Structure and parameter identification of the dynamical model of auxetic foam

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1 Experimental data

The experiments is conducted on ten samples of authentic elastomeric foams prepared in different manufacturing settings. In this report, the influence of two manufacturing parameters on the dynamical behaviour of the foam is evaluated: the length of the foam sample after cutting, L_{cut} (mm), and the relaxed density of the sample, D_{rlx} (g/cm^3). The parameter values for each specimen are presented in Table 1. The random input signal adopted for all foam sample is illustrated in Figure

**Table 1:** Manufacturing parameters of auxetic form used in 10 experiments.

Parameter	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
L_{cut}	74	72	69	61	56	67	66	64	62	57
D_{rlx}	0.123	0.113	0.104	0.095	0.093	0.276	0.255	0.230	0.209	0.193

1. Samples of recorded foam responses are presented in Figure 1. The ability to predict the response
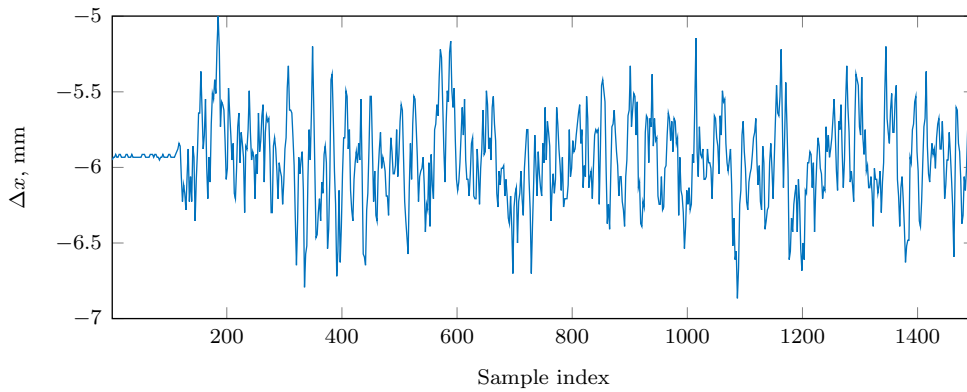
Figure 1: The random input signal for the set of authentic foams is the displacement of the hydraulic actuator of the machine in mm.

of a foam specimen to a known input based on manufacturing parameters is crucial in for optimal design. The aim of this report is to identify the dynamical model that could describe the relationship between measured input and output time series conditioned on the external parameters. Datasets
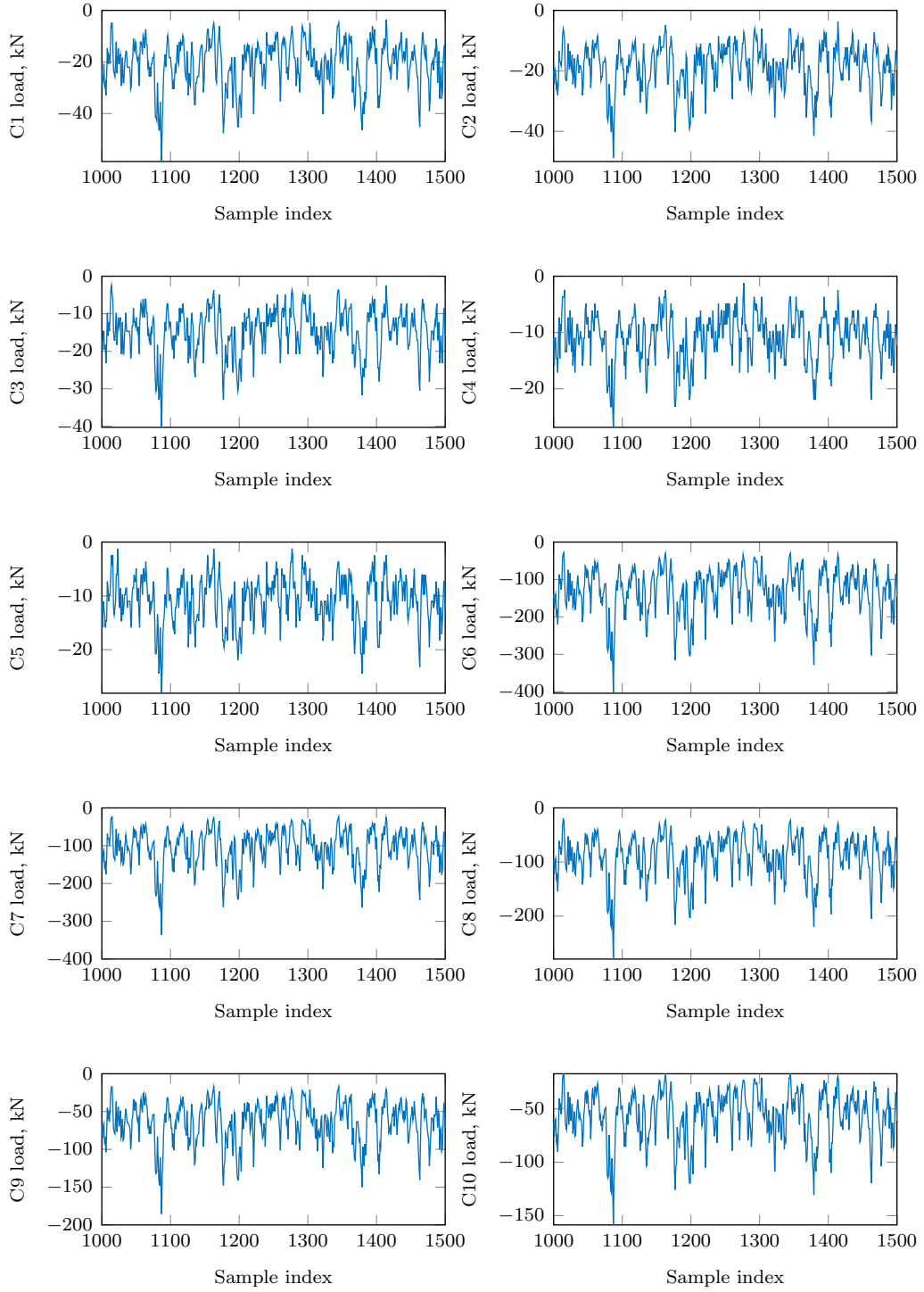
Figure 2: The measured output for each foam sample is the force of the specimen response in kN.

