Structure and parameter identification of the dynamical model of auxetic foam

December 20, 2019

Experimental data is described in the earlier reports. This report summarises the settings used for model identification and presents estimation and modelling results.

1 Structure identification

The following model structure is assumed. The output of the NARX model $\boldsymbol{y}(t)$ is the measured load. The input vector is composed as

$$\boldsymbol{x}(t) = \{x_i(t)\}_{i=1}^{d} = \Big[\{y(t-k)\}_{k=1}^{n_y} \quad \{u(t-k+n_y+1)\}_{k=n_y+1}^{n_y+n_u}\Big]^\top, \tag{1}$$

where n_u is the length of the input lag and n_y is the length of the output lag in discrete time, and where $d = n_u + n_y$. In this case, the identification is performed under the following assumptions:

- the output lag $n_y = 4$.
- the input signal has a lag of length $n_u = 4$.

The resultant input vector of the NARX model then takes the following form:

$$\boldsymbol{x}(t) = \begin{bmatrix} u(t-n_u) & \dots & y(t-1) & u(t-n_u+1) & \dots & u(t) \end{bmatrix}^\top. \tag{2}$$

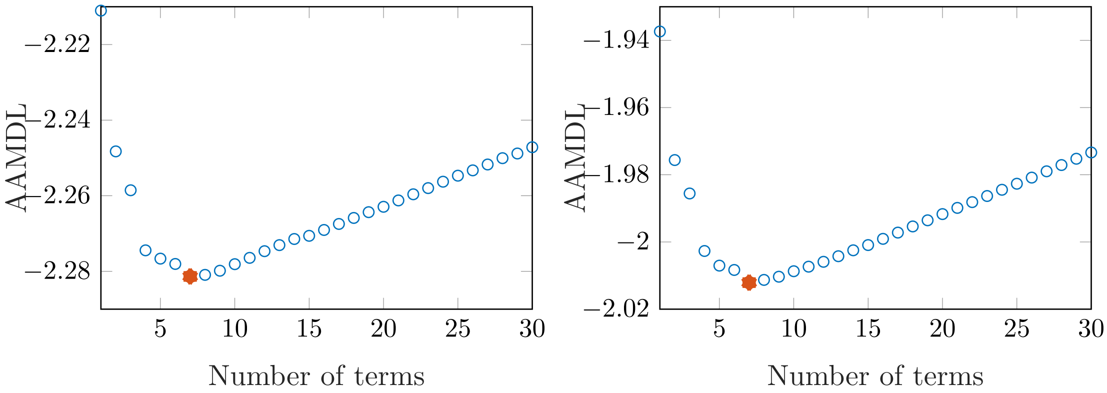
The unknown model is approximated with a sum of polynomial basis functions up to second degree ($\lambda = 3$), rendering the following structure

$$\boldsymbol{y}(t) = \theta^0 + \sum_{i=1}^{d} \theta_i x_i(t) + \sum_{i=1}^{d}\sum_{j=i}^{d} \theta_{i,j} x_i(t) x_j(t) + \sum_{i=1}^{d}\sum_{j=i}^{d}\sum_{k=j}^{d} \theta_{i,j} x_i(t) x_j(t) x_k(t) + e(t). \tag{3}$$

The number and order of significant terms are identified within the EFOR-CMSS algorithm based on the data from 8 out of 10 datasets. Figure 1 illustrates the relationship between the number of model terms and the selected criterion of significance, AAMDL.

2 Parameter estimation

The results of internal parameter estimation via the EFOR-CMSS method for sample sizes of 2000 and 4000 points are presented in Tables 1 and 2, respectively. Figure 2 showing AERRs computed



(a) Sample size 2000. **(b)** Sample size 4000.

Figure 1: Evolution of AAMDL with the respect to the number of terms for samples of different size. The optimal number of terms (✹) increases with the growing sample size.

Table 1: Estimated parameters for the sample length 2000.

