Structure and parameter identification of the dynamical model of auxetic foam

October 21, 2019

1 Experimental data

The experiments is conducted on ten samples of authentic elastomeric foams prepared in different manufacturing settings. In this report, the influence of two manufacturing parameters on the dynamical behaviour of the foam is evaluated: the length of the foam sample after cutting, L_{cut} (mm), and the relaxed density of the sample, D_{rlx} (g/cm^3). The parameter values for each specimen are presented in Table 1. The random input signal is adopted for all foam specimen is illustrated in Figure

Table 1: manufacturing parameters of auxetic form used in 10 experiments.

Parameter	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
L_{cut}	74	72	69	61	56	67	66	64	62	57
D_{rlx}	0.123	0.113	0.104	0.095	0.093	0.276	0.255	0.230	0.209	.193

1. Samples of recorded foam responses are presented in Figure 1. The ability to predict the response

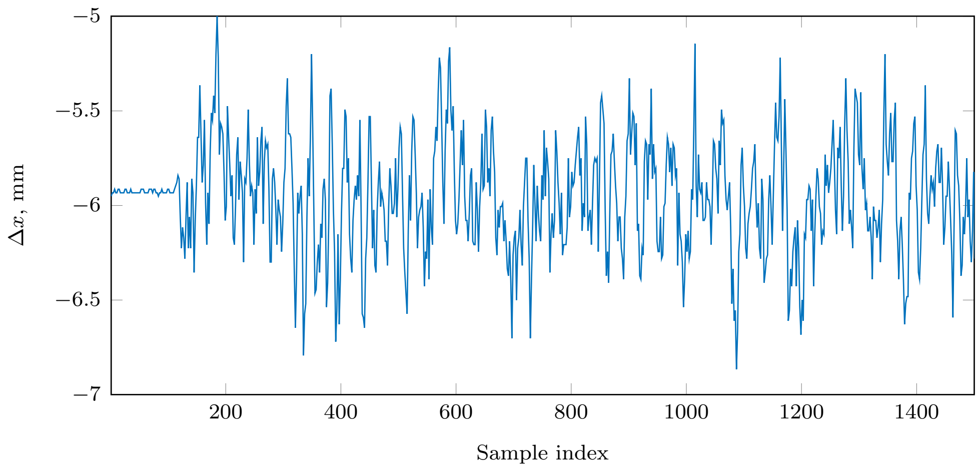


Figure 1: The random input signal for the set of authentic foams is the displacement of the hydraulic actuator of the machine in mm.

of a foam specimen to a known input based on manufacturing parameters is crucial in for optimal design. The aim of this work is to identify the dynamical model that could describe the relationship between measured input and output time series and the external parameters.

