Structure and parameter identification of the dynamical model of auxetic foam

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Experimental data is described in the earlier provided by the University of Bristol. This report summarises the settings used for model identification and presents preliminary results of model structure identification

1 Structure identification

The following model structure is assumed. The output of the NARX model $\boldsymbol{y}(t)$ is the measured load. The input vector is composed as

$$\boldsymbol{x}(t) = \{x_i(t)\}_{i=1}^{d} = \left[\{y(t-k)\}_{k=1}^{n_y} \quad \{u(t-k+n_y)\}_{k=n_y+1}^{n_y+n_u}\right]^\top, \tag{1}$$

where n_u is the length of the input lag and n_y is the length of the output lag in discrete time, and where $d = n_u + n_y$. In this case, the identification is performed under the following assumptions:

- the output lag $n_y = 4$.
- the input signal has a lag of length $n_u = 4$.

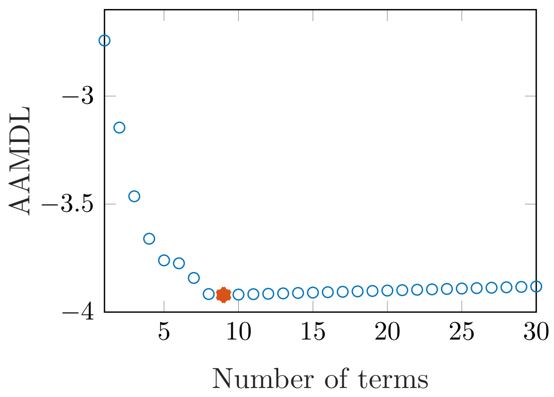
The resultant input vector of the NARX model then takes the following form:

$$\boldsymbol{x}(t) = \left[\begin{array}{cccccc} u(t-n_u) & \dots & y(t-1) & u(t-n_u) & \dots & u(t-1) \end{array}\right]^\top. \tag{2}$$

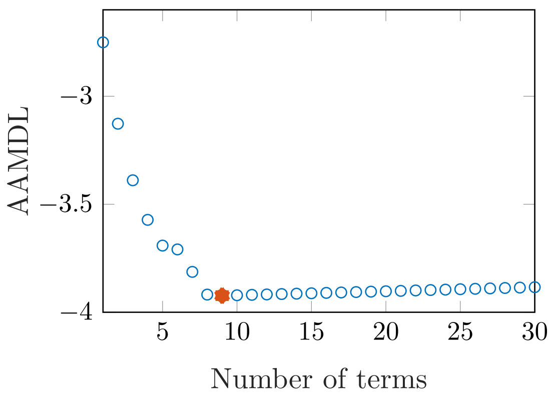
The unknown model is approximated with a sum of polynomial basis functions up to second degree ($\lambda = 3$), rendering the following structure

$$\boldsymbol{y}(t) = \theta^0 + \sum_{i=1}^{d} \theta_i x_i(t) + \sum_{i=1}^{d}\sum_{j=1}^{d} \theta_{i,j} x_i(t) x_j(t) + e(t). \tag{3}$$

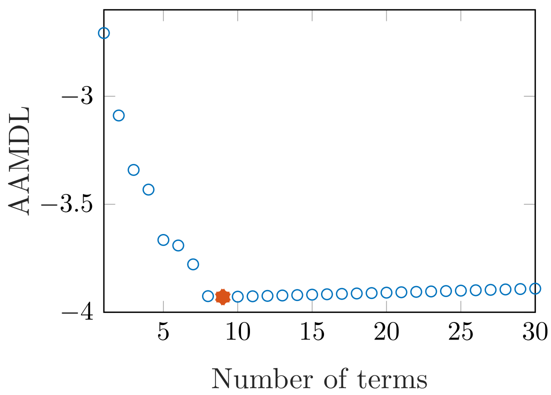
The number and order of significant terms are identified within the EFOR-CMSS algorithm based on the data from 9 datasets for each foam specimen. Figure 1 illustrates the relationship between the number of model terms and the selected criterion of significance, AAMDL.



(a) Square cut.



(b) Y cut.



(c) Z cut.

Figure 1: Evolution of AAMDL with the respect to the number of terms for sample size 4000. The optimal number of terms is highlighted (⬣).

2 Structure identification

The results of internal parameter estimation via the EFOR-CMSS method for sample size of 4000 points for each type of cut are presented in Tables 1-3.

Table 1: Estimated parameters for the square cut dataset.

