most likely value minus σ	0.45	$F^- = 1.35$	0.30
Backfill unit weight, γ_{bf}			
Most likely value plus σ	127 pcf	$F^+ = 1.56$	
Most likely value minus σ	113 pcf	$F^- = 1.44$	0.12
Concrete unit weight, γ_c			
Most likely value plus σ	152 pcf	$F^+ = 1.50$	
Most likely value minus σ	148 pcf	$F^- = 1.49$	0.01

Note: 1 pcf = 0.157 kN/m^3
standard deviation of factor of safety

$$= \sqrt{(0.38/2)^2 + (0.30/2)^2 + (0.12/2)^2 + (0.01/2)^2} = 0.25;$$

coefficient of variation of factor of safety = 0.25/1.50 = 17%.

to be able to assess the reliability of F_{ss}, as well as a best estimate of its value. This can be done using the Taylor series method, which involves these steps:

1. Estimate the standard deviations of the quantities involved in (1). Simple methods for estimating standard deviations are discussed in the next section of the paper. Using those methods, the following values of standard deviation of the parameters involved in this example have been estimated: σ_{efp} = standard deviation of the equivalent fluid pressure = 5 pcf (0.785 kN/m^3); $\sigma_{\tan\delta}$ = standard deviation of tan δ = 0.05; $\sigma_{\gamma bf}$ = standard deviation of the unit weight of backfill = 7 pcf (1.099 kN/m^3); and $\sigma_{\gamma c}$ = standard deviation of the unit weight of concrete = 2 pcf (0.314 kN/m^3).
2. Use the Taylor series technique (Wolff 1994; U.S. Army Corps of Engineers 1997, 1998) to estimate the standard deviation and the coefficient of variation of the factor of safety using these formulas:

$$\sigma_F = \sqrt{\left(\frac{\Delta F_1}{2}\right)^2 + \left(\frac{\Delta F_2}{2}\right)^2 + \left(\frac{\Delta F_3}{2}\right)^2 + \left(\frac{\Delta F_4}{2}\right)^2} \quad (2a)$$

$$V_F = \frac{\sigma_F}{F_{MLV}} \quad (2b)$$

in which $\Delta F_1 = (F_1^+ - F_1^-)$. F_1^+ is the factor of safety calculated with the value of the first parameter (in this case the equivalent fluid pressure) increased by one standard deviation from its best estimate value. F_1^- is the factor of safety calculated with the value of the first parameter decreased by one standard deviation.

In calculating F_1^+ and F_1^-, the values of all of the other variables are kept at their most likely values.

The values of ΔF_2, ΔF_3, and ΔF_4 are calculated by varying the values of the other three variables (footing/sand friction angle, backfill unit weight, and concrete unit weight) by plus and minus one standard deviation from their most likely values. The results of these calculations are shown in Table 1.

F_{MLV} = most likely value of factor of safety, computed using best estimate values for all of the parameters. For this example, F_{MLV} = 1.50.

3. Substituting the value of ΔF into (2*a*), the value of the standard deviation of the factor of safety (σ_F) is found to be 0.25, and the coefficient of variation of the factor of safety (V_F), calculated using (2*b*), is found to be 17%, as shown at the bottom of Table 1.
4. With both F_{MLV} and V_F known, the probability of failure and the reliability of the factor of safety can be determined using Table 2 or one of the methods described in Appendix I. Table 2 assumes a lognormal distribution of factor of safety values, which is usually a reasonable approximation. There is no ''proof'' that factors of safety are lognormally distributed, but the writer believes that it is a reasonable approximation.

The assumption of a lognormal distribution for factor of safety does not imply that the values of the individual variables (γ_{ef}, tan δ, γ_{bf}, and γ_c) must be distributed in the same way. As discussed below, it is not necessary to make any particular assumption concerning the distri-

TABLE 2. Probabilities That Factor of Safety Is Smaller than 1.0, Based on Lognormal Distribution of Factor of Safety

	Coefficient of Variation of Factor of Safety (V_F)														
F_{MLV}	2%	4%	6%	8%	10%	12%	14%	16%	20%	25%	30%	40%	50%	60%	80%
1.05	0.8%	12%	22%	28%	33%	36%	39%	41%	44%	47%	49%	53%	55%	58%	61%
1.10	0.00%	0.9%	6%	12%	18%	23%	27%	30%	35%	40%	43%	48%	51%	54%	59%
1.15	0.00%	0.03%	1.1%	4%	9%	13%	18%	21%	27%	33%	37%	43%	48%	51%	56%
1.16	0.00%	0.01%	0.7%	3%	8%	12%	16%	20%	26%	32%	36%	42%	47%	50%	56%
1.18	0.00%	0.00%	0.3%	2%	5%	9%	13%	17%	23%	29%	34%	41%	45%	49%	55%
1.20	0.00%	0.00%	0.13%	1.2%	4%	7%	11%	14%	21%	27%	32%	39%	44%	48%	54%
1.25	0.00%	0.00%	0.01%	0.3%	1.4%	4%	6%	9%	15%	22%	27%	35%	41%	45%	51%
1.30	0.00%	0.00%	0.00%	0.06%	0.5%	1.6%	3%	6%	11%	17%	23%	31%	37%	42%	49%
1.35	0.00%	0.00%	0.00%	0.01%	0.2%	0.7%	1.9%	4%	8%	14%	19%	28%	34%	40%	47%
1.40	0.00%	0.00%	0.00%	0.00%	0.04%	0.3%	1.0%	2%	5%	11%	16%	25%	32%	37%	45%
1.50	0.00%	0.00%	0.00%	0.00%	0.00%	0.04%	0.2%	0.7%	3%	6%	11%	19%	27%	32%	41%
1.60	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.05%	0.2%	1.1%	4%	7%	15%	22%	28%	38%
1.70	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.06%	0.5%	2%	5%	12%	19%	25%	34%
1.80	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.2%	1.2%	3%	9%	16%	22%	31%
1.90	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.08%	0.65%	2%	7%	13%	19%	29%
2.00	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.03%	0.36%	1.3%	5%	11%	17%	26%
2.20	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.10%	0.56%	1.3%	8%	13%	22%
2.40	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.03%	0.23%	1.9%	5%	10%	19%
2.60	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.09%	1.1%	4%	7%	16%
2.80	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.04%	0.66%	3%	6%	13%
3.00	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.02%	0.39%	1.8%	4%	11%

Note: F_{MLV} = factor of safety computed using most likely values of parameters.