C1-C2, C4-C7, and C9-C10 are used for model identification while the datasets C3 and C8 are used to validate the identified model.

2 Structure identification

The following model structure is assumed. The output of the NARX model $\boldsymbol{y}(t)$ is the measured load. The input vector is composed as

$$\boldsymbol{x}(t) = \{x_i(t)\}_{i=1}^d = \left[\{y(t-k+1)\}_{k=1}^{n_y} \quad \{u(t-k+n_y+1)\}_{k=n_y+1}^{n_y+n_u} \right]^\top, \tag{1}$$

where n_u is the length of the input lag and n_y is the length of the output lag in discrete time, and where $d = n_u + n_y$. In this case, the identification is performed under the following assumptions:

- only the input signal affects the output ($n_y = 0$).
- the input signal has a lag of length $n_u = 4$.

The resultant input vector of the NARX model then takes the following form:

$$\boldsymbol{x}(t) = \left[\begin{array}{cccc} u(t-3) & u(t-2) & u(t-1) & u(t) \end{array} \right]^\top. \tag{2}$$

The unknown model is approximated with a sum of polynomial basis functions up to second degree ($\lambda = 2$), rendering the following structure

$$\boldsymbol{y}(t) = \theta^0 + \sum_{i=1}^d \theta_i x_i(t) + \sum_{i=1}^d \sum_{j=1}^d \theta_{i,j} x_i(t) x_j(t) + e(t). \tag{3}$$

The number and order of significant terms are identified within the EFOR-CMSS algorithm based on the data from 8 out of 10 datasets. Figure 3 illustrates the relationship between the number of model terms and the selected criterion of significance, AAMDL.

(a) Sample size 2000.

(b) Sample size 4000.
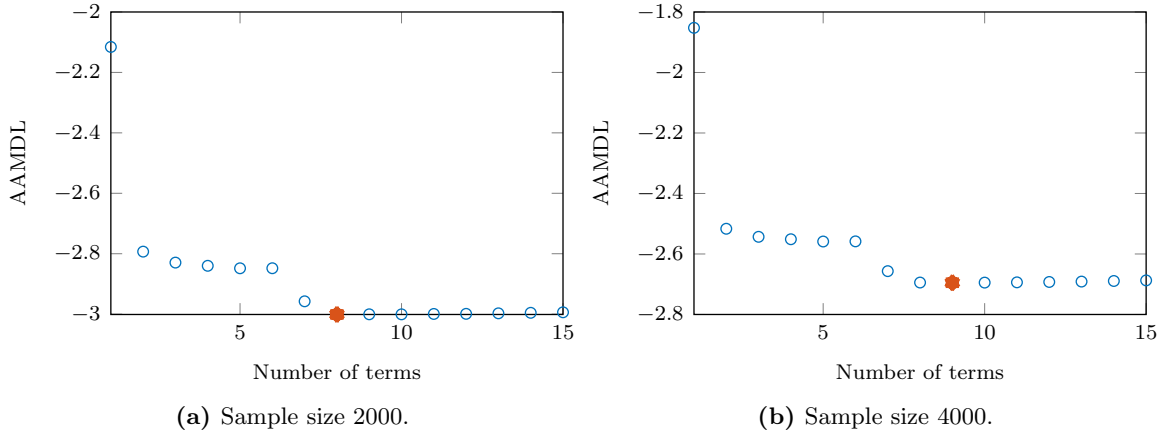
Figure 3: Evolution of AAMDL with the respect to the number of terms for samples of different size. The optimal number of terms (⬢) increases with the growing sample size.

