PMIS model — notation, explanations and MILP formulation

based on the uploaded manuscript (PMIS).

1. Symbols, sets, parameters and variables (English descriptions)

Sets and indices

- I : set of all Mobile Network Operators (MNOs). Index $i \in I$.
- $\tau \in I$: the operator of interest (the decision-maker whose strategy we optimize).
- T : discrete time horizon (set of time periods), typically $T = \{1, \dots, |T|\}$. Index $t \in T$.
- A : set of geographical areas. Index $a \in A$.
- S_i : set of sites owned by operator i; in particular S_τ are the sites of operator τ. A generic site is $s \in S_i$ (or $s \in S_\tau$ when relevant).
- G : set of legacy technologies (e.g., $\{2G, 3G, 4G\}$).
- NG : the next-generation technology (e.g., 5G).
- O_i : set of legacy commercial offers of operator i (one per legacy technology); NO_i : offer of operator i based on the NG technology.
- O : union of all offers $\bigcup_{i \in I}(O_i \cup \{NO_i\})$.

Parameters

- u_a : number of potential customers (population) in area $a \in A$.
- D_{NG} : average traffic demand (per user) for NG users.
- CAPANG : capacity (bandwidth units) of a site equipped with NG.
- Z^t : maximum number of sites that can be newly equipped (deployed) by operator τ at time t (budgetary / roll-out constraint).
- $QAt \in [0, 1]$: regulatory / strategic threshold: minimal fraction of the total population that must be under NG coverage of operator τ at time t.
- Sa : set of sites of operator τ that can cover area a (i.e., $Sa \subseteq S_\tau$).
- As : set of areas covered by site s of operator τ (i.e., $As \subseteq A$).
- $R^t_{a,i} \in \{0, 1\}$: parameter equal to 1 if competitor i has NG coverage of area a at time t (known/fixed for competitors).
- Any migration function values $f^a_{C,o',o} \in [0, 1]$: percent of customers in area a who, under coverage configuration C, move from offer o' to offer o between two consecutive periods. (These are parameters built from data.)

Decision variables (operator τ's variables)

- $z_{ts} \in \{0,1\}$: equals 1 if operator τ has installed NG on site $s \in S_\tau$ by (or at) time period t (0 otherwise). This represents the deployment decision timeline.
- $r_{ta} \in \{0,1\}$: equals 1 if area a is covered by operator τ's NG at time t (i.e., at least one site of τ covering a has NG installed).
- $\delta_{taC} \in \{0,1\}$: equals 1 if, at time t in area a, the coverage configuration across all operators equals the Boolean vector $C = (c_i)_{i\in I} \in \{0,1\}^{|I|}$ (where $c_i = 1$ means operator i covers area a with NG). These are auxiliary indicators used to select the correct migration function f.
- $u_{taio} \in \mathbb{N}$: number of customers in area a subscribing at time t to offer o of operator i (for all $i \in I$, $o \in O_i \cup \{NO_i\}$).
- $u_{ta\tau NO_\tau,s} \in \mathbb{N}$: number of NG-users (operator τ's NG-offer) from area a served through site s at time t (routing of NG-user traffic to sites).

Conventions

- We denote the final time period as $|T|$ or T_{end}. The objective maximizes NG subscribers of τ at the final period.
- All summations over offers, operators and configurations are explicitly shown in the formulation below.

2. MILP formulation (objective + constraints)

Below is the mathematical program as a Mixed Integer (originally MINLP but linearized) program.

Objective: maximize number of NG subscribers of operator τ at the final period

$$\max_{z,u,r,\delta} \quad \sum_{a\in A} u^{|T|}_{a,\tau,NO_\tau} \tag{1}$$

where $u^{|T|}_{a,\tau,NO_\tau}$ denotes the number of subscribers of operator τ to its NG-offer in area a at time $|T|$.

Coverage linking constraints

$$r_{ta} \leq \sum_{s\in S_a} z_{ts} \quad \forall a \in A,\ t \in T, \tag{2}$$

$$z_{ts} \leq r_{ta} \quad \forall a \in A,\ s \in S_a,\ t \in T, \tag{3}$$

(These ensure $r_{ta} = 1$ only if some covering site s has NG, and each deployed site $z_{ts} = 1$ implies coverage of its covered areas a.)

