Forecasting Time Series modeled as Brownian Motion

Anastasia Kasara

April 2018

Description

First-order autoregressive model

The main subject of the current project is the modeling of time-series data by assuming a model of the following type:

$$y_k = a_0 + a_1 t_k + W(t_k)$$

where a_0, a_1 are deterministic parameters and $\epsilon(t)$ represents a normally distributed variable with mean value zero, $\mu_w = 0$ and variance, $\sigma_k = \sigma^2 t_k$. The above model is known as **geometric brownian model** (GBM) with applications in a diverse range of domains, eg. physics, economics, biology, environmental, etc.. This model was first developed as a physical model (Einstein 1905) for the explanation of the observed irregular motion of particles (Brown). The rationale behind this model can be found if the above model rewriten as

$$dy(t) = a_1 dt + \sigma dW(t)$$

where, $dy = y_{k+1} - y_k$, $dt = t_{k+1} - t_k$ and $dW(t) = W(t_{k+1} - t_k)$. In principle the above equation says that the change of the modeled quantity, $y(t)$, between t_k and t_{k+1} is due to a deterministic bias ($a_1 dt$), proportional to the the time elapsed, and by a random amount $\sigma dW(t)$. The random increments $dW(t)$ have the following properties:

- the increments are independent each other
- $dW(t) \sim N(0, dt)$

with $N(\mu, \sigma)$, being the normal distribution:

$$f(x|\mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$