This project aims to help you understand the similarities and differences among convolution, cross-correlation, and auto-correlation. Additionally, you will explore the Nyquist sampling theory and aliasing. The assignment aims to help you understand the relationship between maximum frequency of signal and the sampling frequency (or sampling interval T). By using simple single-frequency signals, you will understand how the aliasing is generated. Meanwhile, this project aims to help you understand the hard thresholding and soft thresholding, and their proper application in the denoising using transform.

Topic 1: understanding convolution, cross-correlation and auto-correlation.

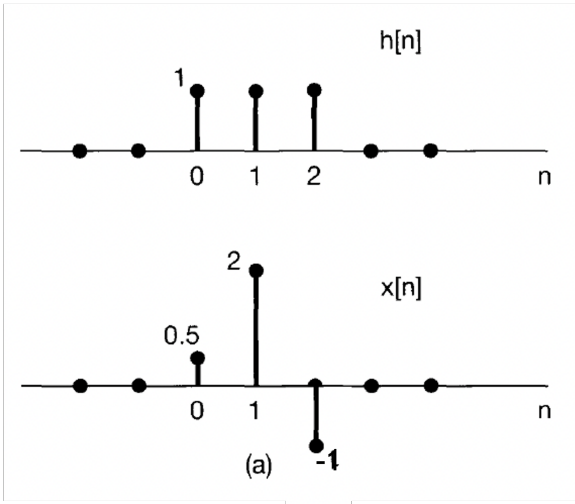

Figure 1: Two discrete signals for doing convolution, cross-correlation, and auto-correlation.

There are two discrete signals $x[n]$ and $h[n]$ in Figure 1.
(1) Plot the convolution results of $x[n] * h[n]$ and $h[n] * x[n]$
(2) Plot the cross-correlation results of $x[n]{\cdot}h[n]$ and $h[n]{\cdot}x[n]$
(3) Plot the auto-correlation results of $x[n]{\cdot}x[n]$ and $h[n]{\cdot}h[n]$
(4) Write one paragraph on your comments/observations of these results.

Topic 2: understanding Nyquist sampling theory and aliasing.

For a given signal, what happens if we change the sampling interval (T) or sampling frequency $\omega_s = 2\pi/T$? please plot your results as the examples in Figure 2.
(1) When the fixed signal is a single frequency $\omega_0 = 2\pi/10$, plot the sampling results if we apply the sampling interval $T = 1s$
(2) When the fixed signal is a single frequency $\omega_0 = 2\pi/10$, plot the sampling results if we apply the sampling interval $T = 3s$
(3) When the fixed signal is a single frequency $\omega_0 = 2\pi/10$, plot the sampling results if we apply the sampling interval $T = 6s$
(4) When the fixed signal is a single frequency $\omega_0 = 2\pi/10$, plot the sampling results if we apply

the sampling interval $T = 8s$
(5) Write one paragraph on your comments/observations of these results.

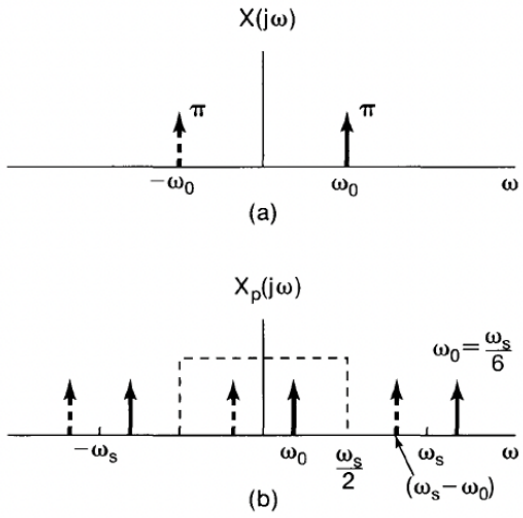

Figure 2: Simple signal with single frequency ω_0 and its Fourier transform.