Step	Terms	C1	C2	C4	C5	C6	C7	C9	C10	AERR(%)
1	$y(t-1)$	0.53	0.4	0.18	0.29	0.82	0.79	0.7	0.72	91.962
2	$u(t-1)$	6.02	6.41	6.46	2.35	39.38	31.99	13.32	13.36	0.836
3	$u(t-2)$	-13.88	-14.64	-12.42	-9.28	-35.75	-34.03	-22.68	-19.78	0.243
4	$y(t-3)$	0.1	0.08	0.07	0	0.16	0.15	0.1	0.14	0.331
5	$y(t-2)u(t-4)u(t-2)$	0	0	0	0	-0.01	-0.01	0	0	0.079
6	$u(t-3)u(t-1)$	-2.67	-2.78	-1.91	-2.19	-0.22	-1.7	-3.67	-2.66	0.061
7	$u(t-4)u(t-2)u(t-2)$	-0.19	-0.19	-0.12	-0.14	0	-0.11	-0.26	-0.19	0.09

Table 2: Estimated parameters for the sample length 4000.

Step	Terms	C1	C2	C4	C5	C6	C7	C9	C10	AERR(%)
1	$y(t-1)$	0.54	0.43	0.19	0.23	0.89	0.89	0.8	0.79	91.067
2	$u(t-1)$	6.45	5.02	4.41	3.52	44.97	33.23	14.98	12.69	0.954
3	$u(t-2)$	-13.2	-12.84	-10.84	-10.5	-31.27	-24.37	-16.92	-15.68	0.263
4	$y(t-3)$	0.13	0.09	0.03	0.03	0.24	0.23	0.18	0.2	0.397
5	$y(t-2)u(t-4)u(t-1)$	0	0	0	0	-0.01	-0.01	-0.01	-0.01	0.13
6	$u(t-3)u(t-1)$	-2.35	-2.67	-2.07	-2.23	2.88	1.68	-1.32	-1.57	0.065
7	$u(t-4)u(t-2)u(t-2)$	-0.17	-0.19	-0.13	-0.15	0.2	0.12	-0.1	-0.12	0.109

for all terms on the first iteration confirms this concern: the AERR of the selected significant term is only slightly higher than the first autoregressive term $y(t-1)$. Scatter plots for $y(t-1)$, $u(t)$, and $y(t-1)u(t)$ are shown in Figures 5-7. The simplest way to avoid this problem is to first estimate the AR(0) relationship and/or current input signal, and use its residual as the first approximation in the EFOR-CMSS algorithm.

In order to link the external and internal parameters, an arbitrary polynomial function of two arguments is formed

$$\theta_i(L_{cut}, D_{rlx}) = \beta_0 + \beta_1 L_{cut} + \beta_2 D_{rlx} + \beta_3 L_{cut}^2 + \beta_4 D_{rlx}^2 + \beta_5 L_{cut} D_{rlx}, \qquad i = 1, \ldots, N_s, \tag{4}$$