3 Parameter estimation

The results of internal parameter estimation via the EFOR-CMSS method for sample sizes of 2000 and 4000 points are presented in Tables 2 and 3, respectively. It can be seen that the first 7 significant

terms are the same for both samples; moreover, the estimated values are of the same order in both cases. The further analysis deals only with the parameter estimates obtained from the smaller data sample.

Table 2: Estimated parameters for the sample length 2000.

Step	Terms	C1	C2	C4	C5	C6	C7	C9	C10	AEER(%)
1	$x_4 x_4$	-26.04	-20.99	-10.69	-10.96	-191.78	-157.64	-87.42	-69.8	89.511
2	x_3	75.42	59.58	33	26.06	508.94	419.54	242.35	195.15	8.849
3	$x_1 x_4$	0.62	0.76	0.32	0.48	8.55	7.83	2.66	1.15	0.139
4	$x_1 x_1$	0.01	-0.19	-0.15	-0.22	0.05	-0.48	0.44	0.76	0.045
5	x_2	0.71	-0.73	-2.24	-0.66	45.94	36.57	18.4	12.68	0.032
6	x_4	-171.24	-139.22	-69.61	-73.69	-1273.02	-1046.38	-579.72	-465.59	0.006
7	c	-233.16	-200.83	-93.74	-119.7	-1805.9	-1488.55	-803.7	-648.8	0.308
8	$x_3 x_4$	15.47	12.1	6.36	5.68	110.13	90.43	51.77	41.43	0.093

Table 3: Estimated parameters for the sample length 4000.

Step	Terms	C1	C2	C4	C5	C6	C7	C9	C10	AEER(%)
1	$x_4 x_4$	-18.97	-15.33	-8.34	-8.94	-138.21	-114.23	-63.58	-50.3	88.667
2	x_3	63.2	53.09	30.09	24.55	426.09	362.14	208.4	168.78	9.494
3	$x_1 x_4$	4.29	3.45	2.03	3.11	34.39	27.1	14.45	10.2	0.12
4	$x_1 x_1$	0.6	0.87	0.31	0.56	9.24	5.54	4.15	4.1	0.042
5	x_2	2.34	0.07	-1.9	-2.13	43.4	34.96	18.62	12.71	0.036
6	x_4	-157.87	-128.81	-69.1	-69.29	-1153.95	-964.72	-539.51	-431.83	0.006
7	c	-226.03	-187.69	-100.5	-116.41	-1722.47	-1432.4	-789.68	-632.3	0.335
8	$x_3 x_3$	9.21	7.92	4.31	4.67	70	55.98	32.65	26.32	0.103
9	$x_1 x_3$	-4.8	-4.8	-2.64	-4.26	-45.05	-31.95	-19.47	-15.95	0.007

In order to link the external and internal parameters, an arbitrary polynomial function of two arguments is formed

$$\theta_i(L_{cut}, D_{rlx}) = \beta_0 + \beta_1 L_{cut} + \beta_2 D_{rlx} + \beta_3 L_{cut}^2 + \beta_4 D_{rlx}^2 + \beta_5 L_{cut} D_{rlx}, \qquad i = 1, \ldots, N_s, \quad (4)$$

L_{cut} and D_{rlx} are the external parameters of manufacturing process, and where N_s is the estimated number of the significant terms. The coefficients $B = [\beta_0 \ldots \beta_5]$ are unknown and must be estimated from the internal parameter values available. The linear relationship between the batch of internal parameter values corresponding to each significant term Θ_i and the surface coefficients can be established in the following form

$$\begin{bmatrix} \theta_i^1 \\ \vdots \\ \theta_i^k \\ \vdots \\ \theta_i^K \end{bmatrix} = \begin{bmatrix} 1 & L & D & L \times L & D \times D & L \times D \end{bmatrix} B, \qquad (5)$$

where $k = 1, \ldots, K$ is the index of the dataset, $i = 1, \ldots, N_s$ is the element index of the internal parameter vector, and where $\times$ denotes the by-element product such that

$$L \times L = \left[L_{cut}^{(j)} L_{cut}^{(j)} \right]_{j=1}^{N}.$$

Given the equation (5), the vector of unknown coefficients B can be identified via the classical least squares (LS) method. The estimated coefficients are presented in Table 4, and the surface fitting results for each internal parameter are illustrated in Figure 4 The figure also shows values of the internal

Table 4: Estimated polynomial coefficients for the sample length 2000.

