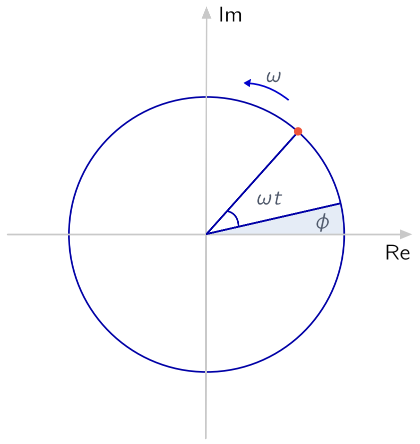

Figure 1.6: Simple harmonic oscillator with phase shift ϕ in the polar plane.

Note that in the description above, the notion of phase shift ϕ was omitted for simplicity. In reality, digital encoding of audio signals can introduce (unwanted) phase shifts ϕ (Figure 1.6):

$$f(t) = \sum_{n=-\infty}^{\infty} A_n e^{i(\omega n t + \phi)}. \tag{1.3}$$

1.1.2 Continuous Fourier Transform

Fourier series is sufficient to describe periodic signals, but for more complicated aperiodic functions, the (continuous) Fourier transform is required. This transform allows the conversion of a continuous time-domain signal into the frequency domain through an invertible linear transformation, defined as follows:

$$\mathcal{F}[f(t)] = \hat{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{i\omega t} dt \qquad t, \omega \in \mathbb{R}, \tag{1.4}$$

and the inverse Fourier transform as

$$\mathcal{F}^{-1}[\hat{f}(\omega)] = f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{f}(\omega) e^{-i\omega t} d\omega \qquad t, \omega \in \mathbb{R}. \tag{1.5}$$

Notice the similarity between the Fourier transform and its inverse. In this case, a symmetric notation is used for both transforms. However, in other literature, the term $\frac{1}{\sqrt{2\pi}}$ may be excluded from the Fourier transform and only included in the inverse Fourier transform as $\frac{1}{2\pi}$ (Baraniuk, 2020). Here, we evenly distribute this term across both transforms, acting as a normalization factor.

1.1.3 Discrete Fourier Transform

Digital computers can only process discrete and finite data, and since audio data is aperiodic and discrete, the Discrete Time Fourier Transform (DTFT) is intuitively applicable. However, in practice, the "Discrete Fourier Transform" (DFT) is mostly used, often extended with various techniques to enhance its performance, such as improving speed. In this equation, the continuous-time signal is represented by discrete values:

$$\mathcal{F}[x[n]] = X[k] = \sum_{n=0}^{N-1} x[n] e^{-i\frac{2\pi}{N}kn} \qquad k = 0, 1, \ldots N-1, \tag{1.6}$$

and the inverse Discrete Fourier transform:

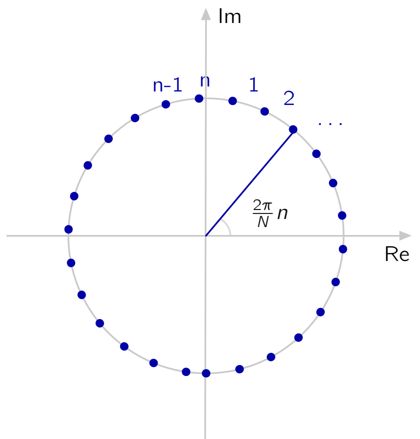

Figure 1.7: Polar representation of phasor in a discrete circle.

$$\mathcal{F}^{-1}[X[k]] = x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{i\frac{2\pi}{N}kn} \qquad k = 0, 1, \ldots N-1. \tag{1.7}$$

N represents the number of samples, often chosen as a power of two, indicating that only specific points n can be captured by the Discrete Fourier Transform (Figure 1.7).