For a fixed sampling interval T, what happens if we change the input signal? please plot your results as the examples in Figure 2.
(1) When the fixed sampling interval $T = 4s$, plot the sampling results if we input a signal with single frequency $\omega_0 = 2\pi/12$
(2) When the fixed sampling interval $T = 4s$, plot the sampling results if we input a signal with single frequency $\omega_0 = 2\pi/10$
(3) When the fixed sampling interval $T = 4s$, plot the sampling results if we input a signal with single frequency $\omega_0 = 2\pi/5$
(4) When the fixed sampling interval $T = 4s$, plot the sampling results if we input a signal with single frequency $\omega_0 = 2\pi/3$
(5) Write one paragraph on your comments/observations of these results.

Topic 3: hard thresholding and soft thresholding and their applications

There are two sets of coefficients in Figure 3, (a) coefficients from accurate signal, (b) coefficients from noisy signal (the accurate signal from (a) plus noise). Besides the two large coefficients, the maximum absolute value of other coefficients in (b) is 0.1 , the definitions of hard thresholding and soft thresholding are in figure 2.
(1) Plot the coefficients of denoising results using hard thresholding when $\delta = 0.11$
(2) Plot the coefficients of denoising results using soft thresholding when $\delta = 0.11$
(3) Plot the coefficients of denoising results using hard thresholding when $\delta = 0.7$
(4) Plot the coefficients of denoising results using soft thresholding when $\delta = 0.7$
(5) Write one paragraph on your comments/observations of these results, and the relationship with

δ.
(6) Write one paragraph on your understanding of δ, and your idea on how to estimate δ accurately.

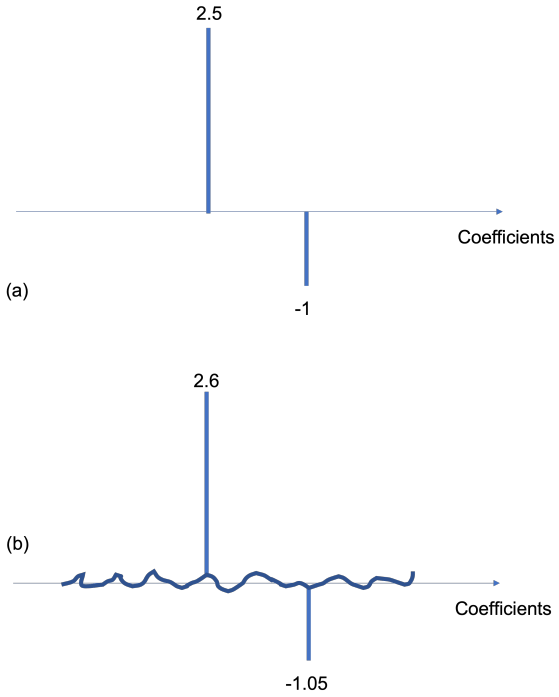

Figure 3: Coefficients in the transform domain (a) Coefficients from the accurate signal (b) Coefficients from the noisy signal.

Topic 4: understanding basic components of Nyquist sampling

(1) Write one paragraph on your understanding of Nyquist sampling.
(2) Ask ChatGPT what its understanding of Nyquist sampling is.
(3) What is the difference of your understanding and the answer from ChatGPT? Please list the three items: (1) the common part (2) what you know but ChatGPT doesn't know (3) what ChatGPT knows but you don't know.
(4) Write one paragraph on your thoughts of these differences.

- hard thresholding:
  - setting all coefficients below a threshold level to zero
  - leave larger coefficients intact

$$\widetilde{v}_i = \begin{cases} \widetilde{v}_i & \text{if } |\widetilde{v}_i| \geq \delta \\ 0 & \text{if } |\widetilde{v}_i| < \delta \end{cases}$$

- soft thresholding (wavelet shrinkage):
  - setting all coefficients below a threshold level to zero
  - decreasing larger coefficients by the threshold amount

$$\widetilde{v}_i = \begin{cases} \widetilde{v}_i + \delta & \text{if } \widetilde{v}_i \leq -\delta \\ 0 & \text{if } |\widetilde{v}_i| < \delta \\ \widetilde{v}_i - \delta & \text{if } \widetilde{v}_i \geq \delta \end{cases}$$

Figure 4: the definitions of hard thresholding and soft thresholding.