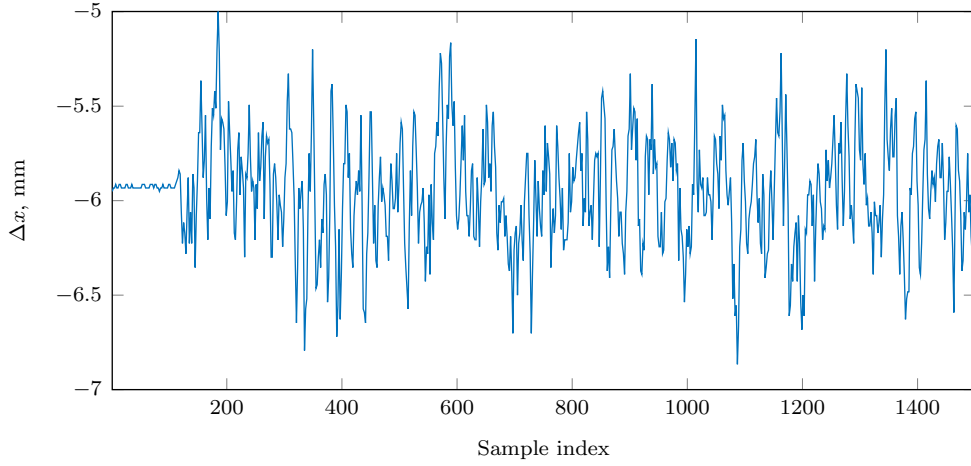


# Structure and parameter identification of au

October 16, 2019

## 1 Experimental data



**Figure 1:** The input for the set C.

## 2 Structure identification

The following model structure is assumed. The output of the NARX model  $\mathbf{y}(t)$  is the measured load. The input vector is composed as

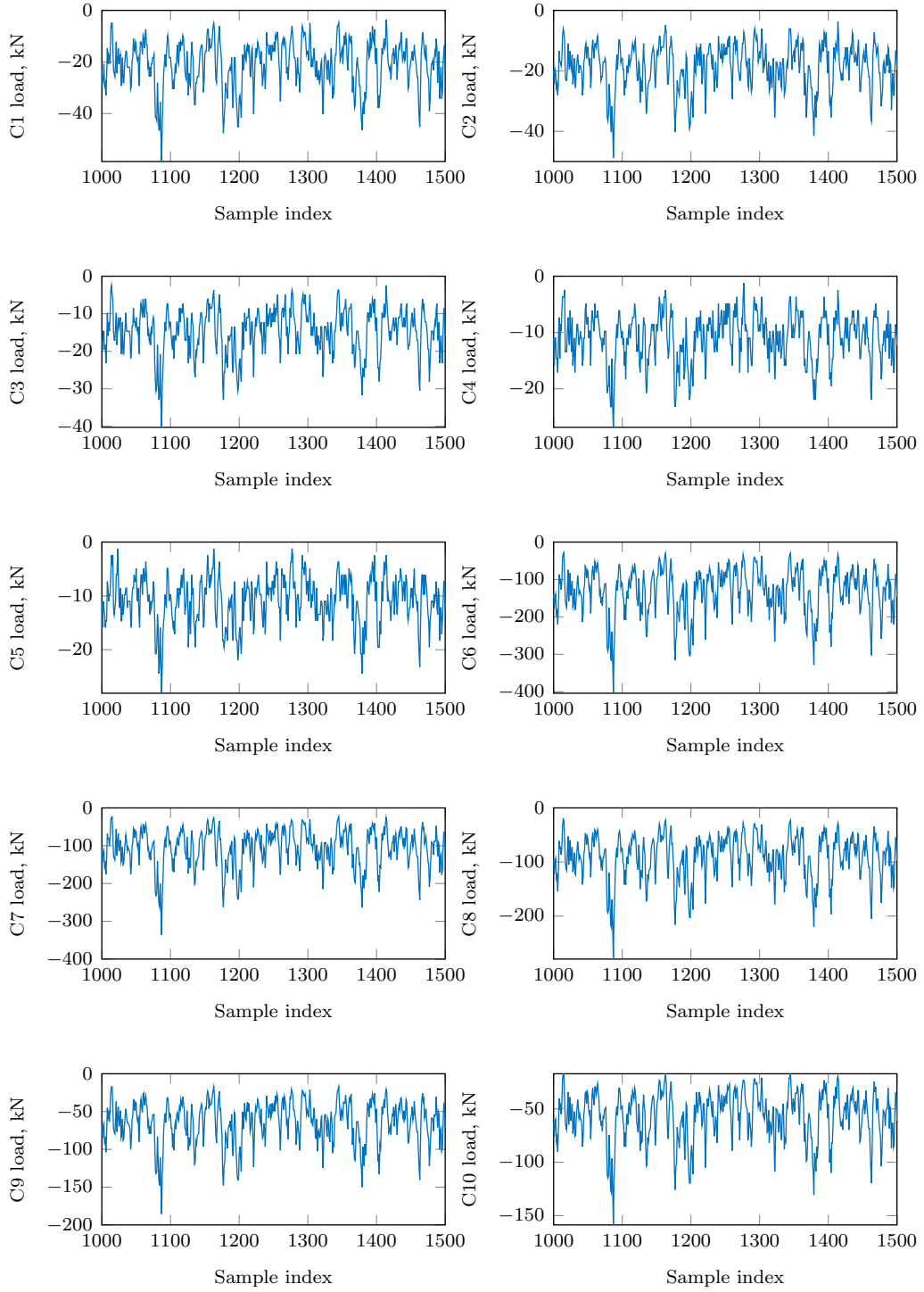
$$\mathbf{x}(t) = \{x_i(t)\}_{i=1}^d = \left[ \{y(t-k+1)\}_{k=1}^{n_y} \quad \{u(t-k+n_y+1)\}_{k=n_y+1}^{n_y+n_u} \right]^\top, \quad (1)$$