Step	Terms	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	AERR(%)
1	$y(t-1)$	1.32	0.55	0.53	0.5	0.49	0.51	1.41	0.6	0.61	1.13	0.41	0.3	55.442
2	$u(t-1)$	0.4	0.46	0.46	0.46	0.45	0.43	0.38	0.46	0.45	0.45	0.55	0.55	29.045
3	$y(t-2)$	-0.91	-0.6	-0.58	-0.65	-0.6	-0.61	-1.14	-0.76	-0.76	-0.93	-0.67	-0.63	10.766
4	c	3.61	1.45	-2.33	7.8	3.71	0.3	3.32	0.92	-2.52	7.01	13.54	2.6	1.086
5	$u(t-3)$	0.08	0.27	0.27	0.36	0.34	0.33	0.13	0.33	0.33	0.18	0.38	0.4	1.05
6	$y(t-4)$	-0.02	-0.46	-0.48	-0.62	-0.59	-0.58	-0.1	-0.51	-0.52	-0.17	-0.52	-0.58	0.153
7	$u(t-2)$	-0.15	0.25	0.25	0.28	0.28	0.26	-0.15	0.25	0.24	-0.11	0.32	0.37	0.521
8	$y(t-3)$	0.25	0.42	0.44	0.55	0.51	0.52	0.46	0.53	0.54	0.42	0.39	0.4	0.438
9	$u(t-4)$	-0.01	0.01	0.01	0.02	0.04	0.04	-0.04	0.02	0.02	-0.07	0.01	0.02	0.023

Table 2: Estimated parameters for the Y cut dataset.

Step	Terms	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9	Y10	Y11	Y12	AERR(%)
1	$y(t-1)$	0.81	0.44	0.42	1.22	0.71	0.73	0.75	0.78	0.81	1.13	0.41	0.3	56.577
2	$u(t-1)$	0.48	0.53	0.52	0.39	0.45	0.44	0.37	0.36	0.35	0.45	0.55	0.55	27.235
3	$y(t-2)$	-0.89	-0.77	-0.77	-1.19	-0.93	-0.91	-0.78	-0.82	-0.83	-0.93	-0.67	-0.63	9.86
4	c	14.56	7.37	-1.59	6.12	4.03	1.09	-1.2	-0.77	-0.2	7.01	13.54	2.6	1.592
5	$u(t-3)$	0.32	0.43	0.43	0.28	0.37	0.35	0.27	0.27	0.26	0.18	0.38	0.4	1.499
6	$y(t-4)$	-0.44	-0.64	-0.66	-0.39	-0.6	-0.59	-0.58	-0.59	-0.57	-0.17	-0.52	-0.58	0.23
7	$y(t-3)$	0.55	0.54	0.56	0.77	0.72	0.7	0.7	0.74	0.74	0.42	0.39	0.4	0.891
8	$u(t-2)$	0.13	0.33	0.34	-0.05	0.18	0.17	0.18	0.17	0.15	-0.11	0.32	0.37	0.708
9	$u(t-4)$	-0.04	0.02	0.03	-0.08	0.01	0.02	0.02	0.02	0.01	-0.07	0.01	0.02	0.017

Table 3: Estimated parameters for the Z cut dataset.

Step	Terms	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Z8	Z9	Z10	Z11	Z12	AERR(%)
1	$y(t-1)$	0.46	0.44	0.48	0.64	0.54	0.58	0.52	0.48	0.5	1.13	0.41	0.3	46.961
2	$u(t-1)$	0.54	0.54	0.53	0.5	0.5	0.49	0.48	0.48	0.47	0.45	0.55	0.55	33.438
3	$y(t-2)$	-0.82	-0.79	-0.8	-0.84	-0.76	-0.78	-0.68	-0.64	-0.66	-0.93	-0.67	-0.63	11.325
4	$u(t-3)$	0.44	0.43	0.41	0.36	0.36	0.35	0.38	0.38	0.37	0.18	0.38	0.4	2.312
5	c	-4.23	-3.73	-2.3	2.04	-0.36	-1.09	2.3	0.41	-0.28	7.01	13.54	2.6	2.061
6	$y(t-4)$	-0.64	-0.63	-0.61	-0.51	-0.56	-0.54	-0.58	-0.59	-0.58	-0.17	-0.52	-0.58	0.338
7	$y(t-3)$	0.55	0.54	0.55	0.53	0.53	0.53	0.51	0.5	0.52	0.42	0.39	0.4	0.867
8	$u(t-2)$	0.31	0.31	0.28	0.2	0.26	0.23	0.23	0.24	0.23	-0.11	0.32	0.37	1.009
9	$u(t-4)$	0.02	0.02	0.02	0	0.01	0.01	0.03	0.04	0.04	-0.07	0.01	0.02	0.029