```
{a > 0, c > 0, gamma > 0}
```

$$\{a > 0,\ c > 0,\ \text{gamma} > 0\}$$

```
f11 = -s_-1 == a * y_-2
```

$$-s_{-1} == a\, y_{-2}$$

```
f12 = -2 * y_-2 == x_-2 * s_-1
```

$$-2\, y_{-2} == s_{-1}\, x_{-2}$$

```
f13 = -2 * x_-2 == y_-2 * s_-1
```

$$-2\, x_{-2} == s_{-1}\, y_{-2}$$

```
{y_-2, x_-2, s_-1} /. Solve[{f11, f12, f13}, {y_-2, x_-2, s_-1}]
```

$$\left\{\{0, 0, 0\},\ \left\{-\frac{2}{a},\ \frac{2}{a},\ 2\right\},\ \left\{\frac{2}{a},\ \frac{2}{a},\ -2\right\}\right\}$$

```
f21 = 0 == a * y_-1
```

$$0 == a\, y_{-1}$$

```
f22 = -y_-1 == -c * y_-2 - x_-2 * s_0 - x_-1 * s_-1
```

$$-y_{-1} == -s_0\, x_{-2} - s_{-1}\, x_{-1} - c\, y_{-2}$$

```
f23 = -x_-1 == -c * x_-2 + y_-2 * s_0 + y_-1 * s_-1
```

$$-x_{-1} == -c\, x_{-2} + s_0\, y_{-2} + s_{-1}\, y_{-1}$$

```
sis = {y_-1, x_-1, s_0} /. Solve[{f21, f22, f23}, {y_-1, x_-1, s_0}]
```

$$\left\{\left\{0,\ -\frac{-c\, x_{-2}^2 - c\, y_{-2}^2}{x_{-2} - s_{-1}\, y_{-2}},\ -\frac{-c\, s_{-1}\, x_{-2} - c\, y_{-2}}{-x_{-2} + s_{-1}\, y_{-2}}\right\}\right\}$$

$$\left\{\left\{0,\ -\frac{-c\, x_{-2}^2 - c\, y_{-2}^2}{x_{-2} - s_{-1}\, y_{-2}},\ -\frac{-c\, s_{-1}\, x_{-2} - c\, y_{-2}}{-x_{-2} + s_{-1}\, y_{-2}}\right\}\right\}$$

```
z = % /. {y_-2 → 2 / a, x_-2 → 2 / a, s_-1 → -2}
```

$$\left\{\left\{0,\ \frac{4\, c}{3\, a},\ \frac{c}{3}\right\}\right\}$$

In[3]:=
```
f31 = s_1 == a * y_0 + gamma
f32 = 0 == -c * y_-1 - s_-1 - x_-2 * s_1 - x_-1 * s_0 - x_0 * s_-1
f33 = 0 == -c * x_-1 + y_-2 * s_1 - y_-1 * s_0 - y_0 * s_-1
```

Out[3]= $s_1 == \text{gamma} + a\, y_0$

Out[4]= $0 == -s_{-1} - s_1\, x_{-2} - s_0\, x_{-1} - s_{-1}\, x_0 - c\, y_{-1}$

Out[5]= $0 == -c\, x_{-1} + s_1\, y_{-2} - s_0\, y_{-1} - s_{-1}\, y_0$

In[25]:=
```
r = {y0, x0, s1} /. Solve[{f31, f32, f33}, {y0, x0, s1}]
```

Out[25]=
$$\left\{\left\{-\frac{-c\,x_{-1}+\text{gamma}\,y_{-2}-s_0\,y_{-1}}{-s_{-1}+a\,y_{-2}},\ -\frac{s_{-1}+s_0\,x_{-1}+c\,y_{-1}}{s_{-1}}+\frac{x_{-2}\,(-\text{gamma}\,s_{-1}+a\,c\,x_{-1}+a\,s_0\,y_{-1})}{s_{-1}\,(s_{-1}-a\,y_{-2})},\ -\frac{-\text{gamma}\,s_{-1}+a\,c\,x_{-1}+a\,s_0\,y_{-1}}{s_{-1}-a\,y_{-2}}\right\}\right\}$$