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butions of the variables to use the methods described here.

The retaining wall shown in Fig. 1 has a most likely value of factor of safety against sliding (F_{MLV}) equal to 1.50, and a coefficient of variation of factor of safety (V_F) equal to 17%. At the intersection of these values in Table 2, it can be seen that the probability of failure is about 1%. This equates to a reliability of about 99%.

Interpretation of ''Probability of Failure''

The event whose probability is described as the ''probability of failure'' is not necessarily a catastrophic failure. In the case of retaining wall sliding, for example, ''failure'' would not be catastrophic. If the wall slid a small distance away from the backfill, the earth pressure on the wall would decrease, and sliding would stop. Subsequently, if the earth pressure increased again because of backfill creep, another episode of sliding might ensue. Eventually, if repeated episodes of sliding resulted in significant displacement of the wall, this behavior could constitute unsatisfactory performance of the wall, but not catastrophic failure.

In recognition of this important distinction between catastrophic failure and less significant performance problems, the Corps of Engineers uses the term ''probability of unsatisfactory performance'' (U.S. Army Corps of Engineers 1998). Whatever terminology is used, it is important to keep in mind the real consequences of the event analyzed and not to be blinded by the word ''failure'' where the term ''probability of failure'' is used.

SUMMARY OF TAYLOR SERIES METHOD

The steps involved in using the Taylor series method are these:

1. Determine the most likely values of the parameters involved and compute the factor of safety by the normal (deterministic) method. This is F_{MLV}.
2. Estimate the standard deviations of the parameters that involve uncertainty, using the methods discussed later in this paper.
3. Compute the factor of safety with each parameter increased by one standard deviation and then decreased by one standard deviation from its most likely value, with the values of the other parameters equal to their most likely values. This involves $2N$ calculations, where N is the number of parameters whose values are being varied. These calculations result in N values of F^+ and N values of F^-. Using these values of F^+ and F^-, compute the values of ΔF for each parameter and compute the standard deviation of the factor of safety (σ_F) using (2*a*) and the coefficient of variation of the factor of safety (V_F) using (2*b*).
4. Use the value of F_{MLV} from the first step and the value of V_F from the third step to determine the value of P_f, by means of Table 2 or one of the methods given in the Appendix I. Table 2 was developed using an Excel spreadsheet, following the method described in the appendix.

Today, when virtually all calculations of factor of safety are performed using spreadsheets or other computer programs, the $2N$ calculations in step 3 require little extra effort and little additional engineering time. These calculations can be done about as quickly as new parameter values can be entered into a spreadsheet or data file.

The bulk of the analysis effort is required in evaluating the data for the first calculation, in step 1. Thus, although additional calculations must be performed, they involve little time and effort beyond estimating values of the standard deviations of the parameters. As discussed in the following section, values of standard deviation of geotechnical parameters can be estimated using available data and applying engineering judgment. The use of prudent and informed judgment is as important in estimating values of standard deviations of parameters as it is in estimating most likely values of parameters.

The great advantage of computing P_f (the probability that the factor of safety could be less than 1.0) is that it provides an overall measure of the uncertainty in the results of the analysis. Computing both F_{MLV} and P_f, adds little to the time and effort required for the analysis, but adds greatly to the value of the result.

In addition, the values of ΔF computed for the different parameters afford a measure of their contributions to the probability that the factor of safety could be less than 1.0. For example, in Table 1 it can be seen that the unit weight of concrete has little effect on the result, and that γ_{ef} and $\tan\delta$ have large effects. In this sense the Taylor's series method can be viewed as a structured sensitivity analysis or parametric study.

METHODS OF ESTIMATING STANDARD DEVIATION

An essential component of the art of geotechnical engineering is the ability to estimate reasonable values of parameters based on meager data, or based on correlations with results of in situ and index tests. In order to be able to estimate P_f, it is necessary to estimate the standard deviations of the parameters involved in computing the factor of safety. This can be done using the same types of judgment and experience used to estimate average values of parameters.

Depending on the amount of data available, various methods can be used to estimate the standard deviations of geotechnical parameters. Four methods that are applicable to various situations are described in the following paragraphs.

Computation from Data

If sufficient data are available, the formula definition of σ can be used to calculate its value:

$$\sigma = \sqrt{\frac{\sum [(x_i - \bar{x})^2]}{N - 1}} \tag{3}$$

in which σ = standard deviation; x_i = ith value of the parameter (x); $\bar{x}$ = average value of the parameter x; and N = number of values of x (the size of the sample).

Most scientific calculators and spreadsheet computer programs have facilities for calculating standard deviation using (3).

If the only method of determining values of standard deviation was (3), reliability analyses could not be used much in geotechnical engineering, because in most cases the amount of data is insufficient for use in (3). In order to be able to apply reliability analyses to the common situation, in which limited amounts of data are available and many properties are estimated using correlations, it is necessary to use other methods to estimate values of standard deviation. Three such methods are described in the following paragraphs.

Published Values

One approach to estimating values of standard deviation when sufficient data is not available to calculate σ using (3) is to use estimates based on published values, which are most conveniently expressed in terms of the coefficient of variation, V:

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$$V = \frac{\text{Standard deviation}}{\text{Average value}} = \frac{\sigma}{\bar{x}} \qquad (4a)$$

from which the standard deviation can be computed:

$$\sigma = (V)(\bar{x}) \qquad (4b)$$

Values of V for a number of geotechnical engineering parameters and in situ tests, compiled by the writer and by Harr (1984), Kulhawy (1992), and Lacasse and Nadim (1997), are listed in Table 3. While the values in Table 3 represent a considerable number of tests, the value of V quoted in various references cover extremely wide ranges of values for the same parameter, and the conditions of sampling and testing are not specified. The values compiled in Table 3 therefore provide only a rough guide for estimating values of V for any given case. It is important to use judgment in applying values of V from published sources, and to consider as well as possible the degree of uncertainty in the particular case at hand.

