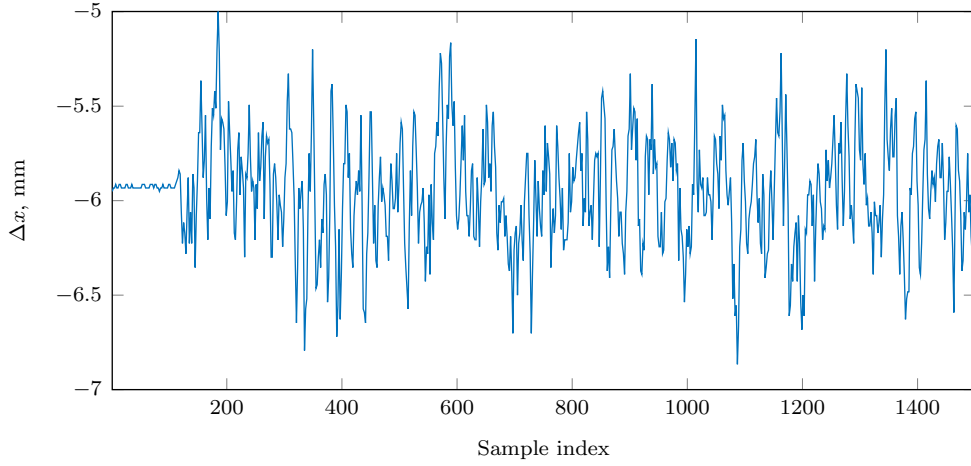


# Structure and parameter identification of au

October 8, 2019

## 1 Experimental data



**Figure 1:** The input for the set C.

## 2 Structure identification

The following model structure is assumed. The output of the NARX model  $\mathbf{y}(t)$  is the measured load. The input vector is composed as

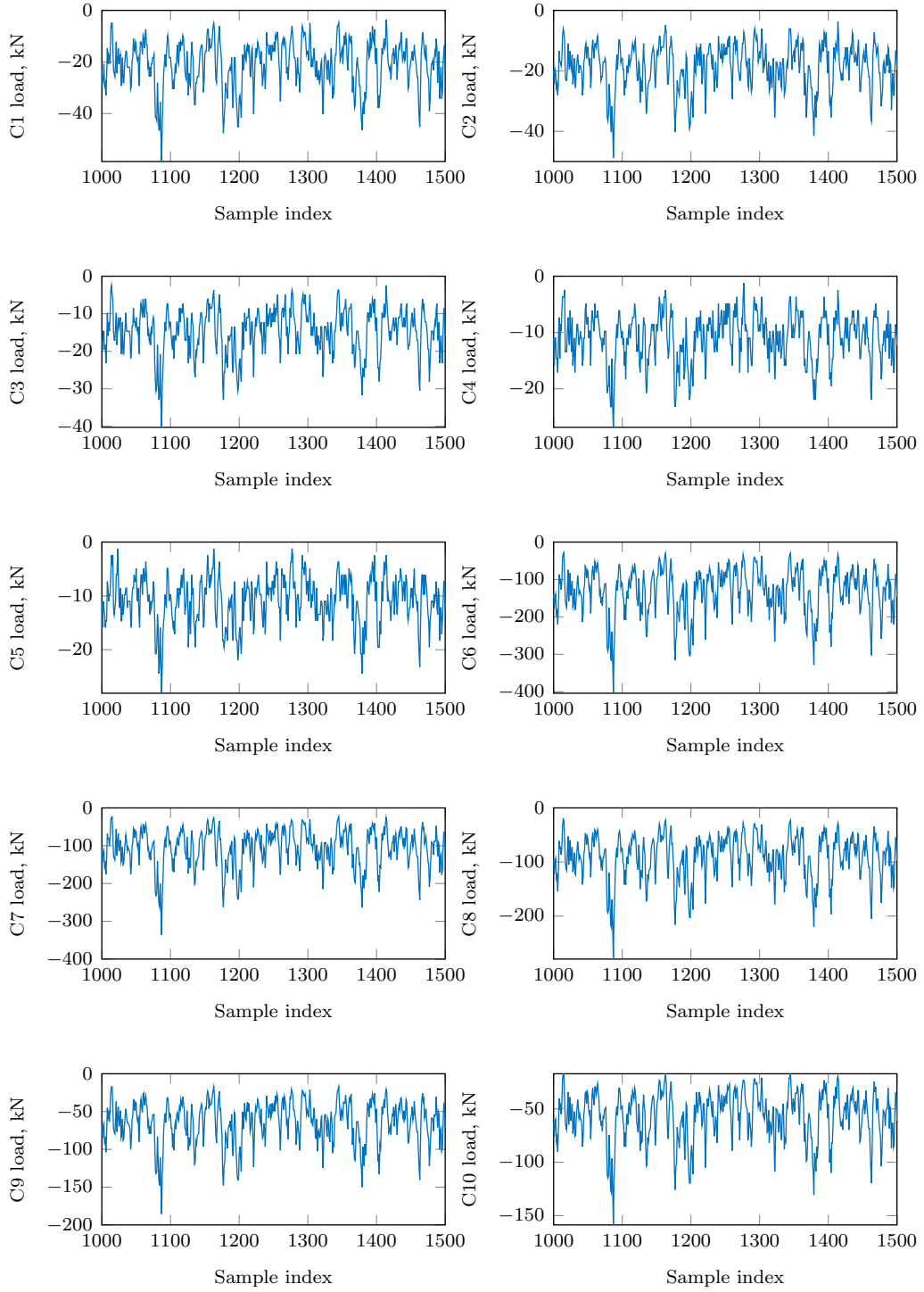
$$\mathbf{x}(t) = \{x_i(t)\}_{i=1}^d = \left[ \{y(t-k+1)\}_{k=1}^{n_y} \quad \{u(t-k+n_y+1)\}_{k=n_y+1}^{n_y+n_u} \right]^\top, \quad (1)$$