In[26]:=
```
z = % /. {y-2 → 2 / a, x-2 → 2 / a, s-1 → -2,
    y-1 → 0, x-1 → 4 * c / (3 * a), s0 → c / 3,
    gamma → 2 * c^2 / 3}
```

Out[26]=
$$\left\{\left\{0,\ \frac{2\,c^2}{3\,a}+\frac{1}{2}\left(-2+\frac{4\,c^2}{9\,a}\right),\ \frac{2\,c^2}{3}\right\}\right\}$$

In[19]:=
```
Simplify[%]
```

Out[19]=
$$\left\{\left\{0,\ -1+\frac{8\,c^2}{9\,a},\ \frac{2\,c^2}{3}\right\}\right\}$$

In[27]:=
```
f41 = 2 * s2 == a * y1
```

Out[27]=
$$2\,s_2 == a\,y_1$$

In[28]:=
```
f42 = y1 == -c * y0 - s0 - s2 * x-2 - s1 * x-1 - s0 * x0 + x1 * s-1
f43 = x1 == -c * x0 + s2 * y-2 + s1 * y-1 + y0 * s0 + y1 * s-1
```

Out[28]=
$$y_1 == -s_0 - s_2\,x_{-2} - s_1\,x_{-1} - s_0\,x_0 + s_{-1}\,x_1 - c\,y_0$$

Out[29]=
$$x_1 == -c\,x_0 + s_2\,y_{-2} + s_1\,y_{-1} + s_0\,y_0 + s_{-1}\,y_1$$

In[34]:=
```
v = {y1, x1, s2} /. Solve[{f41, f42, f43}, {y1, x1, s2}]
```

Out[34]=
$$\left\{\left\{-\frac{2\,(-s_0-s_1\,x_{-1}-c\,s_{-1}\,x_0-s_0\,x_0+s_{-1}\,s_1\,y_{-1}-c\,y_0+s_{-1}\,s_0\,y_0)}{-2+2\,s_{-1}^2-a\,x_{-2}+a\,s_{-1}\,y_{-2}},\right.\right.$$
$$-(-2\,s_{-1}\,s_0-2\,s_{-1}\,s_1\,x_{-1}-2\,c\,x_0-2\,s_{-1}\,s_0\,x_0-a\,c\,x_{-2}\,x_0-a\,s_0\,y_{-2}-a\,s_1\,x_{-1}\,y_{-2}-a\,s_0\,x_0\,y_{-2}+2\,s_1\,y_{-1}+$$
$$a\,s_1\,x_{-2}\,y_{-1}-2\,c\,s_{-1}\,y_0+2\,s_0\,y_0+a\,s_0\,x_{-2}\,y_0-a\,c\,y_{-2}\,y_0)\,/\,(-2+2\,s_{-1}^2-a\,x_{-2}+a\,s_{-1}\,y_{-2}),$$
$$\left.\left.-\frac{a\,(-s_0-s_1\,x_{-1}-c\,s_{-1}\,x_0-s_0\,x_0+s_{-1}\,s_1\,y_{-1}-c\,y_0+s_{-1}\,s_0\,y_0)}{-2+2\,s_{-1}^2-a\,x_{-2}+a\,s_{-1}\,y_{-2}}\right\}\right\}$$

In[35]:=
```
z = % /. {y_-2 → 2 / a, x_-2 → 2 / a,
    y_-1 → 0, x_-1 → 4 * c / (3 * a), s_0 → c / 3,
    x_0 -> (-1 + 8 c^2 / (9 a)) + y_0,
    s_1 -> 2 c^2 / 3 + a * y_0,
    gamma → 2 * c^2 / 3}
```