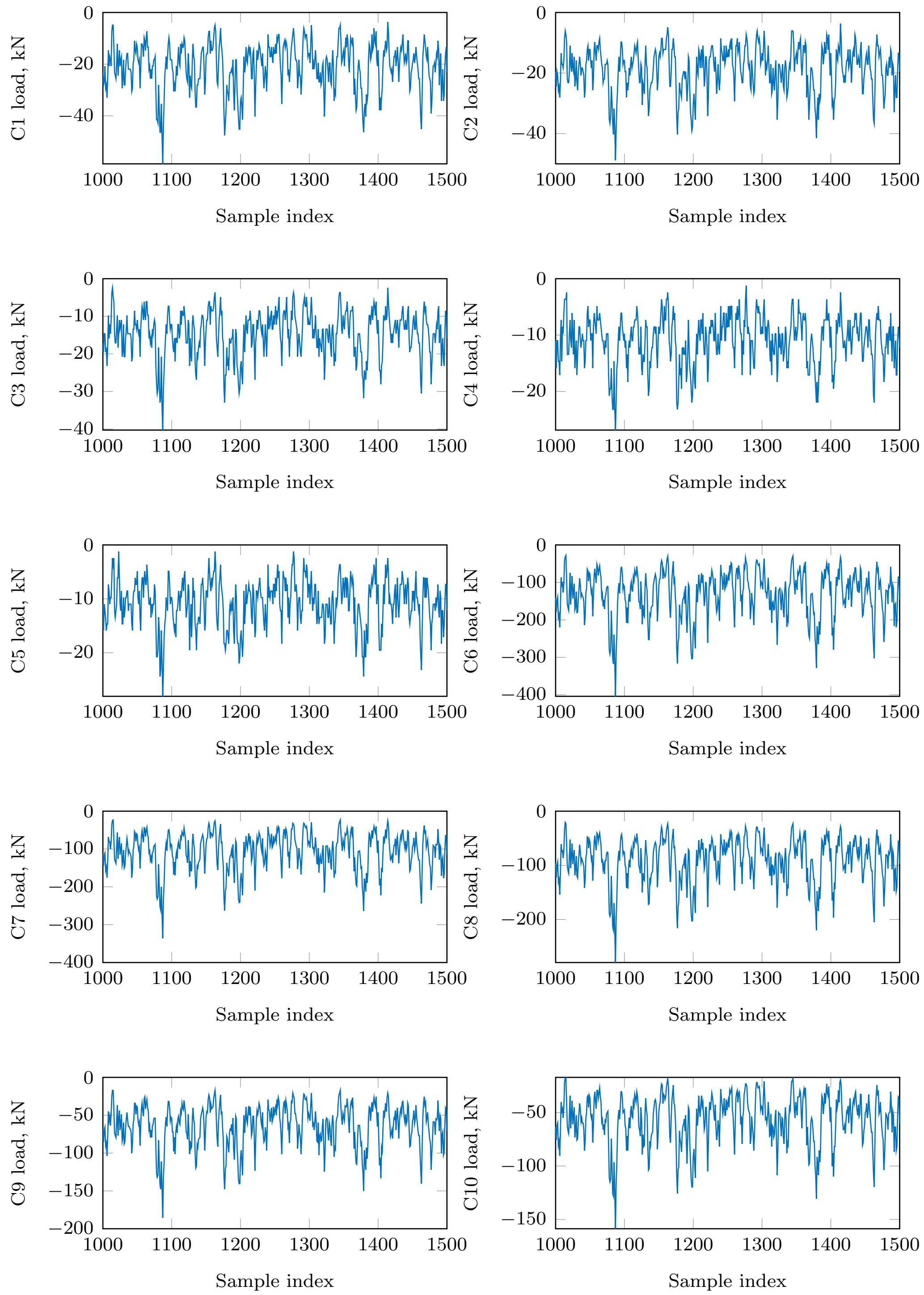


Figure 2: The measured output for each foam sample is the force of the specimen response in kN.

2 Structure identification

The following model structure is assumed. The output of the NARX model $\boldsymbol{y}(t)$ is the measured load. The input vector is composed as

$$\boldsymbol{x}(t) = \{x_i(t)\}_{i=1}^{d} = \left[\{y(t-k+1)\}_{k=1}^{n_y} \quad \{u(t-k+n_y+1)\}_{k=n_y+1}^{n_y+n_u} \right]^\top, \tag{1}$$

where n_u is the length of the input lag and n_y is the length of the output lag in discrete time, and where $d = n_u + n_y$. In this case, the identification is performed under the following assumptions:

- only the input signal affects the output ($n_y = 0$).
- the input signal has a lag of length $n_u = 4$.

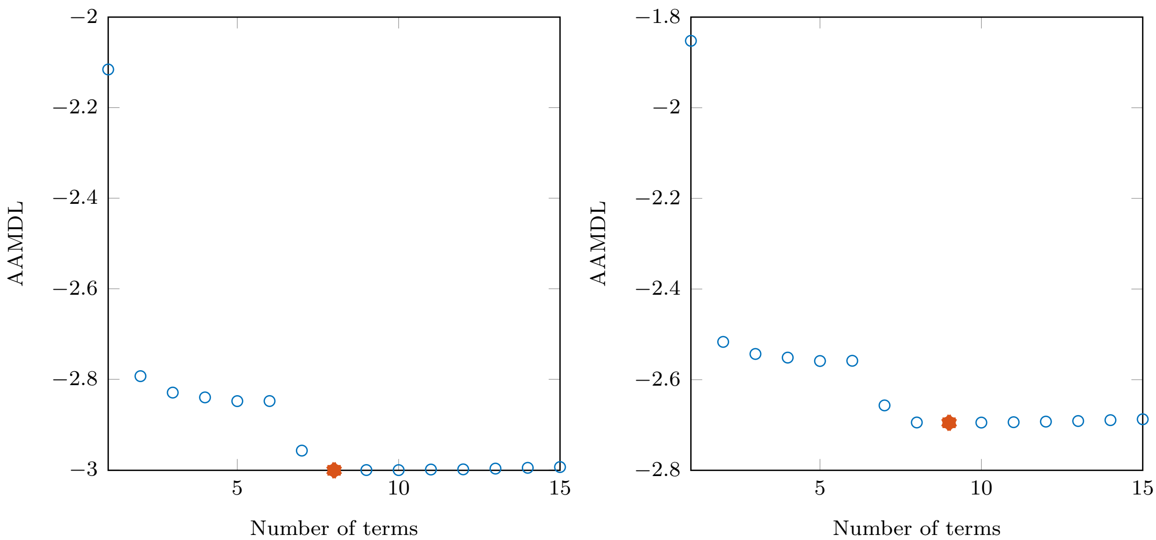
The resultant input vector of the NARX model then takes the following form:

$$\boldsymbol{x}(t) = \left[\; u(t-3) \quad u(t-2) \quad u(t-1) \quad u(t) \; \right]^\top. \tag{2}$$

The unknown model is approximated with a sum of polynomial basis functions up to second degree ($\lambda = 2$), rendering the following structure

$$\boldsymbol{y}(t) = \theta^0 + \sum_{i=1}^{d} \theta_i x_i(t) + \sum_{i=1}^{d} \sum_{j=1}^{d} \theta_{i,j} x_i(t) x_j(t) + e(t). \tag{3}$$

The number and order of significant terms are identified within the EFOR-CMSS algorithm based on the data from 8 out of 10 datasets. Figure 3 illustrates the relationship between the number of model terms and the selected criterion of significance, AAMDL.



(a) Sample size 2000.

(b) Sample size 4000.

Figure 3: AAMDL evolution with the growing number of terms for samples of different size. The optimal number of terms (⬣) increases with the growing sample size.

3 Parameter estimation

The results of internal parameter estimation via the EFOR-CMSS method for sample sizes of 2000 and 4000 points are presented in Tables 2 and 3, respectively. It can be seen that the first 7 significant terms are the same for both samples; moreover, the estimated values are of the same order in both cases. The further analysis deals only with the parameter estimates obtained from the smaller data sample.

Table 2: Estimated parameters for the sample length 2000.

Step	Terms	C1	C2	C4	C5	C6	C7	C9	C10	AEER(%)
1	x_4x_4	-26.04	-20.99	-10.69	-10.96	-191.78	-157.64	-87.42	-69.8	89.511
2	x_3	75.42	59.58	33	26.06	508.94	419.54	242.35	195.15	8.849
3	x_1x_4	0.62	0.76	0.32	0.48	8.55	7.83	2.66	1.15	0.139
4	x_1x_1	0.01	-0.19	-0.15	-0.22	0.05	-0.48	0.44	0.76	0.045
5	x_2	0.71	-0.73	-2.24	-0.66	45.94	36.57	18.4	12.68	0.032
6	x_4	-171.24	-139.22	-69.61	-73.69	-1273.02	-1046.38	-579.72	-465.59	0.006
7	c	-233.16	-200.83	-93.74	-119.7	-1805.9	-1488.55	-803.7	-648.8	0.308
8	x_3x_4	15.47	12.1	6.36	5.68	110.13	90.43	51.77	41.43	0.093

Table 3: Estimated parameters for the sample length 4000.

Step	Terms	C1	C2	C4	C5	C6	C7	C9	C10	AEER(%)
1	x_4x_4	-18.97	-15.33	-8.34	-8.94	-138.21	-114.23	-63.58	-50.3	88.667
2	x_3	63.2	53.09	30.09	24.55	426.09	362.14	208.4	168.78	9.494
3	x_1x_4	4.29	3.45	2.03	3.11	34.39	27.1	14.45	10.2	0.12
4	x_1x_1	0.6	0.87	0.31	0.56	9.24	5.54	4.15	4.1	0.042
5	x_2	2.34	0.07	-1.9	-2.13	43.4	34.96	18.62	12.71	0.036
6	x_4	-157.87	-128.81	-69.1	-69.29	-1153.95	-964.72	-539.51	-431.83	0.006
7	c	-226.03	-187.69	-100.5	-116.41	-1722.47	-1432.4	-789.68	-632.3	0.335
8	x_3x_3	9.21	7.92	4.31	4.67	70	55.98	32.65	26.32	0.103
9	x_1x_3	-4.8	-4.8	-2.64	-4.26	-45.05	-31.95	-19.47	-15.95	0.007