"Three-Sigma Rule"

This rule of thumb, described by Dai and Wang (1992), uses the fact that 99.73% of all values of a normally distributed parameter fall within three standard deviations of the average. Therefore, if HCV = highest conceivable value of the parameter, and LCV = lowest conceivable value of the parameter, these are approximately three standard deviations above and below the average value.

The Three-Sigma Rule can be used to estimate a value of standard deviation by first estimating the highest and the lowest conceivable values of the parameter and then dividing the difference between them by six:

$$\sigma = \frac{HCV - LCV}{6} \qquad (5)$$

in which HCV = highest conceivable value of the parameter, and LCV = lowest conceivable value of the parameter.

For example, the value of standard deviation of equivalent fluid unit weight (γ_{ef}) can be estimated as follows. First, the most likely value of γ_{ef} is estimated using experience, tables, or charts of the type found in Terzaghi et al. (1996). As shown in Fig. 1, the writer has estimated γ_{ef} = 40 pcf (6.28 kN/m^3) as the most likely value of γ_{ef} for the silty sand backfill. It seems reasonable that the highest conceivable value of γ_{ef} for this backfill might be about 55 pcf (8.635 kN/m^3), and the lowest conceivable value might be about 25 pcf (3.925 kN/m^3). These values are based on judgment. With HCV = 55 pcf, and LCV = 25 pcf, the standard deviation of γ_{ef} is computed as σ_{ef} = (55-25)/6 = 5 pcf (0.785 kN/m^3).

Studies have shown that there is a tendency to estimate a range of values between HCV and LCV that is too small. One such study, described by Folayen et al. (1970), involved asking a number of geotechnical engineers to estimate the possible range of values of $C_c/(1 + e)$ for San Francisco Bay mud, with which they all had experience. The results of this exercise are summarized in Table 4. It can be seen that, on average, these experienced engineers were able to estimate the value of $C_c/(1 + e)$ reasonably accurately but that they underestimated the standard deviation by about a factor of two as compared with the results of 45 laboratory tests.

The writer believes that the tendency to underestimate coefficients of variation results mainly from the fact that, while most geotechnical engineers have honed their ability to estimate average values of soil properties, they have had little experience in estimating coefficients of variation. With practice and experience, it should be possible to estimate coefficients of variation as accurately as average values of properties. When using the 3σ rule to estimate standard deviations and coefficients of variation, a conscious effort should be made to make the range between HCV and LCV as wide as seemingly possible, or even wider, to overcome the natural tendency to make the range too small.

With the 3σ rule it is possible to estimate values of standard deviation using the same amounts and types of data that are used for conventional geotechnical analyses. The three-sigma rule can be applied when only limited data are available and when no data is available. It can also be used to judge the reasonableness of values of coefficients of variation from published sources, considering that the lowest conceivable value would be three standard deviations below the mean and the highest conceivable value would be three standard deviations above the mean. If these values seem unreasonable some adjustment of the values is called for.

The 3σ rule uses the simple normal distribution as a basis for estimating that a range of three standard deviations covers virtually the entire population. However, the same is true of

TABLE 3. Values of Coefficient of Variation (V) for Geotechnical Properties and In Situ Tests

Property or in situ test result (1)	Coefficient of variation—V (%) (2)	Source (3)
Unit weight (γ)	3–7%	Harr (1984), Kulhawy (1992)
Buoyant unit weight (γ_b)	0–10%	Lacasse and Nadim (1997), Duncan (2000)[a]
Effective stress friction angle (ϕ')	2–13%	Harr (1984), Kulhawy (1992)
Undrained shear strength (S_u)	13–40%	Harr (1984), Kulhawy (1992), Lacasse and Nadim (1997), Duncan (2000)[a]
Undrained strength ratio (S_u/σ'_v)	5–15%	Lacasse and Nadim (1997), Duncan (2000)[a]
Compression index (C_c)	10–37%	Harr (1984), Kulhawy (1992), Duncan (2000)[a]
Preconsolidation pressure (p_p)	10–35%	Harr (1984), Lacasse and Nadim (1997), Duncan (2000)[a]
Coefficient of permeability of saturated clay (k)	68–90%	Harr (1984), Duncan (2000)[a]
Coefficient of permeability of partly saturated clay (k)	130–240%	Harr (1984), Benson et al. (1999)
Coefficient of consolidation (c_v)	33–68%	Duncan (2000)[a]
Standard penetration test blow count (N)	15–45%	Harr (1984), Kulhawy (1992)
Electric cone penetration test (q_c)	5–15%	Kulhawy (1992)
Mechanical cone penetration test (q_c)	15–37%	Harr (1984), Kulhawy (1992)
Dilatometer test tip resistance (q_{DMT})	5–15%	Kulhawy (1992)
Vane shear test undrained strength (S_v)	10–20%	Kulhawy (1992)

[a]Duncan (2000) refers to the present paper.

TABLE 4. Estimated and Measured Values of $C_c/(1 + e)$ and Its Coefficient of Variation, for San Francisco Bay Mud

Estimated by (1)	Estimated $C_c/(1 + e)$ (2)	Estimated V (3)
Geotechnical engineer #1	0.30	10%
Geotechnical engineer #2	0.275	5%
Geotechnical engineer #3	0.275	5.5%
Geotechnical engineer #4	0.30	10%
Average of #1–#4	0.29	8%
Measured	0.34	18%

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other distributions (Harr 1987), and the 3σ rule is not rigidly tied to an assumed distribution of the variable.

