Cuisinart
WÓDKA
[VODKA]
VISIONS of JAPAN TOMATSU Shomei
SNOW CRYSTALS
FOTOGRAFIE UND DAS UNSICHTBARE
DER KLEINE STOWASSER
MATHEMATICS
MAN AND SPACE
DRUGS
STACK-ON


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SNOW CRYSTALS
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STOWASSER
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Class Today

- Project 1 is going out today
- Read Szeliski 2.1, especially 2.1.5
- More about projections
- Light, color, biological vision

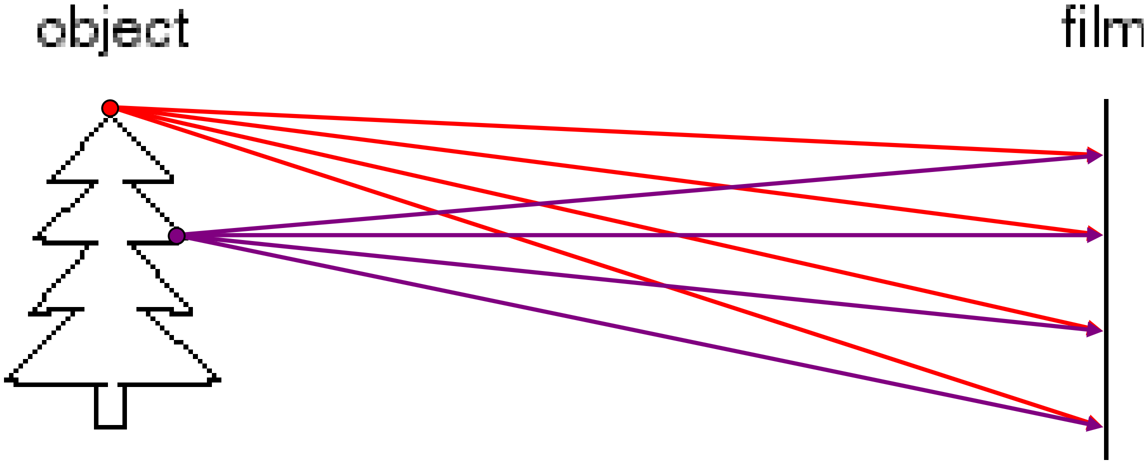
The Geometry of Image Formation

Mapping between image and world coordinates

- Pinhole camera model
- Projective geometry
  - Vanishing points and lines
- Projection matrix

What do you need to make a camera from scratch?

Image formation

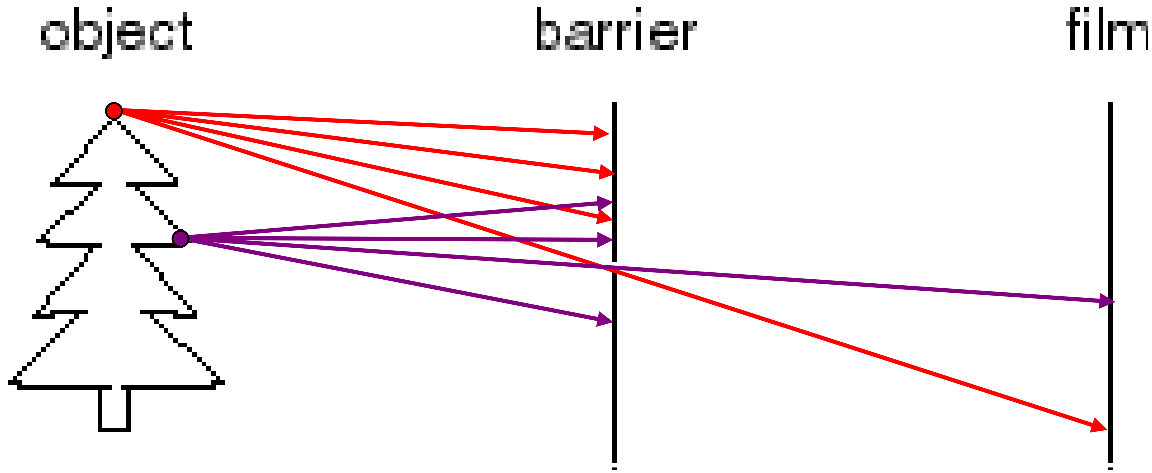


Let's design a camera

- Idea 1: put a piece of film in front of an object
- Do we get a reasonable image?

Slide source: Seitz

Pinhole camera

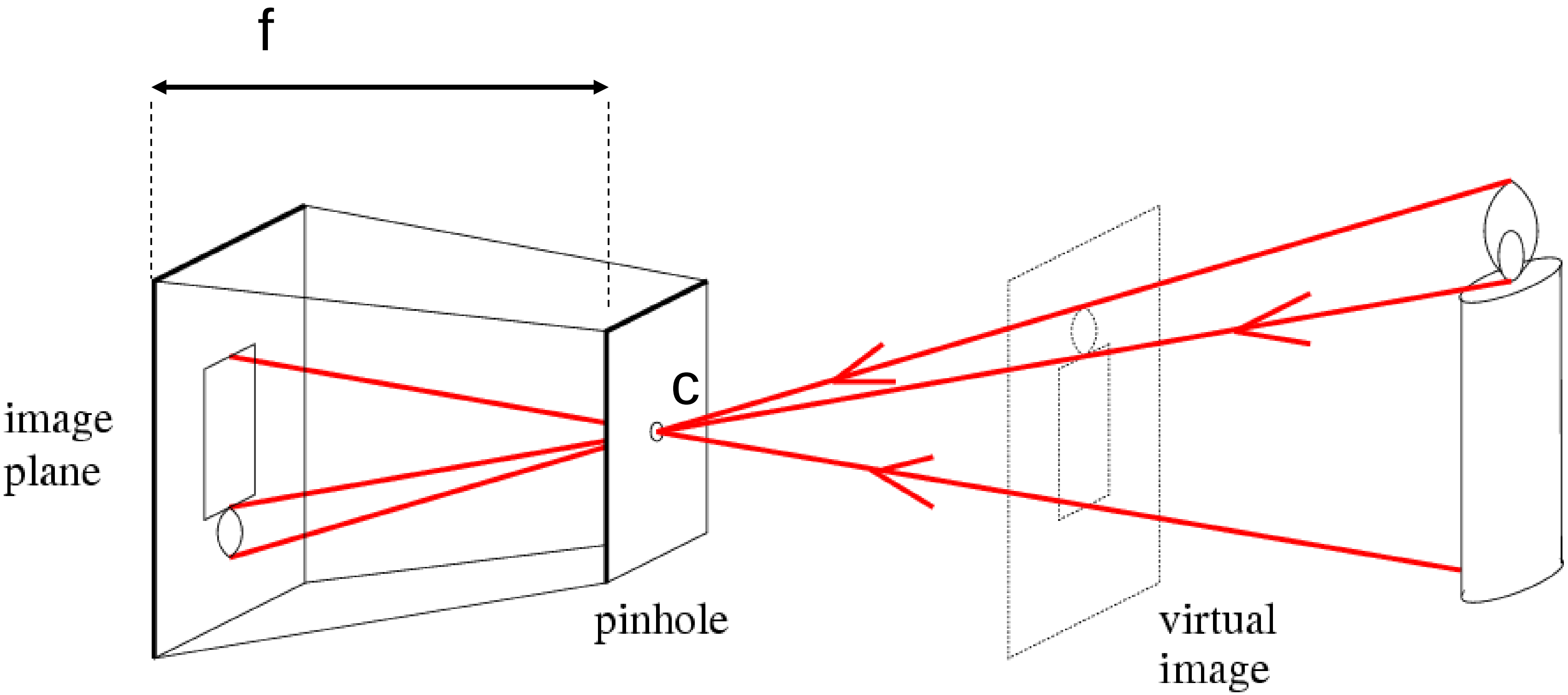


Idea 2: add a barrier to block off most of the rays

- This reduces blurring
- The opening known as the **aperture**

Slide source: Seitz

Pinhole camera



f = focal length
c = center of the camera

Figure from Forsyth

Camera obscura: the pre-camera

- Known during classical period in China and Greece (e.g. Mo-Ti, China, 470BC to 390BC)

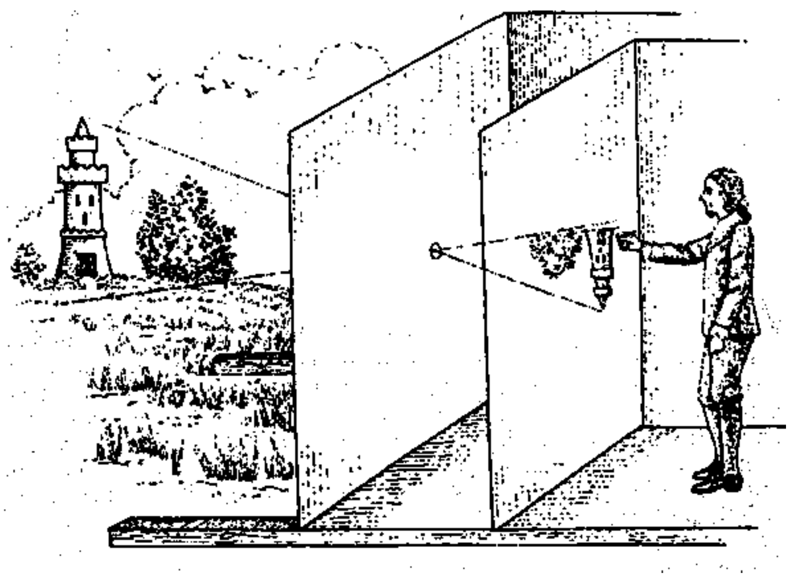
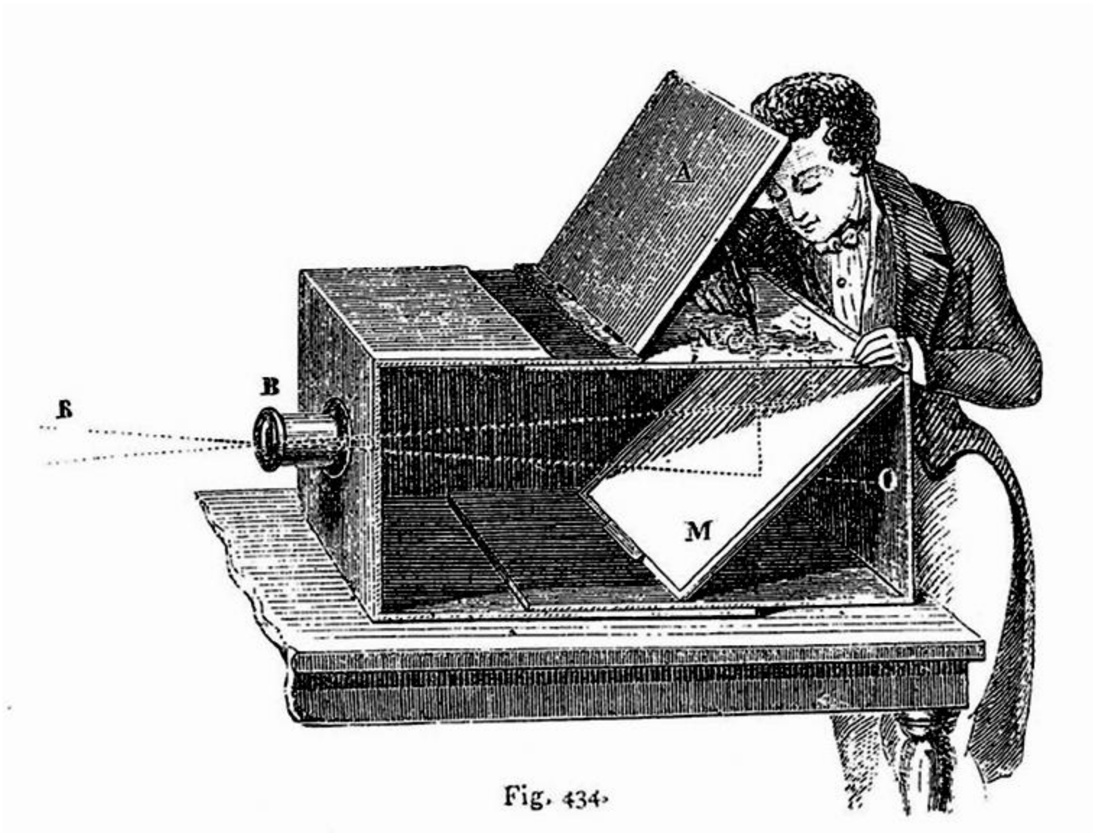


Illustration of Camera Obscura

Freestanding camera obscura at UNC Chapel Hill

Photo by Seth Ilys

Camera Obscura used for Tracing

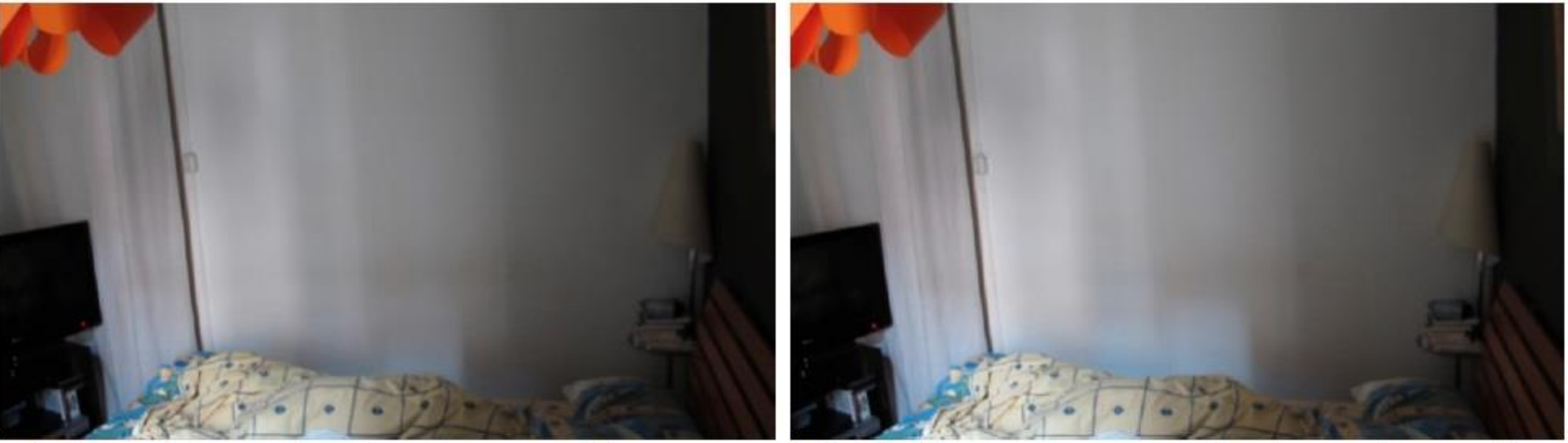


Lens Based Camera Obscura, 1568

Accidental Cameras

Accidental Pinhole and Pinspeck Cameras
Revealing the scene outside the picture.
Antonio Torralba, William T. Freeman