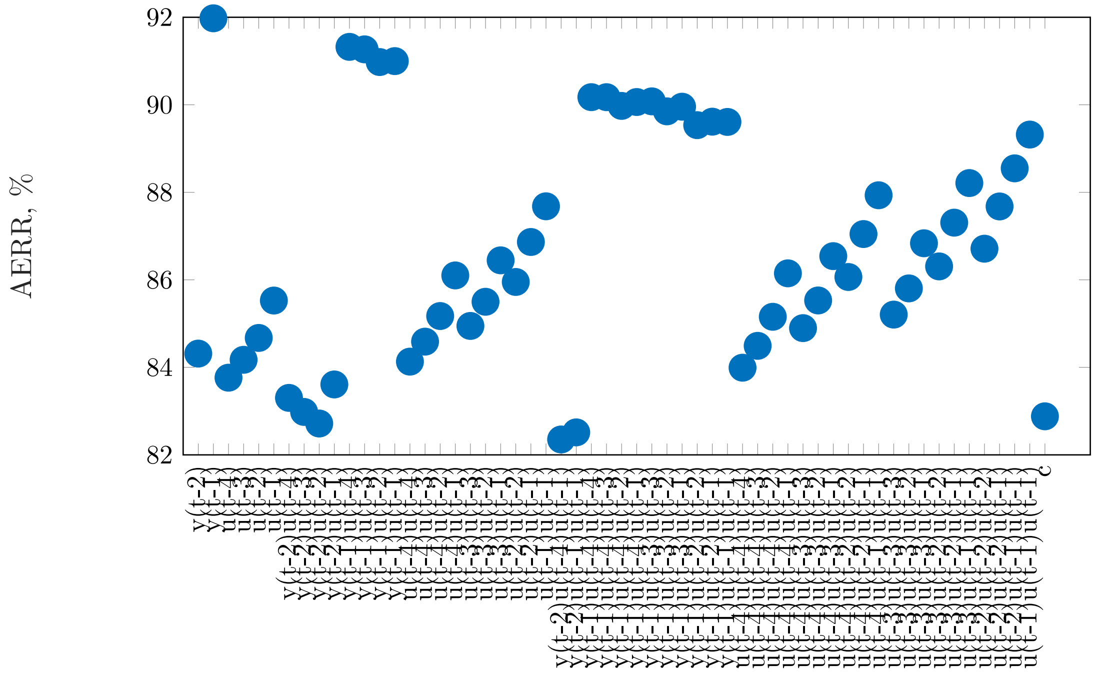


Figure 2: AERR computed for all terms in the dictionary during the first iteration of the forward selection algorithm. While autoregressive and linear input terms are strongly correlated with the output, the maximum criterion achieved by their product.

where L_{cut} and D_{rlx} are the external parameters of manufacturing process, and where N_s is the estimated number of the significant terms. The coefficients $B = [\beta_0 \dots \beta_5]$ are unknown and must be estimated from the internal parameter values available. The linear relationship between the batch of internal parameter values corresponding to each significant term Θ_i and the surface coefficients can be established in the following form

$$\begin{bmatrix} \theta_i^1 \\ \vdots \\ \theta_i^k \\ \vdots \\ \theta_i^K \end{bmatrix} = \begin{bmatrix} 1 & L & D & L \odot L & D \odot D & L \odot D \end{bmatrix} B, \tag{5}$$

where $k = 1, \dots, K$ is the index of the dataset, $i = 1, \dots, N_s$ is the element index of the internal parameter vector, and where $\odot$ denotes Hadamard entry-wise product:

$$L \odot L = \left[L_{cut}^{(k)} L_{cut}^{(k)} \right]_{k=1}^{K}.$$

Given the equation (5), the vector of unknown coefficients B can be identified via the classical least squares (LS) method. The estimated coefficients are presented in Table 3, and the surface fitting results for each internal parameter are illustrated in Figure 3 The figure also shows values of the internal parameters computed for the external settings of experiments C3 and C8. The obtained internal parameters are substituted in the modified version of model (3) that only includes the identified significant polynomial terms to validate the identified model structure. The simulation results are

Table 3: Estimated polynomial coefficients for the sample length 2000.

Terms	β_0	β_1	β_2	β_3	β_4	β_5
$y(t-1)$	7.71	-0.25	7.13	0.05	0	-19.06
$u(t-1)$	-71.79	2.02	123.11	-12.55	0	2397.36
$u(t-2)$	132.27	-4.17	-42.32	2.07	0.03	-596.63
$y(t-3)$	-1.54	0.03	8.51	-0.19	0	12.83
$y(t-2)u(t-4)u(t-2)$	-0.08	0	-0.16	0	0	0.04
$u(t-3)u(t-1)$	19.71	-0.67	13.78	-3.01	0.01	528.39
$u(t-4)u(t-2)u(t-2)$	1.35	-0.04	-0.13	-0.2	0	38.65

compared with true system outputs for C3 and C8 in Figure 4a and Figure 4b, respectively. Computed RMSEs for both lengths of the sample are presented in Table 4.

Table 4: RMSE of the system output generated by the identified model.

Sample size 2000	Sample size 4000
`RMSE for set 3 is 3.89`	`RMSE for set 3 is 1.78`
`RMSE for set 8 is 23.71`	`RMSE for set 8 is 4.71`