Graphical Three-Sigma Rule

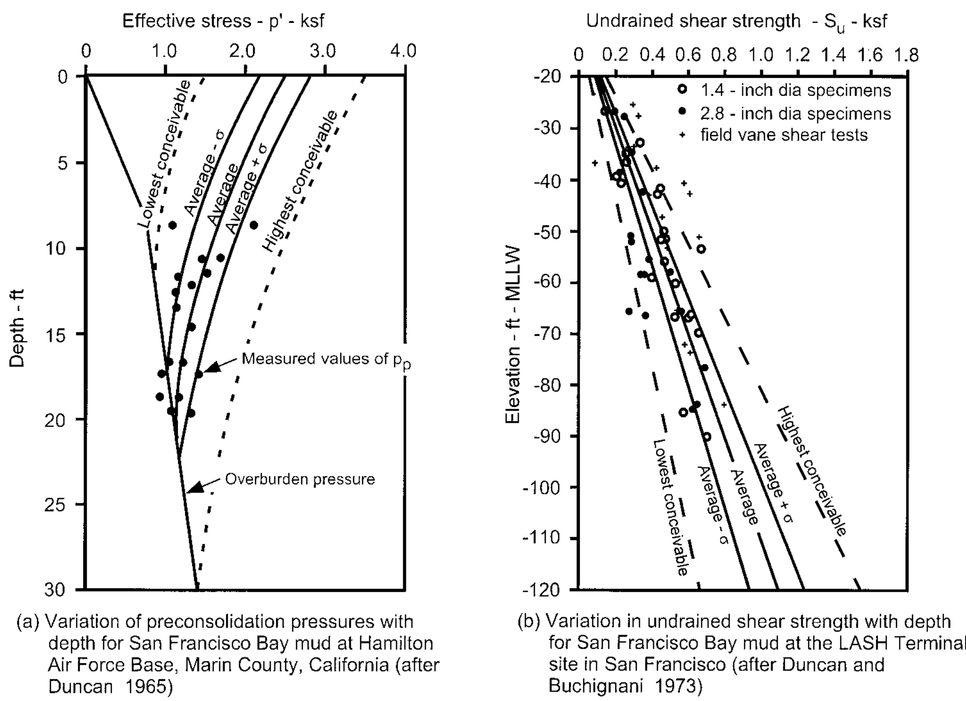
The concept behind the three-sigma rule of Dai and Wang (1992) can be extended to a graphical procedure that is applicable to many situations in geotechnical engineering, where the parameter of interest, such as preconsolidation pressure or undrained shear strength, varies with depth. Examples are shown in Fig. 2.

The graphical three-sigma rule is applied as follows:

1. Draw a straight line or curve through the data that represents the most likely average variation of the parameter with depth.
2. Draw straight lines or curves that represent the highest and lowest conceivable bounds on the data. These should be wide enough to include all valid data and an allowance for the fact that the natural tendency is to estimate such bounds too narrowly, as discussed previously. Note that some points in Fig. 2(b) are outside the estimated highest and lowest conceivable, indicating that these data points are believed to be erroneous.
3. Draw straight lines or curves that represent the average-plus-one standard deviation and the average-minus-one standard deviation. These are one-third of the distance from the average line to the highest and lowest conceivable bounds.

The average-plus-one-sigma and average-minus-one-sigma curves or lines, such as the preconsolidation pressure and undrained strength profiles in Fig. 2, are used in the Taylor series method in the same way as are parameters that can be represented by single values.

This same concept is useful in characterizing strength envelopes for soils. In this case the quantity (shear strength) var-

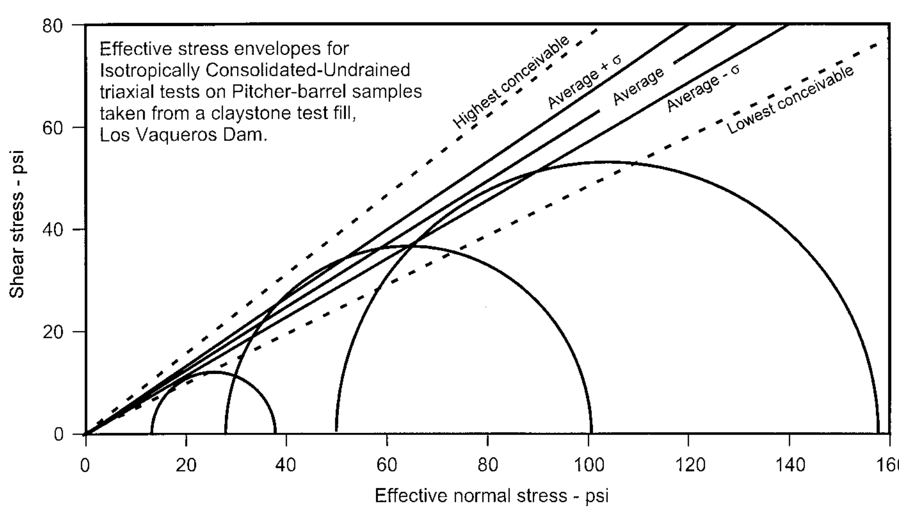


(a) Variation of preconsolidation pressures with depth for San Francisco Bay mud at Hamilton Air Force Base, Marin County, California (after Duncan 1965)

(b) Variation in undrained shear strength with depth for San Francisco Bay mud at the LASH Terminal site in San Francisco (after Duncan and Buchignani 1973)

1 ft = 0.305 m, 1 ksf = 47.9 kPa, 1 inch = 25.4 mm

FIG. 2. Examples of "Graphical Three-Sigma Rule" for Estimating Standard Deviation Limits for Parameters That Vary with Depth



1 psi = 6.90 kPa

FIG. 3. "Graphical Three-Sigma Rule" for Estimating Standard Deviation Limits for Strength Envelope

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ies with normal stress rather than depth, but the procedure is the same. Strength envelopes are drawn that represent the average and the highest and lowest conceivable bounds on the data, as shown in Fig. 3. Then average-plus-one-sigma and average-minus-one-sigma envelopes are drawn one-third of the distance from the average envelope to the highest and lowest conceivable bounds. The average-plus-one-sigma envelope is used to compute the value of F^+, and the average-minus-one-sigma envelope is used to compute the value of F^-.

Using the graphical three-sigma rule to establish average-plus-one-sigma and average-minus-one-sigma strength envelopes is preferable to using separate standard deviations for the strength parameters c and ϕ. Strength parameters (c and ϕ) are useful empirical coefficients that characterize the variation of shear strength with normal stress, but they are not of fundamental significance or interest by themselves. The important parameter is shear strength, and the graphical three-sigma rule provides a straightforward means for characterizing the uncertainty in shear strength.

EXAMPLE—A SLOPE THAT FAILED

In August 1970, during construction of a new Lighter Aboard Ship (LASH) terminal at the Port of San Francisco, a 250-ft (75 m)-long portion of an underwater slope about 100 ft (30 m) high failed (Duncan and Buchignani 1973). A cross section through the failed area is shown in Fig. 4. The trench, which failed as it was being excavated, was to be filled with sand to provide a stability berm for the LASH Terminal.