Coverage configuration indicator (definition of δ) For every Boolean configuration $C = (c_i)_{i\in I} \in \{0,1\}^{|I|}$, area a and time t:

$$\delta_{taC} = \Big(c_\tau r_{ta} + (1-c_\tau)(1-r_{ta})\Big) \prod_{i\in I\setminus\{\tau\}} \Big(c_i R^t_{a,i} + (1-c_i)(1-R^t_{a,i})\Big). \tag{4}$$

(Equality means $\delta_{taC} = 1$ iff the observed coverage vector equals C; linearization procedures are required to convert the product into linear constraints – see remark below.)

Temporal user dynamics (migration / churn) — for all $a \in A$, $i \in I$, $o \in O_i \cup \{NO_i\}$, $t \in T$ (with $t \geq 1$)

$$u_{taio} \;=\; \sum_{C \in \{0,1\}^{|I|}} \delta_{taC} \sum_{i' \in I} \sum_{o' \in O_{i'} \cup \{NO_{i'}\}} f^{a}_{C,o',o}\, u_{t-1,\,a,i',o'}. \tag{5}$$

(Each period the distribution of users follows the migration fractions $f^{a}_{C,o',o}$ conditional on the coverage configuration C.)

Flow decomposition of NG users onto sites — for all $a \in A$, $t \in T$

$$u_{ta\tau NO_\tau} \;=\; \sum_{s \in S_a} u_{ta\tau NO_\tau,s}. \tag{6}$$

Site capacity constraints — for all sites $s \in S_\tau$, $t \in T$

$$\sum_{a \in A_s} D_{NG}\, u_{ta\tau NO_\tau,s} \;\leq\; \text{CAPANG}\; z_{ts}. \tag{7}$$

(If site s is not equipped with NG at time t ($z_{ts} = 0$), its capacity for NG users is 0; if $z_{ts} = 1$, total NG-demand assigned to s must be $\leq$ its capacity.)

Deployment budget (per-period limit on new sites) — for all $t \in T$

$$\sum_{s \in S_\tau} (z_{ts} - z_{t-1,s}) \;\leq\; Z^t, \tag{8}$$

(with the usual convention $z_{0s} = 0$ if nothing is deployed before period 1).

Regulatory coverage threshold — for all $t \in T$

$$\sum_{a \in A} u_a\, r_{ta} \;\geq\; QAt \sum_{a \in A} u_a. \tag{9}$$

(At time t, at least a fraction QAt of the total population must be under NG coverage.)

Domain constraints (integrality and binary)

$$r_{ta} \in \{0,1\} \quad \forall a \in A,\; t \in T, \tag{10}$$
$$z_{ts} \in \{0,1\} \quad \forall s \in S_\tau,\; t \in T, \tag{11}$$
$$\delta_{taC} \in \{0,1\} \quad \forall C \in \{0,1\}^{|I|},\; a \in A,\; t \in T, \tag{12}$$
$$u_{taio} \in \mathbb{N} \quad \forall a \in A,\; i \in I,\; o \in O_i \cup \{NO_i\},\; t \in T, \tag{13}$$
$$u_{taios} \in \mathbb{N} \quad \forall a \in A,\; i \in I,\; o \in O_i \cup \{NO_i\},\; s \in S_i,\; t \in T. \tag{14}$$

3. Remarks on linearization and implementation

- Equation (4) uses products of binary variables and parameters. In practice you linearize each δ_{taC} by replacing this equality with a set of linear constraints:

  $$\delta_{taC} \leq c_\tau r_{ta} + (1 - c_\tau)(1 - r_{ta}), \quad \delta_{taC} \leq c_i R^t_{a,i} + (1 - c_i)(1 - R^t_{a,i}) \;\; \forall i \neq \tau,$$

  and a linking inequality to force $\delta_{taC} = 1$ when all components match. Standard 'big-M' or logical constraints may be used; see literature on linearizing Boolean products (e.g., Glover & Woolsey).

- Equation (5) contains multiplication of δ_{taC} with $u_{t-1,a,i',o'}$ (integer variable). This product is bilinear and must be linearized (introduce auxiliary variables $w_{t,a,C,i',o'} = \delta_{taC} \cdot u_{t-1,a,i',o'}$ and impose McCormick / integer linearization constraints, exploiting integrality bounds on u).
- The model is originally a MINLP (due to products). The paper proposes linearization techniques to obtain a MILP. When coding (Pyomo, Gurobi, CPLEX...), you must implement the linearization steps above.
- Initial conditions: u_{0aio} are given (data). Also z_{0s} may be set to the initial deployment state (or zero).

Reference: This formulation follows the PMIS problem description and equations (1)–(14) in the uploaded manuscript. :contentReference[oaicite:1]index=1