NARMAX terms	Input terms	β_0	β_1	β_2	β_3	β_4	β_5
x_4, x_4	$u^2(t)$	-170.31	4.27	831.86	-2.02	-0.03	-4562.38
x_3	$u(t-1)$	143.25	-3.37	-1382.17	-3.9	0.04	11446.44
x_1, x_4	$u(t-3)u(t)$	5.32	0.17	-203.84	2.88	0	161.25
x_1, x_1	$u^2(t-3)$	1.85	-0.2	78.58	-1.4	0	39.42
x_2	$u(t-2)$	76.05	-1.76	-434.89	5.27	0.01	953.45
x_4	$u(t)$	-1216.06	31.23	5342.46	-8.72	-0.25	-30672.05
c	$const$	-2427.76	63.16	8882.74	-21.5	-0.49	-45523.06
x_3, x_4	$u(t-1)u(t)$	61.26	-1.53	-370.55	0.3	0.01	2478.05

parameters computed for the external settings of experiments C3 and C8. The obtained internal parameters are substituted in the modified version of model (3) that only includes the identified significant polynomial terms to validate the identified model structure. The simulation results are compared with true system outputs for C3 and C8 in Figure 5a and Figure 5b, respectively.

4 Estimating from the separated experimental data

The above report replicates the results of [1] for a different pair of manufacturing parameters. The authors consider all datasets in model identification, however they are collected from two different foam types: sets C1-C5 are obtained from , while sets C6-C10 are obtained from . It is therefore reasonable to consider individual dynamical models for different foam types. For the first foam type, F1, sets C1, C2, C4, and C5 are used for identification, and C3 is used for validation of the dynamical model. For the second foam type, F2, set C8 is used for validation while the rest are used in the identification process. Application of the EFOR-CMSS framework is applied to two different foam types produces similar results, as can be seen in Tables 5-6. However, difficulties may arise during the stage of mapping the external parameters onto the internal parameter space. Since only 4 data points are estimated for each foam type, a bilinear model is used for surface fitting

$$\theta_i(L_{cut}, D_{rlx}) = \beta_0 + \beta_1 L_{cut} + \beta_2 D_{rlx} + \beta_3 L_{cut} D_{rlx}, \qquad i = 1, \ldots, N_s. \tag{6}$$

Model coefficients are estimated with LS method and are presented in Tables 7-8.

Remark 4.1: *It may be possible to perform the direct estimation of the polynomial coefficients from the time series instead of estimating internal parameters in an intermediate step. This would ensure identifiability of higher order polynomials. However, a batch problem of estimating internal parameters*

