Characterization of geotechnical variability

Kok-Kwang Phoon and Fred H. Kulhawy

Abstract: Geotechnical variability is a complex attribute that results from many disparate sources of uncertainties. The three primary sources of geotechnical uncertainties are inherent variability, measurement error, and transformation uncertainty. Inherent soil variability is modeled as a random field, which can be described concisely by the coefficient of variation (COV) and scale of fluctuation. Measurement error is extracted from field measurements using a simple additive probabilistic model or is determined directly from comparative laboratory testing programs. Based on an extensive literature review, the COV of inherent variability, scale of fluctuation, and COV of measurement error are evaluated in detail, along with the general soil type and the approximate range of mean value for which the COVs are applicable. Transformation uncertainty and overall property uncertainty are quantified in a companion paper.

Key words: inherent soil variability, measurement error, coefficient of variation, scale of fluctuation, geotechnical variability.

Résumé : La variabilité géotechnique est un caractère complexe qui résulte de nombreuses sources d'incertitudes. Les trois causes principales d'incertitude géotechnique sont la variabilité intrinsèque, l'erreur de mesure et l'incertitude de transformation. La variabilité intrinsèque peut-être modélisée par un champ aléatoire pouvant être décrit succinctement par le coefficient de variation (COV) et l'échelle de fluctuation. L'erreur de mesure est extraite des relevés en place, en utilisant un modèle probabiliste simple additif. Elle peut aussi être déterminée directement par des programmes d'essais comparatifs de laboratoire. A partir d'un examen fouillé de la littérature, le COV de variabilité intrinsèque, l'échelle de fluctuation et le COV de l'erreur de mesure ont été évalués en détail, de même que le type général de sol et la plage approximative des valeurs moyennes sur laquelle on peut appliquer les COV. L'incertitude de transformation et l'incertitude générale sur la propriété étudiée sont quantifiées dans un papier conjoint.

Mots clés : variabilité intrinsèque su sol, erreur de mesure, coefficient de variation, échelle de fluctuation, variabilité géotechnique.

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Introduction

Since the early 1980s, an extensive research study to develop a sound reliability-based design (RBD) approach for foundations has been in progress at Cornell University under the sponsorship of the Electric Power Research Institute. As a part of this RBD methodology, it was necessary to establish realistic statistical estimates of the variability of design soil properties. A series of five studies on geotechnical variabilities (Spry et al. 1988; Orchant et al. 1988; Filippas et al. 1988; Kulhawy et al. 1992; Phoon et al. 1995) was conducted to quantify realistic "best case" and "worst case" scenarios and provide property guidelines for the calibration of the RBD equations. These results are useful for all types of RBD studies. For foundations, extensive calibration studies by Phoon et al. (1995) indicated that the foundation resistance factors in the RBD equations are functions of the design soil property coefficient of variation (COV).

Ideally, a designer should select the appropriate resistance factors based on the variability of the soil data at a specific site. In the absence of site-specific data, or where the soil data are too limited for meaningful statistical analyses to be performed, guidelines on the probable range of soil property COV are useful as first-order approximations. Even when there is sufficient information for statistical analyses, a more robust estimate of geotechnical variability can be obtained by combining the site-specific data with prior information from these generalized guidelines using Bayesian updating techniques. Details on the application of Bayesian techniques to site characterization are described elsewhere (e.g., Spry et al. 1988; Filippas et al. 1988) and are not repeated herein. Finally, the establishment of typical soil property COV values would help design engineers develop an appreciation for the probable range of variability inherent in the overall estimation of common design soil properties and therefore identify atypical geotechnical variabilities.

Unfortunately, a number of the soil property statistics reported in the geotechnical literature are not suitable for this general use, primarily because they were determined from total variability analyses that implicitly assume a uniform

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Fig. 1. Uncertainty in soil property estimates (source: Kulhawy 1992, p. 101).

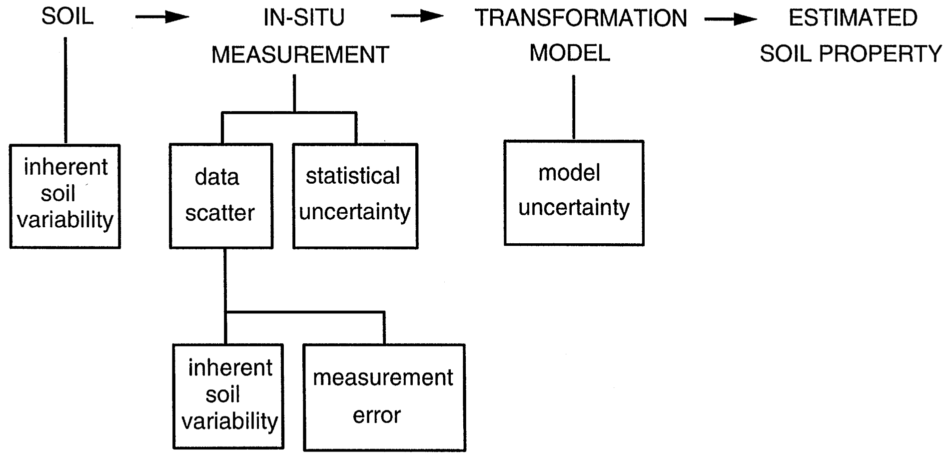
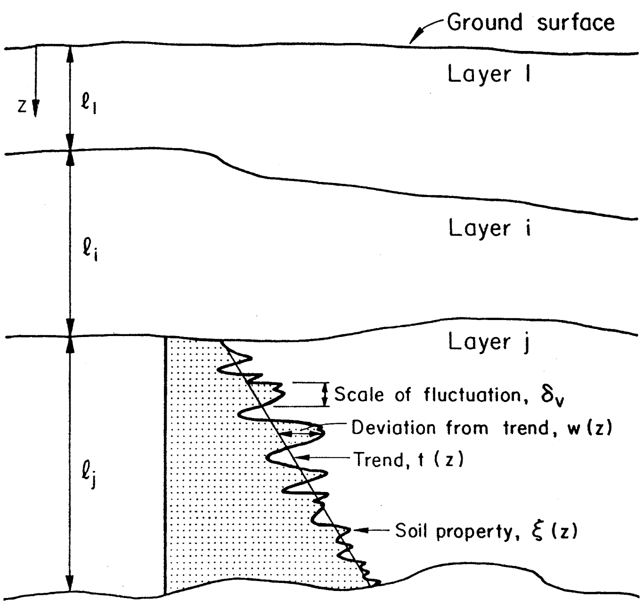


Fig. 2. Inherent soil variability.



source of uncertainty. However, geotechnical variability is more complex and results from many disparate sources of uncertainties, as illustrated in Fig. 1. As shown, the three primary sources of geotechnical uncertainty are inherent variability, measurement error, and transformation uncertainty. The first results primarily from the natural geologic processes that produced and continually modify the soil mass in situ. The second is caused by equipment, procedural–operator, and random testing effects. Collectively, these two sources can be described as data scatter. In situ measurements also are influenced by statistical uncertainty or sampling error that result from limited amounts of information. This uncertainty can be minimized by taking more samples, but it is commonly included within the measurement error at this time (Kulhawy 1992). The third source of uncertainty is introduced when field or laboratory measurements are transformed into design soil properties using empirical or other correlation models. The relative contribution of these three sources to the overall uncertainty in the design soil property clearly depends on the site conditions, degree of equipment and procedural control, and precision of the correlation model. Therefore, soil property statistics that are determined from total variability analyses only can be applied to the specific set of circumstances (site conditions, measurement techniques, correlation models) for which the design soil properties were derived.