Accidental Cameras



a) Input (occluder present) b) Reference (occluder absent)

c) Difference image (b-a) d) Crop upside down e) True view

First Photograph

Oldest surviving photograph

- Took 8 hours on pewter plate

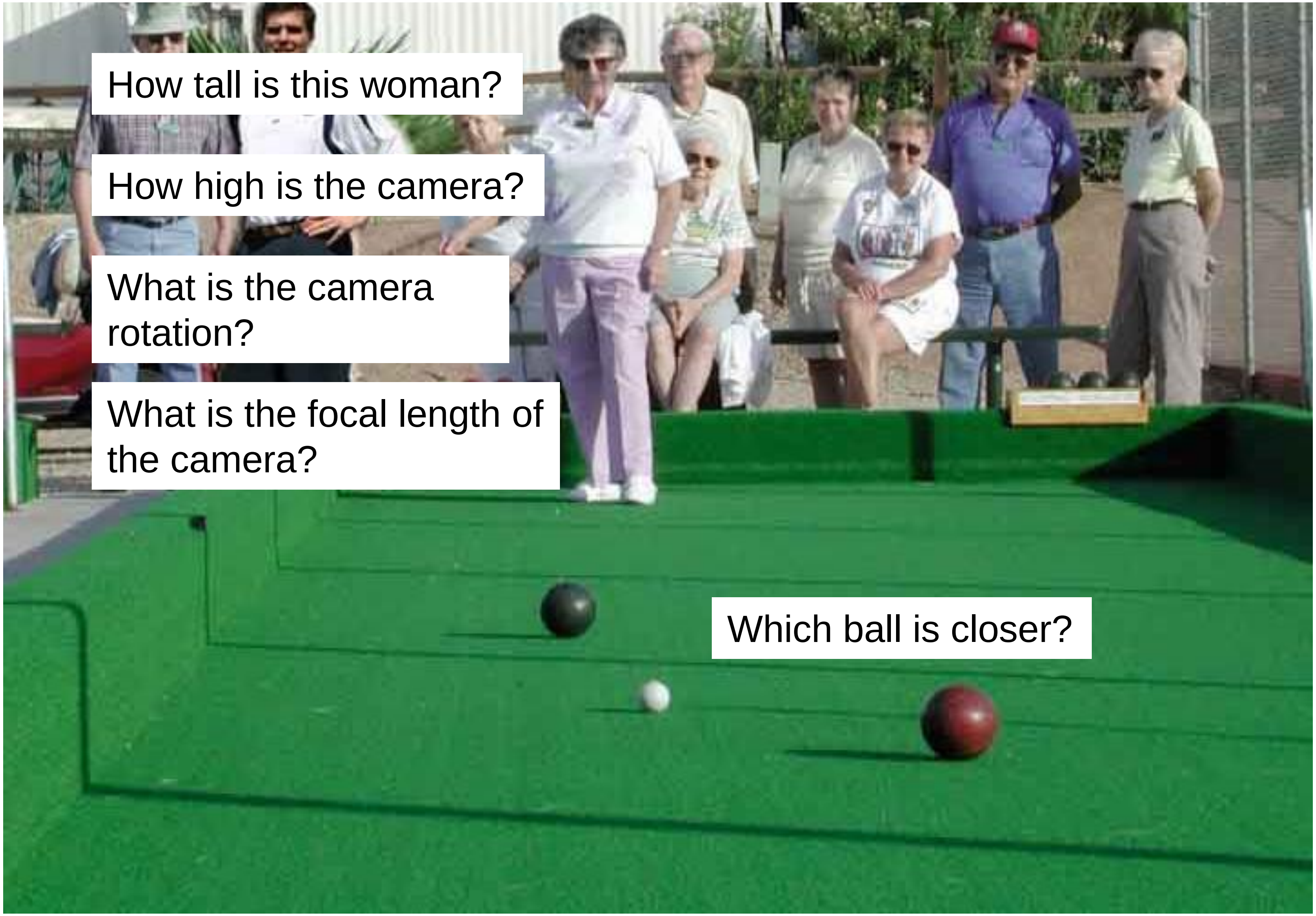
Photograph of the first photograph

Joseph Niepce, 1826

Stored at UT Austin

Niepce later teamed up with Daguerre, who eventually created Daguerrotypes

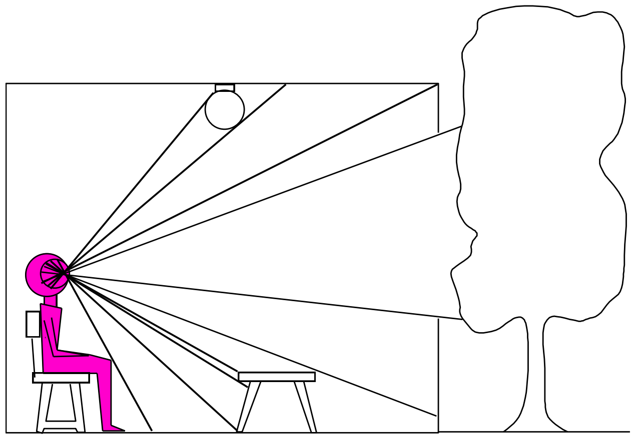
Camera and World Geometry

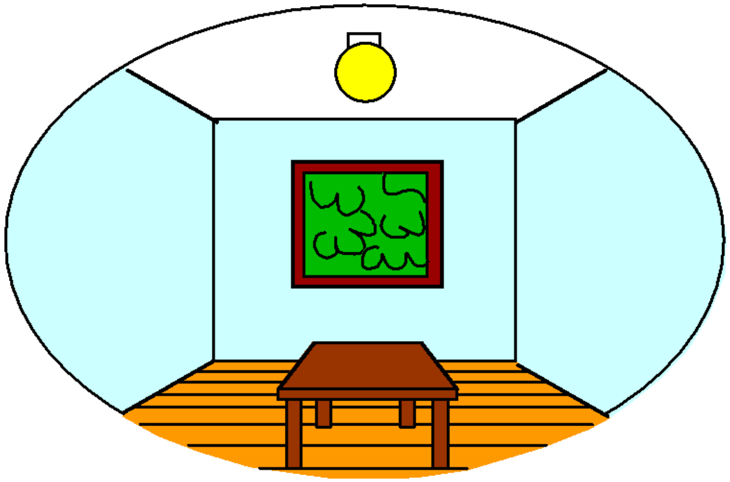


Dimensionality Reduction Machine (3D to 2D)

3D world

2D image





Point of observation

Slide source: Seitz

Projection can be tricky…

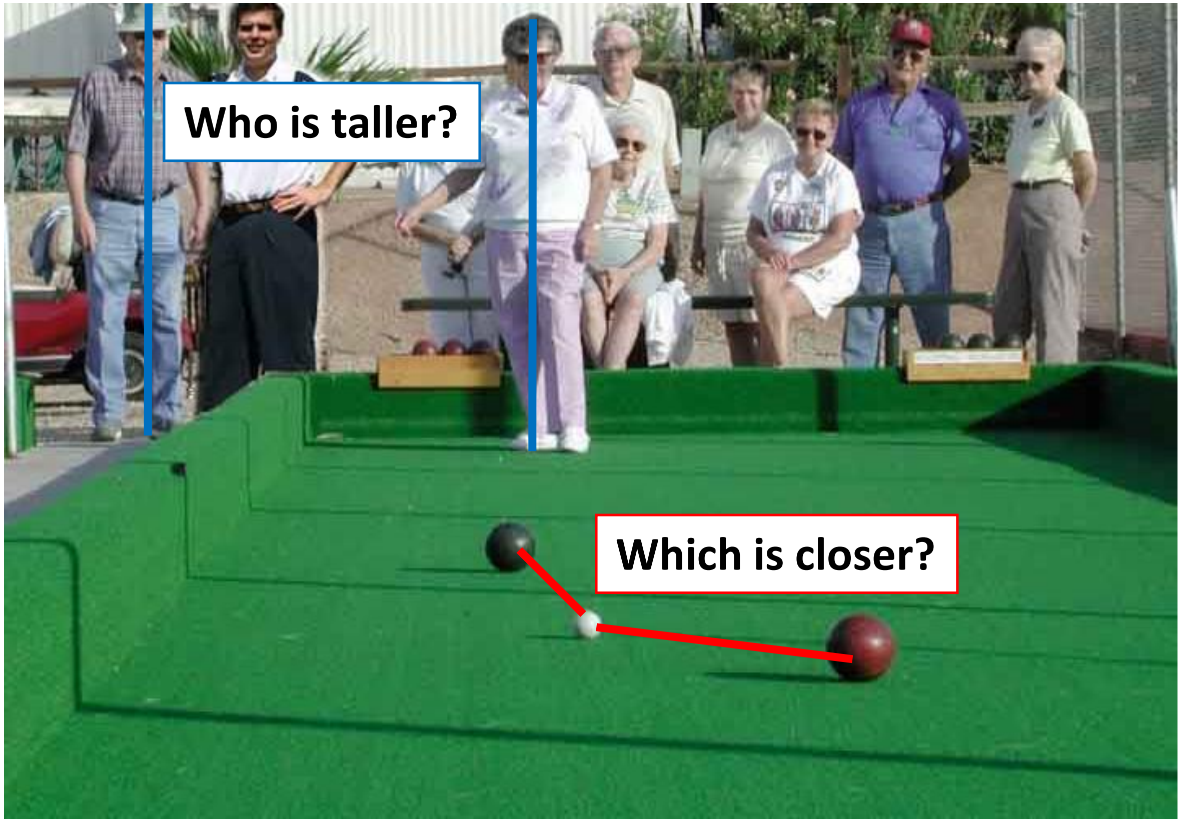
Slide source: Seitz

Projection can be tricky…

Projective Geometry

What is lost?

- Length



Length and area are not preserved

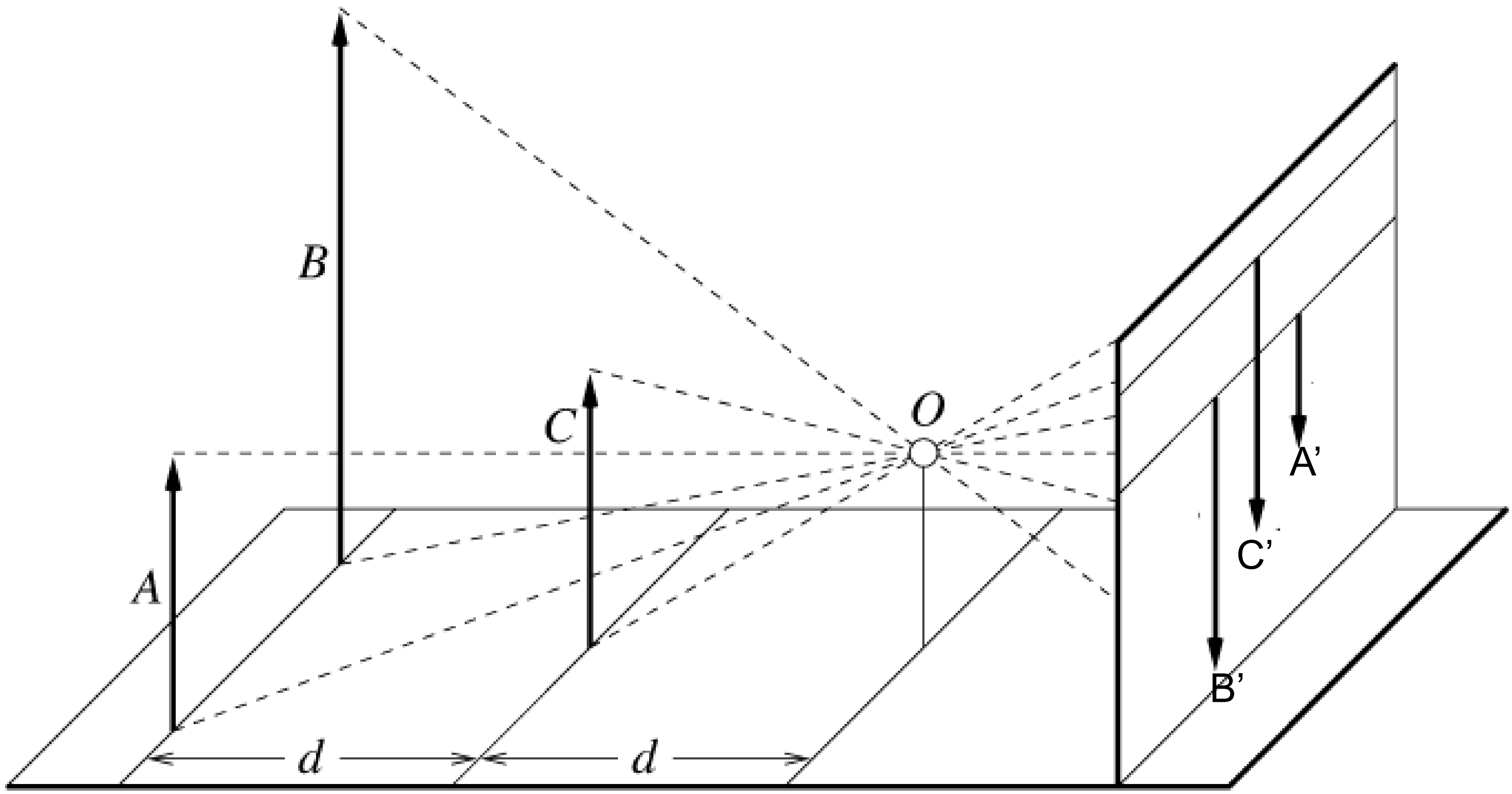
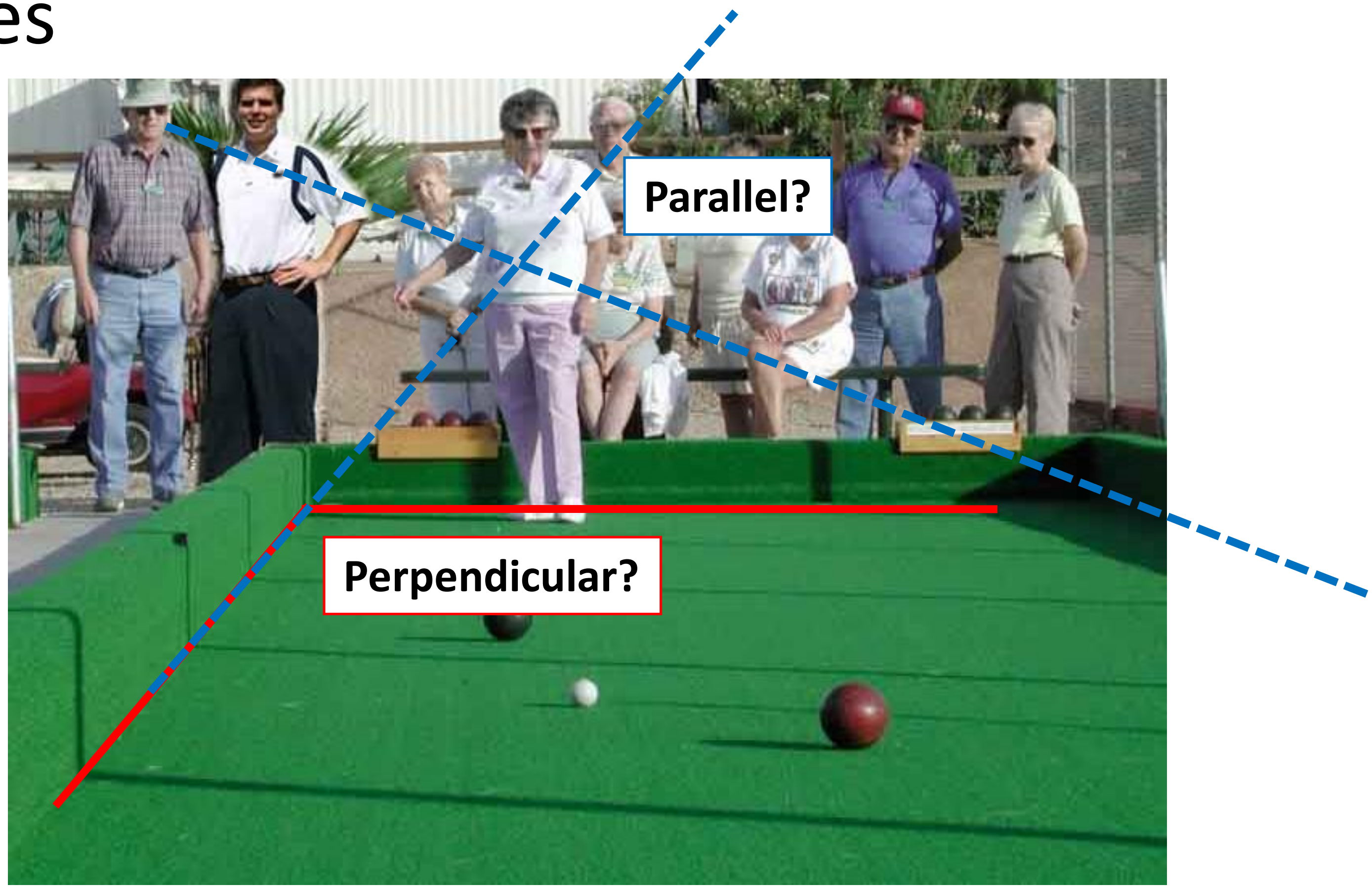


Figure by David Forsyth

Projective Geometry

What is lost?

