

Q3.) Proof of cost function properties :

Given, $\text{cost}(S) = \max(S)$, where S = set of (+)ve integers.

proof of Monotonicity : show S_1 is a subset of S_2

$$\therefore \max(S_1) \leq \max(S_2)$$

\hookrightarrow since $S_1 \subseteq S_2$, all elements in S_1 are also in S_2 .

\therefore maximum elements of S_1 cannot be greater than the maximum element of S_2 .

This means $\max(S_1) \leq \max(S_2)$

Thus, monotonicity property holds.

Proof of Subadditivity : show $\max(S_1 \cup S_2) \leq \max(S_1) + \max(S_2)$

• max value of the union = largest element in either S_1 or S_2 .

• $\max(a, b) \leq a + b$, for any 2 positive integers a & b , we get $\max(S_1 \cup S_2) \leq \max(S_1) + \max(S_2)$

Thus, subadditivity property holds.