In this paper, the inherent soil variability is modeled as a random field, which can be described concisely by the COV and the scale of fluctuation. Measurement error is extracted from field measurements using a simple additive probabilistic model or is determined directly from comparative laboratory test results. Based on an extensive literature review, the COV of inherent variability, the scale of fluctuation, and the COV of measurement error are evaluated in detail, along with the general soil type and the approximate range of mean value for which the COVs are applicable. A companion paper (Phoon and Kulhawy 1999) discusses the transformation uncertainty and illustrates how these component uncertainties can be combined consistently, for a variety of common soil parameters, to quantify the variability of design soil properties for general geotechnical use.

Modeling inherent soil variability

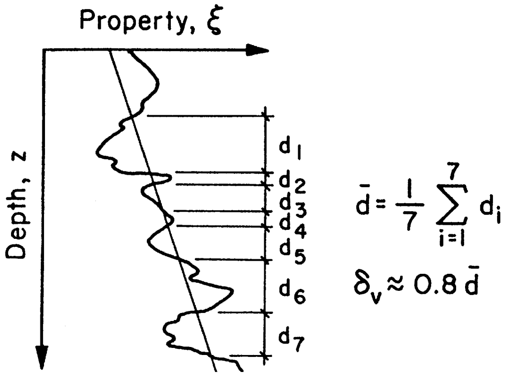
Soil is a complex engineering material that has been formed by a combination of various geologic, environmental, and physical–chemical processes. Many of these processes are continuing and can be modifying the soil in situ. Because of these natural processes, all soil properties in situ will vary vertically and horizontally. As shown in Fig. 2, this spatial variation can be decomposed conveniently into a smoothly varying trend function [$t(z)$] and a fluctuating component [$w(z)$] as follows:

[1] $\xi(z) = t(z) + w(z)$

in which ξ is the in situ soil property, and z is the depth. The fluctuating component defined in eq. [1] represents the inherent soil variability.

A rational means of quantifying inherent variability is to model $w(z)$ as a homogeneous random function or field (Vanmarcke 1983). The function $w(z)$ is considered to be statistically homogeneous if (*i*) the mean and variance of w do not change with depth; and (*ii*) the correlation between

Fig. 3. Estimation of vertical scale of fluctuation (source: Spry et al. 1988, p. 2-12).



the deviations at two different depths is a function only of their separation distance, rather than their absolute positions. Note that this correlation is a measure of linear dependence between two random quantities and varies between +1 and –1. A correlation of +1 or –1 implies that a perfect linear relationship exists between the two random quantities, with the sign indicating a positive or negative slope. The condition of constant mean can be satisfied if the data are detrended as shown in Fig. 2. In fact, the mean of *w* is a constant value of zero, because it is fluctuating equally about the trend line. Aside from a constant mean, the fluctuations also should be approximately uniform to satisfy the variance and correlation conditions given above. Fluctuations in the soil property profile are likely to be uniform if the data are extracted from a homogeneous soil layer.

For data sets that satisfy the above conditions, the inherent soil variability can be evaluated in a straightforward manner. First, the standard deviation of the inherent soil variability (SD_w) is evaluated as

$$[2] \qquad SD_w = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}[w(z_i)]^2}$$

in which n is the number of data points, and $w(z_i)$ is the fluctuation at depth z_i. Then a more useful dimensionless representation of inherent variability can be obtained by normalizing SD_w with respect to the mean soil property trend (t) as follows:

$$[3] \qquad COV_w = \frac{SD_w}{t}$$

in which COV_w is the coefficient of variation of inherent variability.

Another statistical parameter that is needed to describe inherent variability is the correlation distance or scale of fluctuation (Fig. 2), which provides an indication of the distance within which the property values show relatively strong correlation. A simple but approximate method of determining the scale of fluctuation is given by Vanmarcke (1977) as

$$[4] \qquad \delta_v \approx 0.8\bar{d}$$

in which δ_v is the vertical scale of fluctuation, and $\bar{d}$ is the average distance between intersections of the fluctuating property and its trend function, as shown in Fig. 3. Additional details on modeling inherent soil variability can be found elsewhere (e.g., Vanmarcke 1977; Baecher 1985; Spry et al. 1988; Filippas et al. 1988).

Coefficient of variation of inherent soil variability

An extensive literature review was conducted to estimate the typical COV values of inherent soil variability. However, this task was complicated because most COVs reported in the geotechnical literature are based on total variability analyses, as noted above. Therefore, the reported COVs may be considerably larger than the actual inherent soil variability because of four potential problems: (*i*) soil data from different geologic units are mixed, (*ii*) equipment and procedural controls generally are insufficient, (*iii*) deterministic trends in the soil data are not removed, and (*iv*) soil data are taken over a long time period.

The first problem can be minimized by ensuring that the soil data are classified properly into geologic units before statistical analyses are performed (Morse 1971). In the absence of relevant documentation, it is reasonable to assume that soil data taken over restricted locales and limited depth intervals are sufficiently homogeneous for the evaluation of inherent variability.

The second problem is related to measurement errors, which should be separated from inherent variability if the statistical results are to be extended for general use (e.g., Baecher 1985; Orchant et al. 1988). A detailed discussion on measurement errors is given later in this paper. Documentation on equipment and procedural controls during soil testing usually is not detailed sufficiently to permit a quantitative evaluation of measurement errors. However, it is reasonable to assume that measurement errors are minimal for soil data obtained in research programs, where good equipment and procedural controls are likely to be maintained (Orchant et al. 1988).

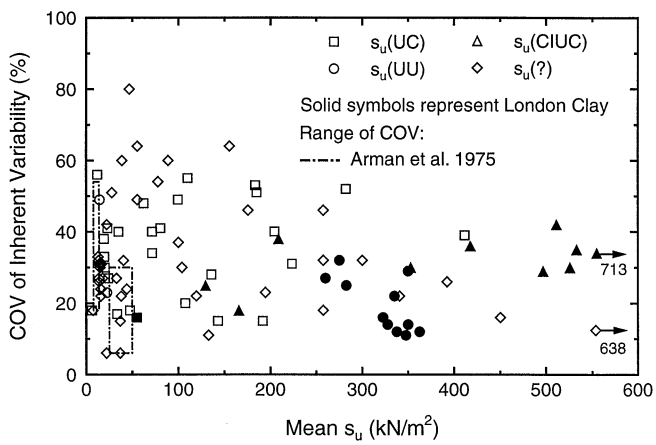
The third problem, which concerns the removal of deterministic trends from the soil data, is not well recognized. Consider a hypothetical soil property that varies linearly with depth with no random fluctuations about the linear trend. The soil property takes on the values of 10, 20, and 30, at depths of 1, 2, and 3, respectively. If the obvious linear trend is not removed, the sample mean and standard deviation of the data set would be evaluated as 20 and 10, respectively. Therefore, the COV of the soil data is 50%. A properly detrended data set, however, would reveal that the fluctuations are zero at all three depths, and the inherent variability clearly is zero. A number of the statistical analyses reported in the geotechnical literature are based on the original data set, rather than the detrended data set. From a rigorous statistical point of view, the results from such analyses do not represent inherent soil variability, unless the original data set contains no obvious trends. This situation is not likely, because most soil properties exhibit variations with depth to some degree. However, the depth variation might not be significant if the sampling interval is sufficiently small. Under this condition, the COV of the data set would be a valid, albeit approximate, indicator of inherent variability.