In order to link the external and internal parameters, an arbitrary polynomial function of two arguments is formed

$$\theta_i(L_{cut}, D_{rlx}) = \beta_0 + \beta_1 L_{cut} + \beta_2 D_{rlx} + \beta_3 L_{cut}^2 + \beta_4 D_{rlx}^2 + \beta_5 L_{cut} D_{rlx}, \qquad i = 1, \dots, N_s, \qquad (4)$$

L_{cut} and D_{rlx} are the external parameters of manufacturing process, and where N_s is the estimated number of the significant terms. The coefficients $B = [\beta_0 \dots \beta_5]$ are unknown and must be estimated from the internal parameter values available. The linear relationship between the batch of internal parameter values corresponding to each significant term Θ_i and the surface coefficients can be established

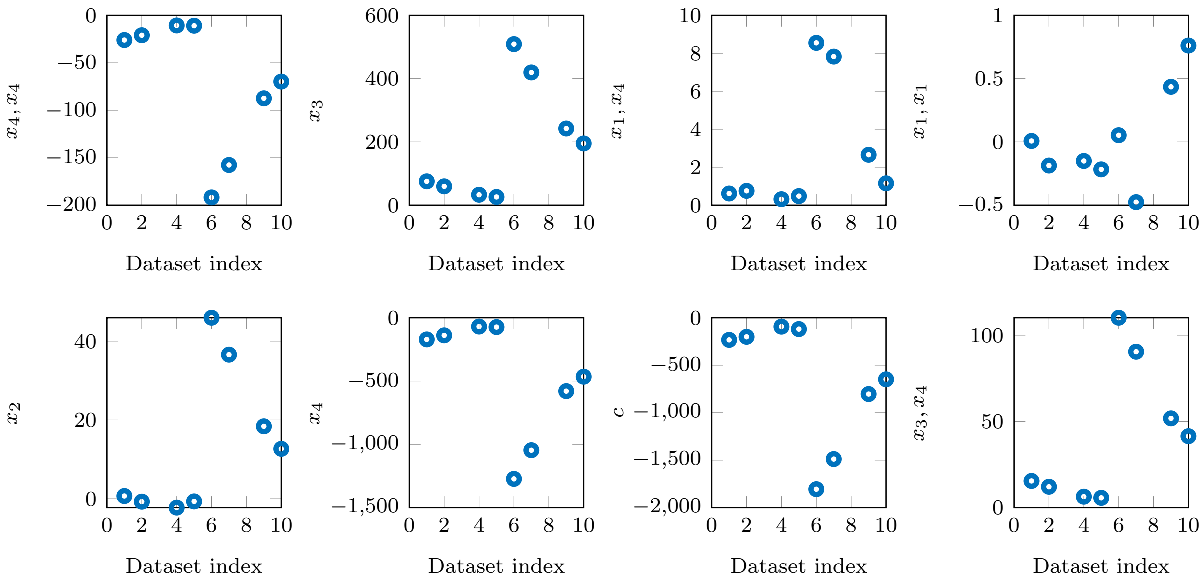


Figure 4: Estimated values of internal parameters.

in the following form:

$$\begin{bmatrix} \theta_i^1 \\ \vdots \\ \theta_i^j \\ \vdots \\ \theta_i^N \end{bmatrix} = \begin{bmatrix} 1 & L & D & L \times L & D \times D & L \times D \end{bmatrix} B, \tag{5}$$

where $j = 1, \dots, N$ is the index of the dataset, and where $\times$ denotes the by-element product such that

$$L \times L = \left[L_{cut}^{(j)} L_{cut}^{(j)} \right]_{j=1}^{N}.$$

Given the equation (5), the vector of unknown coefficients B can be identified via the classical least squares (LS) method. Curve fitting results are presented in Table

Table 4: Estimated polynomial coefficients for the sample length 2000.