where  $n_u$  is the length of the input lag and  $n_y$  is the length of the output lag in discrete time, and where  $d = n_u + n_y$ . In this case, the identification is performed under the following assumptions:

- only the input signal affects the output ( $n_y = 0$ ).
- the input signal has a lag of length  $n_u = 4$ .

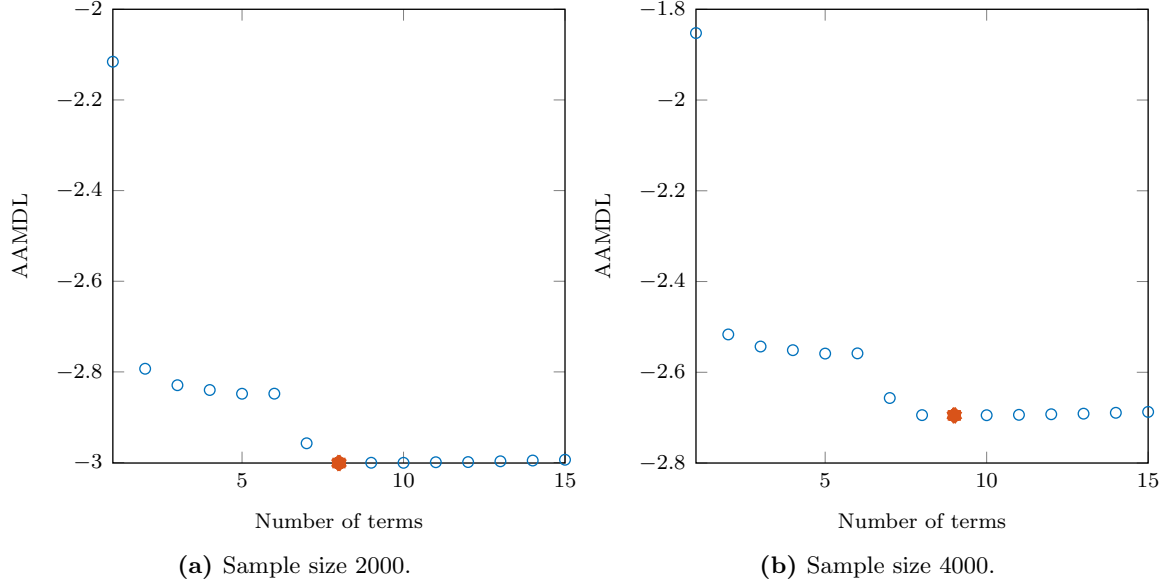
The unknown model is approximated with a sum of polynomial basis functions up to second degree ( $\lambda = 2$ ), rendering the following structure