Table 1. Summary of inherent variability of strength properties (source: Phoon et al. 1995, p. 4-7).

Property[a]	Soil type	No. of data groups	No. of tests per group		Property value		Property COV (%)	
			Range	Mean	Range	Mean	Range	Mean
s_u(UC) (kN/m^2)	Fine grained	38	2–538	101	6–412	100	6–56	33
s_u(UU) (kN/m^2)	Clay, silt	13	14–82	33	15–363	276	11–49	22
s_u(CIUC) (kN/m^2)	Clay	10	12–86	47	130–713	405	18–42	32
s_u (kN/m^2)[b]	Clay	42	24–124	48	8–638	112	6–80	32
$\bar{\phi}$ (°)	Sand	7	29–136	62	35–41	37.6	5–11	9
$\bar{\phi}$ (°)	Clay, silt	12	5–51	16	9–33	15.3	10–50	21
$\bar{\phi}$ (°)	Clay, silt	9	—	—	17–41	33.3	4–12	9
tan $\bar{\phi}$ (TC)	Clay, silt	4	—	—	0.24–0.69	0.509	6–46	20
tan $\bar{\phi}$ (DS)	Clay, silt	3	—	—	—	0.615	6–46	23
tan $\bar{\phi}$[b]	Sand	13	6–111	45	0.65–0.92	0.744	5–14	9

[a]s_u, undrained shear strength; $\bar{\phi}$, effective stress friction angle; TC, triaxial compression test; UC, unconfined compression test; UU, unconsolidated–undrained triaxial compression test; CIUC, consolidated isotropic undrained triaxial compression test; DS, direct shear test.
[b]Laboratory test type not reported.

Fig. 4. COV of inherent variability of s_u versus mean s_u.



The fourth problem is associated with the variation of soil properties with time. If the samples were collected over a period of 1–2 weeks, the soil properties may be regarded as time-invariant (Réthàti 1988). However, over longer time periods, additional variability could be introduced into the data set because of the changes in the soil mass occurring with time. This variation with time can be rather significant (e.g., McCormack and Wilding 1979; Reyna and Chameau 1991). Unfortunately, most studies do not report the time frame over which the soil data are collected. Therefore, it is not possible to determine if the temporal changes in the soil mass are significant.

The reported COVs in the geotechnical literature were examined critically based on the considerations given above. In general, only the COVs of soil data from similar geologic origins that were collected over limited spatial extents with good equipment and procedural controls were considered to be representative of inherent soil variability. However, the problems associated with extraneous sources of variability could not be removed completely because of limited documentation. Therefore, the COVs should be somewhat higher than those representing inherent soil variability alone. This limitation applies to all the inherent variability results presented later in the paper.

Laboratory strength properties

Table 1 summarizes the available data on the inherent variability of the undrained shear strength, effective stress friction angle, and tangent of the effective stress friction angle. Full details are given elsewhere (Phoon et al. 1995). Where possible, the test types are reported because the test boundary conditions can have a considerable effect on the undrained shear strength and the friction angle (e.g., Kulhawy and Mayne 1990). Unfortunately, not all the data can be classified properly, because the importance of reporting test types with the strength properties is only gradually being recognized. Table 1 also summarizes the general soil type, the number of data groups and tests per group, and the mean and COV of the soil property. A description of soil type is useful because the site-specific COVs tabulated can be extrapolated to other locations, provided the soil deposits are of similar geologic formation and environmental history (Kay and Krizek 1971; Vanmarcke 1978, 1989; Tang 1984). The number of tests is a useful indicator of the accuracy of the mean and COV estimates. The number of tests per group typically is fairly large, which implies that the errors in the statistical estimates are minimal. The presentation of the mean in conjunction with the COV also is important to ensure that the COV is not misinterpreted as being applicable to all possible mean values.

Undrained shear strength

The variation of the COV of inherent variability of the undrained shear strength (s_u) is plotted versus the mean s_u in Fig. 4. The lower bound of the COV remains relatively constant at about 10% over the range of mean values shown. However, the upper bound on the COV seems to decrease with increasing mean. The effect of test type on the COV also can be seen. The typical ranges of COV for unconfined compression tests (UC), unconsolidated–undrained triaxial compression tests (UU), and consolidated isotropic undrained triaxial compression tests (CIUC) are 20–55%, 10–30%, and 20–40%, respectively. The differences between UU and CIUC tests are particularly evident because the soil types corresponding to these two tests are primarily London Clays. Therefore, the differences in COV cannot be attributed to the differences in the soil types used in the various tests. More

Table 2. Summary of inherent variability of index parameters (source: Phoon et al. 1995, p. 4-16).

Property[a]	Soil type[b]	No. of data groups	No. of tests per group		Property value		Property COV (%)	
			Range	Mean	Range	Mean	Range	Mean
w_n (%)	Fine grained	40	17–439	252	13–105	29	7–46	18
w_L (%)	Fine grained	38	15–299	129	27–89	51	7–39	18
w_P (%)	Fine grained	23	32–299	201	14–27	22	6–34	16
PI (%)	Fine grained	33	15–299	120	12–44	25	9–57	29
LI	Clay, silt	2	32–118	75	—	0.094	60–88	74
γ (kN/m^3)	Fine grained	6	5–3200	564	14–20	17.5	3–20	9
γ_d (kN/m^3)	Fine grained	8	4–315	122	13–18	15.7	2–13	7
D_r (%)[c]	Sand	5	—	—	30–70	50	11–36	19
D_r (%)[d]	Sand	5	—	—	30–70	50	49–74	61

[a] w_n, natural water content; w_L, liquid limit; w_P, plastic limit; PI, plasticity index; LI, liquidity index; γ, total unit weight; γ_d, dry unit weight; D_r, relative density.

[b] Fine-grained materials derived from a variety of geologic origins, e.g., glacial deposits, tropical soils, and loess.

[c] Total variability for direct method of determination.

[d] Total variability for indirect determination using standard penetration test (SPT) values.

Fig. 5. COV of inherent variability of $\bar{\phi}$ versus mean $\bar{\phi}$.