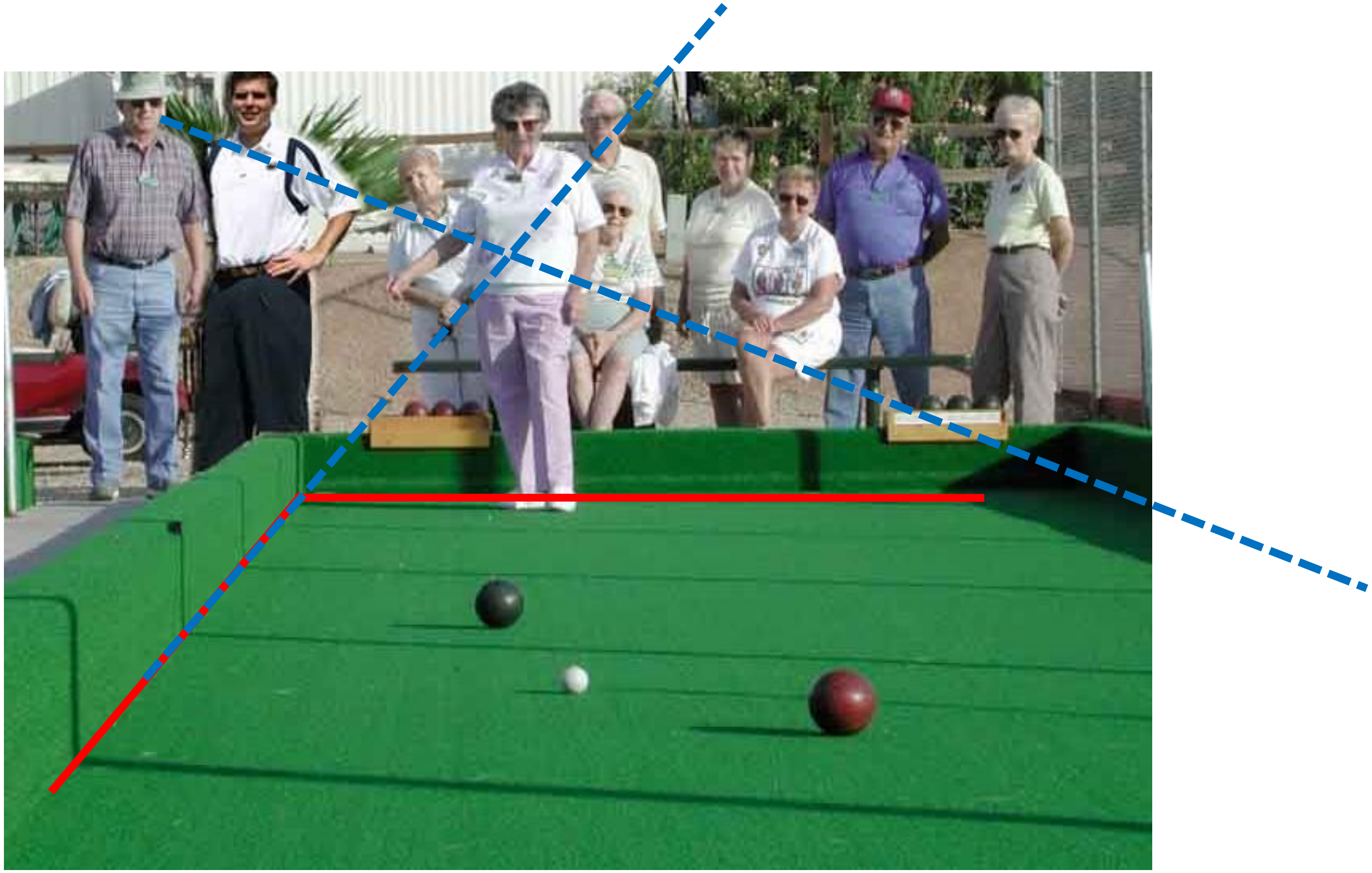
- Length
- Angles



Projective Geometry

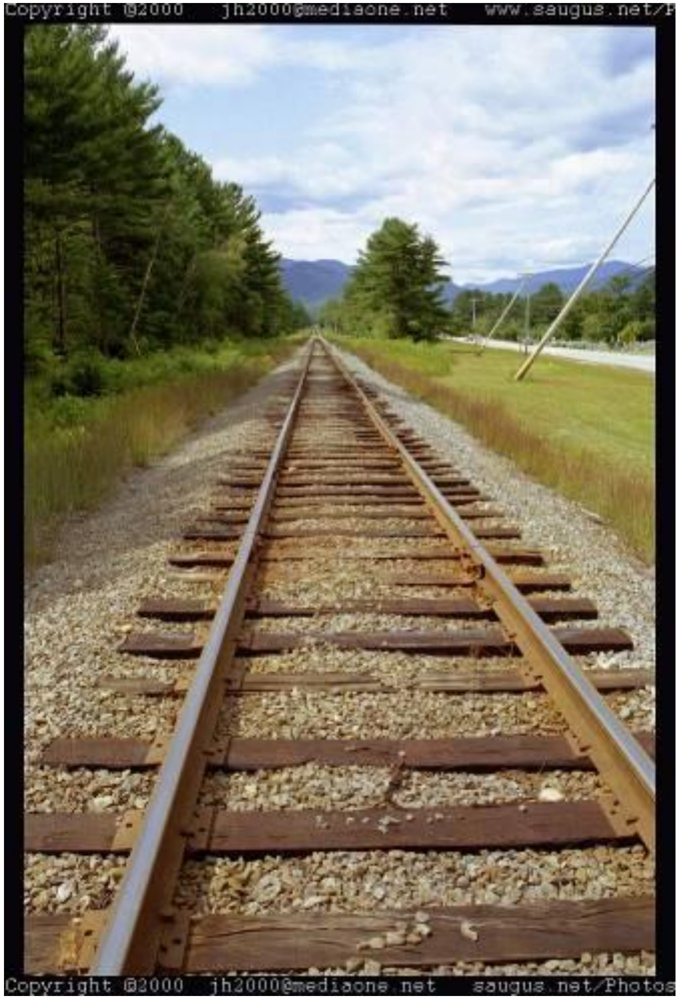
What is preserved?

- Straight lines are still straight

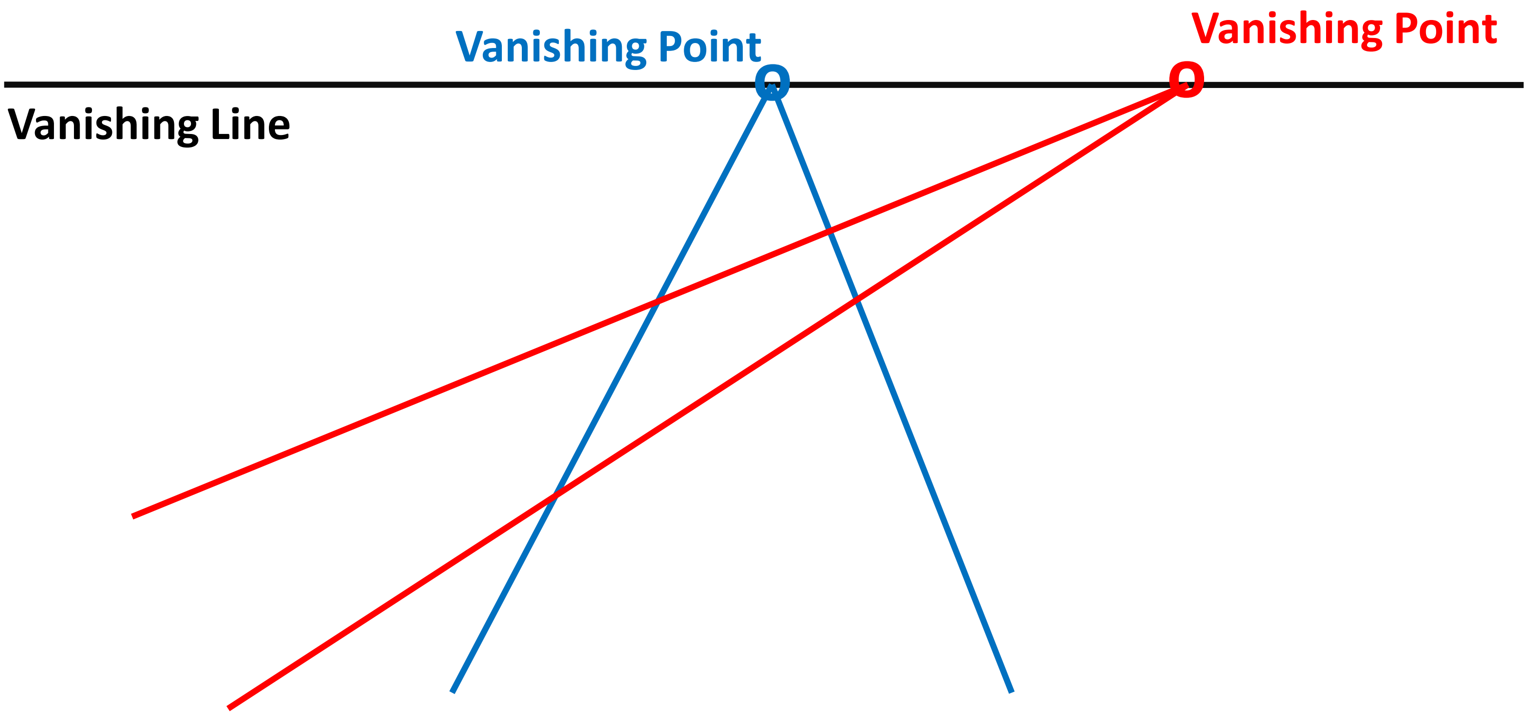


Vanishing points and lines

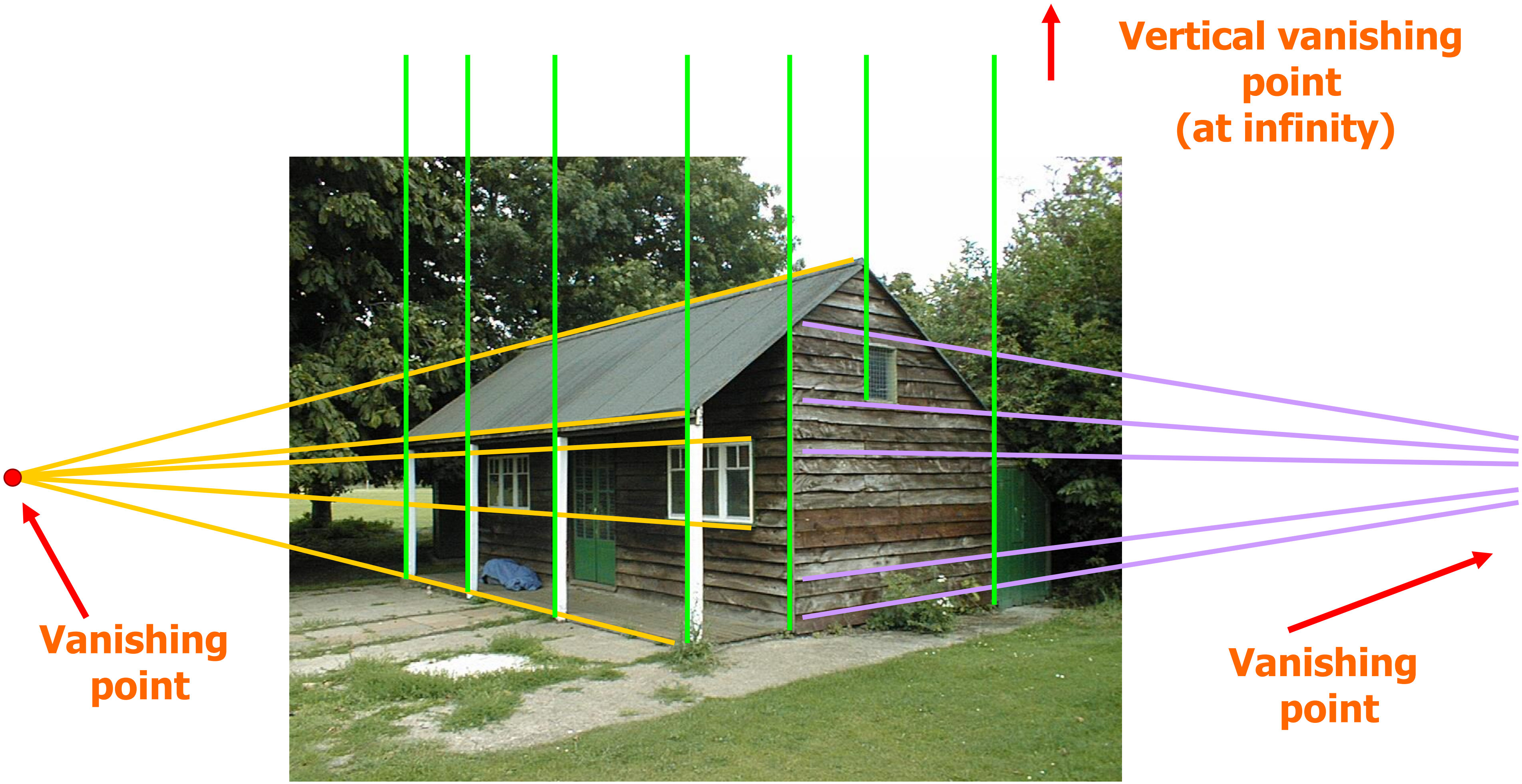
Parallel lines in the world intersect in the image at a "vanishing point"



Vanishing points and lines

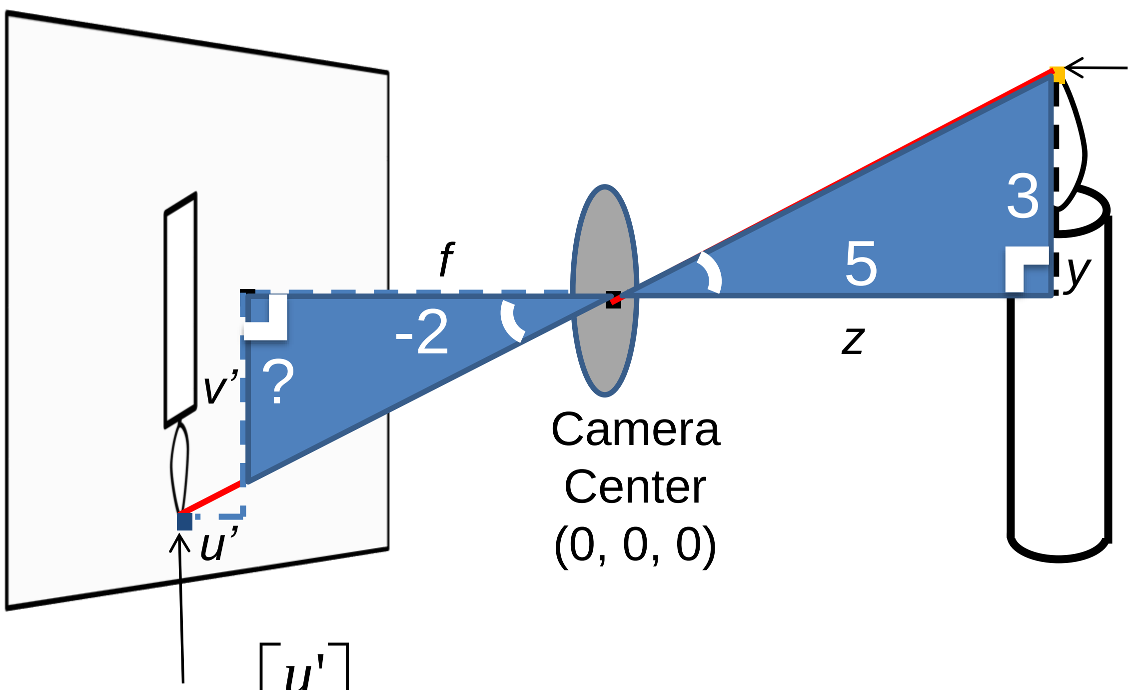


Vanishing points and lines



Slide from Efros, Photo from Criminisi

Projection: world coordinates→image coordinates



$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$x = \begin{bmatrix} u' \\ v' \end{bmatrix}$$

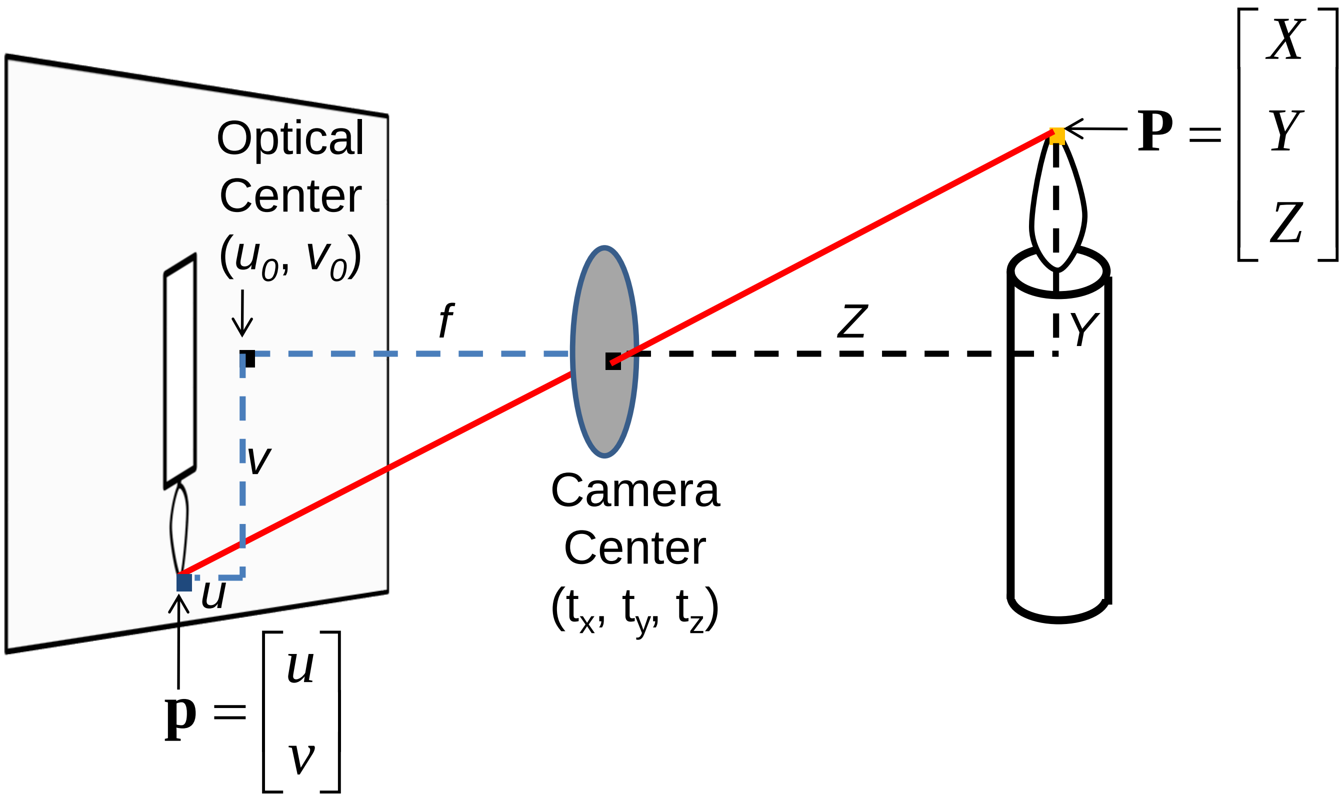
If X = 2, Y = 3,
Z = 5, and f = 2
What are U and V?

$$\frac{v'}{-f} = \frac{y}{z}$$

$$u' = -x * \frac{f}{z} \qquad u' = -2 * \frac{2}{5}$$

$$v' = -y * \frac{f}{z} \qquad v' = -3 * \frac{2}{5}$$

Projection: world coordinates→image coordinates



How do we handle the general case?