Terms	β_0	β_1	β_2	β_3	β_4	β_5
x_4, x_4	-170.31	4.27	831.86	-2.02	-0.03	-4562.38
x_3	143.25	-3.37	-1382.17	-3.9	0.04	11446.44
x_1, x_4	5.32	0.17	-203.84	2.88	0	161.25
x_1, x_1	1.85	-0.2	78.58	-1.4	0	39.42
x_2	76.05	-1.76	-434.89	5.27	0.01	953.45
x_4	-1216.06	31.23	5342.46	-8.72	-0.25	-30672.05
c	-2427.76	63.16	8882.74	-21.5	-0.49	-45523.06
x_3, x_4	61.26	-1.53	-370.55	0.3	0.01	2478.05

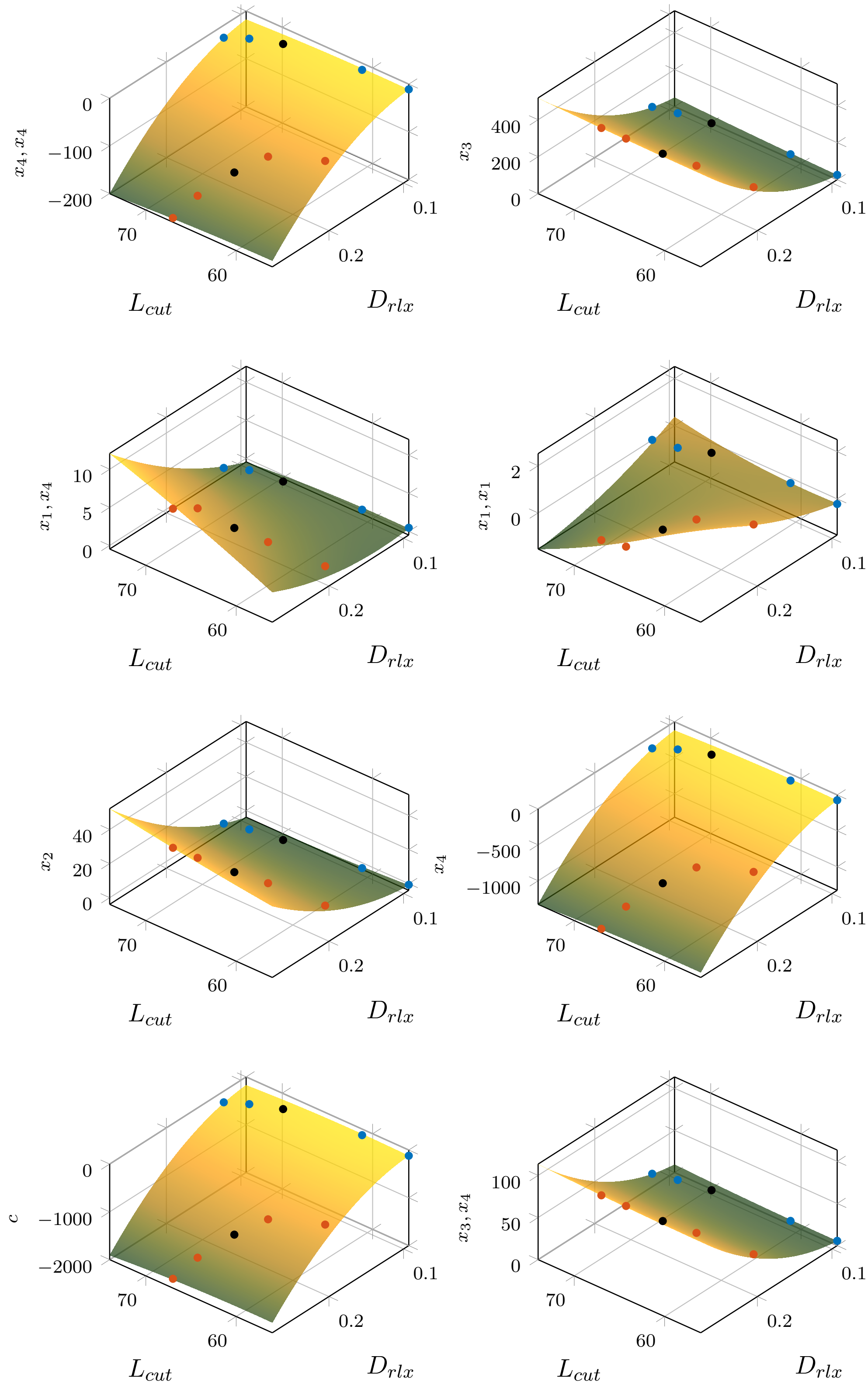


Figure 5: The quantifiable relationship between external parameters (L_{cut}, D_{rlx}) and internal parameters θ is obtained by fitting a polynomial surface to the internal parameter values estimated for the set F1 (•) and set F2 (•). The fitted surfaces ( ) are then used to compute the internal model parameters corresponding to the settings of choice (•).