Out[35]=
$$\left\{\left\{-\frac{1}{-4+2 s_{-1}+2 s_{-1}^2}\right.\right.$$
$$2\left(-\frac{c}{3}-c\,y_0+\frac{1}{3}c\,s_{-1}\,y_0-\frac{1}{3}c\left(-1+\frac{8c^2}{9a}+y_0\right)-c\,s_{-1}\left(-1+\frac{8c^2}{9a}+y_0\right)-\frac{4c\left(\frac{2c^2}{3}+a\,y_0\right)}{3a}\right),$$
$$-\frac{1}{-4+2 s_{-1}+2 s_{-1}^2}\left(-\frac{2c}{3}-\frac{2c\,s_{-1}}{3}-\frac{2c\,y_0}{3}-2c\,s_{-1}\,y_0-\frac{14}{3}c\left(-1+\frac{8c^2}{9a}+y_0\right)-\right.$$
$$\left.\frac{2}{3}c\,s_{-1}\left(-1+\frac{8c^2}{9a}+y_0\right)-\frac{8c\left(\frac{2c^2}{3}+a\,y_0\right)}{3a}-\frac{8c\,s_{-1}\left(\frac{2c^2}{3}+a\,y_0\right)}{3a}\right),\ -\frac{1}{-4+2 s_{-1}+2 s_{-1}^2}$$
$$\left.a\left(-\frac{c}{3}-c\,y_0+\frac{1}{3}c\,s_{-1}\,y_0-\frac{1}{3}c\left(-1+\frac{8c^2}{9a}+y_0\right)-c\,s_{-1}\left(-1+\frac{8c^2}{9a}+y_0\right)-\frac{4c\left(\frac{2c^2}{3}+a\,y_0\right)}{3a}\right)\right\}\right\}$$

In[37]:=
```
FullSimplify[%]
```

Out[37]=
$$\left\{\left\{\frac{c\left(32c^2+72a\,y_0+3s_{-1}\left(-9a+8c^2+6a\,y_0\right)\right)}{27a\left(-2+s_{-1}+s_{-1}^2\right)},\right.$$
$$\left.\frac{2c\left(-27a+40c^2+54a\,y_0+4s_{-1}\left(4c^2+9a\,y_0\right)\right)}{27a\left(-2+s_{-1}+s_{-1}^2\right)},\ \frac{c\left(32c^2+72a\,y_0+3s_{-1}\left(-9a+8c^2+6a\,y_0\right)\right)}{54\left(-2+s_{-1}+s_{-1}^2\right)}\right\}\right\}$$

In[41]:=
```
f51 = 3 * s_3 == a * y_2
f52 = 2 * y_2 == -c * y_1 - s_1 - x_-2 * s_3 - x_-1 * s_2 - x_0 * s_1 - x_1 * s_0 - x_2 * s_-1
f53 = 2 * x_2 == -c * x_1 + y_-2 * s_3 - y_-1 * s_2 - y_0 * s_1 - y_1 * s_0 - y_2 * s_-1
```

Out[41]=
$$3\,s_3 == a\,y_2$$

Out[42]=
$$2\,y_2 == -s_1 - s_3\,x_{-2} - s_2\,x_{-1} - s_1\,x_0 - s_0\,x_1 - s_{-1}\,x_2 - c\,y_1$$

Out[43]=
$$2\,x_2 == -c\,x_1 + s_3\,y_{-2} - s_2\,y_{-1} - s_1\,y_0 - s_0\,y_1 - s_{-1}\,y_2$$