Interlude: why does this matter?

Relating multiple views

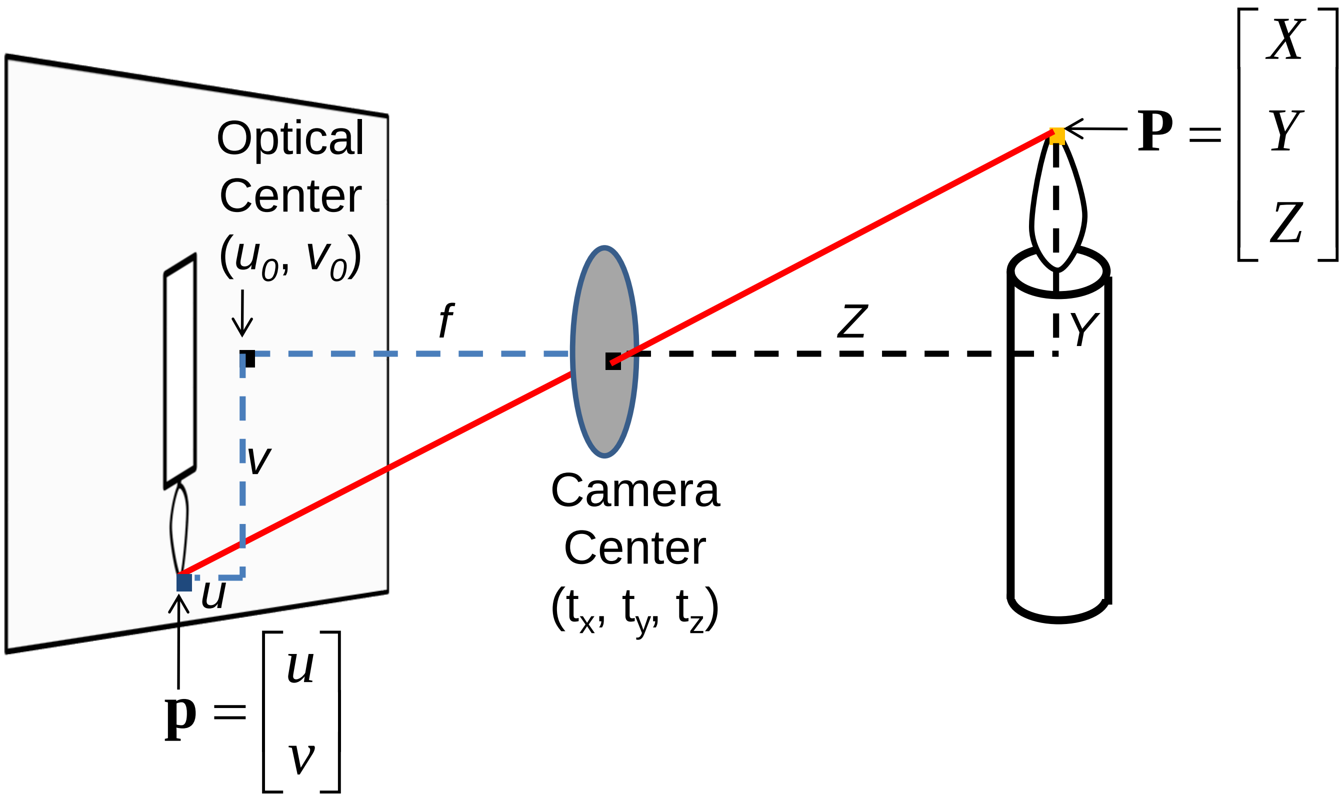
Photo Tourism

Exploring photo collections in 3D

Noah Snavely Steven M. Seitz Richard Szeliski
University of Washington *Microsoft Research*

SIGGRAPH 2006

Projection: world coordinates→image coordinates



How do we handle the general case?

Homogeneous coordinates

Conversion

Converting to *homogeneous* coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Homogeneous coordinates

Invariant to scaling

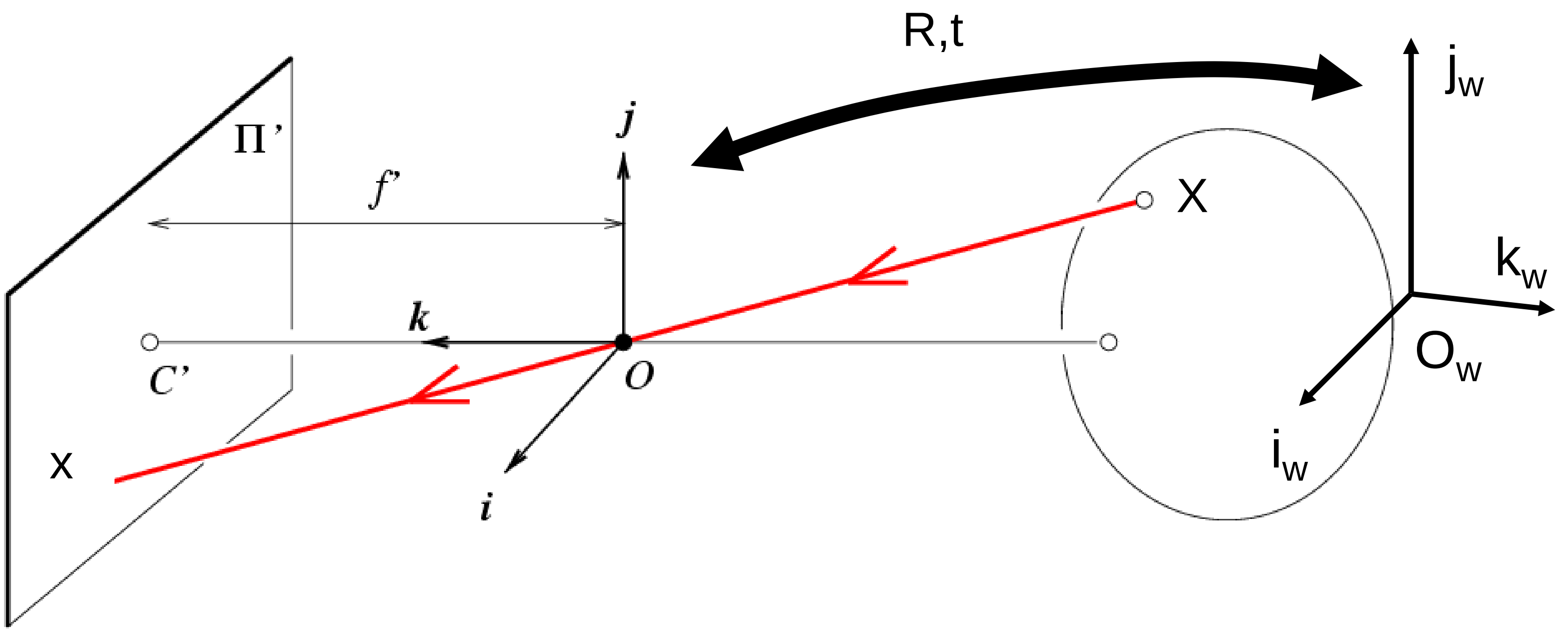
$$k\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} kx \\ ky \\ kw \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{kx}{kw} \\ \frac{ky}{kw} \end{bmatrix} = \begin{bmatrix} \frac{x}{w} \\ \frac{y}{w} \end{bmatrix}$$

Homogeneous Coordinates

Cartesian Coordinates

Point in Cartesian is ray in Homogeneous

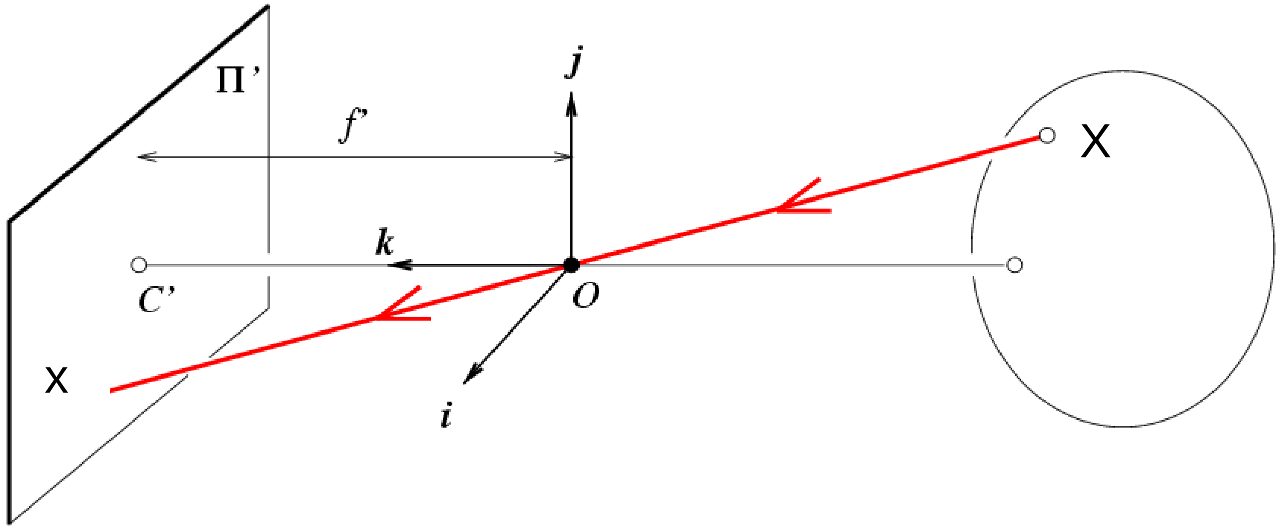
Projection matrix



$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

x: Image Coordinates: (u,v,1)
K: Intrinsic Matrix (3x3)
R: Rotation (3x3)
t: Translation (3x1)
X: World Coordinates: (X,Y,Z,1)

Projection matrix



Intrinsic Assumptions

- Unit aspect ratio
- Optical center at (0,0)
- No skew

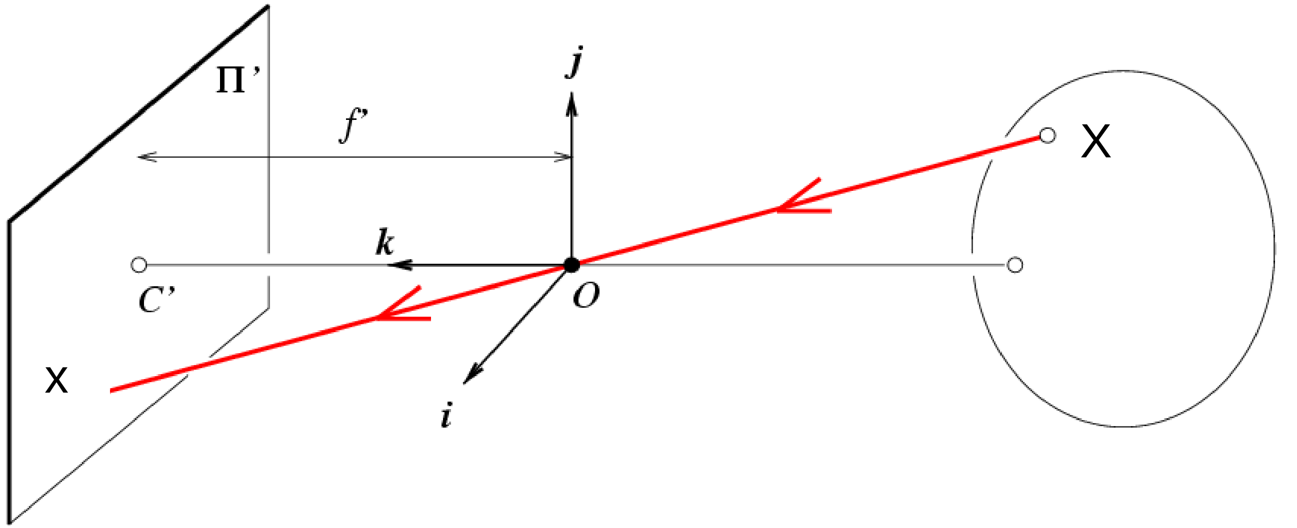
Extrinsic Assumptions

- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{X} \Rightarrow w\begin{bmatrix}u \\ v \\ 1\end{bmatrix} = \begin{bmatrix}\underbrace{\boxed{\begin{matrix}f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1\end{matrix}}}_{\mathbf{K}} & \begin{matrix}0 \\ 0 \\ 0\end{matrix}\end{bmatrix}\begin{bmatrix}x \\ y \\ z \\ 1\end{bmatrix}$$

Slide Credit: Savarese

Projection matrix



Intrinsic Assumptions

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- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{X} \Rightarrow w\begin{bmatrix}u\\v\\1\end{bmatrix} = \begin{bmatrix}f & 0 & 0 & 0\\0 & f & 0 & 0\\0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}$$

Remove assumption: known optical center

Intrinsic Assumptions

- Unit aspect ratio
- No skew

Extrinsic Assumptions

- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{X} \quad \Rightarrow \quad w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & u_0 & 0 \\ 0 & f & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Remove assumption: square pixels

Intrinsic Assumptions
- No skew

Extrinsic Assumptions
- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{X} \quad \Rightarrow \quad w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & 0 & u_0 & 0 \\ 0 & \beta & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Remove assumption: non-skewed pixels

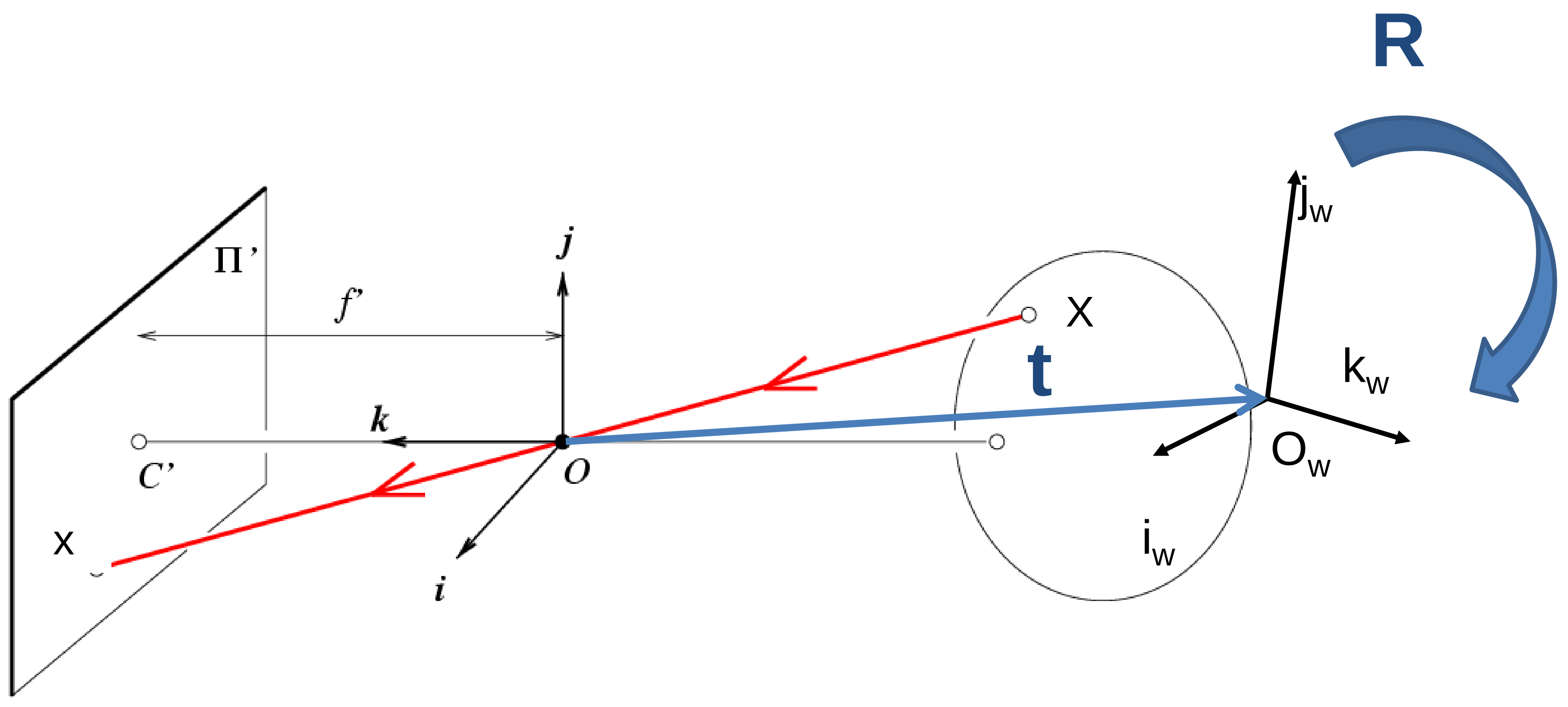
Intrinsic Assumptions

Extrinsic Assumptions
- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{X} \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 & 0 \\ 0 & \beta & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Note: different books use different notation for parameters

