Numerical Methods of Thermo-Fluid Dynamics I

Winter Semester 2022-2023

DELIVERABLE TASK III:
Numerical Solution of 2D Lid-driven Cavity Flow

Given: Monday, 19/12/2022

Deadline: 30/01/2023

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Deliverable Task III

In the third deliverable task you will write a MATLAB code to solve the two-dimensional Navier–Stokes equations for lid-driven cavity flow. The computational domain is square $(x, y) = [0, 1] \times [0, 1]$ and the dimensionless Navier–Stokes equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + \frac{\partial (u^2)}{\partial x} + \frac{\partial (uv)}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), \tag{2}$$

$$\frac{\partial v}{\partial t} + \frac{\partial (uv)}{\partial x} + \frac{\partial (v^2)}{\partial y} = -\frac{\partial p}{\partial y} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right), \tag{3}$$

must advance in time $t \in [0, 0.1]$. The following boundary conditions will be used:

$$\begin{aligned} u(x,1,t) &= 1, \quad u(x,0,t) = u(0,y,t) = u(1,y,t) = 0, \\ v(x,0,t) &= v(x,1,t) = v(0,y,t) = v(1,y,t) = 0, \end{aligned}$$

and initially the fluid is at rest

$$u(x, y, 0) = v(x, y, 0) = 0.$$

The deliverable task should contain the source codes (50p) and a written report (50p) describing the following results:

- (10p) Choose a grid with proper resolution and run the code with several time-steps, what is the maximum time-step you can use at Re=0.1? Explain this phenomenon in terms of the stability requirements we discussed for advection-diffusion equations.
- (20p) Run the simulation with $\mathrm{Re} = 0.1$, generate 5 figures for the streamlines and 5 figures for the pressure contours of the whole domain at $t = 0 : 0.02 : 0.1$, explain the properties of the flow.

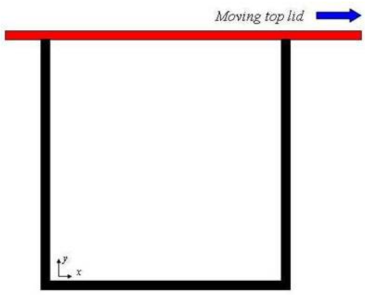

Figure 1: lid-driven cavity flow.

- (20p) Change the Reynolds number to $\mathrm{Re} = 1$. Generate two figures comparing $u(0.5, y)$ and $v(0.5, y)$ for $\mathrm{Re} = 0.1, 1$ at $t = 0.1$, explain the differences.

The Deliverable Task III must be submitted to suharto.saha@fau.de
until January 30th, 2023, before the practicals session as .zip/.rar file.