where  $n_u$  is the length of the input lag and  $n_y$  is the length of the output lag in discrete time, and where  $d = n_u + n_y$ . In this case, the identification is performed under the following assumptions:

- only the input signal affects the output ( $n_y = 0$ ).
- the input signal has a lag of length  $n_u = 4$ .

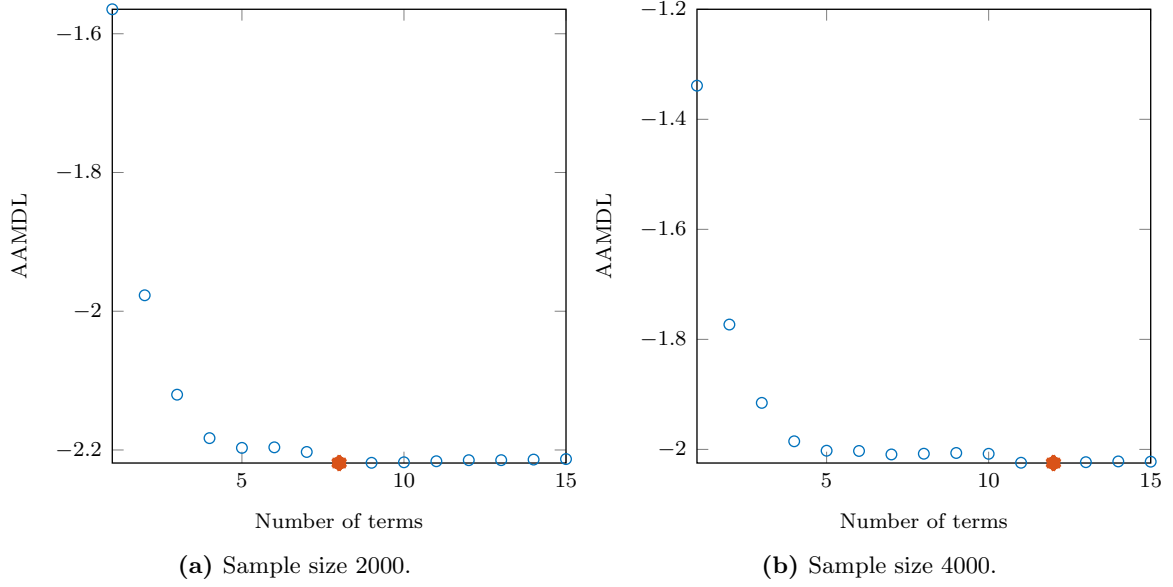
The unknown model is approximated with a sum of polynomial basis functions up to second degree ( $\lambda = 2$ ), rendering the following structure

