Maximum Weighted Independent Set Problem

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1 Problem Statement

Let $G = (V, E)$ be an arbitrary undirected and weighted graph, where $V = \{1, 2, \dots, n\}$ is the vertex set of G and $E \subseteq V \times V$ is e edge set. For each vertex $i \in V$ a positive weight w_i is assosiated with i, collected in the *weight vector* $w \in \mathbb{R}^n$.

The symmetric $n \times n$ matrix

$$A_G = (a_{ij})_{(i,j) \in V \times V},$$

where $a_{ij} = 1$ if $(i, j) \in E$ is an edge f G and $a_{ij} = 0$ if $(i, j) \notin E$, is called the *adjacency matrix* of G.

The *complement graph* of $G = (V, E)$ is the graph $\bar{G} = (V, \bar{E})$, where

$$\bar{E} = \{(i, j) \mid i, j \in V,\ i \neq j \text{ and } (i, j) \notin E\}.$$

For a subset $S \subseteq V$, we define the weight of S to be $W(S) = \sum\limits_{i \in S} w_i$, and call $G(S) = (S, E \cap (S \times S))$ *the subgraph* induced by S.

A graph $G = (V, E)$ is *complete* if all its vertices are pairwise adjacent, i. e. $\forall i, j \in V$ with $i \neq j$, we have $(i, j) \in E$. A *clique* C is a subset f V such that $G(C)$ is complete. The *weighted clique number* of G is the total weight of hte maximum weight clique:

$$\omega(G, w) = \max\{W(S) \mid S \text{ is a clique in } G\}.$$

An *independent set* is a subset of V, whose elements are pairwise nonadjacent. The maximum weight independent set problem asks for an independent set of maximum weight. It is easy to see that S is a clique of G if and only if S is an independent set of $\bar{G}$. Then, the problems of maximum weighted clique and maximum weighted independent set are equivalent.

2 Integer Programming Formulation

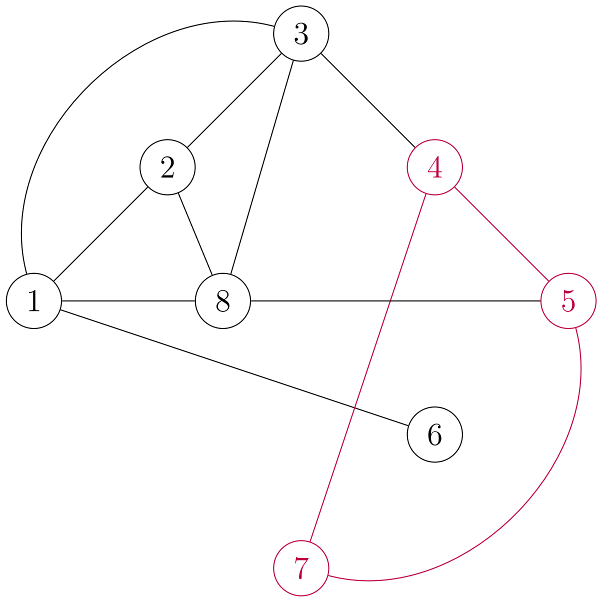
Let x_i denote if the vetrex i is included in the maximum clique for all $1 \leqslant i \leqslant n$. Then, the maximum weight clique problem admits the following formulation:

$$\max \sum_{i=1}^{n} x_i,$$

$$x_i + x_j - a_{ij} \leqslant 1,\ \ 1 \leqslant i \leqslant n,\ 1 \leqslant j \leqslant n,$$

$$x_i \in \{0, 1\},\ \ 1 \leqslant i \leqslant n.$$

3 Data



4 Solution

solver	solution	time
glpk	16.0	0.027
couenne	16.0	0.268
cbc	16.0	0.528
scip	16.0	0.542
bonmin	16.0	0.701
ipopt	18.0	0.377

Since 'ipopt' is the interior point optimizer for nonlinear continuous systems, it does not solve integer problems and obtained the wrong solution.