Homework 1

Stat 345 - Spring 2020

Problem 1

For each of the following experiments, describe the sample space S

a) Picking 2 marbles, one at a time, from a bag that contains many yellow and green marbles

$S = \{YG, YY, GG, GY\}$

b) Getting two odd faces from rolling two dice

$S = \{(1,1),(1,3),(1,5),(3,1),(3,3),(3,5),(5,1),(5,3),(5,5)\}$

c) Count the proportion of defectives items in a shipment

$S = \{n : 0, \frac{1}{n}, \frac{2}{n}, \frac{3}{n}, ...\}$ or $S\{n : 0 \leq n \leq 1\}$

d) Count the number of hail damaged spots on some car

$S = \{0, 1, 2, 3, ...\}$ or $S = \{x : x \in \{\mathbb{N} \cup 0\}\}$

Problem 2

Pick a card from the standard deck of 52 cards. Consider the following events:

A = {card has a Red Suit}

B = {card is a Heart}

C = {card is a Queen}

D = {card is a King}

Describe these events in terms of problem.

a) $A \backslash B =$ {card is a Diamond}

b) $(B \cap A) \cup (A \cap B^c) = A =$ {card is red}

c) $(C \cup D)^c =$ {card is not a Queen and card is not a King}

d) $(A \cup B) \cap C =$ {card is a Queen of Hearts or card is a Queen of Diamonds}

e) $(C \cup D) \cap B =$ {card is a Queen of Hearts or card is a King of Hearts}

Problem 3

Jane has 11 friends. She is planning to meet with some of her friends each day of a certain week, Thursday through Sunday (one friend per day).

a) How many possibilities are there for Jane's schedule for that week, if she is not willing to meet with the same friend more than once?

She has 11 choices to meet with one of her friends on Thursday. On Friday she certainly will not meet with a friend whom she met on Thursday (10 choices now), and so on.

$$11 \cdot 10 \cdot 9 \cdot 8 = 7920$$

b) How many possibilities are there for Jane's schedule for that week, if she is willing to meet with the same friend more than once, but not twice in a row (or more)?

Note that on Saturday she can meet with the same friend she met on Thursday but not with the friend she met on Friday.

$$11 \cdot 10 \cdot 10 \cdot 10 = 11000$$

Problem 4

You have 7 rock music CDs, 5 indie music CDs, and 2 pop music CDs.

a) In how many ways can you arrange them?

$(7 + 5 + 2) = 14!$, usual permutation of n objects is $n!$

b) In how many different ways can you arrange them so that the CDs of the same type are contiguous?

We break down the problem in two stages, where we first select the order of the CD types, and then the order of the CDs of each type. There are 3! ordered sequences of the types of CDs (such as indie/rock/pop, rock/pop/indie, etc.), and there are 7! permutations of the rock CDs, 5! permutations of the indie CDs, 2! permutations of the pop CDs. Thus for each of the 3! CD type sequences, there are 7!5!2! arrangements of CDs.

$$3!7!5!2!$$

c) Suppose that you want to give your *rock music CDs* to your friends. You want to give 3 to Max, 2 to John, and 2 to Alice. In how many ways you can do that?

$\binom{7}{3}\binom{4}{2}\binom{2}{2} = 210$ ways, choose 3 CDs out of 7 for Max, then choose 2 out of the remaining $7 - 3 = 4$ for John, and give the last 2 to Alice. Of course, you can first choose 2 out of 7 discs for Alice, 3 out of the remaining 5 for Max, and give the last 2 for John. In this case, you can do that in $\binom{7}{2}\binom{5}{3}\binom{2}{2} = 210$ ways. Assuming that you don't care which of your friends gets the CDs first, the answer is 210.

Alternatively, the problem can be viewed as the number of permutations of n objects of which n_1 are of one type, n_2 are of a second type, n_3 are of the third type. Using the formula $\frac{n!}{n_1!n_2!n_3!} = \frac{7!}{3!2!2!} = 120$.