$$\mathbf{y}(t) = \theta^0 + \sum_{i=1}^d \theta_i x_i(t) + \sum_{i=1}^d \sum_{j=1}^d \theta_{i,j} x_i(t) x_j(t) + e(t). \quad (2)$$



**Figure 2:** Experimental data.

The number and order of significant terms are identified within the EFOR-CMSS algorithm based on the data from 8 out of 10 datasets. Figure 3 illustrates the relationship between the number of model terms and the selected criterion of significance, AAMD<sub>L</sub>.



**Figure 3:** AAMD L evolution with the growing number of terms for samples of different size.

### 3 Parameter estimation

Results of internal parameter estimation via EFOR-CMSS for different sample sizes are presented in Tables 2 and 3. Visualised

**Table 1:** Estimated parameters for the sample length 2000.

Step	Terms	C1	C2	C4	C5	C6	C7	C9	C10	AEER(%)
1	$x_4, x_4$	-26.04	-20.99	-10.69	-10.96	-191.78	-157.64	-87.42	-69.8	89.511
2	$x_3$	75.42	59.58	33	26.06	508.94	419.54	242.35	195.15	8.849
3	$x_1, x_4$	0.62	0.76	0.32	0.48	8.55	7.83	2.66	1.15	0.139
4	$x_1, x_1$	0.01	-0.19	-0.15	-0.22	0.05	-0.48	0.44	0.76	0.045
5	$x_2$	0.71	-0.73	-2.24	-0.66	45.94	36.57	18.4	12.68	0.032
6	$x_4$	-171.24	-139.22	-69.61	-73.69	-1273.02	-1046.38	-579.72	-465.59	0.006
7	$c$	-233.16	-200.83	-93.74	-119.7	-1805.9	-1488.55	-803.7	-648.8	0.308
8	$x_3, x_4$	15.47	12.1	6.36	5.68	110.13	90.43	51.77	41.43	0.093

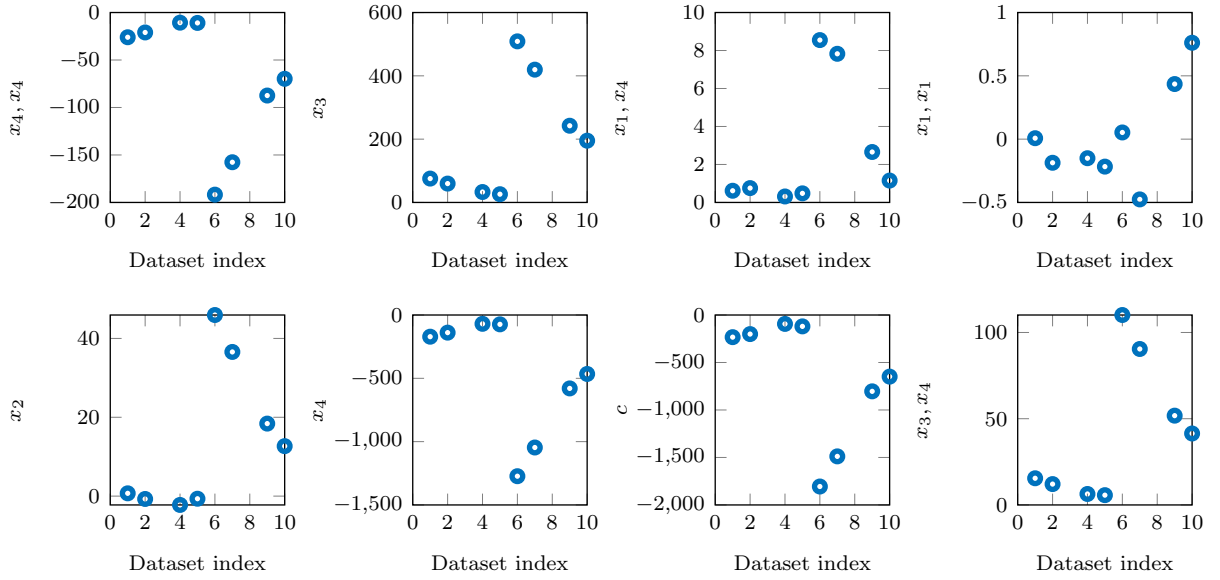
In order to link the external and internal parameters an arbitrary polynomial function is selected for two arguments