The failure took place entirely within San Francisco Bay mud, a normally consolidated, slightly organic clayey silt or silty clay of marine origin. The clay at the site has moderate plasticity, with a Liquid Limit of about 50 and a Plastic Limit of about 30. The undrained shear strength of the Bay mud was measured using unconsolidated-undrained (UU) triaxial compression tests and in situ vane shear tests, the results of which are shown in Fig. 2(b).

Previous experience in the San Francisco Bay area indicated that underwater slopes in Bay mud could be excavated at least as steep as 1(H) on 1(V). Slope stability analyses performed during design of the LASH Terminal showed that, with the average strength profile shown in Fig. 2(b), the factor of safety of a 1(H) on 1(V) slope would have been 1.25.

Using steeper trench slopes would reduce the volume of excavation and fill and would reduce costs. It was estimated that, if the trench slopes could be excavated at 0.875(H) on 1(V), the cost of the stability trench would be reduced by about $200,000. Considerable effort was devoted to evaluating the undrained strength of the Bay mud, and the stability of the trench slope, as accurately as possible. Using the average strength profile shown in Fig. 2(b), it was found that the factor of safety of the slopes would be 1.17 if they were excavated at 0.875(H) on 1(V). Because the analyses were based on a considerable amount of high-quality data, it was decided to excavate the slopes at 0.875(H0 on 1(V), as shown in Fig. 4.

On August 20, 1970, after a section of the trench about 500 ft (150 m) long had been excavated, the dredge operator found that the clamshell bucket could not be lowered to the depth from which mud had been excavated only hours before. Using the side-scanning sonar with which the dredge was equipped, four cross sections were made within 2 h, which showed that a failure had occurred that involved a 250-ft (75 m) -long section of the trench. The cross section is shown in Fig. 4. Later a second failure occurred, involving an additional 200 ft (60 m) of length along the trench. The rest of the 2,000-ft (600 m) -long trench slope remained stable for about four months, at which time the trench was backfilled with sand.

The cost of excavating the mud that slid into the trench, plus the cost of extra sand backfill, was approximately the same as the savings resulting from the use of steeper slopes. Given the fact that the expected savings were not realized, that the failure caused great alarm among all concerned, and that the confidence of the owner was diminished as a result of the failure, it is now clear that using 0.875(H) on 1(V) slopes was not a good idea.

A detailed investigation after the failure indicated that the undrained strength of the Bay mud decreased, due to creep strength loss, to values smaller than measured in the laboratory UU tests and field vane shear tests, which were performed at normal rates of shearing and are quite rapid compared to rates of shearing in the field (Duncan and Buchignani 1973).

Reliability analyses of the slope have been performed recently using the Taylor series method, with the undrained shear strengths shown in Fig. 2(b). The results of this analysis are shown in Table 5. The greatest contributor to the standard deviation of the factor of safety (the largest value of ΔF) is the undrained shear strength of the Bay mud. However, the

TABLE 5. Taylor Series Reliability Analysis for LASH Terminal Cut Slope (with All Variables Assigned Their Most Likely Values, F_{ss} = 1.17)

Variable (1)	Values (2)	Factors of safety (3)	ΔF (4)
Bay mud undrained strength			
Most likely value plus σ	see Fig. 2	$F^+ = 1.33$	
Most likely value minus σ	see Fig. 2	$F^- = 1.02$	0.31
Bay mud buoyant unit weight			
Most likely value plus σ	41.3 pcf	$F^+ = 1.08$	
Most likely value minus σ	34.7 pcf	$F^- = 1.28$	0.20

Note:

standard deviation of factor of safety

$$= \sqrt{(0.31/2)^2 + (0.20/2)^2} = 0.18;$$

coefficient of variation of factor of safety = 0.18/1.17 = 16%.

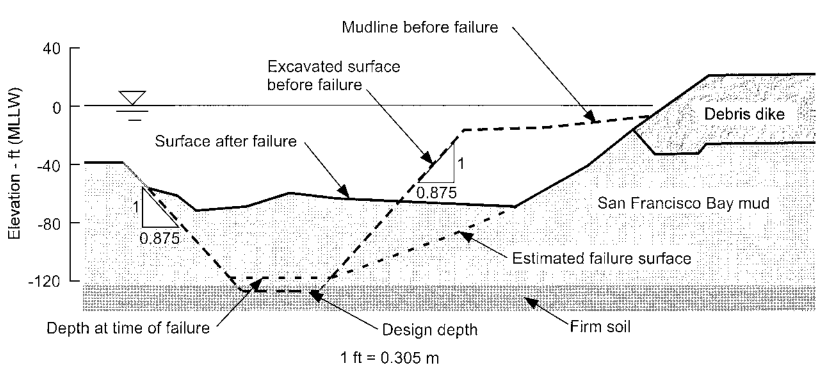

FIG. 4. Cross Section through Excavated Trench at LASH Terminal after Failure

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buoyant unit weight of the Bay mud is also a significant factor. Although the standard deviation of the buoyant unit weight is only 3.3 pcf (0.518 kN/m^3), the average buoyant unit weight is only 38 pcf (5.97 kN/m^3) and the coefficient of variation is 9%. Coefficients of variation for buoyant unit weight are larger than for moist unit weight, because values of buoyant unit weight are smaller.

With a most likely value of factor of safety equal to 1.17 and a coefficient of variation of factor of safety equal to 0.16, as shown in Table 5, the probability of failure, found from Table 2, is 18%. The two sections that failed involved about 450 ft (140 m), or about 22% of the total 2,000 ft (600 m) length of cut slope. This close agreement between the computed probability of failure and the percentage of the slope that failed is probably fortuitous.

In retrospect, it is clear that a factor of safety equal to 1.17 was too low and a probability of failure equal to 18% was too high to be acceptable. At the time the slope was designed, however, we believed that $F = 1.17$ provided a real margin of safety. The failure showed that the real margin of safety was zero in some parts of the slope, and it must have been extremely small in the rest.

