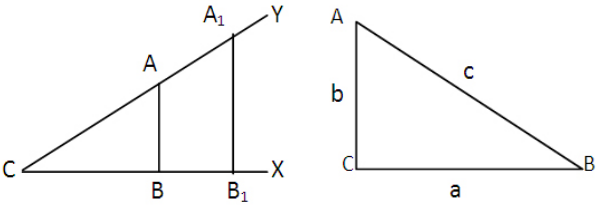
wÎ‡KvYwgwZi c_g cwV

wÎ‡KvYwgwZ MwY‡Zi AZ¨š¿i“ZcY GKwU kvLv|we‡kl K‡i R¨wgwZ‡Z †Kvb wKQi gvb wbY‡qi †¶‡Î wÎ‡KvYwgwZ AZ¨š¿i“ZcY|AvR Avgiv wÎ‡KvYwgwZi cv_wgK weIq Ges wKQ mgm¨v wb‡q Av‡jvPbv Kie|

‡Kv‡Yi wÎ‡KvYwgwZK AbcvZ:



g‡b Kwi, $\angle YCX$ GKwU m¶‡KvY (wPÎ-1)|GLb CY evûi †h‡Kvb we›` A n‡Z CX Gi Dci AB j¤ AwK|Zvn‡j ΔACB Gi 3wU evûi 6wU AbcvZ cvIqv hvq|G¿‡jvB nj $\angle ACB$ Gi wÎ‡KvYwgwZK AbcvZ| awi,$\angle ACB = \theta$ (GLv‡b θ †Kv‡Yi wecixZ evûi‡K j¤ aiv nq)

$sin\ \theta$ =j¤/AwZfR=$\frac{AB}{AC} = \frac{1}{csc\,\theta}$

$cos\ \theta$ =fwg/AwZfR=$\frac{BC}{AC} = \frac{1}{sec\,\theta}$

$\tan\theta$ =j¤/fwg=$\frac{AB}{BC} = \frac{1}{csc\theta} = \frac{\sin\theta}{\cos\theta}$

$csc\ \theta$ =AwZfR/j¤=$\frac{AC}{AB} = \frac{1}{sin\,\theta}$ (GwU‡K $cosec$ I ejv nq)

$sec\ \theta$ =AwZfR/fwg=$\frac{AC}{BC} = \frac{1}{cos\,\theta}$

$cot\ \theta$ =fwg/j¤=$\frac{BC}{AB} = \frac{1}{tan\,\theta} = \frac{\cos\theta}{\sin\theta}$

GB AbcvZ¿‡jvi GKU fvjfv‡e j¶¨ Ki‡jB eS‡Z cvi‡e †h, †Kej sin , cos Gi AbcvZ g‡b ivL‡jB Ab¨¿‡jv ‡ei Kiv hvq|

(wKš mveavb! gL¯‡Ki‡e bv|Avgiv gL¯‡K bv ewj!)

GB AbcvZ evûi‡Z A we›`i Ae¯v‡bi Dci wbfi K‡i bv (wPÎ-1)| KviY, $\Delta ACB, \Delta A_1B_1C$ m`k|

ZvB, $\frac{AB}{A_1B_1} = \frac{AC}{A_1C} \Longrightarrow \frac{AB}{AC} = \frac{A_1B_1}{A_1C} = \sin\theta$

Abi¡cfv‡e evKx AbcvwZi Rb¨I cgvY Kiv hvq †h, GB AbcvZ ïagvÎ †Kv‡Yi gv‡bi Dci wbfi K‡i| A_vr †Kvb †Kv‡Yi gvb Rvbv _vK‡j Zvi wÎ‡KvYwgwZK AbcvZ wbYq Kiv m¤e|

GLb Avgiv †`Le †Kvb mg‡KvYx wÎf‡R (cieZx‡Z Ab¨vb¨ wÎf‡R) GB AbcvZ¿‡jv wKfv‡e KvR jvMv‡bv hvq| wPÎ-2 j¶¨ Ki|(GLv‡b,$\angle C = 90°$ ,$\sin\angle A$ †K $sinA$ †jLv n‡q‡Q Ges $A, B, C$ we›`i wecixZ evûi‡K a, b, c bvg †`Iqv n‡q‡Q|)Avgiv AbcvZ¿‡jv‡K Kv‡R jvM‡q evûû‡jvi gvb †ei Ki‡Z cwi|

sin A= a/c	cos A= b/c	tan A=a/b
sin B= b/c	cos B= a/c	tan B= b/a

Ges,

a= c sin A	a= c cos B	a= b tan A	
b= c sin B	b= c cos A	b= a tan B	
c= a csc A	c= a sec B	c=b csc B	c= b sec A

c_g QK †_‡K Avi I GKwU ˆewkó¨ †`Lv hvq; Zv nj †Kvb `wU †KvY A,B hw` m¶‡KvY nq Ges $A + B = 90°$ nq, Zvn‡j, sin A=cos B, cosA= sin B, tan A=cot B,cot A=tan B ; Abi¡cfv‡e Ab¨vb¨ Abcv‡Zi Rb¨I GB ˆewkó¨ KvhKi|