$$\text{content} \dots \quad (3)$$

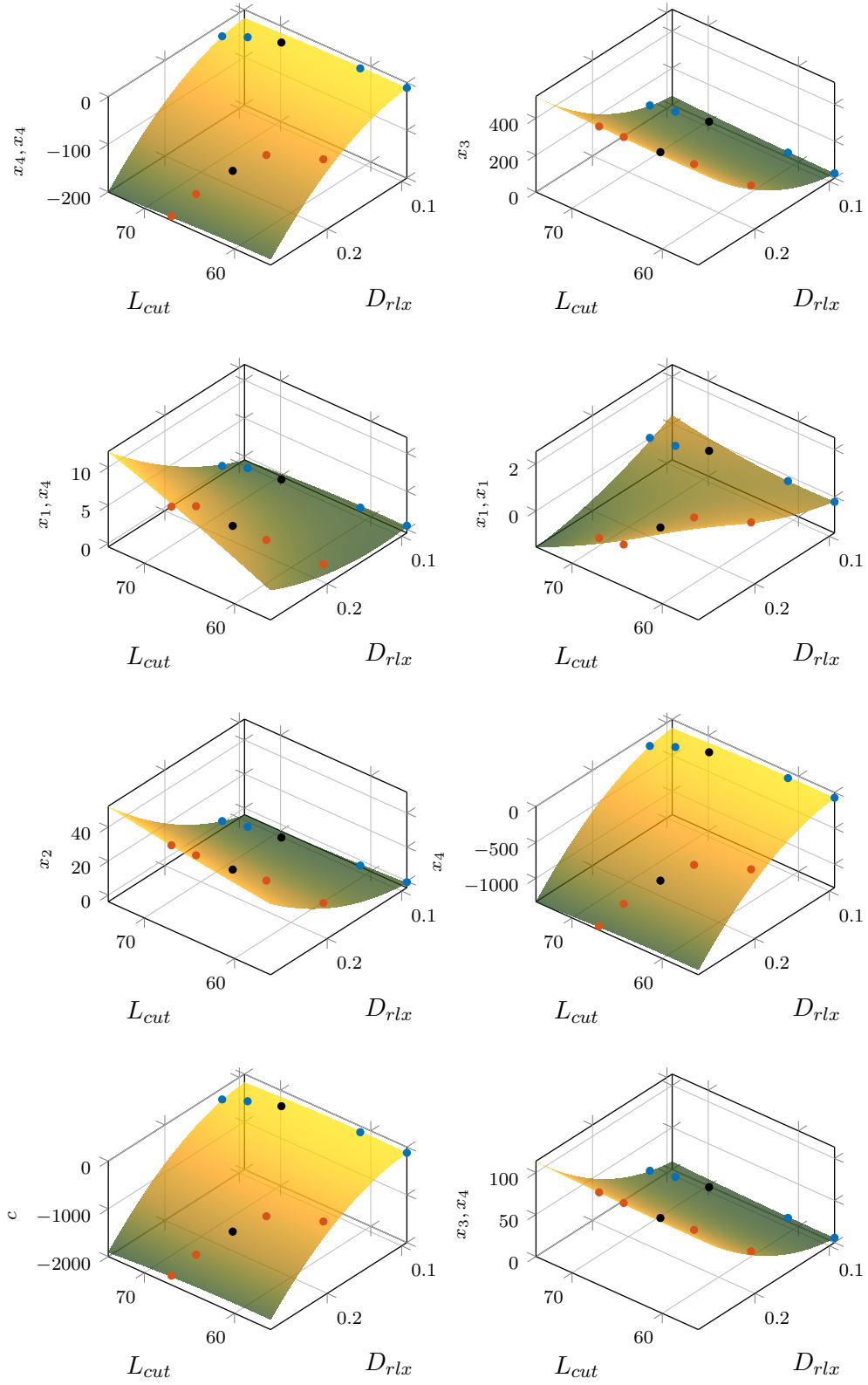
Curve fitting results are presented in Tables 4 and 5.