Oriented and Translated Camera



Allow camera translation

Intrinsic Assumptions

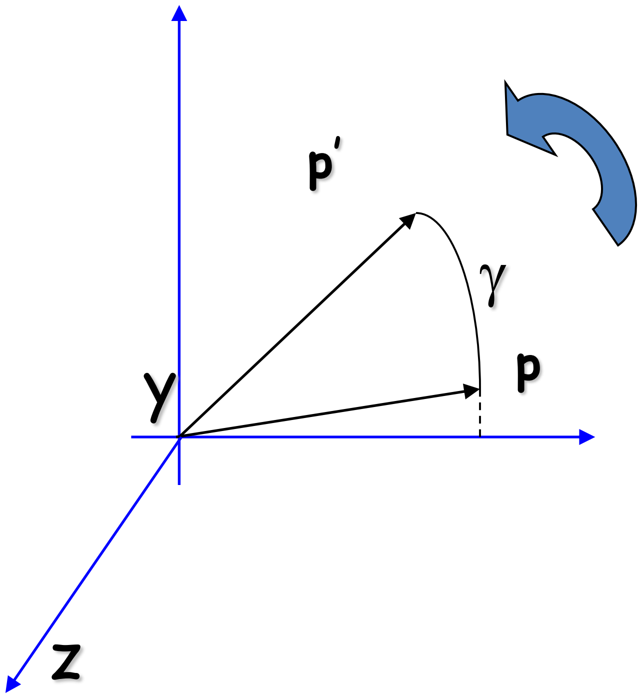
Extrinsic Assumptions
- No rotation

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{t}\end{bmatrix}\mathbf{X} \Rightarrow w\begin{bmatrix}u \\ v \\ 1\end{bmatrix} = \begin{bmatrix}\alpha & 0 & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z\end{bmatrix}\begin{bmatrix}x \\ y \\ z \\ 1\end{bmatrix}$$

Slide Credit: Saverese

3D Rotation of Points

Rotation around the coordinate axes, counter-clockwise:



$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$

$$R_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

$$R_z(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Allow camera rotation

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

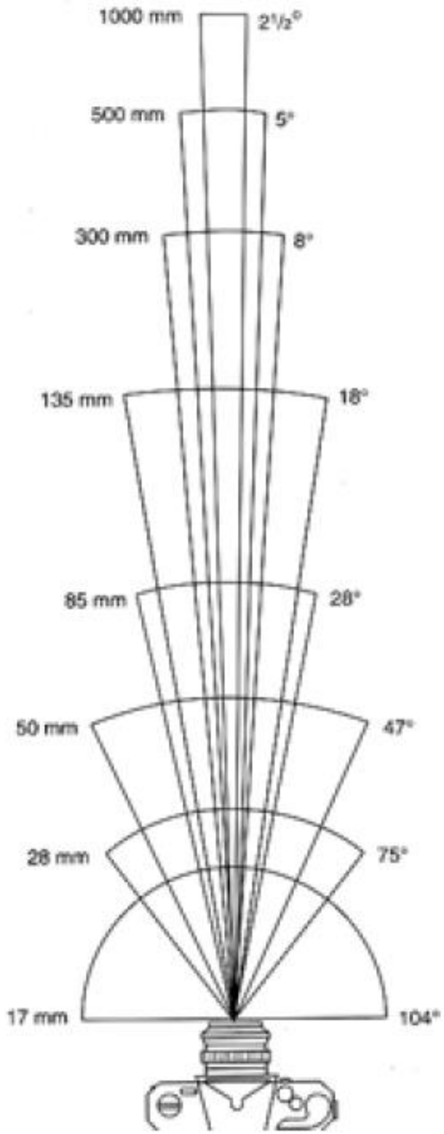
$$w\begin{bmatrix}u\\v\\1\end{bmatrix} = \begin{bmatrix}\alpha & s & u_0\\0 & \beta & v_0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}r_{11} & r_{12} & r_{13} & t_x\\r_{21} & r_{22} & r_{23} & t_y\\r_{31} & r_{32} & r_{33} & t_z\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}$$

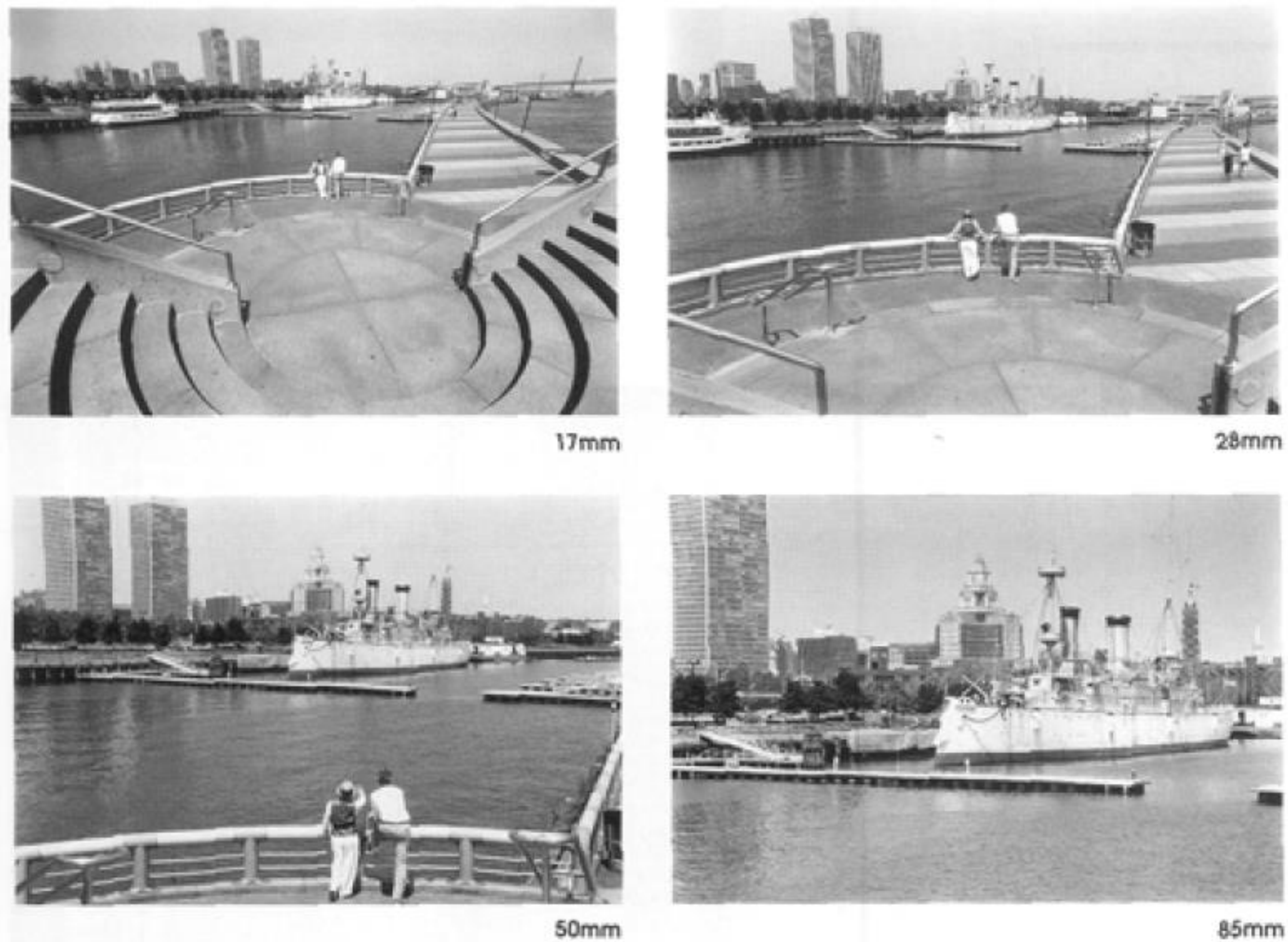
Degrees of freedom

$$\mathbf{x} = \mathbf{K}\left[\mathbf{R} \quad \mathbf{t}\right]\mathbf{X}$$

$$w\begin{bmatrix}u\\v\\1\end{bmatrix} = \overset{5}{\begin{bmatrix}\alpha & s & u_0\\0 & \beta & v_0\\0 & 0 & 1\end{bmatrix}}\overset{6}{\begin{bmatrix}r_{11} & r_{12} & r_{13} & t_x\\r_{21} & r_{22} & r_{23} & t_y\\r_{31} & r_{32} & r_{33} & t_z\end{bmatrix}}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}$$

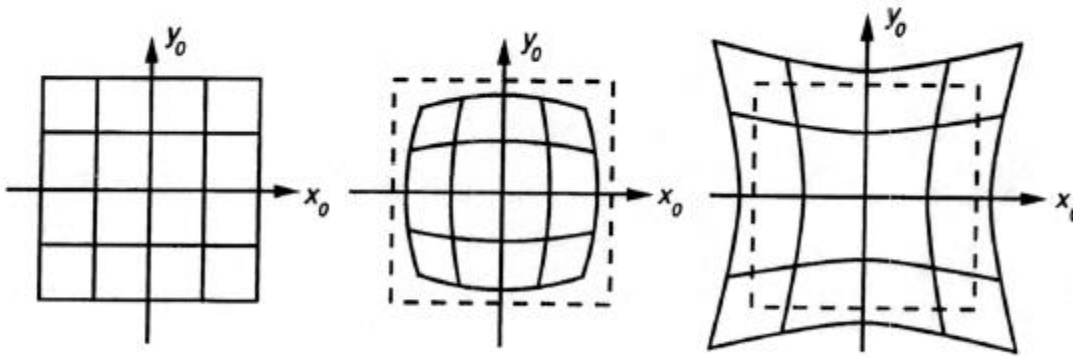
Field of View (Zoom, focal length)





From London and Upton

Beyond Pinholes: Radial Distortion



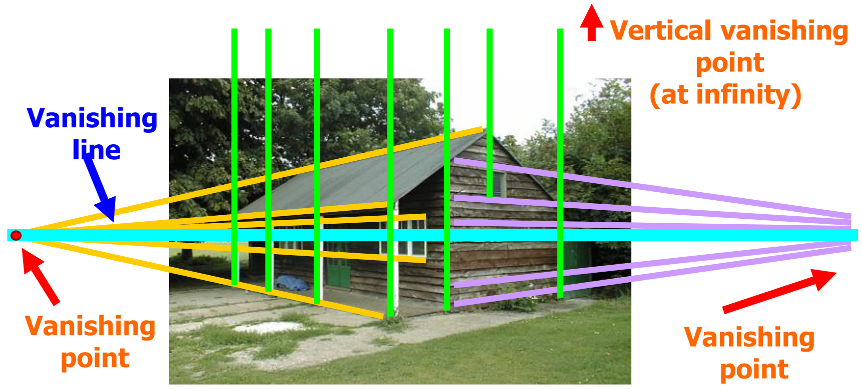
No Distortion   Barrel Distortion   Pincushion Distortion

Corrected Barrel Distortion

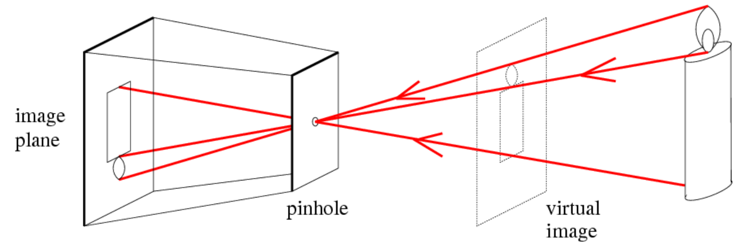
Image from Martin Habbecke

Things to remember

- Vanishing points and vanishing lines



- Pinhole camera model and camera projection matrix



$$\mathbf{x} = \mathbf{K}[\mathbf{R} \quad \mathbf{t}]\mathbf{X}$$

- Homogeneous coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Reminder: read your book

- Lectures have assigned readings
- Szeliski 2.1 and especially 2.1.5 cover the geometry of image formation

5 minute break

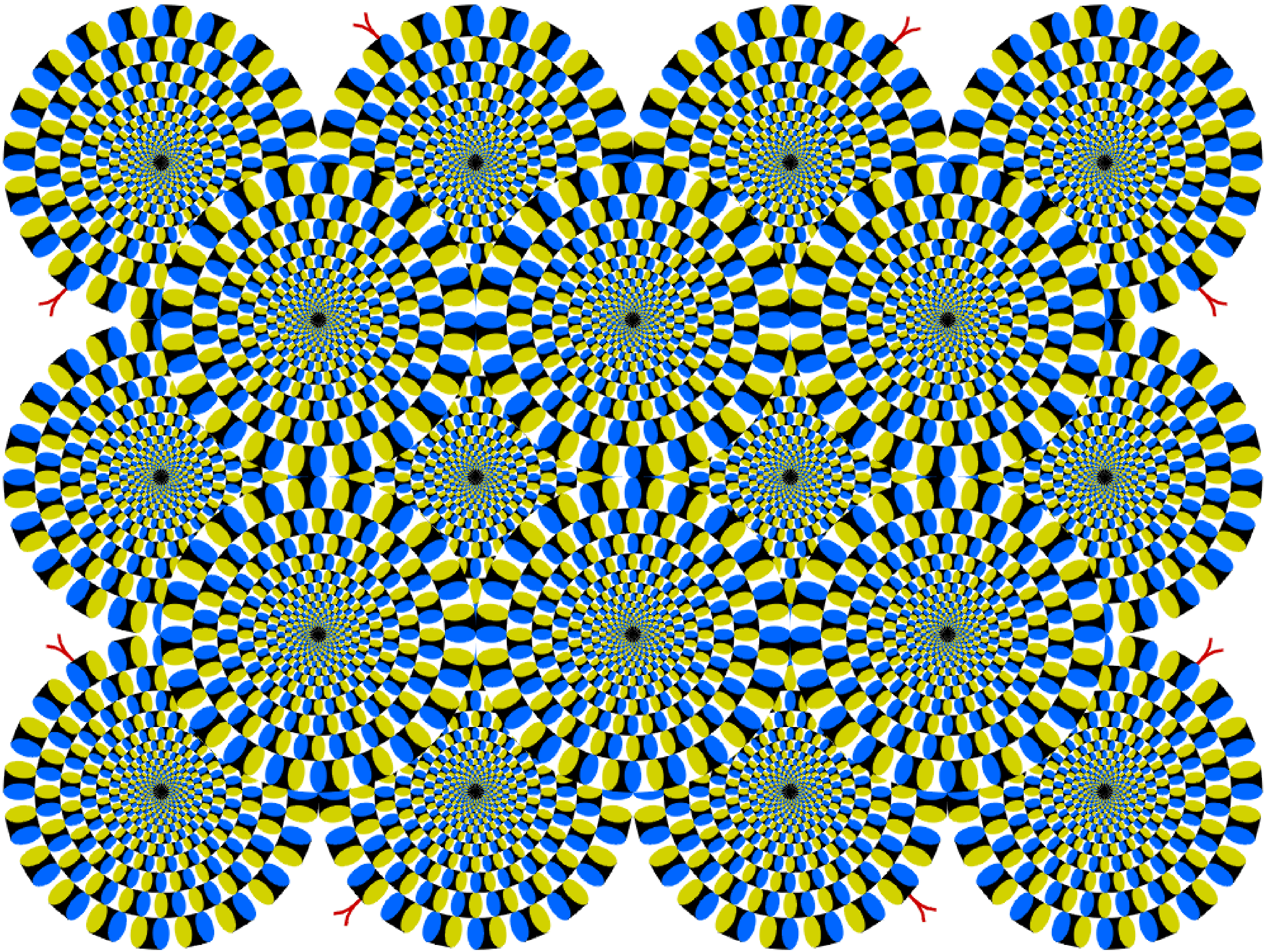
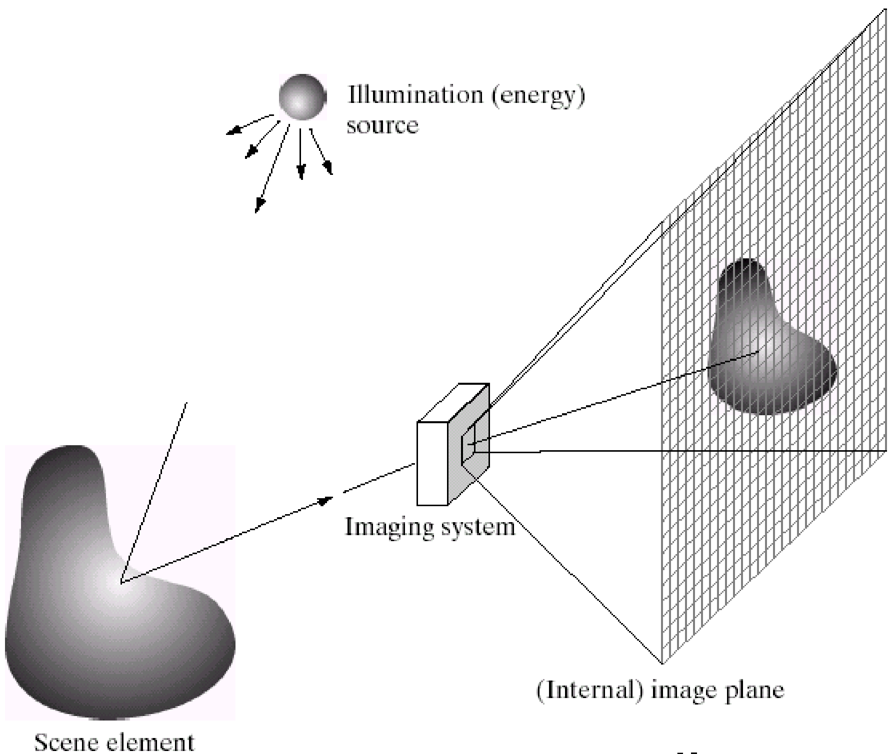
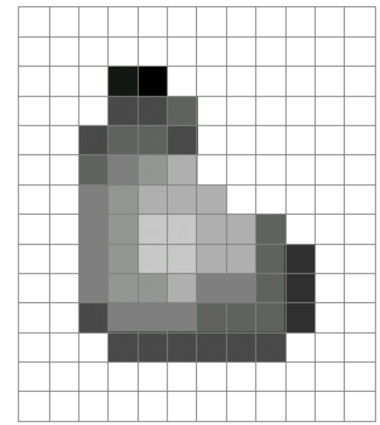