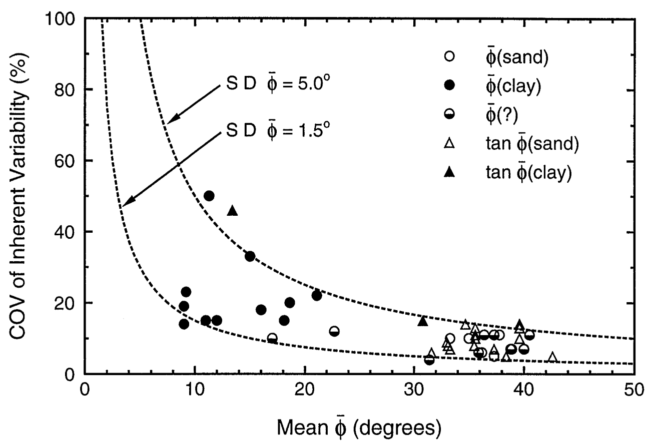
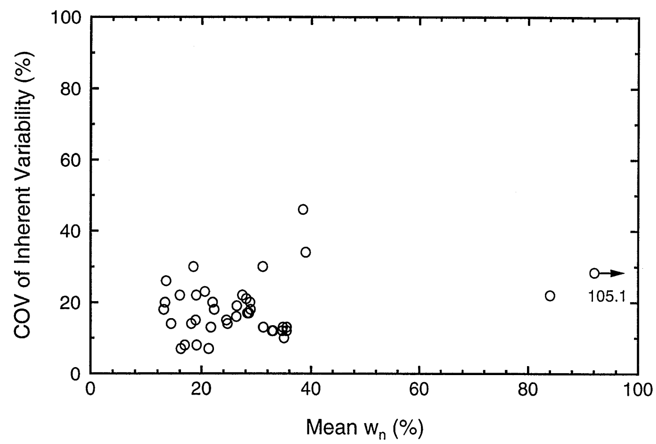


Fig. 6. COV of inherent variability of w_n versus mean w_n.



data on other soil types are needed to confirm this observation.

Friction angle

The variation of the COV of inherent variability of the effective stress friction angle ($\bar{\phi}$) is plotted versus the mean $\bar{\phi}$ in Fig. 5. Both the COV of $\bar{\phi}$ and tan $\bar{\phi}$ are plotted. There is no apparent difference between the COV of these two parameters. The effect of soil type on the COV also is illustrated in Fig. 5, in which the soil type is classified broadly into sand and clay. The COV for clay generally is higher than that for sand. A possible reason for this effect can be found in eq. [3], which states that the COV is inversely proportional to the mean if the standard deviation (SD) is a constant. This relationship is plotted in Fig. 5 for two constant SD values of 1.5° and 5.0°. It is evident that this range of standard deviation is applicable to both sand and clay. Therefore, the differences in the COV for sand and clay primarily are caused by the differences in the mean friction angle. Note that the differences in the COV for sand and clay only are apparent because the mean friction angles of the clays shown in Fig. 5 are very low. For most soils, the mean friction angle typically is between 20° and 40°. The COV within this range of mean friction angle essentially is 5–15%.

Laboratory index parameters

The inherent variability of some common index parameters is summarized in Table 2, along with the soil type, number of data groups and tests per group, and the mean and COV of the index parameter. Full details are given elsewhere (Phoon et al. 1995).

Natural water content

The variation of the COV of inherent variability for the natural water content (w_n) is plotted versus the mean w_n in Fig. 6. No trends in the COV are present as the mean varies from 13 to 105%. The typical range of COV for w_n is between 8 and 30%.

Liquid and plastic limits

The variations of the COV of inherent variability for the liquid limit (w_L) and plastic limit (w_P) are plotted versus the mean w_L and w_P in Fig. 7. As with w_n, no trends in the COV are present as the mean w_L and w_P vary from 27 to 89% and 14 to 27%, respectively. The typical range of COV is between 6 and 30% for both index parameters, which is comparable to that for w_n.

Plasticity and liquidity indices

The variations of the COV of inherent variability for the

Fig. 7. COV of inherent variability of w_L and w_P versus mean w_L and w_P.

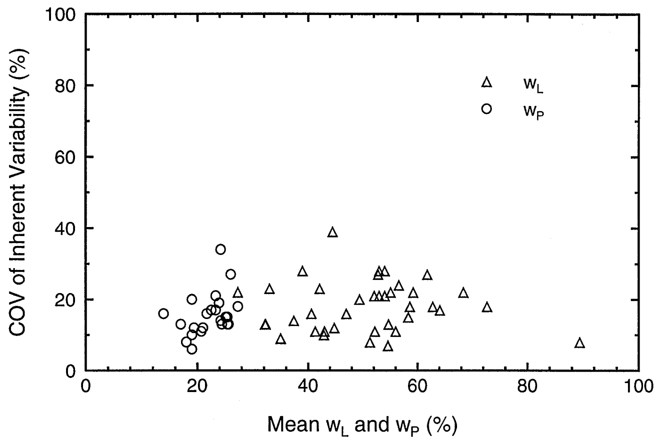


Fig. 8. COV of inherent variability of PI and LI versus mean PI and LI.

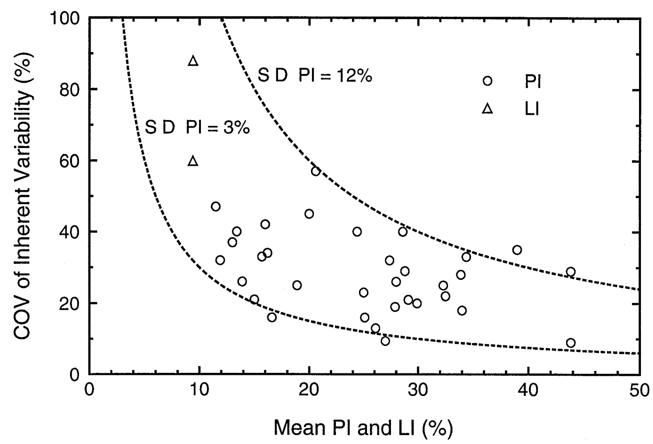


Fig. 9. COV of inherent variability of γ and γ_d versus mean γ and γ_d.