**Table 2:** Estimated parameters for the sample length 4000.

Step	Terms	C1	C2	C4	C5	C6	C7	C9	C10	AEER(%)
1	$x_4, x_4$	-18.97	-15.33	-8.34	-8.94	-138.21	-114.23	-63.58	-50.3	88.667
2	$x_3$	63.2	53.09	30.09	24.55	426.09	362.14	208.4	168.78	9.494
3	$x_1, x_4$	4.29	3.45	2.03	3.11	34.39	27.1	14.45	10.2	0.12
4	$x_1, x_1$	0.6	0.87	0.31	0.56	9.24	5.54	4.15	4.1	0.042
5	$x_2$	2.34	0.07	-1.9	-2.13	43.4	34.96	18.62	12.71	0.036
6	$x_4$	-157.87	-128.81	-69.1	-69.29	-1153.95	-964.72	-539.51	-431.83	0.006
7	$c$	-226.03	-187.69	-100.5	-116.41	-1722.47	-1432.4	-789.68	-632.3	0.335
8	$x_3, x_3$	9.21	7.92	4.31	4.67	70	55.98	32.65	26.32	0.103
9	$x_1, x_3$	-4.8	-4.8	-2.64	-4.26	-45.05	-31.95	-19.47	-15.95	0.007

**Figure 4:** Estimated values of internal parameters.**Table 3:** Estimated polynomial coefficients for the sample length 2000.