A reliability analysis was not performed when the slope was designed, and it is not possible to say 29 years later whether or not a calculated probability of failure of 18% would have altered our views about the advisability of excavating the slope at 0.875(H) on 1(V). However, it seems likely that knowing the probability of failure was 18% would have caused us to consider the steep slope to be less stable than we thought when we designed it, and we might well have changed the design had we evaluated it from the point of view of reliability. It is safe to say that our subjective perceptions at the time the slope was designed would have been that the probability of failure was considerably smaller than 18%.

EXAMPLE—CONSOLIDATION SETTLEMENT

Reliability theory can also be used to evaluate the effects of uncertainties in settlement calculations. As an example, consider the conditions shown in Fig. 5, where a 30-ft (9 m) -thick layer of San Francisco Bay mud is loaded by four feet of fill, imposing a surcharge of 500 psf (24 kPa). The overburden pressures and preconsolidation pressures for the layer are shown in Fig. 2(a).

For the conditions shown in Fig. 5, the computed most likely ultimate settlement is 1.07 ft (0.326 m). The results of a Taylor series reliability analysis of the settlement are shown in Table 6. Varying p_p, C_c, and C_r by plus and minus one standard deviation leads to the values of S^+ and S^- shown in Table 6. Although the coefficient of consolidation c_v also varies, it has no effect on the ultimate settlement. Using the values

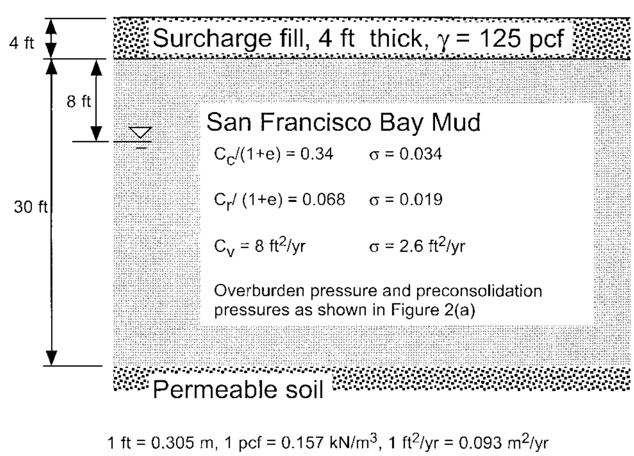

FIG. 5. Consolidation Settlement Example

TABLE 6. Taylor Series Reliability Analysis for Ultimate Consolidation Settlement (with All Variables Assigned Their Most Likely Values, S = 1.07 ft)

Variable (1)	Values (2)	Settlement (3)	ΔS (4)
Preconsolidation pressure			
Most likely value plus σ	see Fig. 2	S^+ = 0.90 ft	
Most likely value minus σ	see Fig. 2	S^- = 1.21 ft	0.31 ft
$C_c/(1+e)$			
Most likely value plus σ	0.374	S^+ = 1.18 ft	
Most likely value minus σ	0.306	S^- = 0.94 ft	0.24 ft
$C_r/(1+e)$			
Most likely value plus σ	0.087	S^+ = 1.17 ft	
Most likely value minus σ	0.049	S^- = 0.97 ft	0.20 ft

Note: 1 ft = 0.305 m;

standard deviation of ultimate settlement

$= \sqrt{(0.31\ \text{ft}/2)^2 + (0.24\ \text{ft}/2)^2 + (0.20\ \text{ft}/2)^2} = 0.22$ ft;

coefficient of variation of ultimate settlement = 0.22 ft/1.07 ft = 21%.

of ΔS shown in Table 6, the standard deviation of ultimate settlement is 0.22 ft (0.067 m), and the coefficient of variation of ultimate settlement is 21%.

The probability that the settlement will be larger than some factor times the computed mostly likely settlement can be determined using Table 7. This table shows probabilities that the settlement ratio, SR, will be larger than the values listed in the left-hand column. SR is defined as

$$SR = \frac{\text{Possible settlement}}{\text{Most likely settlement}} \quad (6)$$

Table 7 provides a means of estimating an effective upper limit on settlement, based on the most likely settlement and the coefficient of variation. By choosing a small probability, the corresponding settlement ratio can be found in Table 7, and the possible settlement can be computed using the formula

$$\text{Possible settlement} = (SR) \times (\text{Most likely settlement}) \quad (7)$$

For example, Table 7 can be used to determine the settlement corresponding to a 1% probability of occurrence. With a coefficient of variation of settlement equal to 21%, the value of SR corresponding to 1% probability is about 1.6. In other words, there is a 1% chance that the ultimate settlement could be larger than (1.6) × (1.07 ft) = 1.7 ft, or, in metric units, the settlement would be larger than (1.6) × (0.326 m) = 0.522 m. Thus, 1.7 ft (or 0.522 m) can be viewed as an effective upper limit on settlement (with only 1% chance of being exceeded), considering possible variations in p_p, C_c, and C_r.

The same procedure can be used to determine the possible settlement at any time. For the conditions shown in Fig. 5, the computed most likely settlement at t = 2 years is 0.59 ft (0.18 m). A Taylor series reliability analysis of the settlement at 2 years is shown in Table 8. In this case four variables (p_p, C_c, and C_r, and c_v) influence the standard deviation and coefficient of variation of settlement. The standard deviation of settlement at two years is 0.12 ft (0.037 m), and the coefficient of variation is 21%.

As noted previously, a coefficient of variation equal to 21%, together with a 1% probability of being exceeded, corresponds to SR = 1.6. Thus the two-year settlement could possibly be as large as (1.6) × (0.59 ft) = 0.94 ft, or (1.6) × (0.18 m) = 0.288 m, with a probability of 1%. Thus 0.94 ft (0.288 m) can be viewed as an effective upper limit on settlement at two years (with a probability of 1%), considering variations in p_p, C_c, and C_r, and c_v.