In[64]:=
```
v = {y_2, x_2, s_3} /. Solve[{f51, f52, f53}, {y_2, x_2, s_3}]
```

Out[64]=
$$\left\{\left\{\frac{3\left(2\left(s_1+s_2\,x_{-1}+s_1\,x_0+s_0\,x_1+c\,y_1\right)-s_{-1}\left(c\,x_1+s_2\,y_{-1}+s_1\,y_0+s_0\,y_1\right)\right)}{-3\left(4-s_{-1}^2\right)-a\left(2\,x_{-2}+s_{-1}\,y_{-2}\right)},\right.\right.$$
$$-\left(3\,s_{-1}\,s_1+3\,s_{-1}\,s_2\,x_{-1}+3\,s_{-1}\,s_1\,x_0-6\,c\,x_1+3\,s_{-1}\,s_0\,x_1-a\,c\,x_{-2}\,x_1-a\,s_1\,y_{-2}-\right.$$
$$a\,s_2\,x_{-1}\,y_{-2}-a\,s_1\,x_0\,y_{-2}-a\,s_0\,x_1\,y_{-2}-6\,s_2\,y_{-1}-a\,s_2\,x_{-2}\,y_{-1}-6\,s_1\,y_0-a\,s_1\,x_{-2}\,y_0+$$
$$\left.3\,c\,s_{-1}\,y_1-6\,s_0\,y_1-a\,s_0\,x_{-2}\,y_1-a\,c\,y_{-2}\,y_1\right)\Big/\left(-12+3\,s_{-1}^2-2\,a\,x_{-2}-a\,s_{-1}\,y_{-2}\right),$$
$$\left.\left.\frac{a\left(2\left(s_1+s_2\,x_{-1}+s_1\,x_0+s_0\,x_1+c\,y_1\right)-s_{-1}\left(c\,x_1+s_2\,y_{-1}+s_1\,y_0+s_0\,y_1\right)\right)}{-3\left(4-s_{-1}^2\right)-a\left(2\,x_{-2}+s_{-1}\,y_{-2}\right)}\right\}\right\}$$

In[65]:=
```
FullSimplify[v]
```

Out[65]=
$$\left\{\left\{\frac{3\,(2\,(s_2\,x_{-1}+s_1\,(1+x_0)+s_0\,x_1+c\,y_1)-s_{-1}\,(c\,x_1+s_2\,y_{-1}+s_1\,y_0+s_0\,y_1))}{3\,s_{-1}^2-2\,(6+a\,x_{-2})-a\,s_{-1}\,y_{-2}},\right.\right.$$
$$(6\,c\,x_1+a\,s_1\,y_{-2}+a\,s_2\,x_{-1}\,y_{-2}+a\,s_1\,x_0\,y_{-2}+a\,s_0\,x_1\,y_{-2}+6\,s_2\,y_{-1}+$$
$$6\,s_1\,y_0+(6\,s_0+a\,c\,y_{-2})\,y_1-3\,s_{-1}\,(s_2\,x_{-1}+s_1\,(1+x_0)+s_0\,x_1+c\,y_1)+$$
$$a\,x_{-2}\,(c\,x_1+s_2\,y_{-1}+s_1\,y_0+s_0\,y_1))\Big/\left(3\,s_{-1}^2-2\,(6+a\,x_{-2})-a\,s_{-1}\,y_{-2}\right),$$
$$\left.\left.\frac{a\,(2\,(s_2\,x_{-1}+s_1\,(1+x_0)+s_0\,x_1+c\,y_1)-s_{-1}\,(c\,x_1+s_2\,y_{-1}+s_1\,y_0+s_0\,y_1))}{3\,s_{-1}^2-2\,(6+a\,x_{-2})-a\,s_{-1}\,y_{-2}}\right\}\right\}$$

In[66]:=
```
z = v /. {y_-2 → 2 / a, x_-2 → 2 / a,
  y_-1 → 0, x_-1 → 4 * c / (3 * a), s_0 → c / 3,
  x_0 -> (-1 + 8 c^2 / (9 a)) + y_0,
  s_1 -> 2 c^2 / 3 + a * y_0,
  gamma → 2 * c^2 / 3}
```