Avi I wKQ ˆewkó¨: ΔABC, $a^2+b^2=c^2$ Ges,

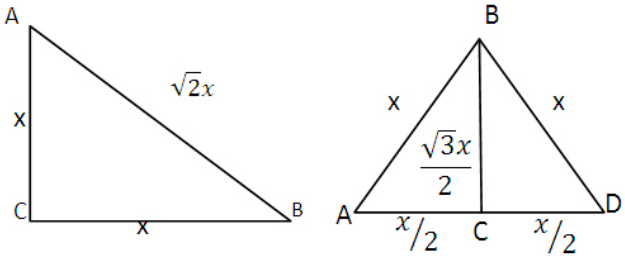
$sin^2A + cos^2A = \frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{a^2+b^2}{c^2} = \frac{c^2}{c^2} = 1 - - - (1)$

A_vr, $sin^2A + cos^2A = 1$ [$(sinA)^2$†K $\sin^2 A$ †jLv nq]

(1) †K h_vµ‡g $\sin^2 A$ I $\cos^2 A$ Øviv fvM K‡i cvB,

$1 + cot^2A = csc^2$ Ges $tan^2A + 1 = sec^2A$

wKQ we‡kl †Kv‡Yi wÎ‡KvYwgwZK AbcvZ:



GLv‡b,3 bs wP‡Î $\angle A = \angle B = 45°$, d‡j $BC = CA = x$, Ges $AB^2 = x^2 + x^2 \Rightarrow AB = \sqrt{2}x$ A_vr, $\sin A = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$

$cosA = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$

GLv‡b,3 bs wP‡Î $\angle A = 60°, \angle B = 30°$,
d‡j $AB = x, AC = {}^{x}/_{2}$,
(Gi cgv‡Yi Rb¨ C †K D chš ewaZ Kwi hv‡Z $AC = CD$ nq I cgvY Ki $\Delta ACB \cong \Delta DCB$ Ges ΔABD mgevû d‡j, $AC = CD = \frac{x}{2}$|)

Ges $BC^2 = x^2 - \frac{x^2}{4} \Rightarrow BC = \frac{\sqrt{3}x}{2}$ A_vr, $\sin A = \frac{\frac{\sqrt{3}x}{2}}{x} = \frac{\sqrt{3}}{2} =$ $\cos B = \frac{\sqrt{3}}{2}$; $cosA = \frac{x/2}{x} = \frac{1}{2} = sinB = \frac{1}{2}$

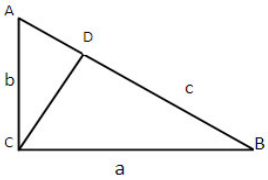
[KviY,এখানে $sinA = cosB, cosA = sinB$]

	0°	30°	45°	60°	90°
sin	0	${}^{1}/_{2}$	${}^{1}/_{\sqrt{2}}$	${}^{\sqrt{3}}/_{2}$	1
cos	1	${}^{\sqrt{3}}/_{2}$	${}^{1}/_{\sqrt{2}}$	${}^{1}/_{2}$	0

0°,90° Gi gvb ¯'vbv¼ c×wZ‡Z †ei Kiv hvq; Avi evKx gvb ¿‡jv GLb †ei Kivi `wqZ †Zvgv‡`iB w`¿jvg|GiKg QK K‡i gvb †ei K‡i me gvb †ei Ki‡j gvb¿‡jvi g‡a¨ wKQ PgrKvi wgj L‡R cv‡e|‡mUv †ei Kivi `wqZ GLb †Zvgv‡`i!

A‡bK wKQB †Zv †kLv nj|GLb wKQ mgm¨v mgvav‡bi cvjv|

mgm¨v:‡Kvb wÎfRABC G
$\angle A = 60°$ I $AB = 20$
n‡j BC = KZ? Le mnR!ZvB bv?
sin A=a/c ⇒a=c sin A
⇒a =20 sin60=$10\sqrt{3}$



mgm¨wU‡K Avi I GKU KwVb Kiv hvK, CD, ABGi Dci j¤ n‡j BD =KZ?

cos B=BD/a ⇒BD=a cos B=$10\sqrt{3} * \frac{\sqrt{3}}{2} = 15$

সমস্যা: $\cos 0° . \cos 1° . \cos 2° \cdots \cos 149° \cos 150° =$?

আশা করি, চালাকিটা ধারতে পেরেছ। $\cos 90° = 0$। ফলে, পুরোটাই 0.

Avi I A‡bK PgrKvi mgm¨vi mgvavb wÎ‡KvYwgwZ e¨envi K‡i Kiv hvq|

Note for Bangladesh Math Olympiad 2009:
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