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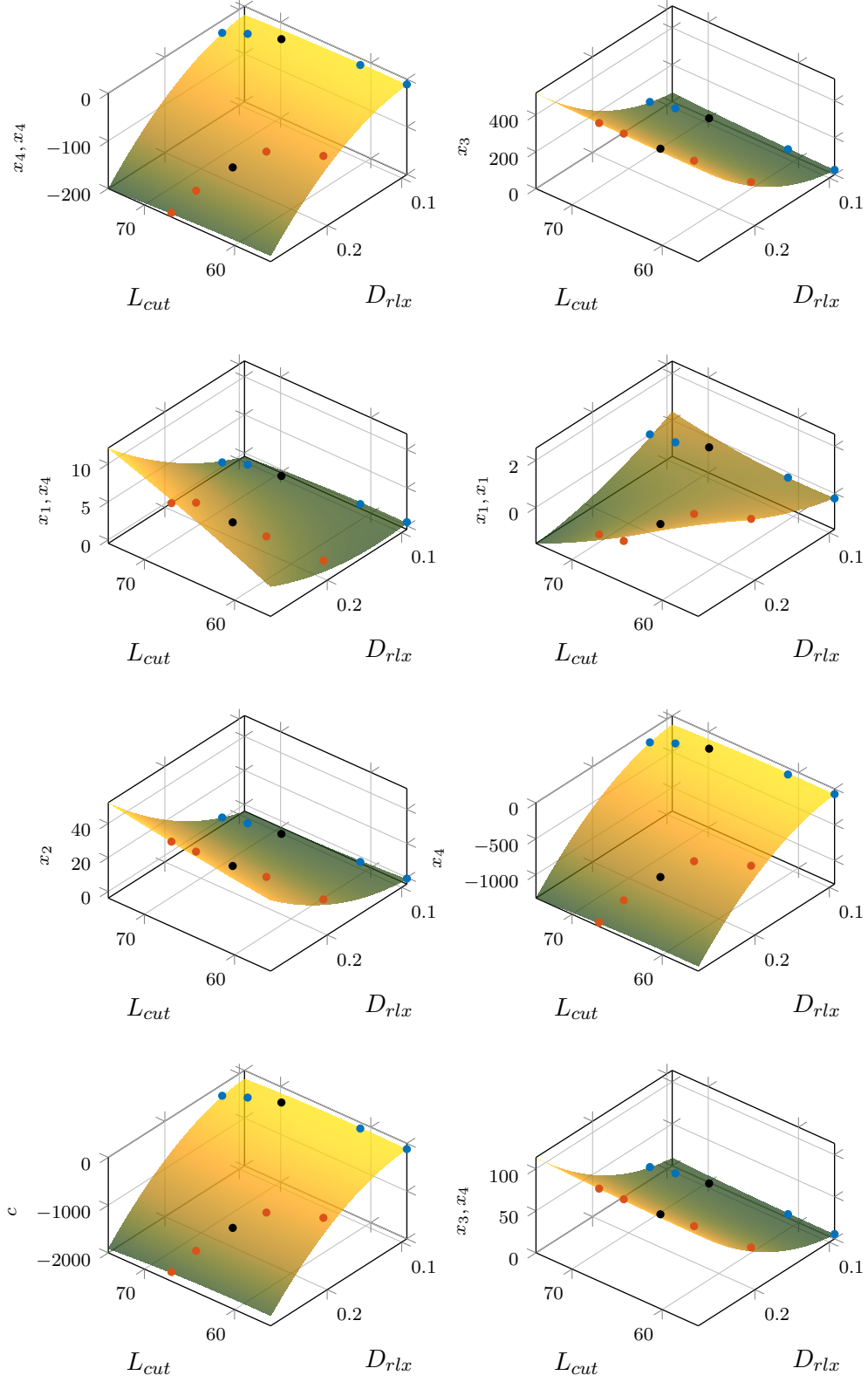
Figure 4: The quantitative relationship between external parameters (L_{cut}, D_{rlx}) and internal parameters θ is obtained by fitting a polynomial surface to the internal parameter values estimated for the sets C1-C5 (•) and sets C6-C10 (•). The fitted surfaces ( ) are then used to compute the internal model parameters corresponding to the settings of choice (•).
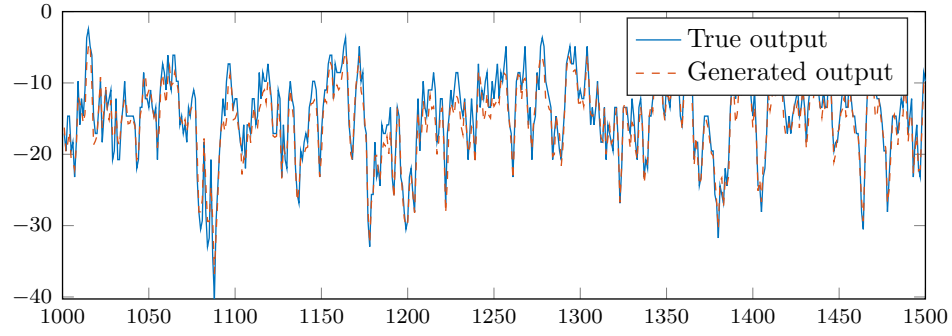
(a) C3
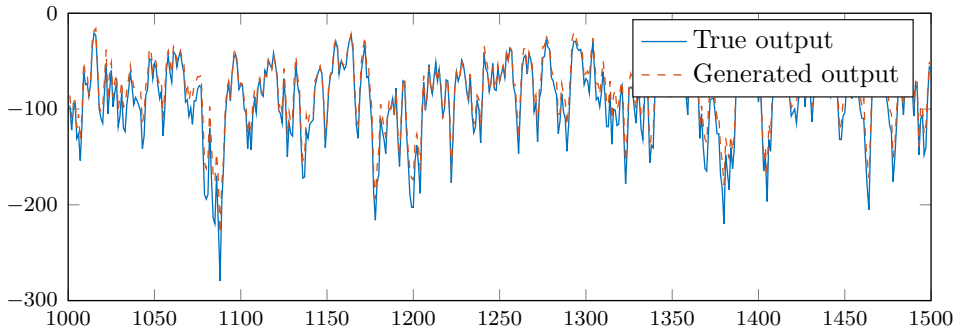
(b) C8

Figure 5: Samples of the output obtained experimentally and the output generated by the identified model.