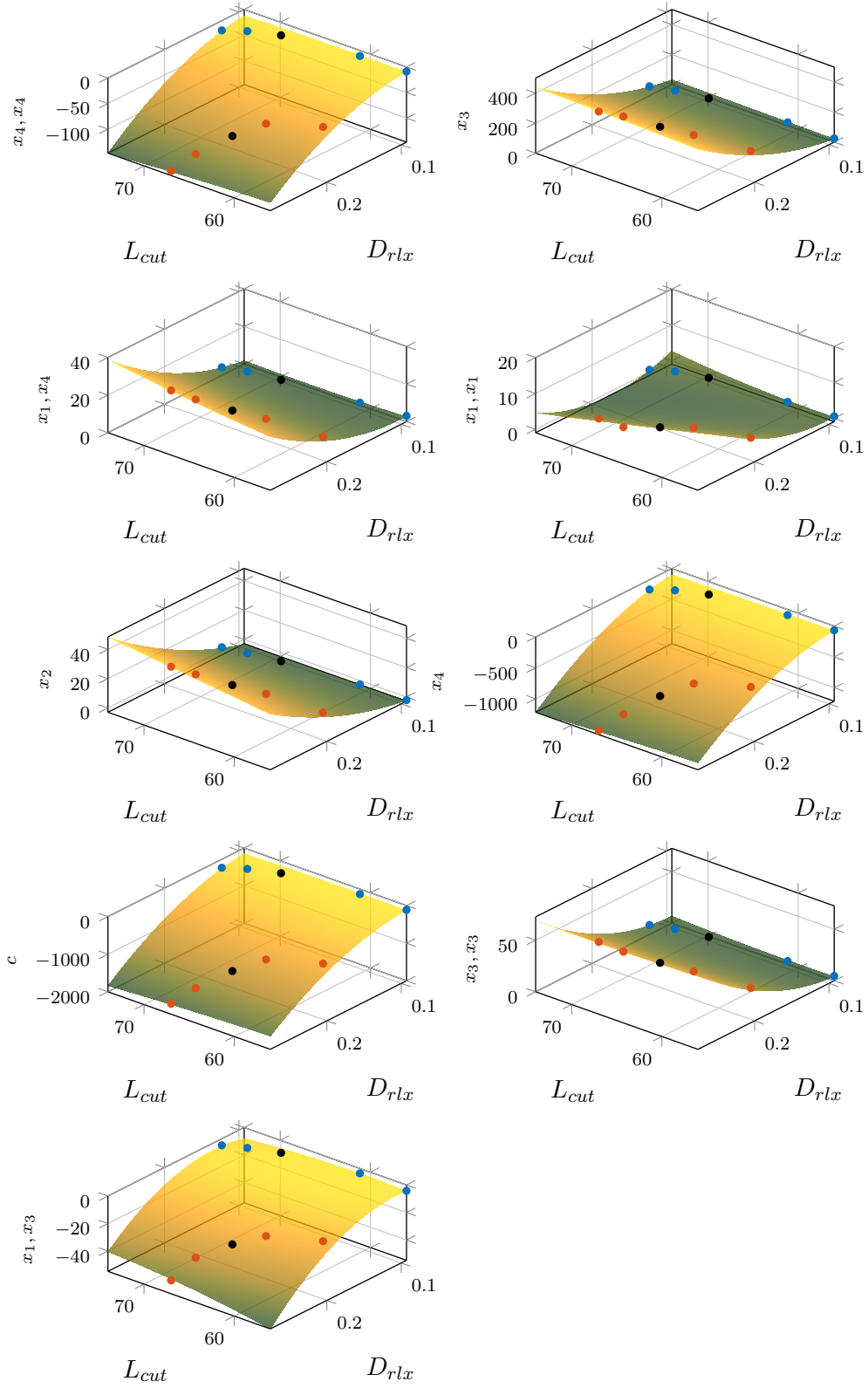
Terms	$\beta_0$	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$\beta_5$
$x_4, x_4$	-170.31	4.27	831.86	-2.02	-0.03	-4562.38
$x_3$	143.25	-3.37	-1382.17	-3.9	0.04	11446.44
$x_1, x_4$	5.32	0.17	-203.84	2.88	0	161.25
$x_1, x_1$	1.85	-0.2	78.58	-1.4	0	39.42
$x_2$	76.05	-1.76	-434.89	5.27	0.01	953.45
$x_4$	-1216.06	31.23	5342.46	-8.72	-0.25	-30672.05
$c$	-2427.76	63.16	8882.74	-21.5	-0.49	-45523.06
$x_3, x_4$	61.26	-1.53	-370.55	0.3	0.01	2478.05



**Figure 5:** Parameter maps reconstructed with LS surface fitting to the internal parameters estimated from the data sample of length 2000.

**Table 4:** Estimated polynomial coefficients for the sample length 4000.

Terms	$\beta_0$	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$\beta_5$
$x_4, x_4$	-125.63	2.93	707.07	-4.21	-0.02	-3082.44
$x_3$	-11.09	1.62	-1300.85	5.5	-0.01	8416.59
$x_1, x_4$	68.38	-1.52	-356.04	3.41	0.01	826.42
$x_1, x_1$	16.4	-0.72	129.23	-4.4	0.01	559.78
$x_2$	19.92	-0.36	-304.69	4.27	0	729.42
$x_4$	-760.18	17	5145.74	-29.05	-0.12	-24830.84
$c$	-1758.23	41.83	8866.02	-51.35	-0.29	-38902.3
$x_3, x_3$	55.85	-1.48	-198.99	-1.75	0.01	1812.12
$x_1, x_3$	-94.37	2.78	58.97	5.49	-0.03	-1745



**Figure 6:** Parameter maps reconstructed with LS surface fitting to the internal parameters estimated from the data sample of length 4000.