Problem 5

From a group of 9 biologists and 5 chemists, a group consisting of 3 biologists and 2 chemists is to be formed. In how many ways can this be done if

a) any biologist and any chemist can be included? By multiplying corresponding combinations (order doesn't matter) $C_3^9 \cdot C_2^5 = \binom{9}{3}\binom{5}{2} = 840$.

b) one particular chemist must be in the group? Because one chemist is already in the group, we need to choose one more from $5 - 1 = 4$, $\binom{9}{3}\binom{4}{1} = 840$.

c) three particular biologists cannot be in the group? Exclude these three biologists from consideration, $\binom{6}{3}\binom{5}{2} = 840$.

Remember that $C_r^n = \binom{n}{r} = \frac{n!}{r!(n-r)!}$ is a binomial coefficient.

Problem 6

Use R statistical software (include your code) to make a well labeled plot $y = \left(1 + \frac{1}{n}\right)^n$.

Show on the plot that y approaches e for large n. This will illustrate that

$$\lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^n = e$$

```
n <- c(1:200)
y <- c()
for(i in 1:length(n))
{
  y <- rbind(y, (1+1/n[i])^n[i])
}
# save the plot to png
png(filename="hw1plot.png")
plot(n, y, type = 'l', xlab="n", ylab="y")
abline(h = exp(1), col="red", lty="dashed")
title(expression(paste("Illustration of y = ",
lim(bgroup("(", 1+frac(1,n), ")")^n,n%->%infinity)," = e")))
dev.off()
```

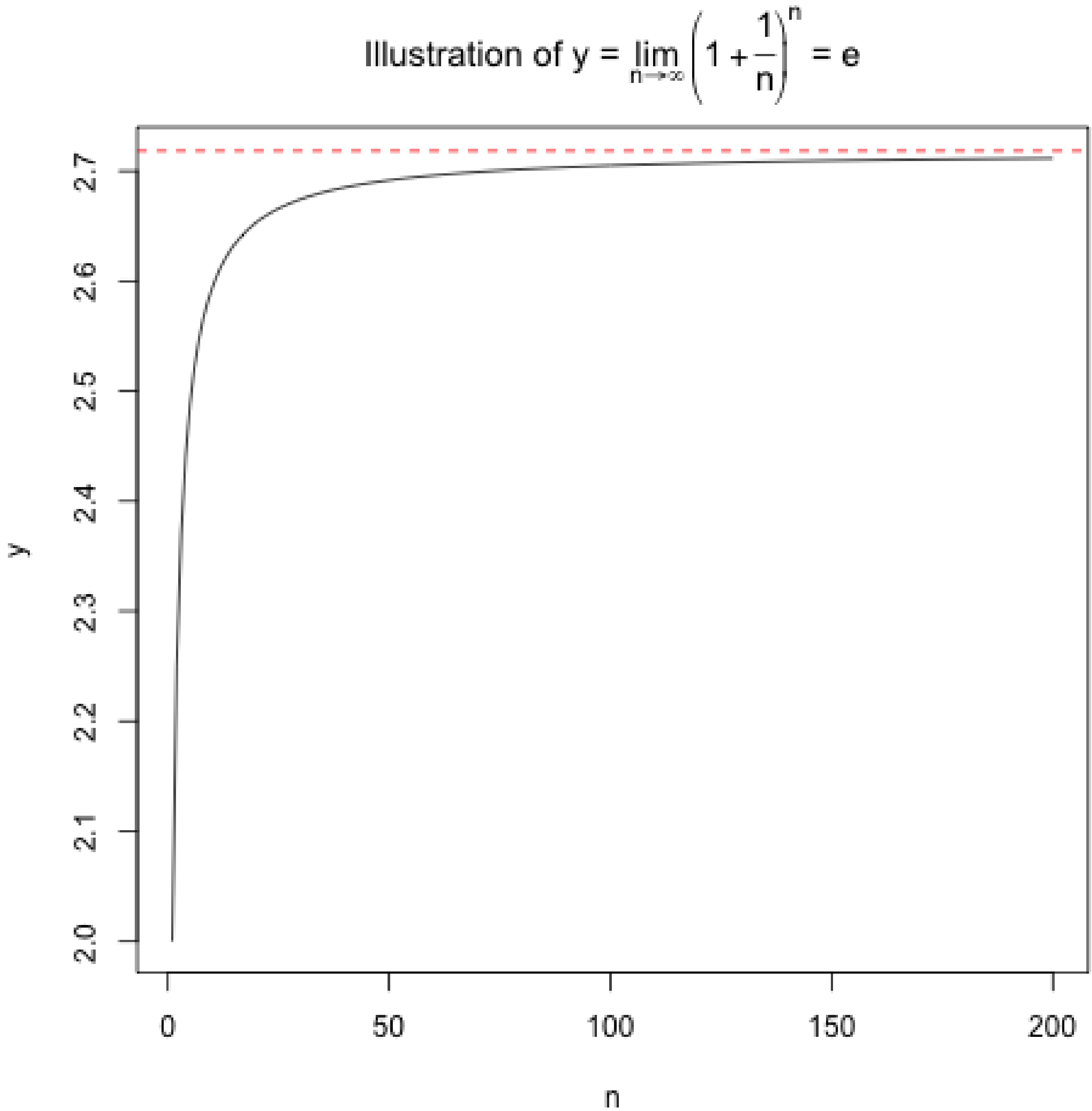
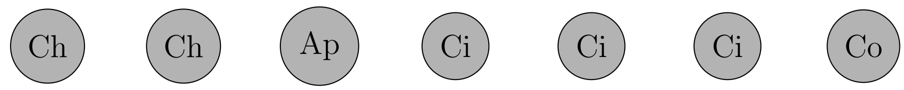


