Boosting

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6.3 Assume that both base classifiers, h_t and h_{t+1} at rounds t and $t+1$, respectively, are the same. At round t, suppose that $\epsilon_t = \frac{1}{3}$. This would produce the following α_t and Z_t:

$$\alpha_t = \frac{1}{2}\ln\left(\frac{1-\epsilon_t}{\epsilon_t}\right) \qquad Z_t = 2\sqrt{\epsilon_t(1-\epsilon_t)}$$

$$= \frac{1}{2}\ln\left(\frac{1-\frac{1}{3}}{\frac{1}{3}}\right) = \boxed{.347} \qquad = 2\sqrt{\frac{1}{3}\left(\frac{2}{3}\right)} = \boxed{.943}$$

The distribution for our misclassified point/points, J, at round t+1:

$$D_{t+1}(J) = \frac{D_t(J)\exp(-\alpha_t y_J h(x_J))}{Z_t} = \frac{1/m\, e^{.347}}{.943} = \boxed{\frac{1.5}{m}}$$

And so, the error at round t+1, $\epsilon_{t+1} = \sum_{i=1}^{m} D_t(i) 1_{h(x_i)\neq y_i}$

$$\epsilon_{t+1} = \frac{1.5}{\cancel{m}}\left(\frac{\cancel{m}}{3}\right) = .5$$

ϵ_{t+1} is not less than $\frac{1}{2}$. This proves that h_{t+1} must be different.

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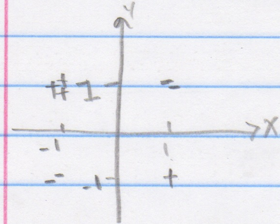
6.6 Fix $\epsilon = \frac{1}{2}$. Assume $m = 8$.



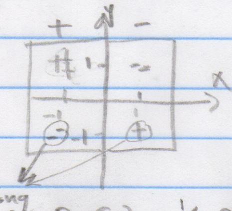
$\frac{m}{4} = \frac{8}{4} = 2$ neg points at $(-1,-1)$

$\frac{m}{4} = \frac{8}{4} = 2$ neg points at $(1,1)$

$\frac{m(1-\epsilon)}{4} = 1$ pos point at $(1,-1)$

$\frac{m(1+\epsilon)}{4} = 3$ pos points at $(-1,1)$

$D_1(i) = \frac{1}{m}$ for all points. At $t=1$:



$\epsilon_1 = \frac{1}{m}\left(\frac{m}{4} + \frac{m(1-\epsilon)}{4}\right) = \frac{1}{m}\left(\frac{m}{4}\, 2-\epsilon\right) = \boxed{.375}$

$\alpha_1 = \frac{1}{2}\ln\left(\frac{1-.375}{.375}\right) = \boxed{.255}$

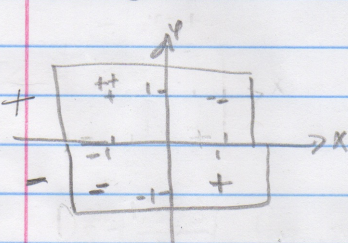
$Z_1 = 2\sqrt{.375(1-.375)} = \boxed{.968}$

wrong

$D_2(i) = \frac{\frac{1}{m}\exp(-\alpha_1 y_i h_1(x_i))}{Z_1} = \boxed{\frac{1.333}{m}}$ for misclassified points

$D_2(i) = \boxed{\frac{.8}{m}}$ for correctly classified points

At $t=2$:



$\epsilon_2 = \frac{1.333}{m}\left(\frac{m}{4}(1-\epsilon)\right) + \frac{.8}{m}\left(\frac{m}{4}\right) = \boxed{.533}$

So, this should be stopped when ϵ_t is at equal or $> \frac{1}{2}$

the final classifier error is at best $\frac{1}{4}(1-\epsilon)$