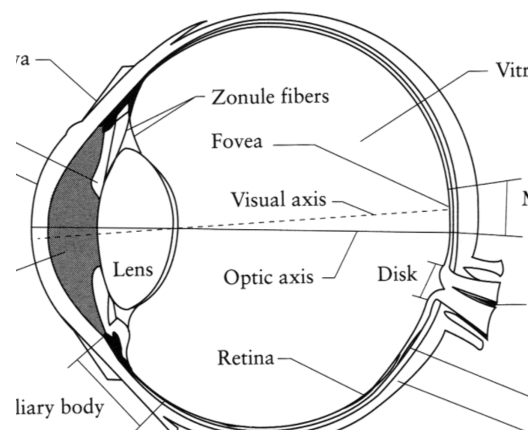
Image Formation



Film



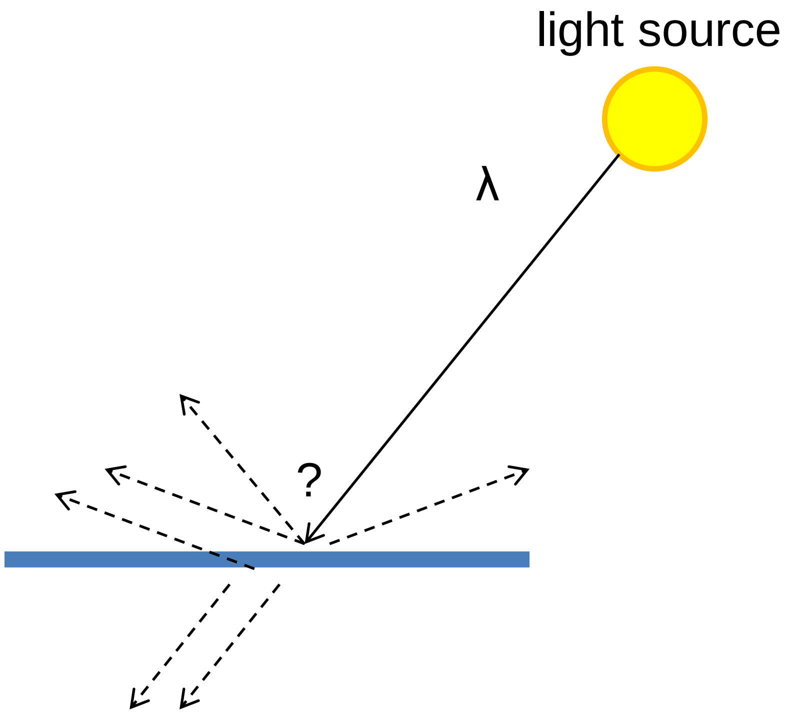
Digital Camera



The Eye

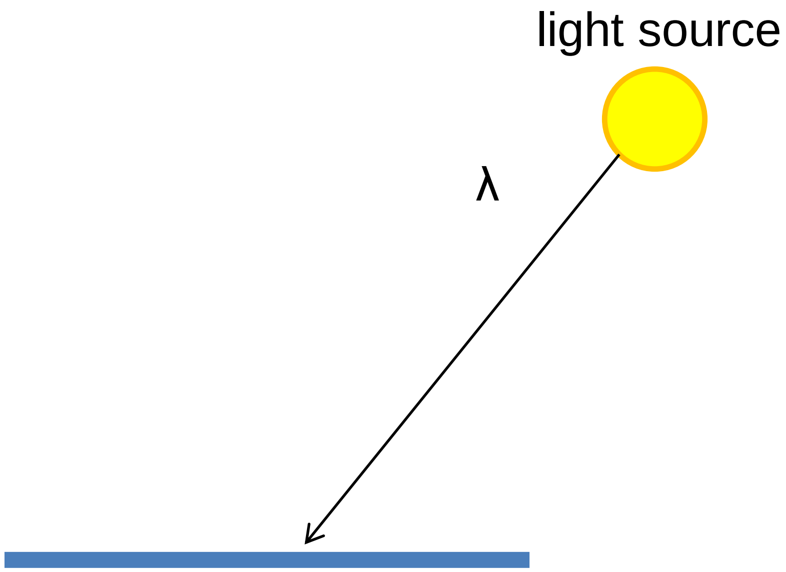
A photon’s life choices

- Absorption
- Diffusion
- Reflection
- Transparency
- Refraction
- Fluorescence
- Subsurface scattering
- Phosphorescence
- Interreflection



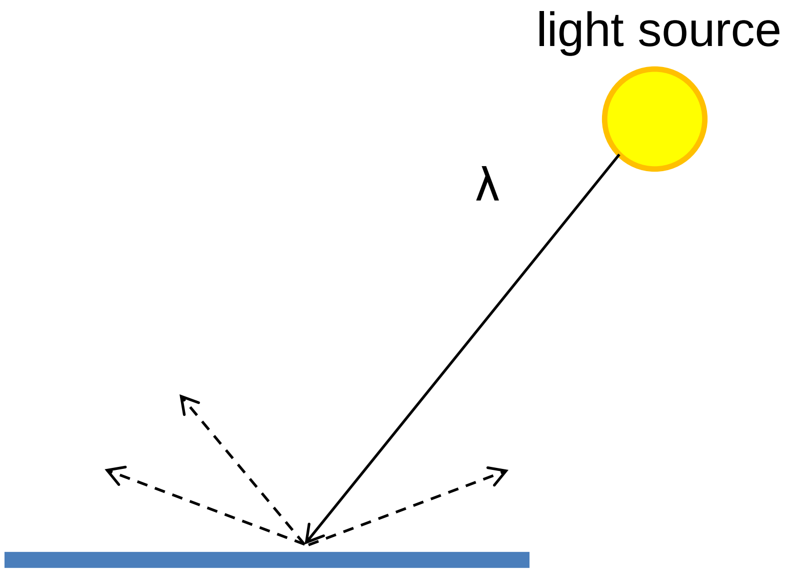
A photon’s life choices

- **Absorption**
- Diffusion
- Reflection
- Transparency
- Refraction
- Fluorescence
- Subsurface scattering
- Phosphorescence
- Interreflection



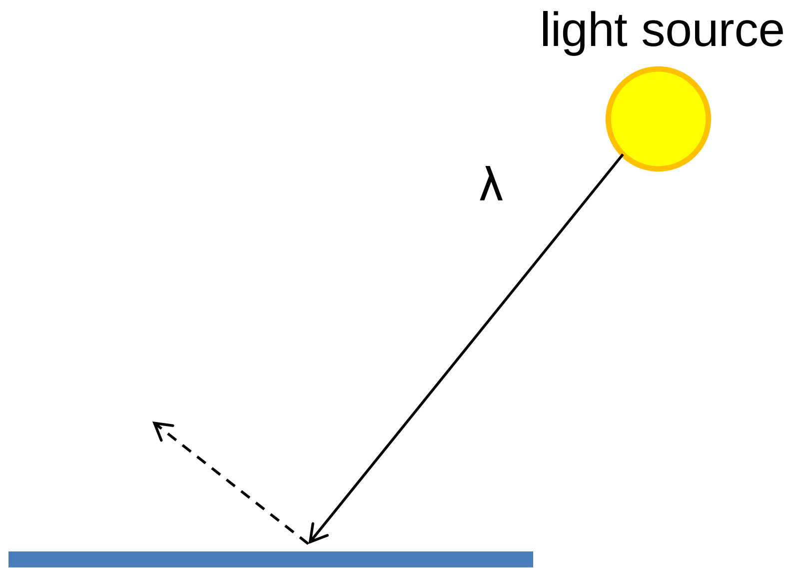
A photon's life choices

- Absorption
- **Diffuse Reflection**
- Reflection
- Transparency
- Refraction
- Fluorescence
- Subsurface scattering
- Phosphorescence
- Interreflection



A photon’s life choices

- Absorption
- Diffusion
- **Specular Reflection**
- Transparency
- Refraction
- Fluorescence
- Subsurface scattering
- Phosphorescence
- Interreflection



A photon's life choices

- Absorption
- Diffusion
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- **Transparency**
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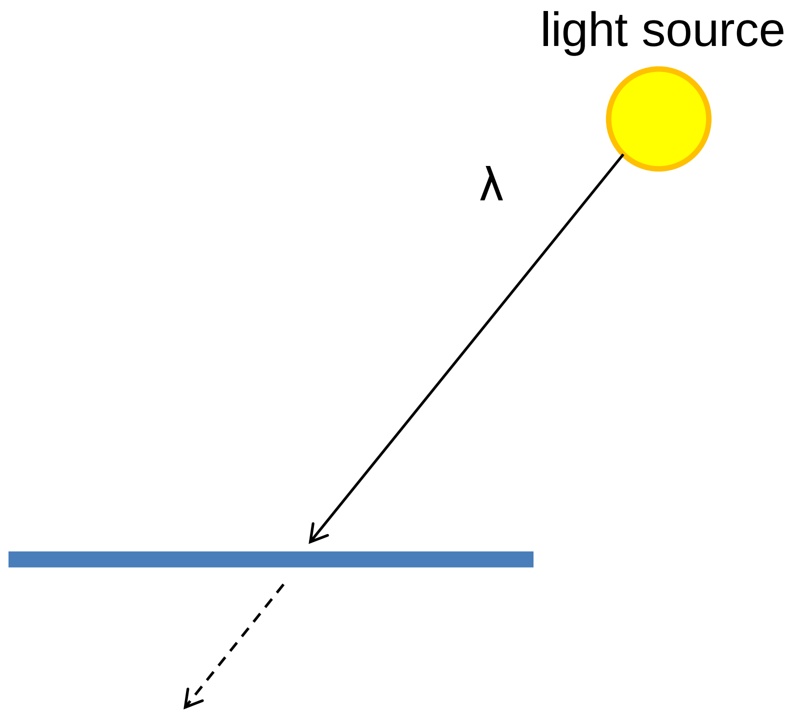

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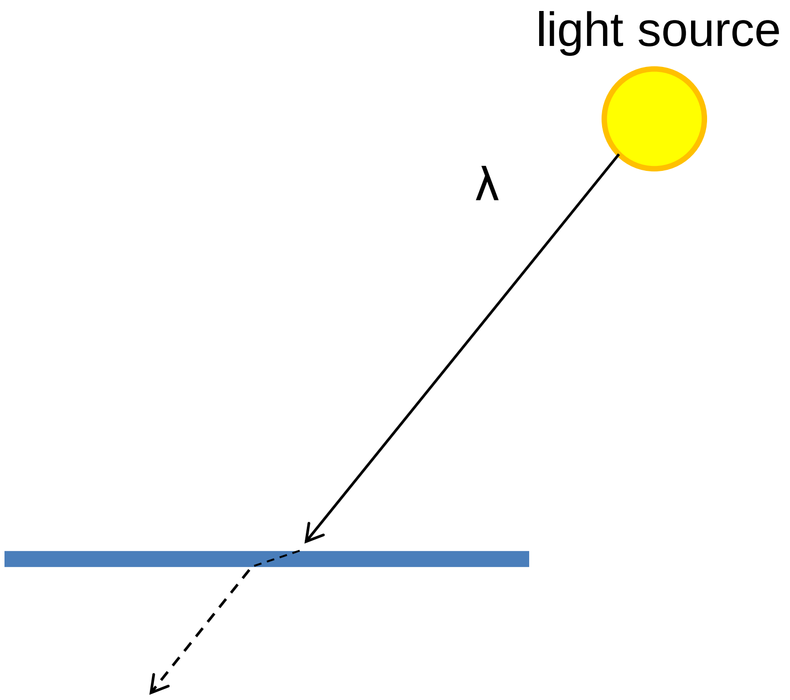

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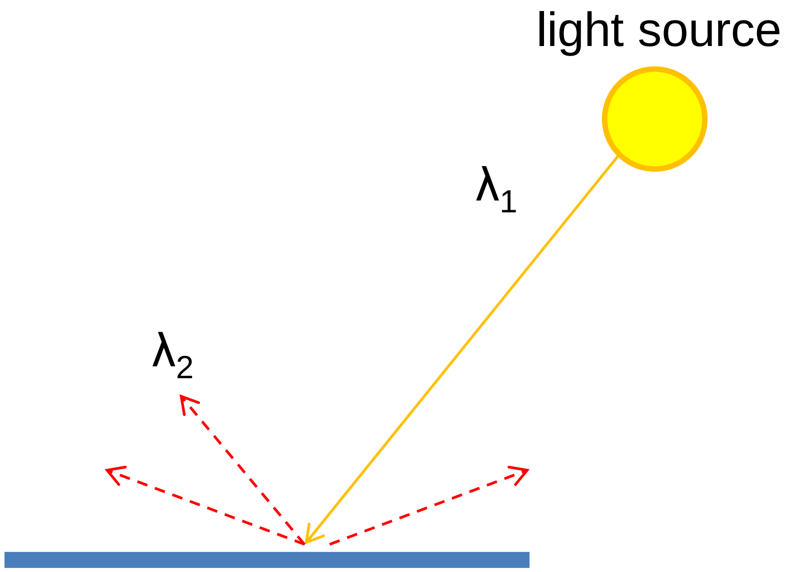

A photon's life choices

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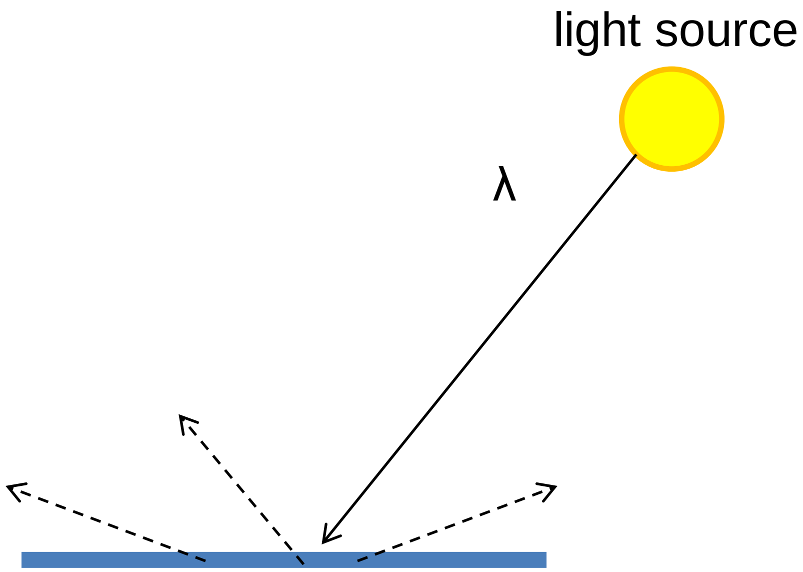