Although in this case the coefficient of variation of settlement is equal to 21% for both the two-year settlement and the

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TABLE 7. Probabilities That Settlement May Be Larger Than Computed Most Likely Settlement, Based on Lognormal Distribution of Settlement

	Coefficient of Variation of Settlement (V_s)											
SR	5%	10%	15%	20%	25%	30%	40%	50%	60%	**67%**[a]	70%	80%
1.10	3%	16%	24%	28%	30%	32%	33%	33%	33%	32%	32%	31%
1.20	0%	3%	10%	15%	19%	22%	25%	27%	27%	27%	27%	27%
1.30	0%	0%	3%	8%	12%	15%	19%	21%	23%	23%	23%	23%
1.40	0%	0%	1%	4%	7%	10%	14%	17%	19%	20%	20%	20%
1.50	0%	0%	0%	2%	4%	6%	11%	14%	16%	17%	17%	18%
1.60	0%	0%	0%	1%	2%	4%	8%	11%	13%	14%	14%	15%
1.70	0%	0%	0%	0%	1%	3%	6%	9%	11%	12%	12%	13%
1.80	0%	0%	0%	0%	1%	2%	4%	7%	9%	10%	11%	12%
1.90	0%	0%	0%	0%	0%	1%	3%	6%	8%	9%	9%	10%
2.00	0%	0%	0%	0%	0%	1%	2%	4%	6%	7%	8%	9%
2.20	0%	0%	0%	0%	0%	0%	1%	3%	4%	5%	6%	7%
2.50	0%	0%	0%	0%	0%	0%	1%	1%	3%	4%	4%	5%
3.00	0%	0%	0%	0%	0%	0%	0%	1%	1%	2%[a]	2%	3%

Note: *SR* = Settlement Ratio = Possible Settlement/Most Likely Settlement.
[a]Settlement of foundations on sand and gravel, computed using the method of Burland and Burbridge (1985), or Terzaghi et al. (1996), has a coefficient of variation of 67%.

TABLE 8. Taylor Series Reliability Analysis for Consolidation Settlement at t = 2 years (with All Variables Assigned Their Most Likely Values, S = 0.59 ft)

Variable (1)	Values (2)	Settlement (3)	ΔS (ft) (4)
Preconsolidation pressure			
Most likely value plus σ	see Fig. 2(a)	$S^+ = 0.58$ ft	
Most likely value minus σ	see Fig. 2(a)	$S^- = 0.60$ ft	0.02 ft
$C_c/(1+e)$			
Most likely value plus σ	0.378	$S^+ = 0.65$ ft	
Most likely value minus σ	0.310	$S^- = 0.52$ ft	0.13 ft
$C_r/(1+e)$			
Most likely value plus σ	0.087	$S^+ = 0.67$ ft	
Most likely value minus σ	0.049	$S^- = 0.50$ ft	0.17 ft
C_V (ft²/year)			
Most likely value plus σ	10.6 ft²/year	$S^+ = 0.64$ ft	
Most likely value minus σ	5.4 ft²/year	$S^- = 0.52$ ft	0.12 ft

Note: 1 ft = 0.305 m, 1 ft²/year = 0.093 m²/year;

standard deviation of settlement at 2 years

$$= \sqrt{(0.02\text{ ft}/2)^2 + (0.13\text{ ft}/2)^2 + (0.17\text{ ft}/2)^2 + (0.12\text{ ft}/2)^2} = 0.12\text{ ft};$$

coefficient of variation of settlement at 2 years = 0.12 ft/0.59 ft = 21%.

ultimate settlement, this will not always be the case. The settlements during consolidation are influenced by c_v but the ultimate settlement is not. Therefore the coefficient of variation for during-consolidation and ultimate settlements will, in general, not be the same.

EXAMPLE—SETTLEMENT OF FOOTINGS ON SAND

Settlements of footings on sand can be estimated based on standard penetration blow count using the method developed by Burland and Burbridge (1985), which has been described in Terzaghi et al. (1996). Settlements are estimated using the formula

$$S_c = B^{0.75}\frac{1.7}{(\bar{N}_{60})^{1.4}}\left(q - \frac{2}{3}\sigma'_{vo}\right) \tag{8}$$

in which S_c = computed settlement (immediate) in mm; B = footing width in meters; q = bearing pressure in kPa; σ'_{vo} = initial effective vertical stress at base of footing level in kPa; and $\bar{N}_{60}$ = average SPT blow count within a depth of $B^{0.75}$ m beneath the footing, corrected to 60% of the theoretical hammer energy (Terzaghi et al. 1996).

The relationship between settlements calculated using this

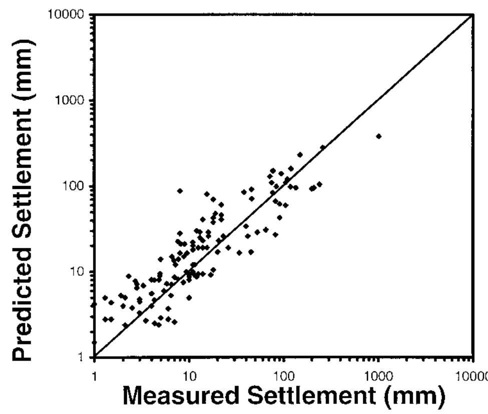

FIG. 6. Comparison of Measured Settlements at End of Construction with Settlements Predicted by Eq. (8)

formula and the measured settlements of 124 footings is shown in Fig. 6. It can be seen that the points scatter about the line of equality, indicating that (8) overestimates settlements in some cases and underestimates them in others.

Of greatest practical interest is the possibility that the actual settlement may be larger than the settlement computed using (8). The coefficient of variation associated with values computed from (8) was determined by computing the standard deviation of the data points around the $S_m = S_c$ line. Footings with measured settlements less than half an inch (13 mm) were not included in the analysis, because it was considered that settlements smaller than half an inch are usually of little practical significance. There were 54 footings with measured settlements greater than 13 mm. It was found that the expression $S_m = S_c$ has a coefficient of variation of 67% in these cases. Thus the column in Table 7 corresponding to V = 67% can be used to estimate how large the settlement might possibly be, based on a value calculated using (8).

As an example, consider the case of an 8 ft- (2.44 m)-square footing carrying a load of 320 kips (1,424 kN), founded 4 ft (1.22 m) below the surface at a site where $\bar{N}_{60}$ = 25. The bearing pressure, q = 320/64 = 5 ksf (240 kPa), and the initial effective vertical pressure, σ'_{vo} = 500 psf (24 kPa). The settlement computed using (8) is S_c = 8 mm, or 0.3 in.

Using the column in Table 7 for V = 67%, it can be seen that a probability of 2% corresponds to SR = 3.0. There is therefore a 2% chance that the settlement could be as large as (3.0) × (0.3 in.) = 0.9 in. Thus, with a probability of 2%, 0.9 in. can be viewed as an upper limit on the settlement of the footing.