Table 5: Estimated parameters for the datasets obtained from foam F1.

Step	Terms	C1	C2	C4	C5	AEER(%)
1	x_4, x_4	-26.04	-20.99	-10.69	-10.96	90.313
2	x_3	75.42	59.58	33	26.06	7.772
3	x_1, x_4	0.62	0.76	0.32	0.48	0.072
4	x_1, x_1	0.01	-0.19	-0.15	-0.22	0.035
5	x_2	0.71	-0.73	-2.24	-0.66	0.005
6	x_4	-171.24	-139.22	-69.61	-73.69	0.002
7	c	-233.16	-200.83	-93.74	-119.7	0.242
8	x_3, x_4	15.47	12.1	6.36	5.68	0.071

from all datasets must be formulated first in order to avoid rank deficiency in the LS matrices. This will be investigated in the future.

References

[1] H. L. Wei, Z. Q. Lang, and S. A. Billings, "Constructing an overall dynamical model for a system with changing design parameter properties," *International Journal of Modelling, Identification and Control*, vol. 5, no. 2, p. 93, 2008, ISSN: 1746-6172. DOI: 10.1504/IJMIC.2008.022014. [Online]. Available: http://www.inderscience.com/link.php?id=22014.

Table 6: Estimated parameters for the datasets obtained from foam F2.

Step	Terms	C6	C7	C9	C10	AEER(%)
1	x_4, x_4	-199.23	-163.93	-90.75	-71.69	88.709
2	x_3	670.73	553.59	311.92	238.03	9.927
3	x_1, x_4	36.26	30.69	13.76	9.54	0.207
4	x_2	-237.96	-196.97	-97.99	-69.39	0.066
5	x_1, x_1	4.36	2.85	2.01	2.31	0.048
6	x_4	-1318.98	-1084.72	-600.14	-477.45	0.009
7	c	-1665.09	-1377.67	-752.29	-598.01	0.374
8	x_3, x_4	137.39	113	63.49	48.62	0.116
9	x_1	215.63	175.27	84.61	68.21	0.003
10	x_2, x_4	-47.82	-39.33	-19.6	-13.79	0.006

Table 7: Estimated polynomial coefficients for the model describing the dynamics of foam F1.

| Terms	β_0	β_1	β_2	β_3
x_4, x_4	752.37	-8.64	-8838.21	104.16
x_3	1634.9	-21.1	-18933.22	256.09
x_1, x_4	-200.43	2.46	2307.82	-29.14
x_1, x_1	93.51	-1.17	-1072.53	13.72
x_2	-429.54	4.88	5130.04	-61.73
x_4	6565.82	-76.57	-76864.82	921.08
c	22443.12	-269.32	-261223.28	3228.31
x_3, x_4	50.76	-0.94	-534.9	10.96

Table 8: Estimated polynomial coefficients for the model describing the dynamics of foam F1. foam F2.