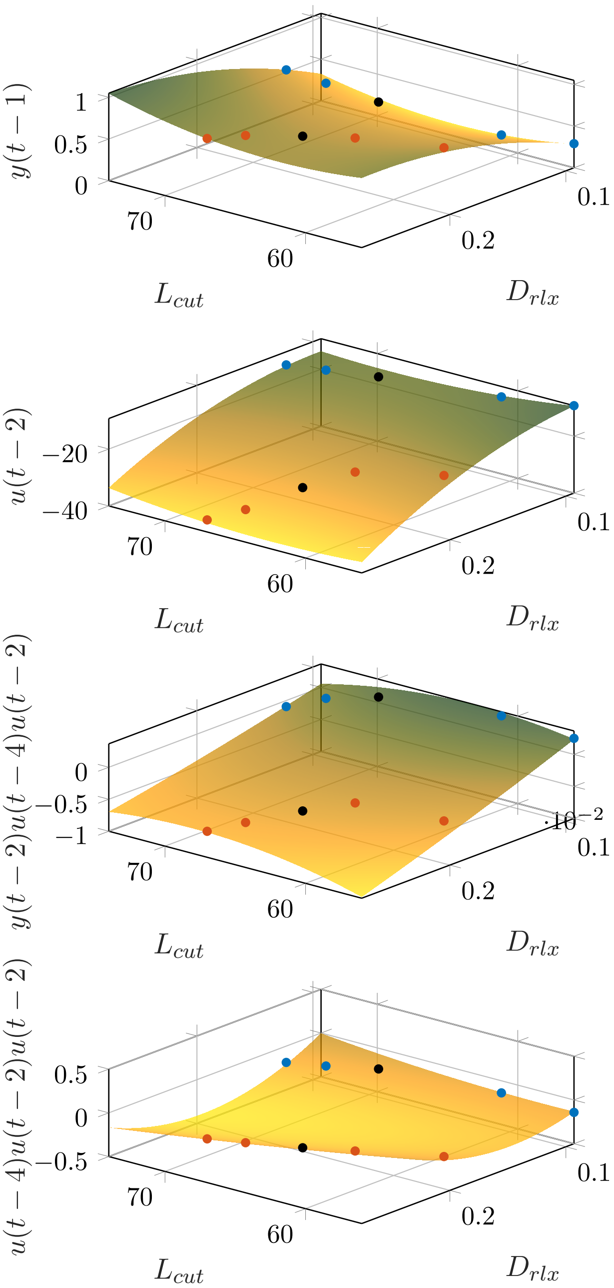
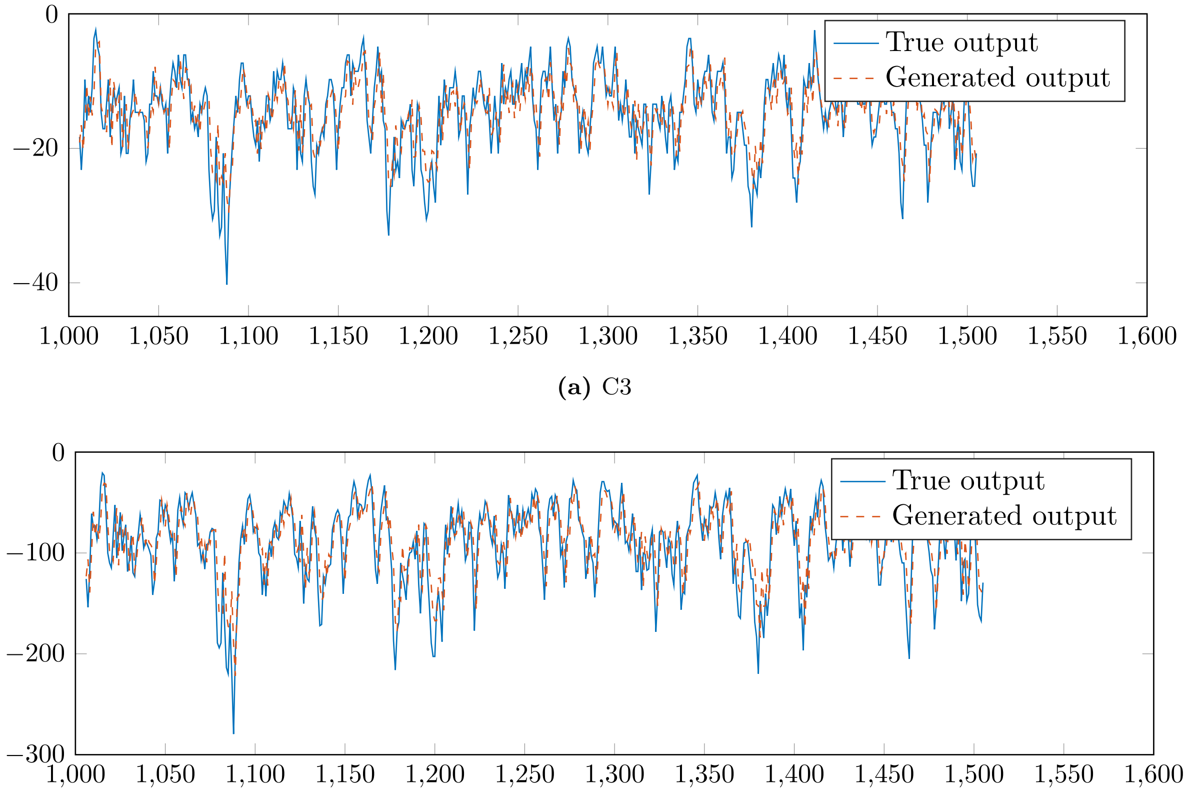


