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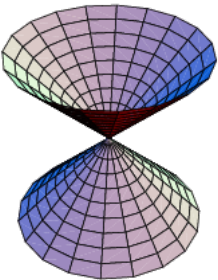
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Cone

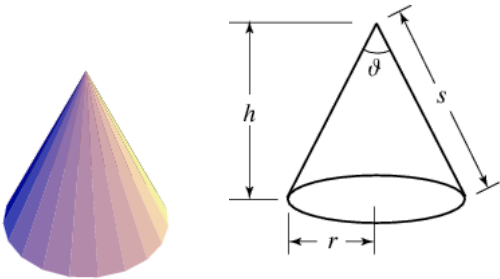
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In general, a cone is a [pyramid](#) with a circular [cross section](#). A right cone is a cone with its vertex above the center of its base. However, when used without qualification, the term "cone" often means "right cone."

In discussions of [conic sections](#), the word "cone" is taken to mean "double cone," i.e., two cones placed apex to apex. The double cone is a [quadratic surface](#), and each single cone is called a "nappe." The [hyperbola](#) can then be defined as the intersection of a [plane](#) with both [nappes](#) of the cone.

Cones are implemented in the [Wolfram Language](#) as `Cone[{x1, y1, z1}, {x2, y2, z2}], r]`.



A right cone of height  $h$  and base radius  $r$  oriented along the  $z$ -axis, with vertex pointing up, and with the base located at  $z = 0$  can be described by the [parametric equations](#)

$$x = \frac{h - u}{h} r \cos \theta$$
$$y = \frac{h - u}{h} r \sin \theta$$
$$z = u$$

(1)

(2)

(3)

for  $u \in [0, h]$  and  $\theta \in [0, 2\pi)$ .

The [opening angle](#) of a right cone is the vertex angle made by a cross section through the apex and center of the base. For a cone of height  $h$  and radius  $r$ , it is given by

$$\vartheta = 2 \tan^{-1} \left( \frac{r}{h} \right).$$

(4)

Adding the squares of (1) and (2) shows that an implicit Cartesian equation for the cone is given by

$$\frac{x^2 + y^2}{c^2} = (z - z_0)^2,$$

(5)

where

$$c \equiv \frac{r}{h}$$

(6)

is the ratio of radius to height at some distance from the vertex, a quantity sometimes called the opening angle, and  $z_0 = h$  is the height of the apex above the  $z = 0$  plane.

The [volume](#) of a cone is

$$V = \frac{1}{3} A_b h,$$

(7)

where  $A_b$  is the base [area](#) and  $h$  is the height. If the base is circular, then

$$V = \pi \int_0^h (x^2 + y^2) dz$$
$$= \pi \int_0^h \left( \frac{(h - z) r}{h} \right)^2 dz$$


(8)


(9)


THINGS TO TRY:


- = cone
- = 125 + 375
- = continuum hypothesis

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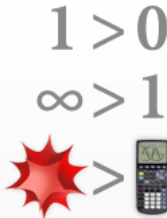
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Sarah Lichtblau
- 

Cone, Tent, and Cylinder  
George Beck
- 

Placing Objects at Predetermined Sets of Points  
S&#225;ndor Kabai
- 

Brake Shoes  
S&#225;ndor Kabai



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$$= \frac{1}{3} \pi r^2 h. \quad (10)$$

This amazing fact was first discovered by Eudoxus, and other proofs were subsequently found by Archimedes in *On the Sphere and Cylinder* (ca. 225 BC) and Euclid in Proposition XII.10 of his *Elements* (Dunham 1990).

The **geometric centroid** can be obtained by setting  $R_2 = 0$  in the equation for the centroid of the **conical frustum**,

$$\bar{z} = \frac{\langle z \rangle}{V} = \frac{h (R_1^2 + 2 R_1 R_2 + 3 R_2^2)}{4 (R_1^2 + R_1 R_2 + R_2^2)}, \quad (11)$$

(Eshbach 1975, p. 453; Beyer 1987, p. 133) yielding

$$\bar{z} = \frac{1}{4} h. \quad (12)$$

The interior of the cone of base radius  $r$ , height  $h$ , and mass  $M$  has moment of inertia tensor about its apex of

$$I = \begin{bmatrix} \frac{1}{20} (2 h^2 + 3 r^2) M & 0 & 0 \\ 0 & \frac{1}{20} (2 h^2 + 3 r^2) M & 0 \\ 0 & 0 & \frac{3}{10} M r^2 \end{bmatrix}. \quad (13)$$

For a right circular cone, the **slant height**  $s$  is

$$s = \sqrt{r^2 + h^2} \quad (14)$$

and the surface **area** (not including the base) is

$$S = \pi r s \quad (15)$$

$$= \pi r \sqrt{r^2 + h^2}. \quad (16)$$

The **locus** of the apex of a variable cone containing an **ellipse** fixed in three-space is a **hyperbola** through the **foci** of the **ellipse**. In addition, the **locus** of the apex of a cone containing that **hyperbola** is the original **ellipse**. Furthermore, the **eccentricities** of the **ellipse** and **hyperbola** are reciprocals.

There are three ways in which a grid can be mapped onto a cone so that it forms a **cone net** (Steinhaus 1999, pp. 225-227).

The equation for a general (infinite, double-napped) cone is given by

$$x = a u \cos v \quad (17)$$

$$y = a u \sin v \quad (18)$$

$$z = u, \quad (19)$$

which gives coefficients of the **first fundamental form**

$$E = 1 + a^2 \quad (20)$$

$$F = 0 \quad (21)$$

$$G = a^2 u^2, \quad (22)$$

**second fundamental form** coefficients

$$e = 0 \quad (23)$$

$$f = 0 \quad (24)$$

$$g = \frac{a |u|}{\sqrt{1 + a^2}}, \quad (25)$$

and **area element**

$$dS = a \sqrt{1 + a^2} |u| du dv. \quad (26)$$

The **Gaussian curvature** is

$$K = 0, \quad (27)$$

and the **mean curvature** is

$$M = \frac{|u|}{2 a \sqrt{1 + a^2} u^2}. \quad (28)$$

Note that writing  $z = v$  instead of  $z = u$  would give a **helicoid** instead of a cone.

#### SEE ALSO:

Bicone, Cone Net, Cone Set, Conic Section, Conical Frustum, Cylinder, Double Cone, Generalized Cone, Helicoid, Nappe, Pyramid, Sphere, Sphericon

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