| Terms	β_0	β_1	β_2	β_3
x_4, x_4	13.44	3.89	-876.45	-12.5
x_3	923.68	-25.09	-3650.85	131.7
x_1, x_4	-943.69	12.18	5604.01	-74.79
x_2	2240.92	-25.9	-13596.07	162.72
x_1, x_1	355.33	-5.08	-1986.17	29.06
x_4	144.45	25.34	-6447.46	-74.73
c	5763.57	-44.84	-40741.32	368.83
x_3, x_4	306.47	-6.82	-1400.59	36.47
x_1	-1733.21	18.12	11504.04	-131.98
x_2, x_4	325.67	-3.48	-2002.72	22.29
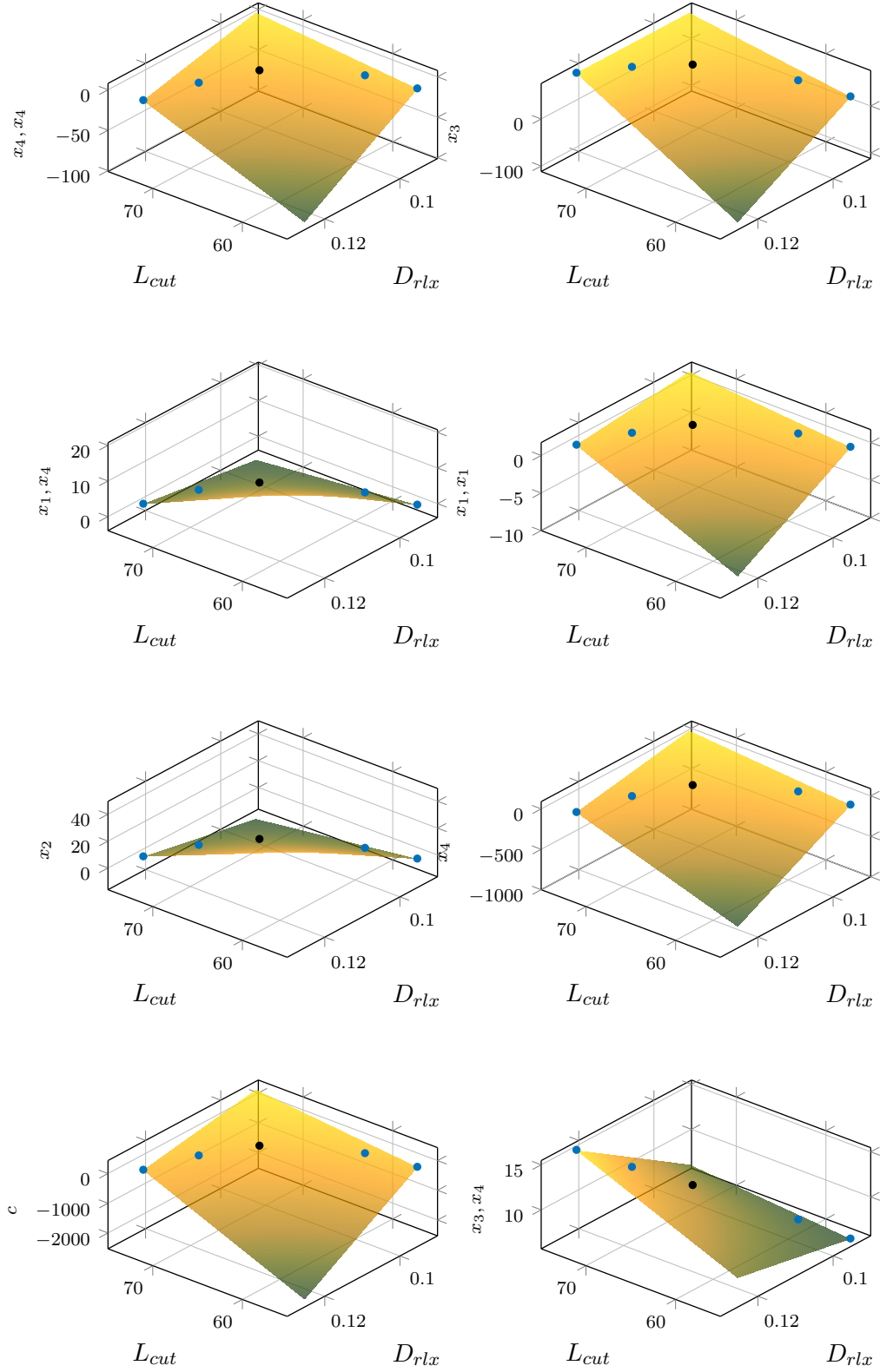
Figure 6: The surfaces ( ) fitted to the internal parameter values estimated for the sets C1-C5 (•) are used to compute the internal model parameters corresponding to the settings of choice (•).