Figure 3: The quantitative relationship between external parameters (L_{cut}, D_{rlx}) and internal parameters θ is obtained by fitting a polynomial surface to the internal parameter values estimated for the sets C1-C5 (•) and sets C6-C10 (•). The fitted surfaces ( ) are then used to compute the internal model parameters corresponding to the settings of choice (•).



(a) C3

(b) C8

Figure 4: Samples of the output obtained experimentally and the output generated by the identified model.

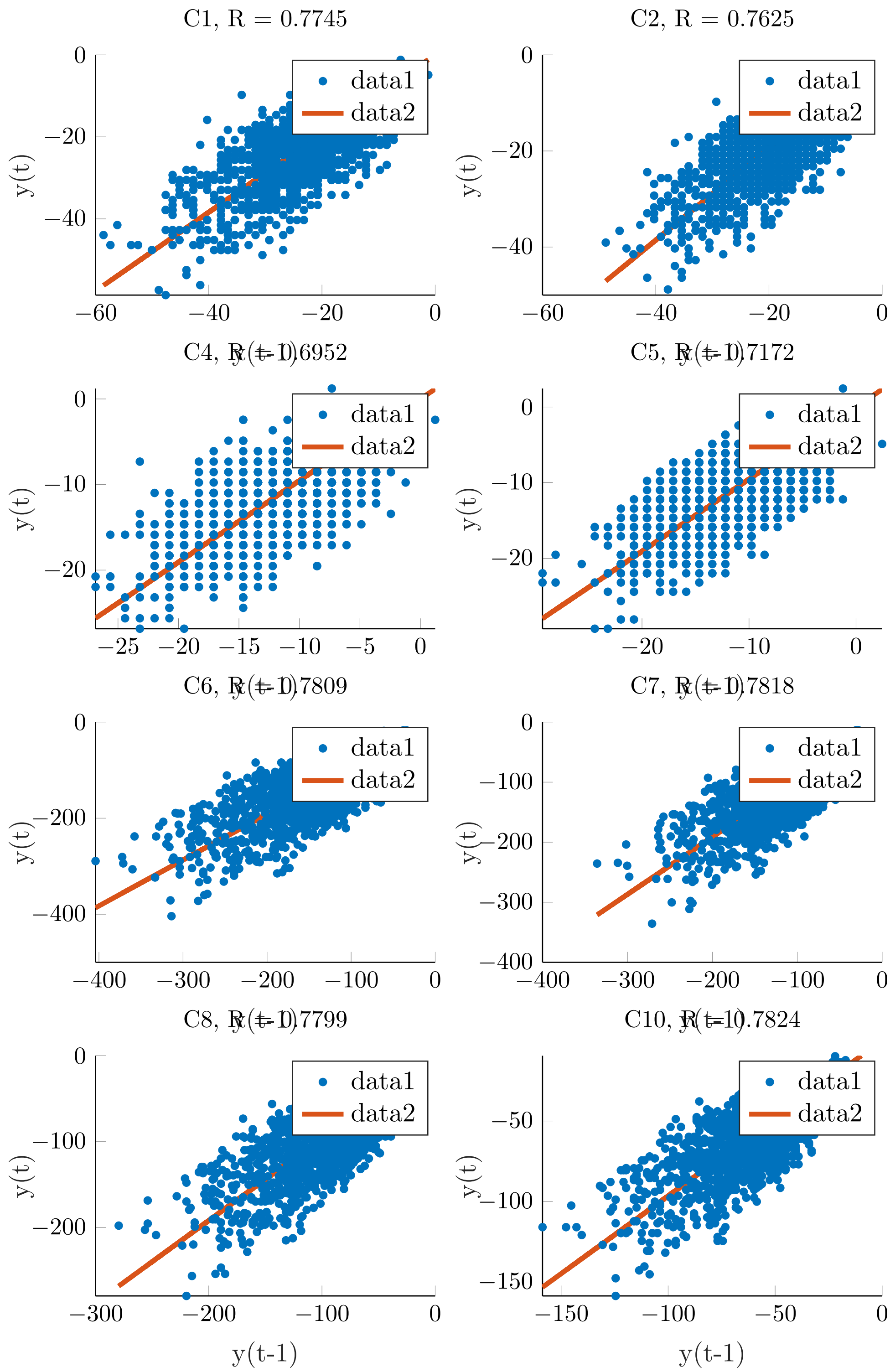


Figure 5: Scatter plots of the AR(0) term $y(t-1)$ for training datasets. Correlation coefficient with the output signal is shown for each figure.