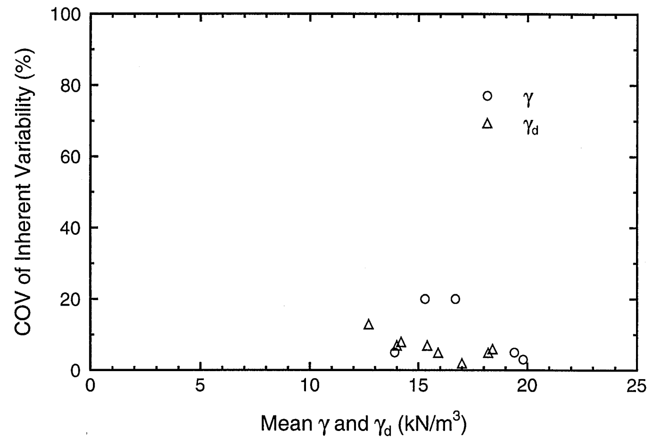
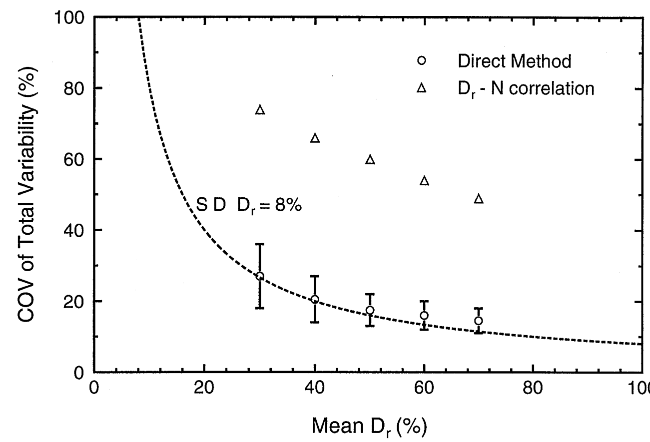


Fig. 10. COV of inherent variability of D_r versus mean D_r.



plasticity index (PI) and liquidity index (LI) are plotted versus the mean PI and LI in Fig. 8. As with $\bar{\phi}$, the smaller COVs at larger mean values are primarily the result of dividing a relatively constant standard deviation by larger means. The larger COVs in Fig. 8 typically are associated with smaller means for the converse reason. By plotting eq. [3] onto Fig. 8, it can be seen that a relatively limited range of standard deviation (3–12%) can explain the large variations in the COV. This typical range of standard deviation appears to be applicable to both PI and LI. However, this observation only can be considered as tentative, because the statistical data on LI are too limited.

Total and dry unit weights

The variations of the COV of inherent variability for the total unit weight (γ) and dry unit weight (γ_d) are plotted versus the mean γ and γ_d in Fig. 9. The typical COVs for both parameters are less than 10%. No trends in the COV can be observed as the mean varies from about 13 to 20 kN/m^3. The two outliers with COV of 20% are associated with the tidal swamp and tropical soils of Nigeria (Ejezie and Harrop-Williams 1984).

Relative density

Statistical information on the inherent variability of relative density D_r appears to be lacking. A statistical analysis on the total variability of D_r suggests that the range of COV is between 11 and 36% for the direct method of determination (Haldar and Tang 1979). However, the COV varies with the mean, as shown in Fig. 10. Most of the variation in the COV can be adequately explained using eq. [3] with a standard deviation of 8%. Figure 10 also presents the results of a statistical analysis on the total variability of D_r for an indirect method of determination using standard penetration test N values (Haldar and Miller 1984). The COVs are much higher, which is to be expected. The COV decreases almost linearly from 74% for a mean D_r of 30% to 49% for a mean D_r of 70%.

Field measurements

The inherent variabilities of some common field measurements are summarized in Table 3. Full details are given elsewhere (Phoon et al. 1995). Note that there are important subdivisions within each field test. For example, cone tip resistance q_c can be measured using a mechanical or electric cone. However, these subdivisions cannot be established at this time because of inadequacies in the data base. The soil type, number of data groups and tests per group, and the mean and COV of the field measurement also are summarized in Table 3.

The COV of inherent variability for these test measure-

Table 3. Summary of inherent variability of field measurements (source: Phoon et al. 1995, p. 4-10).

Test type[a]	Property[b]	Soil type	No. of data groups	No. of tests per group		Property value		Property COV (%)	
				Range	Mean	Range	Mean	Range	Mean
CPT	q_c (MN/m^2)	Sand	57	10–2039	115	0.4–29.2	4.10	10–81	38
CPT	q_c (MN/m^2)	Silty clay	12	30–53	43	0.5–2.1	1.59	5–40	27
CPT	q_T (MN/m^2)	Clay	9	—	—	0.4–2.6	1.32	2–17	8
VST	s_u(VST) (kN/m^2)	Clay	31	4–31	16	6–375	105	4–44	24
SPT	N	Sand	22	2–300	123	7–74	35	19–62	54
SPT	N	Clay, loam	2	2–61	32	7–63	32	37–57	44
DMT	A (kN/m^2)	Sand to clayey sand	15	12–25	17	64–1335	512	20–53	33
DMT	A (kN/m^2)	Clay	13	10–20	17	119–455	358	12–32	20
DMT	B (kN/m^2)	Sand to clayey sand	15	12–25	17	346–2435	1337	13–59	37
DMT	B (kN/m^2)	Clay	13	10–20	17	502–876	690	12–38	20
DMT	E_D (MN/m^2)	Sand to clayey sand	15	10–25	15	9.4–46.1	25.4	9–92	50
DMT	E_D (MN/m^2)	Sand, silt	16	—	—	10.4–53.4	21.6	7–67	36
DMT	I_D	Sand to clayey sand	15	10–25	15	0.8–8.4	2.85	16–130	53
DMT	I_D	Sand, silt	16	—	—	2.1–5.4	3.89	8–48	30
DMT	K_D	Sand to clayey sand	15	10–25	15	1.9–28.3	15.1	20–99	44
DMT	K_D	Sand, silt	16	—	—	1.3–9.3	4.1	17–67	38
PMT	p_L (kN/m^2)	Sand	4	—	17	1617–3566	2284	23–50	40
PMT	p_L (kN/m^2)	Cohesive	5	10–25	—	428–2779	1084	10–32	15
PMT	E_{PMT} (MN/m^2)	Sand	4	—	—	5.2–15.6	8.97	28–68	42

[a]CPT, cone penetration test; VST, vane shear test; SPT, standard penetration test; DMT, dilatometer test; PMT, pressuremeter test.
[b]q_c, CPT tip resistance; q_T, corrected CPT tip resistance; s_u(VST), undrained shear strength from VST; N, SPT blow count (number of blows per foot or per 305 mm); A and B, DMT A and B readings; E_D, DMT modulus; I_D, DMT material index; K_D, DMT horizontal stress index; p_L, PMT limit stress; E_{PMT}, PMT modulus.

ments has been discussed by Phoon and Kulhawy (1996) and will not be repeated herein. However, since these specialty conference proceedings have rather limited circulation internationally, it is wise to repeat key data where pertinent. Therefore, the basic data plots of COV of inherent variability versus the mean in situ tests parameters are given in the Appendix. These data support the interpretations given in Table 3.

Scale of fluctuation

An extensive literature review was conducted to estimate the typical scales of fluctuations for a variety of common geotechnical parameters. The results of this review are summarized in Table 4. Full details are given elsewhere (Phoon et al. 1995). The scales of fluctuation are generally calculated using the method of moments. Information on the soil type and the direction of fluctuation also are included in the table. It is apparent that the amount of information on the scale of fluctuation is relatively limited in comparison to the amount of information on the COV of inherent soil variability. Therefore, the observations given below should be viewed with caution, because there seldom are enough data to establish their generality on a firm basis.

Table 4. Summary of scale of fluctuation of some geotechnical properties (source: Phoon et al. 1995, p. 4-20).