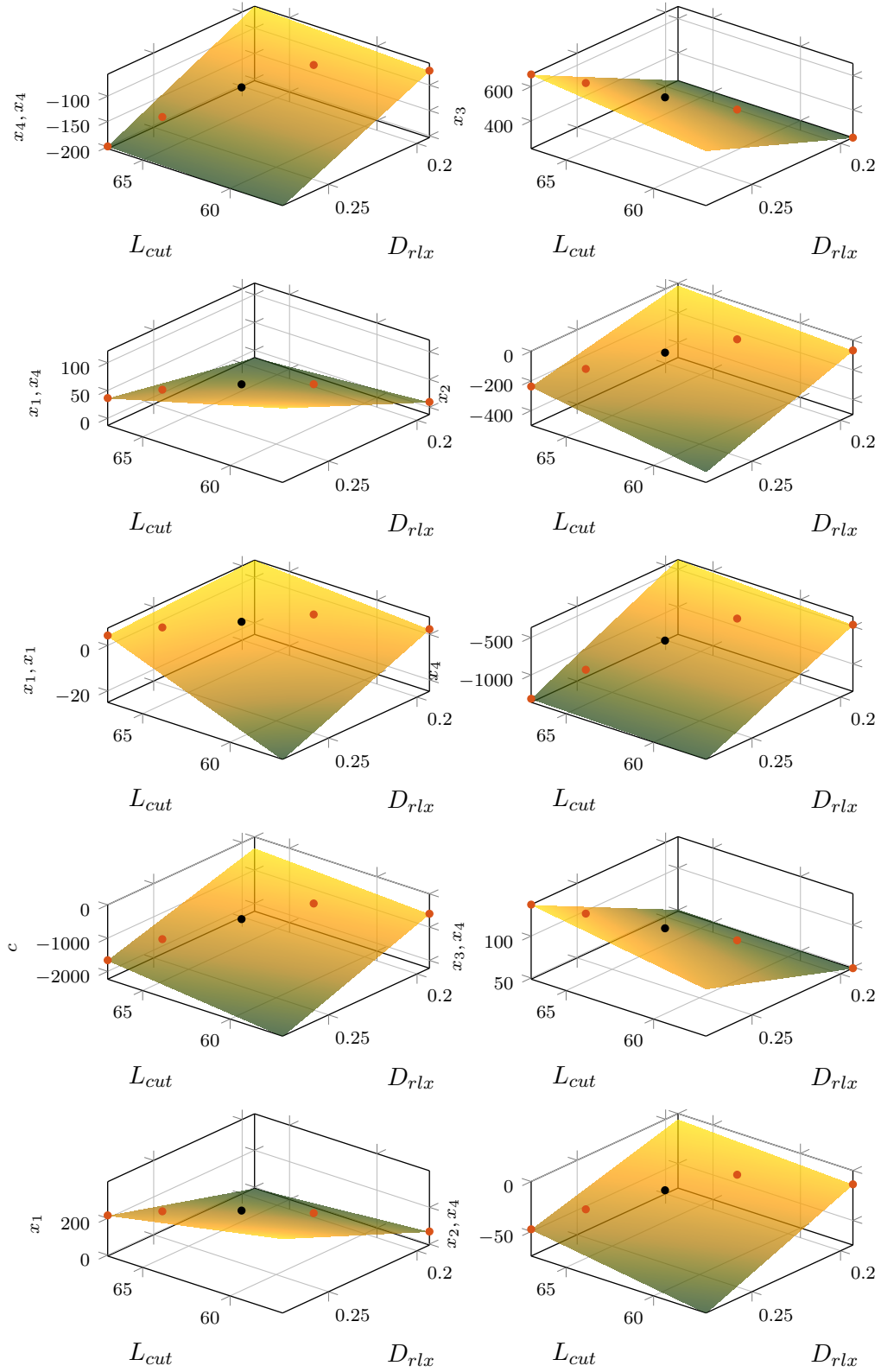
Figure 7: The surfaces (⬦) fitted to the internal parameter values estimated for the sets C1-C5 (•) are used to compute the internal model parameters corresponding to the settings of choice (•).
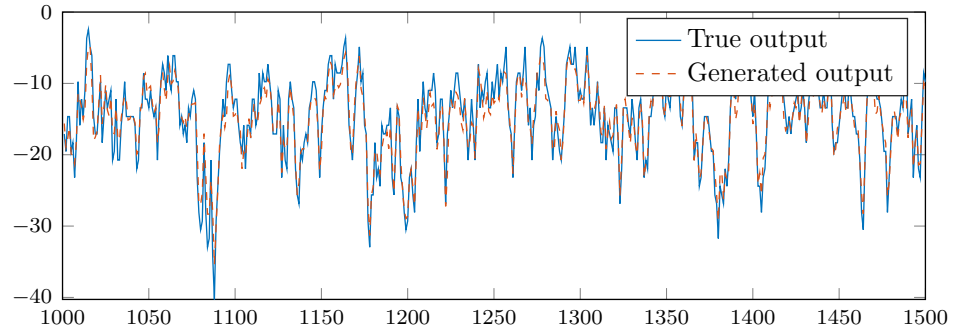
(a) C3
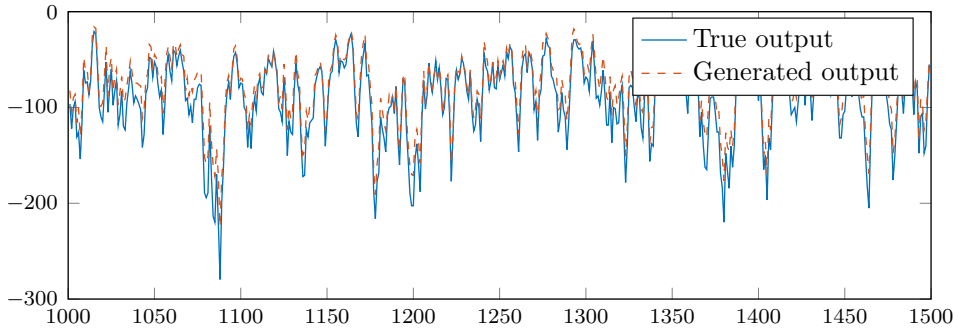
(b) C8

Figure 8: Samples of the output obtained experimentally and the output generated by the models identified from datasets corresponding to Foams (a) F1 and (b) F2.