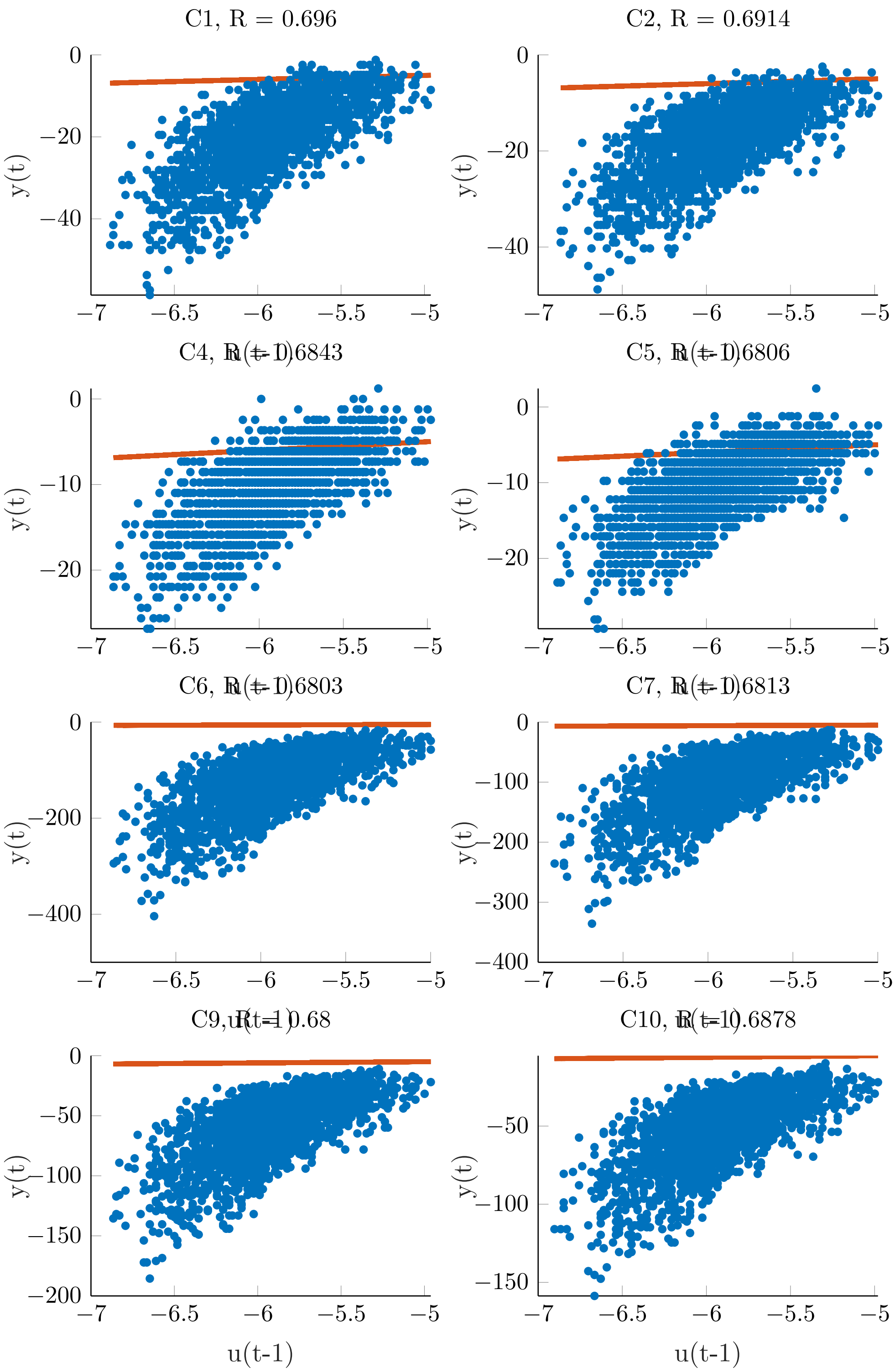


Figure 6: Scatter plots of the input term $u(t)$ for training datasets. Correlation coefficient with the output signal is shown for each figure.