Illustration of $y = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e$
2.0
2.1
2.2
2.3
2.4
2.5
2.6
2.7
y
0
50
100
150
200
n

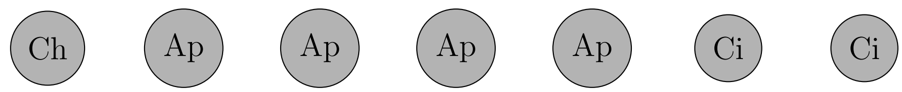
Bonus problem

You want to buy 7 cakes from a local bakery. The store has 4 types of cakes: chocolate, apple, cinnamon, and coconut cakes. How many different selections can you make? Note that cakes of the same type are considered indistinguishable.

It is a problem of putting r (cakes of the same type) indistinguishable items into n (types of cakes) distinguishable boxes. Here we choose with replacement (with repetition because you can select the same type of the cake multiple times) and order doesn't matter. For convenience, let's just list our selections in the same order: chocolate, apple, cinnamon, and coconut cakes.



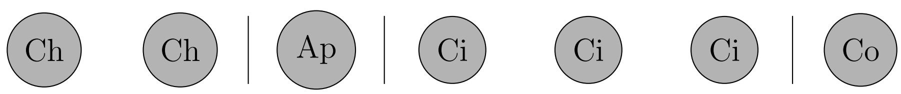
or



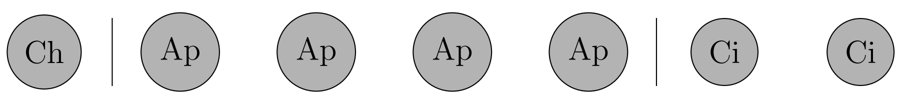
or

...

Let's separate by type by putting | between different types of cakes. Our possible selections separated by type:



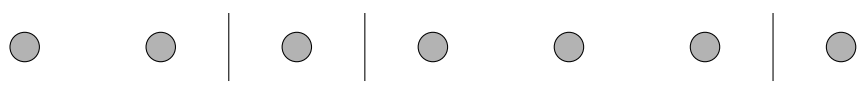
or



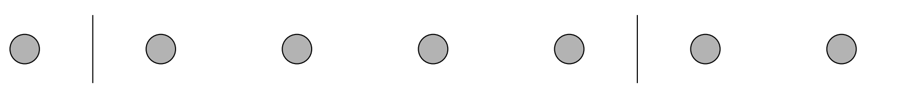
or

...

We don't need the actual names in the diagram above to know what's there:



or



or

...

We have 10 slots and need to decide where to put 3 dividers to separate the cakes by their type. There are $C_3^{10} = 120$ ways to do that. In general, one can use the formula for the number of possible unordered arrangements of size r from n objects with replacement: $\binom{n+r-1}{r} = \binom{10}{7} = \binom{10}{3} = 120$, where $r = 7$ (select 7 cakes) and $n = 4$ (4 types of cakes).