Property[a]	Soil type	No. of studies	Scale of fluctuation (m)	
			Range	Mean
Vertical fluctuation				
s_u	Clay	5	0.8–6.1	2.5
q_c	Sand, clay	7	0.1–2.2	0.9
q_T	Clay	10	0.2–0.5	0.3
s_u(VST)	Clay	6	2.0–6.2	3.8
N	Sand	1	—	2.4
w_n	Clay, loam	3	1.6–12.7	5.7
w_L	Clay, loam	2	1.6–8.7	5.2
$\bar{\gamma}$	Clay	1	—	1.6
γ	Clay, loam	2	2.4–7.9	5.2
Horizontal fluctuation				
q_c	Sand, clay	11	3.0–80.0	47.9
q_T	Clay	2	23.0–66.0	44.5
s_u(VST)	Clay	3	46.0–60.0	50.7
w_n	Clay	1	—	170.0

[a]s_u and s_u (VST), undrained shear strength from laboratory tests and vane shear tests, respectively; $\bar{\gamma}$, effective unit weight.

The vertical scale of fluctuation (δ_v) for the undrained shear strength is on the order of 1–2 m, although it can be as large as 6 m. For the cone tip resistance, δ_v typically is less than 1 m. The value of δ_v for the corrected cone tip resistance seems to be smaller than the corresponding δ_v value for the uncorrected cone tip resistance. The typical value of δ_v for the corrected cone tip resistance is less than 0.5 m. For the vane shear test, the value of δ_v seems to vary between 2 and 6 m. The upper bound of this range appears to be somewhat larger than that for the laboratory measurement of the undrained shear strength. The single reported value of δ_v for the standard penetration test N value falls within the previously noted range of 2–6 m. For the index parameters, most of the δ_v values are within the range of 2–10 m.

The horizontal scale of fluctuation (δ_h) is more than one order of magnitude larger than the vertical scale of fluctua-

Table 5. Summary of total measurement error of some laboratory tests (source: Phoon et al. 1995, p. 4-22).

Property[a]	Soil type	No. of data groups	No. of tests per group		Property value		Property COV (%)	
			Range	Mean	Range	Mean	Range	Mean
s_u(TC) (kN/m^2)	Clay, silt	11	—	13	7–407	125	8–38	19
s_u(DS) (kN/m^2)	Clay, silt	2	13–17	15	108–130	119	19–20	20
s_u(LV) (kN/m^2)	Clay	15	—	—	4–123	29	5–37	13
$\bar{\phi}$ (TC) (°)	Clay, silt	4	9–13	10	2–27	19.1	7–56	24
$\bar{\phi}$ (DS) (°)	Clay, silt	5	9–13	11	24–40	33.3	3–29	13
$\bar{\phi}$ (DS) (°)	Sand	2	26	26	30–35	32.7	13–14	14
tan $\bar{\phi}$ (TC)	Sand, silt	6	—	—	—	—	2–22	8
tan $\bar{\phi}$ (DS)	Clay	2	—	—	—	—	6–22	14
w_n (%)	Fine grained	3	82–88	85	16–21	18	6–12	8
w_L (%)	Fine grained	26	41–89	64	17–113	36	3–11	7
w_P (%)	Fine grained	26	41–89	62	12–35	21	7–18	10
PI (%)	Fine grained	10	41–89	61	4–44	23	5–51	24
γ (kN/m^3)	Fine grained	3	82–88	85	16–17	17.0	1–2	1

[a]LV, laboratory vane shear test.

tion, with a typical range of between 40–60 m. This result is not surprising because soil properties tend to be more variable in the vertical direction than in the horizontal direction. The single reported δ_h value of 170 m for the natural water content is about three to four times larger than the δ_h values for the other soil parameters. This result is consistent with the observation that the δ_v values for index parameters also are the largest. It would appear that index parameters generally are less variable in both vertical and horizontal directions, in comparison with other soil parameters. It is important to note that the scale of fluctuation is strongly influenced by the sampling interval (DeGroot and Baecher 1993). Some of the scales of fluctuation reported in Table 4 possibly might be biased because of sampling limitation.

Measurement error

All soil properties have to be measured by some physical means. This process of measurement introduces additional variability into the soil data. The total variability of a measured property (ξ_m) can be described by the following simple model (Lumb 1971; Orchant et al. 1988):

$$[5] \qquad \xi_m(z) = \xi(z) + e(z)$$

in which ξ is the in situ property, and e is the measurement error. Equation [5] can be expanded by substituting eq. [1] for ξ as follows:

$$[6] \qquad \xi_m(z) = t(z) + w(z) + e(z)$$

in which t is the deterministic trend, and w is the inherent variability. The two uncertain components, w and e, generally are assumed to be uncorrelated because they are derived from unrelated sources (e.g., Lumb 1971; Baecher 1985; Filippas et al. 1988; Kulhawy et al. 1992). As mentioned previously, inherent variability is caused primarily by the natural geologic processes that are involved in soil formation. Measurement error, on the other hand, arises from equipment, procedural–operator, and random testing effects. Equipment effects result from inaccuracies in the measuring devices and variations in equipment geometries and systems employed for routine testing. Procedural–operator effects originate from the limitations in existing test standards and how they are followed. In general, tests that are highly operator dependent and that have complicated test procedures will have greater variability than those with simple procedures and little operator dependency. Random testing error refers to the remaining scatter in the test results that is not assignable to specific testing parameters and is not caused by inherent soil variability. A more complete discussion of measurement error is given elsewhere (Orchant et al. 1988).

Laboratory tests

In principle, measurement error can be determined directly by analyzing the variation of the results obtained by a representative group of soil testing companies performing the “same” test on nominally identical soil samples. Comparative testing programs of this type are available (e.g., Hammitt 1966; Johnston 1969; Sherwood 1970; Singh and Lee 1970; Minty et al. 1979), but they are rather limited. A summary of the total measurement error in terms of the COV is given in Table 5 for a variety of common laboratory tests. Full details are given elsewhere (Phoon et al. 1995). None of the studies reported the contribution from equipment, procedural–operator, and random testing effects separately. Table 5 also summarizes the soil type, the number of data groups and tests per group, and the mean and COV of the measurements.

Undrained shear strength

The variation of the COV of measurement error of the undrained shear strength (s_u) is plotted versus the mean s_u in Fig. 11. The s_u tests can be classified broadly into (*i*) triaxial compression (TC), (*ii*) direct shear (DS), and (*iii*) laboratory vane (LV). No apparent differences in the COV for the different test types can be observed, and most of the COVs are less than 20%. Note that the clayey silt specimens shown in Table 5 were compacted separately by each participant before testing. Therefore, the water content and dry density of the soil specimens were different and could contribute to the larger measurement errors (Singh and Lee 1970). Without the additional variability introduced by compaction, the range of measurement error for these s_u tests probably would be about 5–15%.

Fig. 11. COV of total measurement error of s_u versus mean s_u.