A photon's life choices

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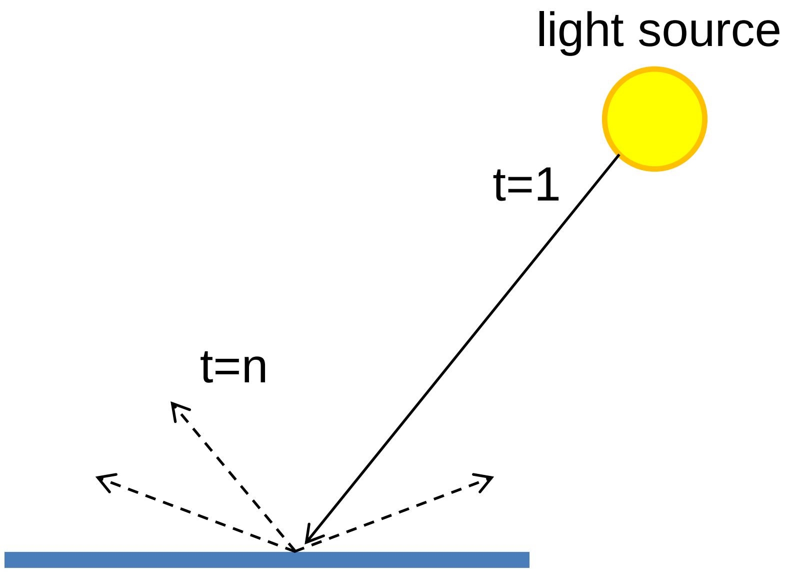
- Subsurface scattering
- **Phosphorescence**
- Interreflection



A photon's life choices

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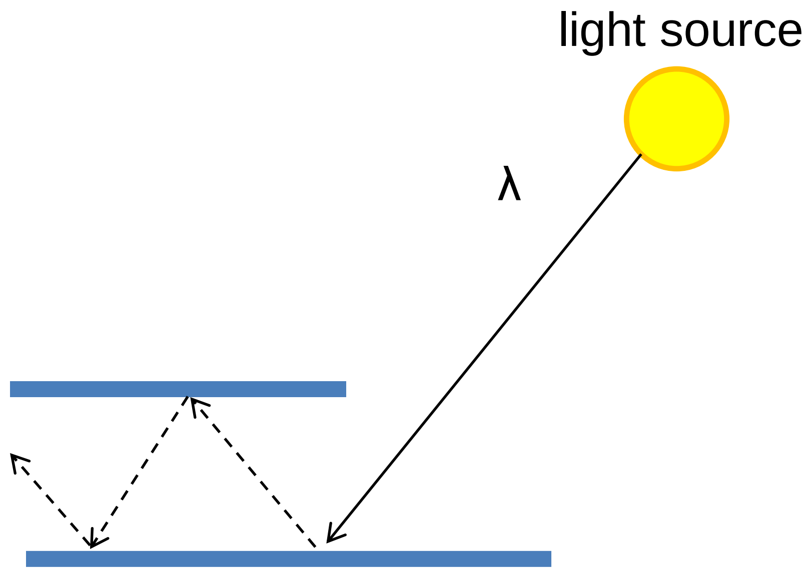
- Phosphorescence
- **Interreflection**



(Specular Interreflection)

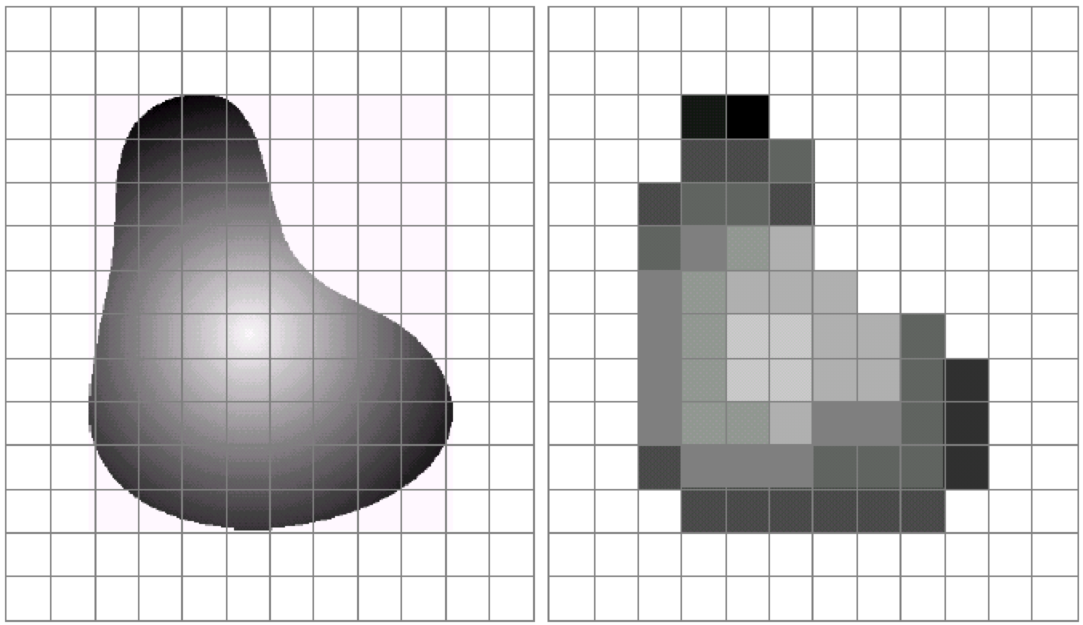
Lambertian Reflectance

- In computer vision, surfaces are often assumed to be ideal diffuse reflectors with no dependence on viewing direction.

Digital camera

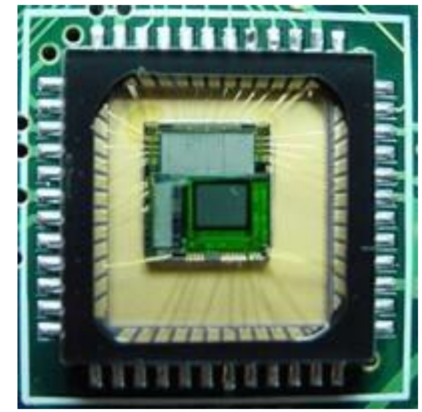
- A digital camera replaces film with a sensor array
  - Each cell in the array is light-sensitive diode that converts photons to electrons
  - Two common types
    - Charge Coupled Device (CCD)
    - CMOS
  - http://electronics.howstuffworks.com/digital-camera.htm

Sensor Array



a b

FIGURE 2.17 (a) Continuos image projected onto a sensor array. (b) Result of image sampling and quantization.



CMOS sensor

Sampling and Quantization

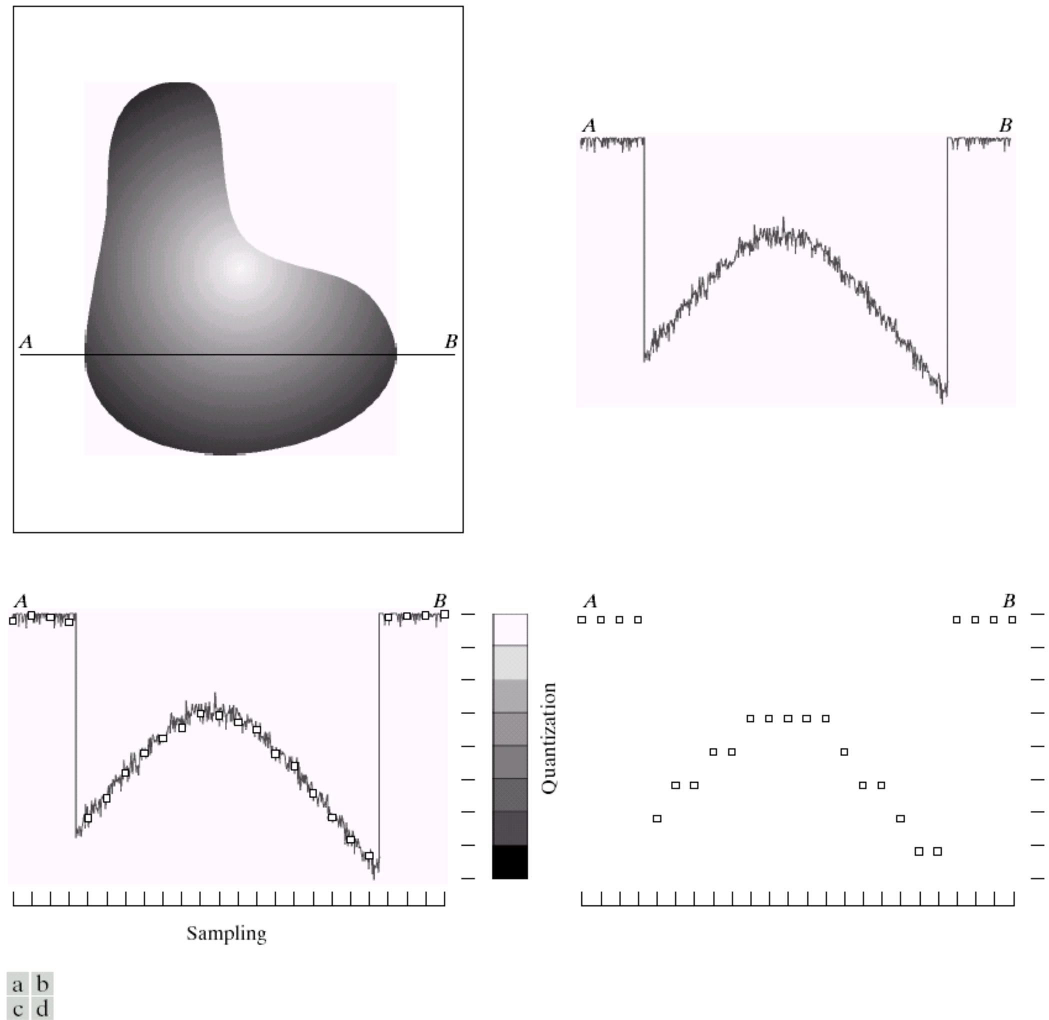
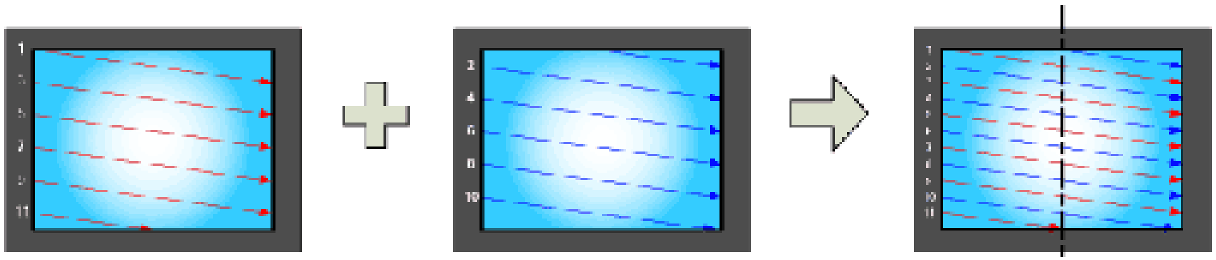


FIGURE 2.16 Generating a digital image. (a) Continuous image. (b) A scan line from A to B in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

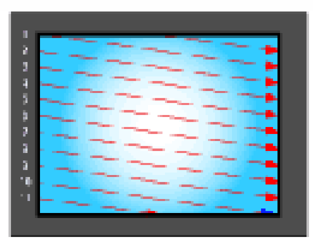
Interlace vs. progressive scan



1st field: Odd field

2nd field: Even field

One complete frame
using interlaced scanning



One complete frame
using progressive scanning

http://www.axis.com/products/video/camera/progressive_scan.htm

Slide by Steve Seitz

Progressive scan

http://www.axis.com/products/video/camera/progressive_scan.htm

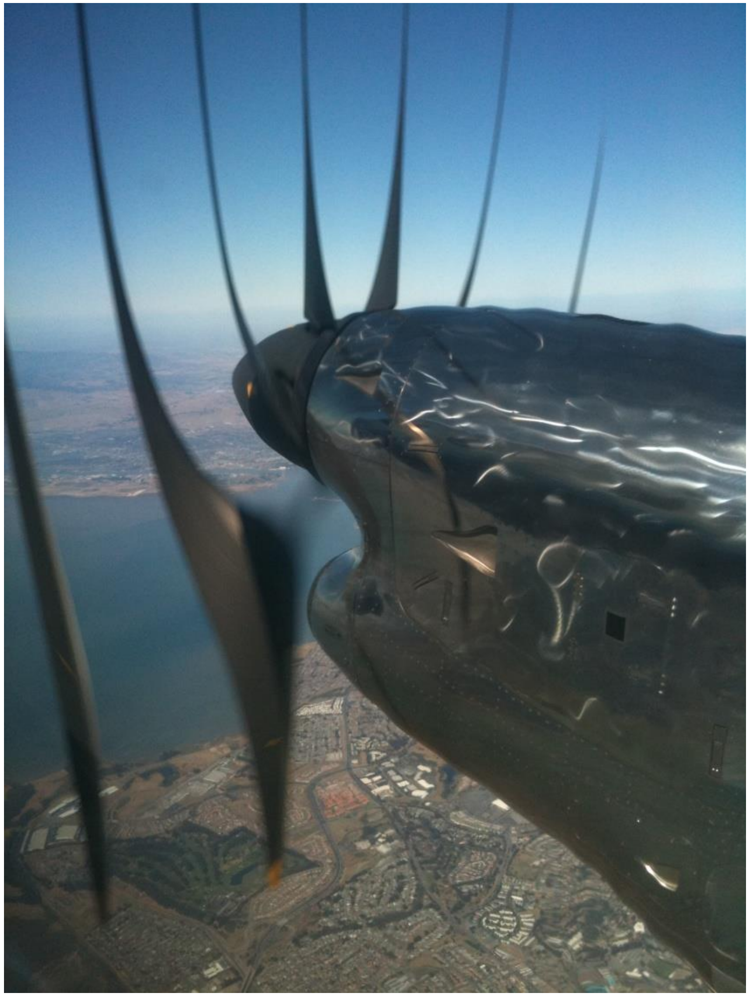
Slide by Steve Seitz

Interlace

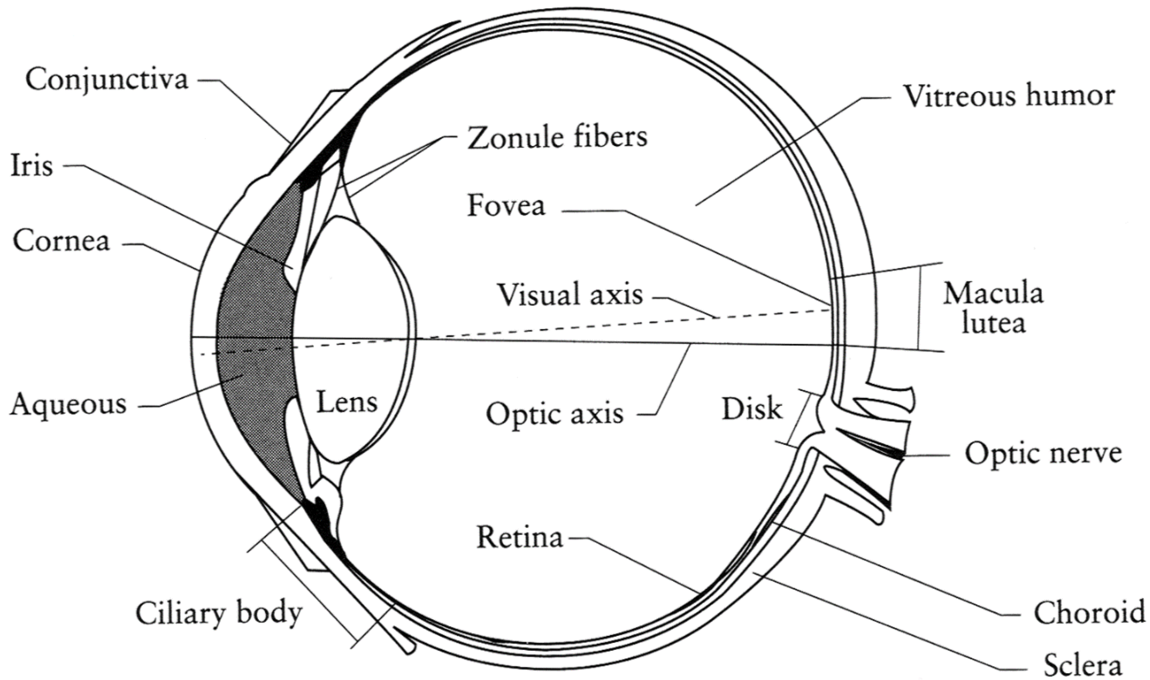
http://www.axis.com/products/video/camera/progressive_scan.htm

Slide by Steve Seitz

Rolling Shutter



The Eye



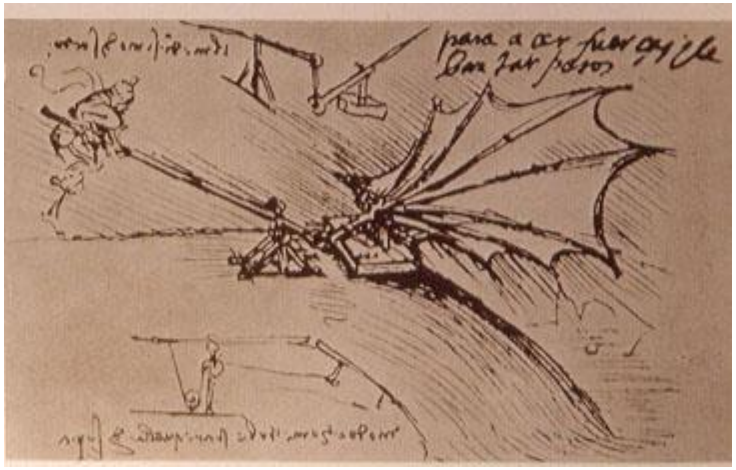
- The human eye is a camera!
  - **Iris** - colored annulus with radial muscles
  - **Pupil** - the hole (aperture) whose size is controlled by the iris
  - What's the "film"?
    - photoreceptor cells (rods and cones) in the **retina**

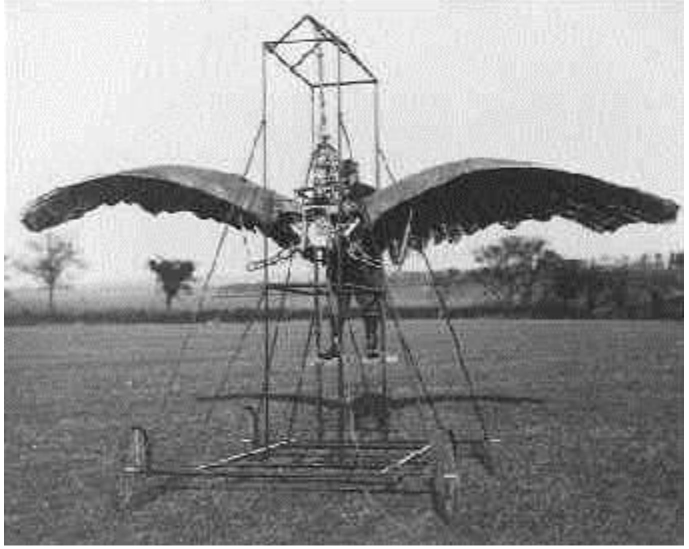
Slide by Steve Seitz

Aside: why do we care about human vision in this class?