$$\mathbf{y}(t) = \theta^0 + \sum_{i=1}^d \theta_i x_i(t) + \sum_{i=1}^d \sum_{j=1}^d \theta_{i,j} x_i(t) x_j(t) + e(t). \quad (2)$$



**Figure 2:** Experimental data.

The number and order of significant terms are identified within the EFOR-CMSS algorithm based on the data from 8 out of 10 datasets. Figure 3 illustrates the relationship between the number of model terms and the selected criterion of significance, AAMD<sub>L</sub>.



**Figure 3:** AAMD L evolution with the growing number of terms for samples of different size.

### 3 Parameter estimation

Results of internal parameter estimation via EFOR-CMSS for different sample sizes are presented in Tables 2 and 3. Visualised

**Table 1:** Estimated parameters for the sample length 2000.

Step	Terms	C1	C2	C4	C5	C6	C7	C9	C10	AEER	AAMD L
1	$c$	-234.54	-204.62	-69.32	-119.47	-1570.61	-1369.94	-810.16	-578.87	0	-1.565
2	$x_4$	-177.03	-144.09	-68.52	-76.68	-1355.31	-1111.81	-630.59	-490.28	0	-1.977
3	$x_3$	84.08	64.3	40.2	28.83	725.98	566.33	311.64	259.45	0	-2.12
4	$x_4, x_4$	-23.32	-17.55	-8.58	-11.12	-150.42	-144.42	-83.78	-56.07	0	-2.183
5	$x_3, x_4$	9.41	4.86	2.53	5.82	19.58	58.54	37.45	9.85	0	-2.197
6	$x_3, x_3$	3.85	4.19	2.53	0.2	63.86	28.86	13.13	21.49	0	-2.196
7	$x_2$	1.42	0.75	-2.7	-0.05	43.65	38.66	17.66	13.68	0	-2.203
8	$x_1$	-4.32	-2.9	0.19	-0.79	-54.57	-45.03	-21.76	-17.7	0	-2.219

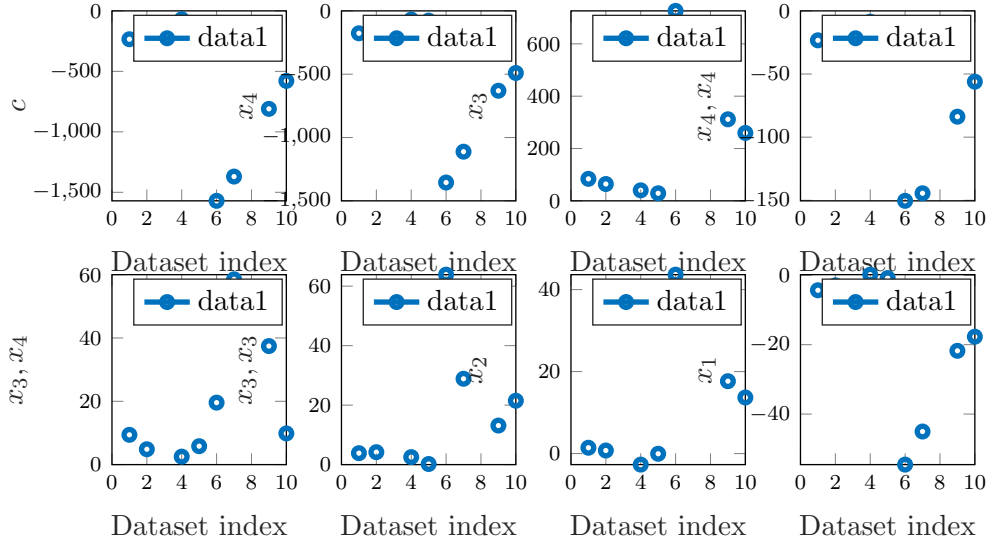
In order to link the external and internal parameters an arbitrary polynomial function is selected for two arguments

$$content... \quad (3)$$

Curve fitting results are presented in Tables 4 and 5.