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Applying reliability concepts in this way provides a means of assessing the effects of the uncertainty involved in using (8), and a simple technique for relating the settlement estimate to the probability that the actual settlement could be larger than the estimated value.

SELECTING APPROPRIATE FACTORS OF SAFETY

Factors of safety provide a hedge against uncertainties in calculations, and the fact that it is never possible to compute with perfect accuracy. Through experience, conventions have developed with regard to what values of factor of safety are suitable for various situations. For example, the U.S. Army Corps of Engineers and many other agencies use F = 1.5 for long-term stability of slopes. Most geotechnical engineers use F = 2.5 to 3.0 for bearing capacity, and the same range of values for safety against erosion and piping.

Requiring the same factor of safety for all long-term slope stability applications, or all bearing capacity applications, is a ''one size fits all'' approach that is certain to result in inappropriate factors of safety in some cases. A more logical approach would consider

- The uncertainties in the quantities that enter the calculations
- The consequences of failure or unsatisfactory performance

This can be achieved, at least approximately, by choosing factors of safety such that the following relationship is satisfied:

$$\begin{pmatrix}\text{Reduction in } P_f\\ \text{associated with more}\\ \text{reliable design}\end{pmatrix} \times \begin{pmatrix}\text{Cost}\\ \text{of}\\ \text{failure}\end{pmatrix} < \begin{pmatrix}\text{Added cost}\\ \text{of more}\\ \text{reliable design}\end{pmatrix} \quad (9)$$

As discussed previously, not all ''failures'' are catastrophic. Some are better described as ''unsatisfactory performance.'' Where the product of probability of failure times the cost of failure or unsatisfactory performance is small, it is justified to use smaller factors of safety. On the contrary, where the product of probability of failure times the cost of failure or unsatisfactory performance is large, higher factors of safety are logical.

The quantities in (9) cannot be evaluated precisely. Nevertheless, the relationship represented by this expression provides a basis for selecting appropriate factors of safety, even though approximations and judgment will be required in applying it to real conditions. In the case of the retaining wall in Fig. 1, for example, a 1% probability of unsatisfactory performance due to sliding would probably not justify the cost of increasing the factor of safety above 1.5 unless the consequences of sliding were unusually large. In the case of the LASH Terminal slope in Fig. 4, however, an 18% probability of failure of the slope, multiplied by the estimated cost of a failure, would have justified higher cost to increase the factor of safety and reduce the probability of failure.

SUMMARY AND CONCLUSIONS

Reliability theory can be applied to geotechnical engineering through simple procedures, and need not require more data than is required for conventional deterministic analyses. With a relatively small additional effort to perform reliability analyses, the value of analyses can be increased considerably.

It is proposed that probability of failure should not be viewed as a replacement for factor of safety, but as a supplement. Computing both factor of safety and probability of failure is better than computing either one alone. Although neither factor of safety nor probability of failure can be computed with high precision, both have value and each enhances the value of the other.

The word ''failure,'' as used in the context of reliability theory, does not necessarily imply catastrophic failure. Some conditions—for example, sliding of a retaining wall—would be more aptly described as ''unsatisfactory performance'' than as ''failure.'' Other conditions, e.g., slope failures that involve large movements, are appropriately described by the word ''failure.'' It is important to bear in mind the likely consequences of the mode of ''failure'' being analyzed, and whether they are catastrophic or more benign.

Reliability concepts can be applied to settlement analyses as well as to factor of safety analyses. They can provide a measure of the accuracy of settlement computations and can be used to estimate the magnitude of settlement that has a very small probability of being exceeded.

Reliability analyses provide a logical framework for choosing factors of safety that are appropriate for the degree of uncertainty and the consequences of failure. While the factors that enter the relationship among probability of failure, consequences of failure, and the added cost of increased factor of safety cannot be evaluated with high accuracy, the relationship does serve to distinguish conditions where lower-than-normal factors of safety are appropriate or where higher-than-normal factors of safety are needed.

APPENDIX I. DETERMINING VALUES OF PROBABILITY OF FAILURE

Table 2 is convenient because it shows values of P_f related to F_{MLV} and V without further computation. Its shortcoming is that only approximate values of P_f can be determined for values of F_{MLV} and V that are intermediate between the values listed in the table. It may sometimes be desirable to have a means of computing more precise values of P_f.

The key to computing more precise values of P_f is to compute the value of the lognormal reliability index, β_{LN}, using the following formula:

$$\beta_{LN} = \frac{\ln\left(\dfrac{F_{MLV}}{\sqrt{1 + V^2}}\right)}{\sqrt{\ln(1 + V^2)}} \quad (10)$$

where β_{LN} = lognormal reliability index; V = coefficient of variation of factor of safety; and F_{MLV} = most likely value of factor of safety.

When β_{LN} has been computed, the value of P_f can be determined accurately in either of two ways:

1. Using tables of the standard cumulative normal distribution function, which can be found in many textbooks on probability and reliability. For example, the value of β_{LN} computed for sliding of the retaining wall in Fig. 1, with F_{MLV} = 1.50 and V = 0.17, is β_{LN} = 2.32. The standard cumulative normal distribution function (the reliability) corresponding to β_{LN} = 2.32 is 0.9898 (Dai and Wang 1992). The probability of failure is one minus the reliability, or $P_f = 1.0 - 0.9898 = 0.0102$.
2. Using the built-in function NORMSDIST in Excel. The argument of this function is the reliability index, β_{LN}. In Excel, under ''Insert Function,'' ''Statistical,'' choose ''NORMSDIST,'' and type the value of β_{LN}. For β_{LN} = 2.32, the result is 0.9898, which corresponds to P_f = 0.0102. Table 2 was developed using this Excel function.

Table 7 has the same use with respect to probabilities of settlement. Values other than the values listed in Table 7 can be determined using the following expression for β_{LN}.

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$$\beta_{LN} = \frac{\ln\{(SR)(\sqrt{1 + V^2})\}}{\sqrt{\ln(1 + V^2)}} \tag{11}$$

Table 7 was developed using this expression for β_{LN} as the argument for the NORMSDIST function in Excel.

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