- We don’t, necessarily.

Ornithopters





Why do we care about human vision?

- We don’t, necessarily.
- But cameras necessarily imitate the frequency response of the human eye, so we should know that much.
- Also, computer vision probably wouldn’t get as much scrutiny if biological vision (especially human vision) hadn’t proved that it was possible to make important judgements from 2d images.

Does computer vision “understand” images?

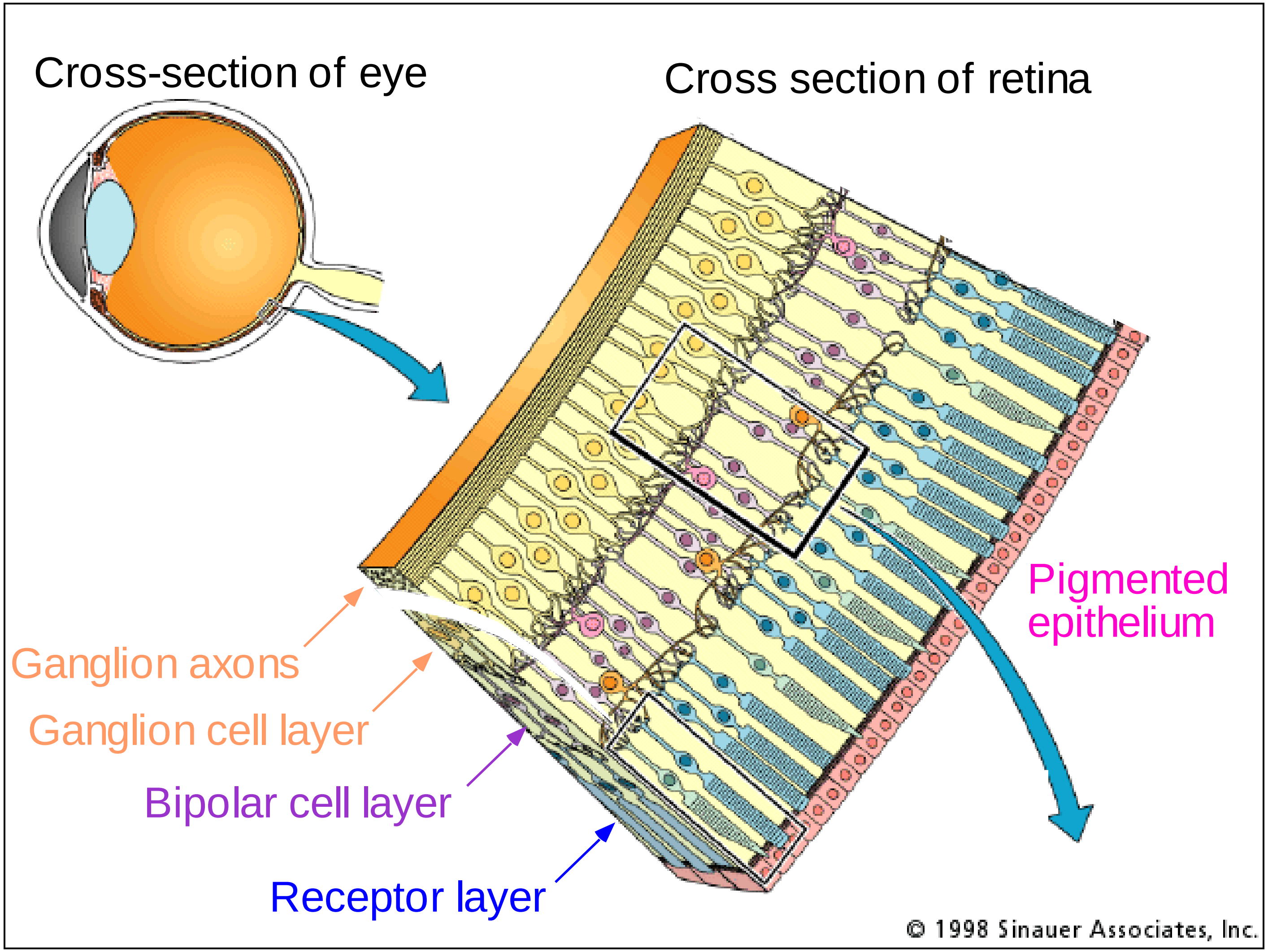
"Can machines fly?" The answer is yes, because airplanes fly.

"Can machines swim?" The answer is no, because submarines don't swim.

"Can machines think?" Is this question like the first, or like the second?

Source: Norvig

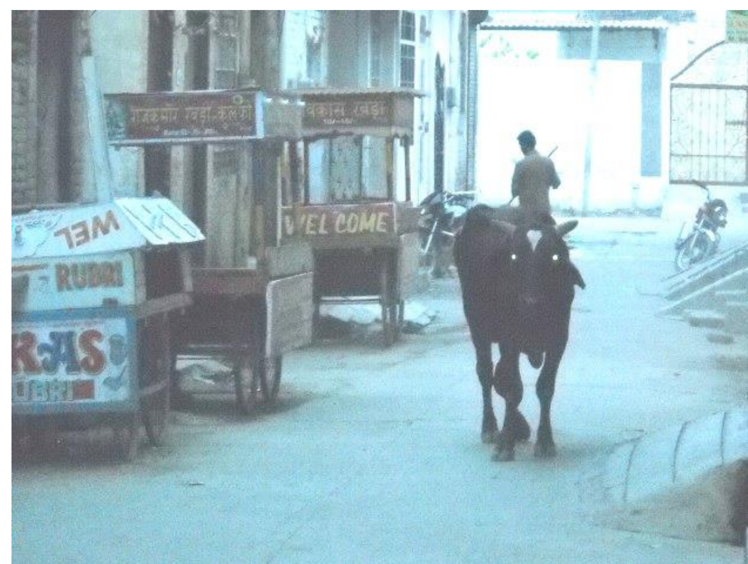
The Retina



Cross-section of eye
Cross section of retina
Pigmented epithelium
Ganglion axons
Ganglion cell layer
Bipolar cell layer
Receptor layer
© 1998 Sinauer Associates, Inc.

What humans don’t have: tapetum lucidum

Human eyes can reflect a tiny bit and blood in the retina makes this reflection red.



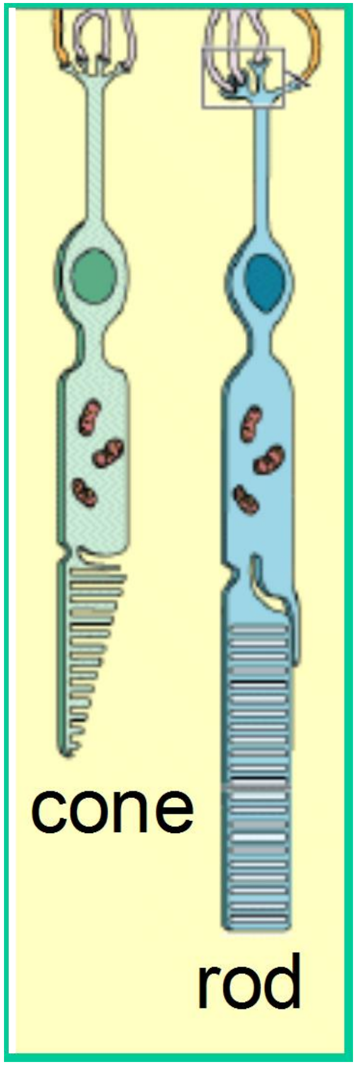
Two types of light-sensitive receptors

Cones

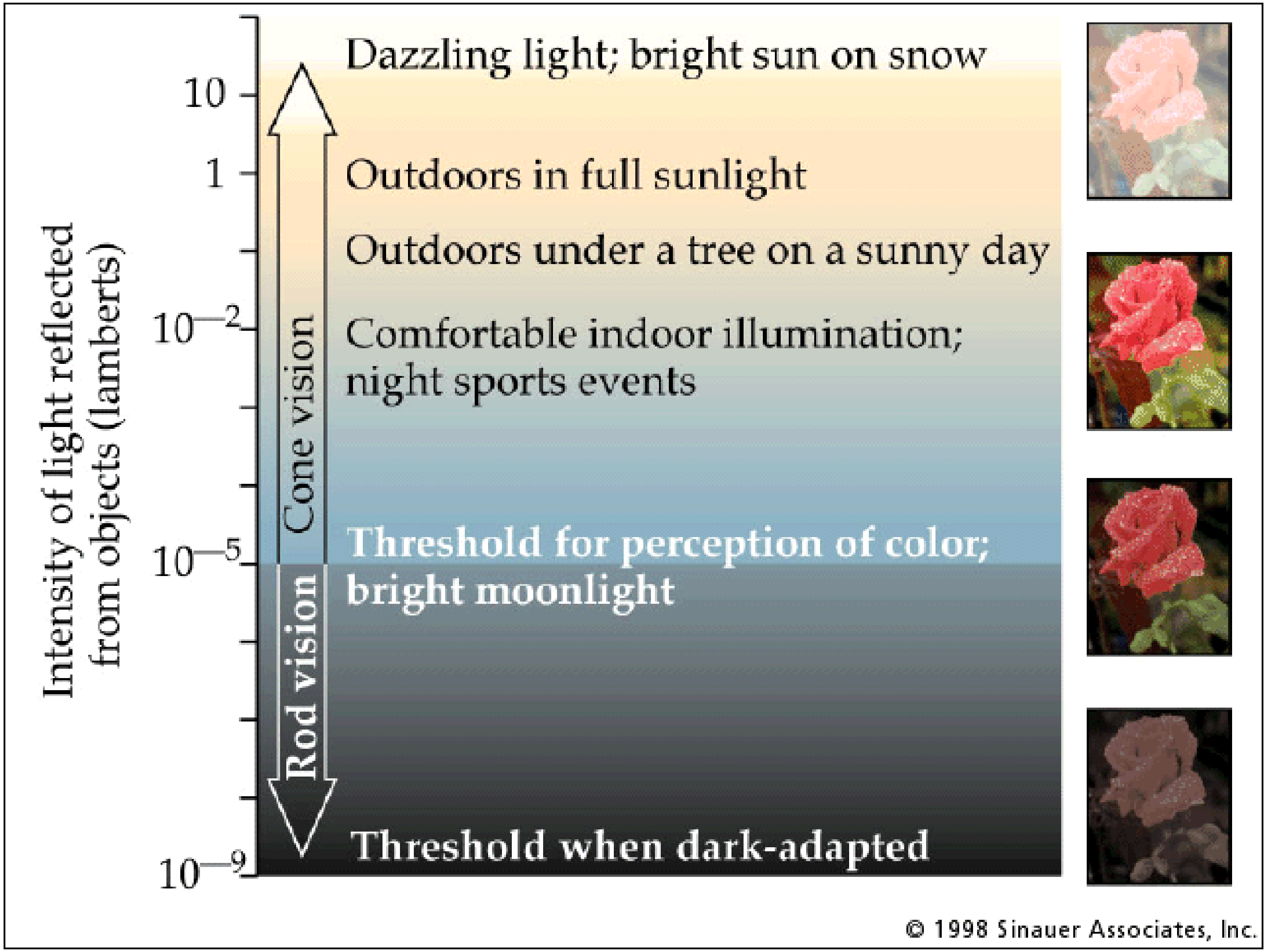
cone-shaped
less sensitive
operate in high light
color vision

Rods

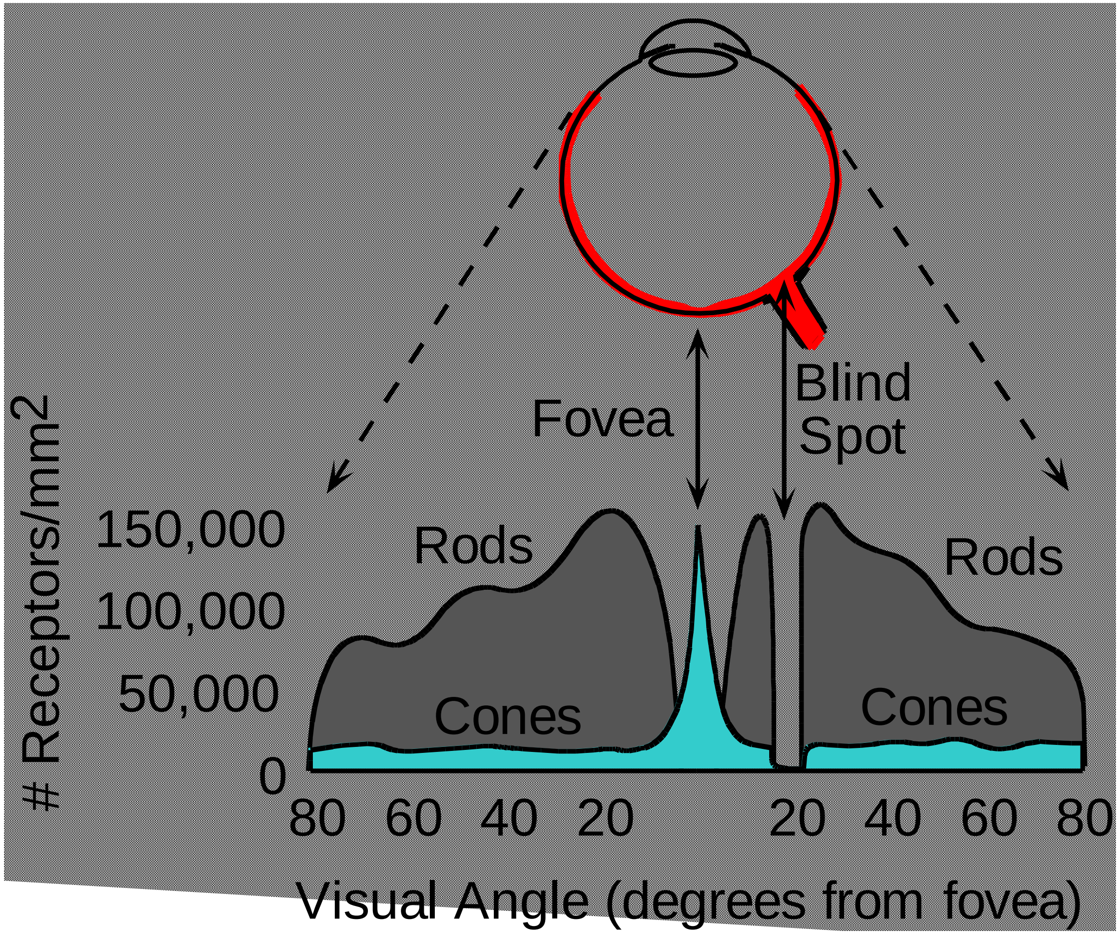
rod-shaped
highly sensitive
operate at night
gray-scale vision



Rod / Cone sensitivity



Distribution of Rods and Cones



Night Sky: why are there more stars off-center?

Averted vision: http://en.wikipedia.org/wiki/Averted_vision

Wait, the blood vessels are in front of the photoreceptors??

https://www.youtube.com/watch?v=L_W-IXqoxHA