Out[66]=
$$\left\{\left\{\left(3\left(-s_{-1}\left(c\,x_1+y_0\left(\frac{2\,c^2}{3}+a\,y_0\right)+\frac{c\,y_1}{3}\right)+\right.\right.\right.$$
$$\left.\left.\left.2\left(\frac{2\,c^2}{3}+\frac{4\,c\,s_2}{3\,a}+\frac{c\,x_1}{3}+a\,y_0+\left(-1+\frac{8\,c^2}{9\,a}+y_0\right)\left(\frac{2\,c^2}{3}+a\,y_0\right)+c\,y_1\right)\right)\right)\Big/$$
$$\left(-a\left(\frac{4}{a}+\frac{2\,s_{-1}}{a}\right)-3\left(4-s_{-1}^2\right)\right),\ -\frac{1}{-16-2\,s_{-1}+3\,s_{-1}^2}$$
$$\left(-\frac{8\,c\,s_2}{3\,a}+\frac{4\,c\,s_{-1}\,s_2}{a}-\frac{26\,c\,x_1}{3}+c\,s_{-1}\,x_1-2\left(\frac{2\,c^2}{3}+a\,y_0\right)+3\,s_{-1}\left(\frac{2\,c^2}{3}+a\,y_0\right)-8\,y_0\left(\frac{2\,c^2}{3}+a\,y_0\right)-\right.$$
$$\left.2\left(-1+\frac{8\,c^2}{9\,a}+y_0\right)\left(\frac{2\,c^2}{3}+a\,y_0\right)+3\,s_{-1}\left(-1+\frac{8\,c^2}{9\,a}+y_0\right)\left(\frac{2\,c^2}{3}+a\,y_0\right)-\frac{14\,c\,y_1}{3}+3\,c\,s_{-1}\,y_1\right),$$
$$\left(a\left(-s_{-1}\left(c\,x_1+y_0\left(\frac{2\,c^2}{3}+a\,y_0\right)+\frac{c\,y_1}{3}\right)+2\left(\frac{2\,c^2}{3}+\frac{4\,c\,s_2}{3\,a}+\frac{c\,x_1}{3}+a\,y_0+\right.\right.\right.$$
$$\left.\left.\left.\left(-1+\frac{8\,c^2}{9\,a}+y_0\right)\left(\frac{2\,c^2}{3}+a\,y_0\right)+c\,y_1\right)\right)\right)\Big/\left(-a\left(\frac{4}{a}+\frac{2\,s_{-1}}{a}\right)-3\left(4-s_{-1}^2\right)\right)\Big\}\Big\}$$

In[67]:=
```
FullSimplify[z]
```

Out[67]=
$$\left\{\left\{\frac{72\,c\,s_2+9\,a\,c\,(2-3\,s_{-1})\,x_1+\left(2\,c^2+3\,a\,y_0\right)\left(16\,c^2-9\,a\,(-2+s_{-1})\,y_0\right)-9\,a\,c\,(-6+s_{-1})\,y_1}{9\,a\,(2+s_{-1})\,(-8+3\,s_{-1})},\right.\right.$$
$$\left(32\,c^4+72\,c\,s_2+6\,a\left(39\,c\,x_1+38\,c^2\,y_0+45\,a\,y_0^2+21\,c\,y_1\right)-\right.$$
$$\left.3\,s_{-1}\left(16\,c^4+36\,c\,s_2+3\,a\left(3\,c\,x_1+14\,c^2\,y_0+9\,a\,y_0^2+9\,c\,y_1\right)\right)\right)\Big/(27\,a\,(2+s_{-1})\,(-8+3\,s_{-1})),$$
$$\left.\left.\frac{72\,c\,s_2+9\,a\,c\,(2-3\,s_{-1})\,x_1+\left(2\,c^2+3\,a\,y_0\right)\left(16\,c^2-9\,a\,(-2+s_{-1})\,y_0\right)-9\,a\,c\,(-6+s_{-1})\,y_1}{27\,(2+s_{-1})\,(-8+3\,s_{-1})}\right\}\right\}$$