**Table 2:** Estimated parameters for the sample length 4000.

Step	Terms	C1	C2	C4	C5	C6	C7	C9	C10	AEER	AAMD
1	$c$	-165.57	-140.83	-57.88	-93.13	-1037.61	-964.13	-559.51	-370.04	0	-1.339
2	$x_4$	-156.14	-123.14	-60.44	-67.23	-1089.72	-931.89	-530.57	-398.02	0	-1.773
3	$x_3$	77.52	56.15	33.75	22.39	466.78	421.82	244.52	171.74	0	-1.915
4	$x_4, x_4$	-19.62	-12.42	-4.16	-7.3	-103.98	-95.74	-55.36	-35.81	0	-1.985
5	$x_3, x_4$	-0.61	-10.91	-11.57	-7.03	-126.87	-80.74	-39.74	-50.79	0	-2.002
6	$x_3, x_3$	16.49	22.71	13.74	12.56	259.29	184.73	99.4	95.7	0	-2.003
7	$x_2$	-19.39	-14.83	-13.43	-5.4	-54.34	-71.89	-35.85	-9.18	0	-2.009
8	$x_2, x_4$	1.31	5.3	4.35	5.21	55.56	39.19	20.47	21.91	0	-2.008
9	$x_2, x_3$	-16.39	-22.92	-9.28	-13.26	-288.73	-197.82	-106.81	-103.02	0	-2.007
10	$x_2, x_2$	5.77	7.59	1.49	3.72	108.29	70.29	38.62	38.72	0	-2.008
11	$x_1$	25.77	21.68	13.42	10.92	220.93	169.96	85.56	72.68	0	-2.024
12	$x_1, x_4$	5.03	4.03	2.28	1.83	45.48	35.21	17.78	14.85	0	-2.025

**Figure 4:** Estimated values of internal parameters.**Table 3:** Estimated polynomial coefficients for the sample length 2000.

Terms	$b_0$	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$
$c$	-2.892	44.069	-35.155	2839.039	158.919	-1549.166
$x_4$	-38.261	77.653	-23.693	1161.223	62.673	-638.548
$x_3$	16.86	-38.672	11.079	-47.723	-0.087	27.378
$x_4, x_4$	9.588	-12.65	-2.58	573.952	32.354	-310.431
$x_3, x_4$	-32.818	47.863	0.365	-910.889	-51.817	489.409
$x_3, x_3$	19.859	-30.015	0.918	433.847	24.892	-232.412
$x_2$	12.195	-18.158	-0.054	-147.455	-8.594	83.188
$x_1$	-8.936	13.586	-0.152	93.013	5.338	-52.772

**Table 4:** Estimated polynomial coefficients for the sample length 4000.

Terms	$b_0$	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$
$c$	44.725	-85.831	-11.425	3326.343	186.521	-1798.02
$x_4$	-8.823	28.746	-18.995	1740.829	96.683	-949.69
$x_3$	-93.592	100.95	10.758	-1132.648	-62.108	600.48
$x_4, x_4$	-14.164	22.884	-1.921	343.79	19.962	-191.685
$x_3, x_4$	15.205	-35.561	0.323	-435.906	-26.83	246.913
$x_3, x_3$	0.885	5.089	2.598	402.931	24.923	-224.536
$x_2$	89.903	-134.412	4.479	671.681	36.296	-346.471
$x_2, x_4$	3.497	4.644	-0.418	95.753	6.059	-54.346
$x_2, x_3$	-30.856	39.053	-3.286	-582.404	-34.682	315.528
$x_2, x_2$	21.221	-32.628	2.174	303.877	17.589	-162.005
$x_1$	11.401	-5.801	-0.918	-48.625	-1.775	26.798
$x_1, x_4$	2.392	-2.224	-0.064	-15.592	-0.71	8.722