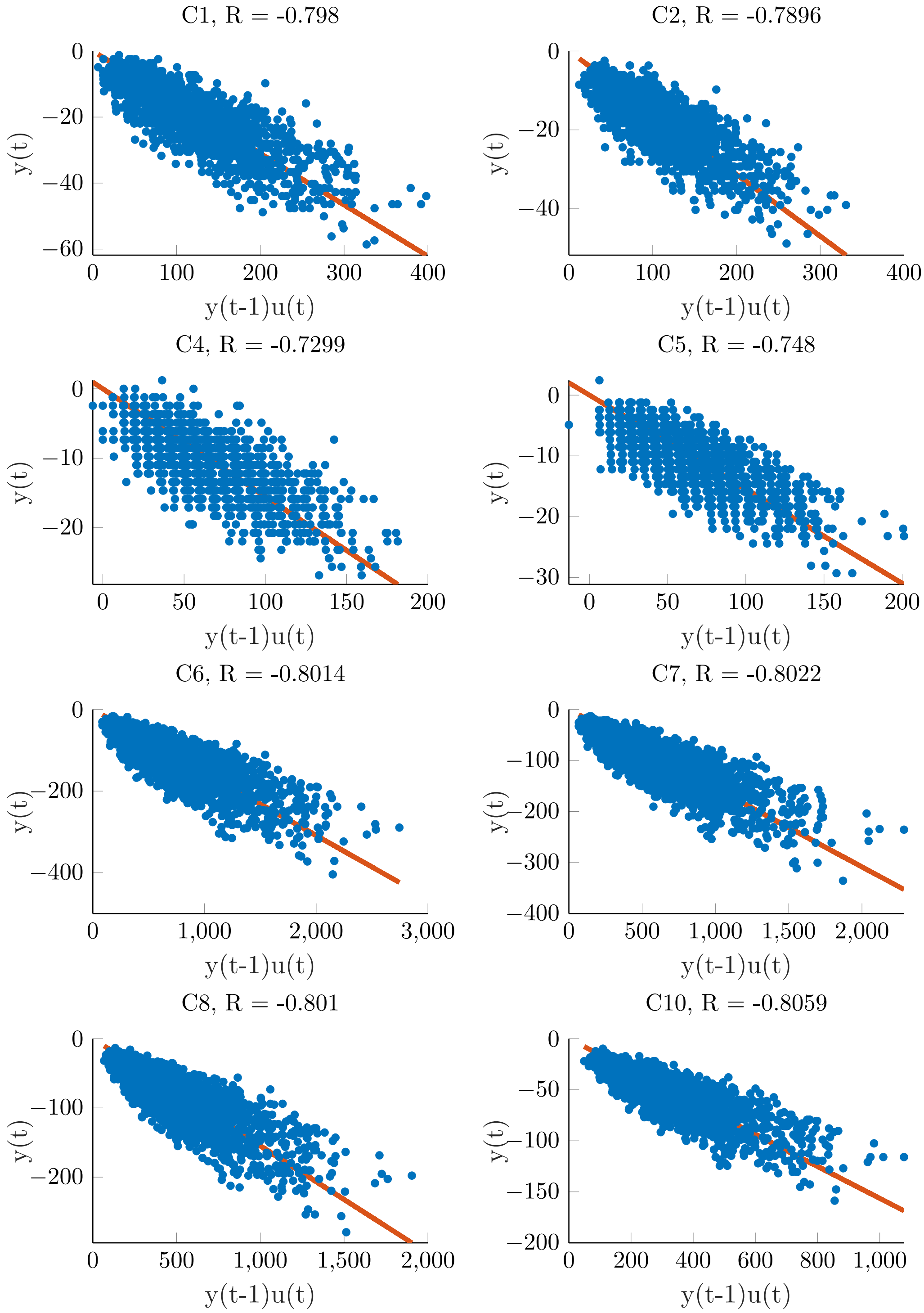


Figure 7: Scatter plots of the identified significant term $y(t-1)u(t)$ for training datasets. Correlation coefficient with the output signal is shown for each figure.