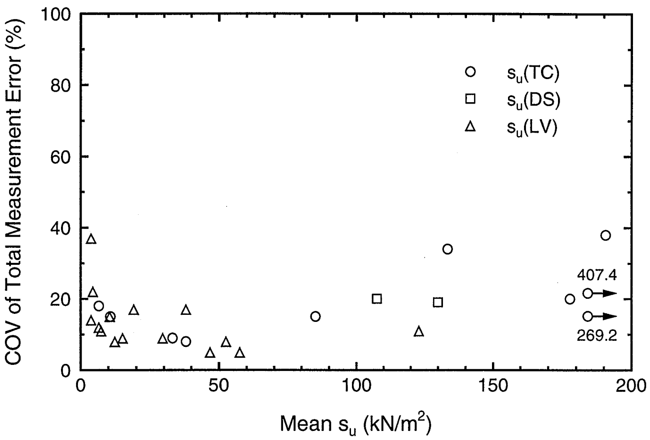


Fig. 12. COV of total measurement error of $\bar{\phi}$ versus mean $\bar{\phi}$.

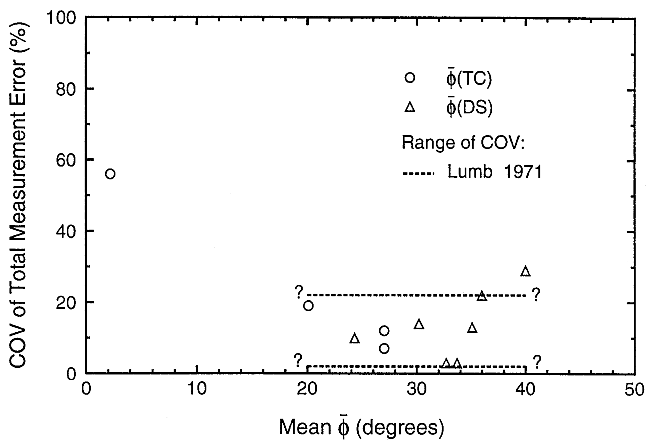


Fig. 13. COV of total measurement error of w_n, w_L, and w_P versus mean w_n, w_L, and w_P.

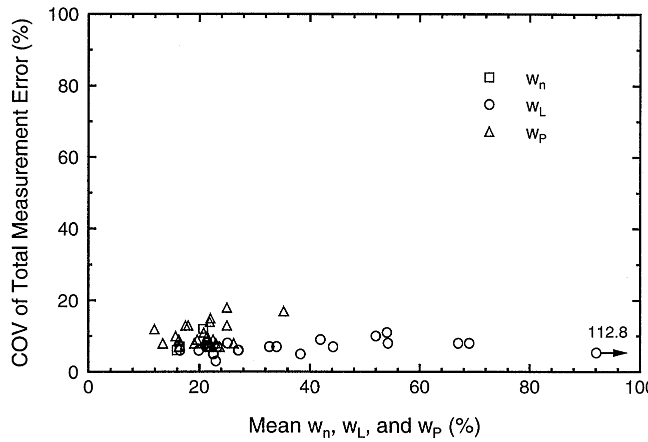
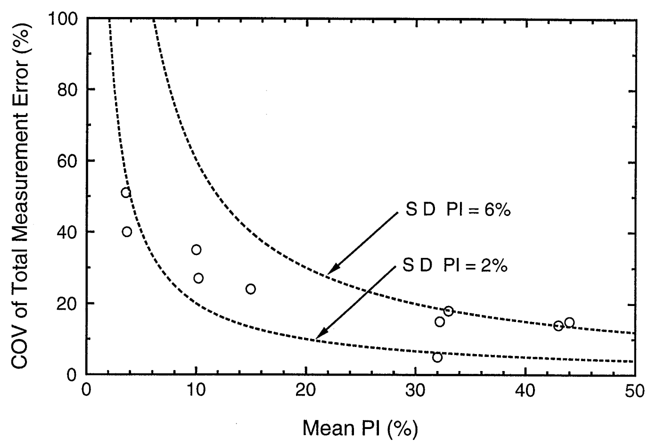


Fig. 14. COV of total measurement error of PI versus mean PI.



Friction angle

The variation of the COV of measurement error of the effective stress friction angle ($\bar{\phi}$) is plotted versus the mean $\bar{\phi}$ in Fig. 12. The friction angle tests can be classified broadly into (*i*) triaxial compression (TC), and (*ii*) direct shear (DS). No apparent differences in the COV for the different test types are evident, and most of the COVs are less than 20%. As for s_u, part of the variability was not related to the strength tests, but to the compaction procedure. In addition, it was observed that significant differences in the friction angles were caused by the linearization of the curved failure envelope over different ranges of confining pressure (Singh and Lee 1970). Based on these considerations, the measurement error for the friction angle tests also is judged to be between 5 and 15%, as for the s_u tests.

Natural water content and liquid and plastic limits

The variation of the COV of measurement error of the natural water content (w_n), liquid limit (w_L), and plastic limit (w_P) are plotted versus the mean w_n, w_L, and w_P in Fig. 13. No trends in the COV can be observed as the mean varies from 12 to 113%. The measurement error for the liquid limit test appears to be somewhat smaller than that for the plastic limit test. The probable ranges of measurement error for the liquid and plastic limit tests are 5–10% and 10–15%, respectively. The limited data for natural water content seem to suggest that the measurement error is intermediate between those of the liquid and plastic limit tests.

Plasticity index

The variation of the COV of measurement error of the plasticity index (PI) is plotted versus the mean PI in Fig. 14. As in Fig. 8, the association of larger COVs with smaller means, or vice versa, can be explained adequately using eq. [3]. Note that the standard deviation for measurement error, which is about 4%, is considerably smaller than the standard deviation for inherent variability shown in Fig. 8, which averages about 8%.

Total unit weight

The COV of measurement error for the total unit weight also is summarized in Table 5. The measurement error of 1–2% is the smallest among all the tests given in the table.

Field tests

For the field tests, a detailed analysis of the measurement error already has been conducted (Kulhawy and Trautmann 1996) and is summarized in Table 6. In the study by Kulhawy and Trautmann (1996), regression analyses were performed on the results of laboratory calibration studies to determine the amount of variation assignable to each test parameter. Where replicate data were available, second-moment statistics (mean and coefficient of variation) were evaluated to estimate random testing error. Inherent soil variability was assumed to

Table 6. Summary of measurement error of common in situ tests (source: Orchant et al. 1988, p. 4-63; Kulhawy and Trautmann 1996, p. 283).

