Task 1 - USD-RUB barrier bond

Bond

Face value = 100%

Interest rate = 4.15% (annual)

Coupon = 0.00002% (semiannual)

T = 0.5 years

Price = 97.99% of capital

Discount = 2.01%

We buy bond

Cash-or-nothing binary put option

S (spot price) = 100%

K (strike) = 95%

T (tenor) = 0.5

r (interest rate) = 4.15%

q (foreign interest rate) = 0.12% (Treasury yield curve, as of 08.10.2020)

σ (volatility) = 14.14% (as of 09.10.2020)

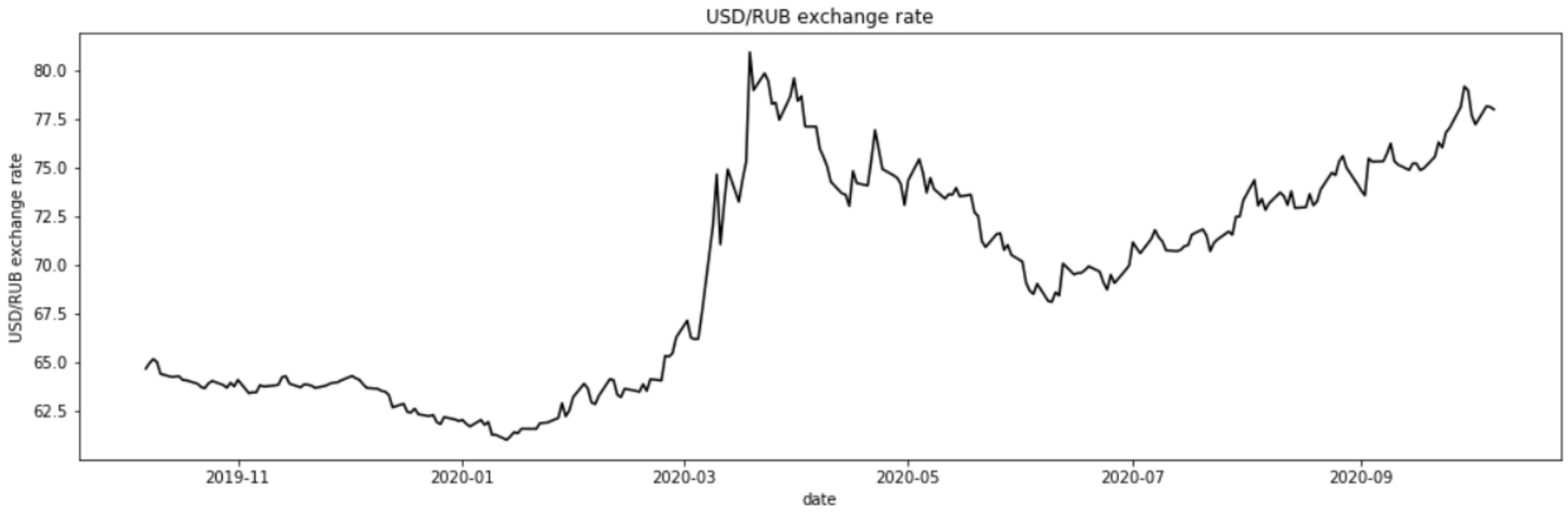
Price = 24.8%

We buy option

Volatility calculation

Volatility was estimated using AR(1)-GARCH(1,1) model with skewed Student`s distribution for residuals. Data for 1 year was used. Procedure:

1. Calculate returns
2. Estimate model
3. Obtain conditional volatility and annualize

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                                Mean Model
============================================================================
                 coef    std err          t      P>|t|     95.0% Conf. Int.
----------------------------------------------------------------------------
Const          0.0800      0.367      0.218      0.827  [ -0.638,  0.798]
Close[1]      -0.0860  5.650e-02     -1.523      0.128 [ -0.197,2.471e-02]
                              Volatility Model
=============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
-----------------------------------------------------------------------------
omega          0.9088      0.788      1.154      0.249   [ -0.635,  2.453]
alpha[1]       0.1259  7.512e-02      1.677  9.362e-02 [-2.128e-02,  0.273]
beta[1]        0.8741  6.520e-02     13.406  5.541e-41   [  0.746,  1.002]
                               Distribution
============================================================================
                 coef    std err          t      P>|t|     95.0% Conf. Int.
----------------------------------------------------------------------------
nu             5.1620      1.753      2.944  3.235e-03  [  1.726,  8.598]
lambda         0.1923  7.194e-02      2.674  7.505e-03 [5.133e-02,  0.333]
```

Calculation of option price

The price 24.8% is for 100% face value, but we need smaller denomination.

After accounting for coupon, put should give us 4.995% of additional income, given that RUB becomes stronger currency.

We multiply 4.995% by 24.8% to get option price of 1.24%. This is 1.26% of bond face value.

Total cost of the instrument is 99.23%.

$$d_1 = \frac{\ln \frac{S}{K} + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}}, \ d_2 = d_1 - \sigma\sqrt{T}$$

$$P = e^{-rT}\Phi(-d_2)$$

Task 2 – Reverse convertible with European put

Bond

Face value = 100%

Interest rate = 4.89% (as of 09.10.2020)

Coupon = 0.01% (as bonds cannot pay no coupon due to legal issues)

T = 3 years

Price = 87% of capital

We buy bond

European put option

S = 100%

K = 70%

T = 3

r = 4.89%

σ = 30%

Price = 3.5% (Black-Scholes model)

We short-sell put option

Task 2 – Reverse convertible with European put

Annual return on option = 1.15%

Annual return on instrument (with coupon) = 1.16%

Total return on instrument = 3.51%

Current cost of instrument = 83.17% of capital.

However, the cost is not final. As we short-sell put, we should pay commission and also return put. So the real cost is higher. Additionally, we may incur further costs if the option is exercised.

Task 3 – Reverse convertible with barrier option

Bond

Face value = 100%

Interest rate = 4.89% (as of 09.10.2020)

Coupon = 0.01% (as bonds cannot pay no coupon due to legal issues)

T = 3 years

Price = 87% of capital

We buy bond

Down-and-in barrier put option

S = 100%

K = 100%

H (barrier) = 70%

T = 3

r = 4.89%

σ = 30%

Price = 12.12%

We short-sell put option

Calculation of option price

We assume that the price is continuously monitored and use Black-Scholes formulae.

$$p_{\mathrm{di}} = -S_0 N(-x_1)e^{-qT} + Ke^{-rT}N(-x_1 + \sigma\sqrt{T}) + S_0 e^{-qT}(H/S_0)^{2\lambda}[N(y) - N(y_1)] - Ke^{-rT}(H/S_0)^{2\lambda-2}[N(y - \sigma\sqrt{T}) - N(y_1 - \sigma\sqrt{T})]$$

$$x_1 = \frac{\ln(S_0/H)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}, \quad y_1 = \frac{\ln(H/S_0)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}$$

$$\lambda = \frac{r - q + \sigma^2/2}{\sigma^2}$$

$$y = \frac{\ln[H^2/(S_0 K)]}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}$$

Annual return on option = 3.88%

Annual return on instrument (with coupon) = 3.89%

Total return on instrument = 12.13%

Current cost of instrument = 75.0% of capital

Again, the cost is not final for the same reasons as previously.

Further considerations

- In the last 2 cases, we do not know q in the formula (dividend yield), so the price is not as accurate as it could be
- For volatility estimation, several models could be tested to find the best one. For instance, for binary options it is important to consider volatility skew
- Generally, Black-Scholes is a simple model that does not provide accurate option price due to underlying assumptions that do not hold in reality (i.e., log-normality of stock prices)
- There should be additional adjustments to option prices (i.e., CVA)