	Coefficient of variation, COV (%)				
Test	Equipment	Procedure	Random	Total[a]	Range[b]
Standard penetration test (SPT)	5–75[c]	5–75[c]	12–15	14–100[c]	15–45
Mechanical cone penetration test (MCPT)	5	10–15[d]	10–15[d]	15–22[d]	15–25
Electric cone penetration test (ECPT)	3	5	5–10[d]	7–12[d]	5–15
Vane shear test (VST)	5	8	10	14	10–20
Dilatometer test (DMT)	5	5	8	11	5–15
Pressuremeter test, prebored (PMT)	5	12	10	16	10–20[e]
Self-boring pressuremeter test (SBPMT)	8	15	8	19	15–25[e]

[a] $\text{COV(Total)} = [\text{COV(Equipment)}^2 + \text{COV(Procedure)}^2 + \text{COV(Random)}^2]^{0.5}$.
[b] Because of limited data and judgment involved in estimating COVs, ranges represent probable magnitudes of field test measurement error.
[c] Best to worst case scenarios, respectively, for SPT.
[d] Tip and side resistances, respectively, for CPT.
[e] It is likely that results may differ for p_o, p_f, and p_L, but the data are insufficient to clarify this issue.

Table 7. Approximate guidelines for inherent soil variability (source: Phoon et al. 1995, p. 4-49).

Test type	Property	Soil type	Mean	COV(%)
Lab strength	s_u(UC)	Clay	10–400 kN/m^2	20–55
	s_u(UU)	Clay	10–350 kN/m^2	10–30
	s_u(CIUC)	Clay	150–700 kN/m^2	20–40
	$\bar{\phi}$	Clay and sand	20–40°	5–15
CPT	q_T	Clay	0.5–2.5 MN/m^2	<20
	q_c	Clay	0.5–2.0 MN/m^2	20–40
	q_c	Sand	0.5–30.0 MN/m^2	20–60
VST	s_u(VST)	Clay	5–400 kN/m^2	10–40
SPT	N	Clay and sand	10–70 blows/ft	25–50
DMT	A	Clay	100–450 kN/m^2	10–35
	A	Sand	60–1300 kN/m^2	20–50
	B	Clay	500–880 kN/m^2	10–35
	B	Sand	350–2400 kN/m^2	20–50
	I_D	Sand	1–8	20–60
	K_D	Sand	2–30	20–60
	E_D	Sand	10–50 MN/m^2	15–65
PMT	p_L	Clay	400–2800 kN/m^2	10–35
	p_L	Sand	1600–3500 kN/m^2	20–50
	E_{PMT}	Sand	5–15 MN/m^2	15–65
Lab index	w_n	Clay and silt	13–100%	8–30
	w_L	Clay and silt	30–90%	6–30
	w_P	Clay and silt	15–25%	6–30
	PI	Clay and silt	10–40%	—[a]
	LI	Clay and silt	10%	—[a]
	γ, γ_d	Clay and silt	13–20 kN/m^3	<10
	D_r	Sand	30–70%	10–40; 50–70[b]

[a] COV = (3–12%)/mean.
[b] The first range of values gives the total variability for the direct method of determination, and the second range of values the total variability for the indirect determination using SPT values.

be small in these laboratory calibration studies, because the soil deposits generally were prepared under controlled laboratory conditions. Field data also were analyzed using second-moment statistical techniques. However, only total variability could be obtained because of the difficulty in separating the inherent variability of the soil deposit from the variability of the test measurements.

As shown in Table 6, the measurement error for the standard penetration test is the largest, and the measurement errors for the electric cone penetration test and the dilatometer test are the smallest. Because of the limited data available and the need to use judgment to estimate these errors, the last column of Table 6 represents the range of probable test measurement variability one can expect in typical field in situ tests. Full details are given elsewhere (Kulhawy and Trautmann 1996).

Summary and conclusions

Realistic estimates of the variability of soil parameters are needed for the development and application of reliability-based design. The variability of design soil parameters should be evaluated as a function of inherent soil variability, measurement error, and transformation uncertainty. The relative contribution of these components to the overall variability in the design parameter depends on the site conditions, degree of equipment and procedural control during testing, and quality of the transformation model.

An extensive literature review was conducted to estimate the statistics of inherent soil variability and measurement error. A summary of the COV of inherent variability for various test measurements is given in Table 7. The general soil type and the approximate range of mean value for which the COV is applicable also are included in the table. With respect to soil type, the COV of inherent variability for sand is higher than that for clay. With respect to measurement type, the COVs of inherent variability for index parameters are the lowest, with the possible exception of the relative density. The highest COVs of inherent variability seem to be associated with measurements in the horizontal direction and measurements of soil modulus.

Another important descriptor of inherent variability is the scale of fluctuation. However, information on this parameter is limited. The vertical scale of fluctuation for laboratory measurement of undrained shear strength is in the range of 1–2 m. For the cone tip resistance and the corrected cone tip resistance, the vertical scale of fluctuation is less than 1 m and 0.5 m, respectively. The vertical scales of fluctuation for vane shear and standard penetration test measurements are in the range of 2–6 m. The horizontal scale of fluctuation for these laboratory and field measurements is on the order of 40–60 m. The vertical and horizontal scales of fluctuation for index parameters were the largest.

Statistical information on measurement error is rather limited. Based on the statistics reported by comparative testing programs, the COVs of measurement error for most laboratory strength tests were estimated to be between 5 and 15%. The COVs of measurement error for the plastic and liquid limit tests were in the range of 10–15% and 5–10%, respectively. The COV of measurement error for the natural water content was intermediate between those of the limit tests. For the plasticity index, the standard deviation of the measurement error was between 2 and 6%. The unit weight determination had the lowest COV of measurement error (~1%). The COVs of measurement error for field tests were given elsewhere.

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List of symbols

A: dilatometer A reading
B: dilatometer B reading
COV: coefficient of variation
$\mathrm{COV_w}$: coefficient of variation of inherent variability
D_r: relative density
E_D: dilatometer modulus
E_PMT: pressuremeter modulus
I_D: dilatometer material index
K_D: dilatometer horizontal stress index
LI: liquidity index
N: standard penetration test value
PI: plasticity index
SD: standard deviation
$\mathrm{SD_w}$: standard deviation of inherent variability
$\bar{d}$: average distance between intersections of the fluctuating property and its trend function
e: measurement error
n: number of data points
l_i, l_j, l_l: layer depths
p_f: pressuremeter yield stress
p_L: pressuremeter limit stress
p_o: pressuremeter seating stress
q_c: cone tip resistance
q_T: corrected q_c
s_u: undrained shear strength
$t(\bullet)$: trend function
t: mean soil property trend
$w(\bullet)$: inherent soil variability
w_L: liquid limit
w_P: plastic limit
w_n: water content
z: depth
z_i: ith depth coordinate
γ: total soil unit weight
$\bar{\gamma}$: effective unit weight
γ_d: dry unit weight
δ_v: vertical scale of fluctuation
δ_h: horizontal scale of fluctuation
ξ: in situ soil property
ξ_m: measured soil property
$\bar{\phi}$: effective stress friction angle

Appendix

This appendix includes basic data plots (Fig. A1, see following page) of COV of inherent variability versus the mean in situ test parameters that support the interpretations given in Table 3.

Fig. A1. COV of inherent variability versus mean in situ test parameters: (*a*) CPT q_c and q_T; (*b*) s_u(VST); (*c*) SPT *N*; (*d*) DMT *A* and *B* readings; (*e*) PMT p_L; (*f*) DMT I_D; (*g*) DMT K_D; (*h*) DMT E_D and PMT E_{PMT}.