Homework 7

Stat 345 - Spring 2020

Name: ____

Problem 1 (20 pts)

Let $X_1, X_2, ..., X_n$ be a random sample of size from a distribution with probability density function

$$f(x) = \lambda x^{\lambda-1},\ 0 < x < 1,\ \lambda > 0$$

a) Get the method of moments estimator of λ. Calculate the estimate when $x_1 = 0.1, x_2 = 0.2, x_3 = 0.3$.

b) Get the maximum likelihood estimator of λ. Calculate the estimate when $x_1 = 0.1, x_2 = 0.2, x_3 = 0.3$.

Problem 2 (30 pts)

Suppose that compressive strength is normally distributed with $\sigma = 1000\ (psi)^2$. A random sample of 12 specimens has a mean compressive strength of $\bar{x} = 3250$ psi.

a) Construct a 95% and 99% two-sided confidence intervals on mean compressive strength. Compare their widths.

b) Suppose it is desired to estimate the compressive strength with an error that is less than 15 psi at 99% confidence. What sample size is required?

Problem 3 (15 pts + bonus 5 pts)

A healthcare provider monitors the number of CAT scans performed each month in each of its clinics. The most recent year of data for a particular clinic follows (the reported variable is the number of CAT scans each month expressed as the number of CAT scans per thousand members of the health plan):

$$2.31, 2.09, 2.36, 1.95, 1.98, 2.25, 2.16, 2.07, 1.88, 1.94, 1.97, 2.02$$

a) Find a 95% two-sided CI on the mean number μ of CAT scans performed each month at this clinic.

b) (bonus) Historically, the mean number of scans performed by all clinics in the system has been $\mu_0 = 1.95$. If there any evidence that this particular clinic performs more CAT scans on average than the overall system average?

Problem 4 (30 pts)

Medical researchers have developed a new artificial heart constructed primarily of titanium and plastic. The heart will last and operate almost indefinitely once it is implanted in the patient's body, but the battery pack needs to be recharged about every four hours. A random sample of 50 battery packs is selected and subjected to a life test. The average life of these batteries is 4.05 hours. Assume that battery life is normally distributed with standard deviation $\sigma = 0.2$ hour.

a) Is there evidence to support the claim that mean battery life differs from 4 hours? Use $\alpha = 0.05$.

b) Compute the power of the test if the true mean battery life is 4.15 hours.

c) What sample size would be required to detect a true mean battery life of 4.15 hours if you wanted the power of the test to be at least 0.99?

Problem 5 (15 pts)

In a random sample of 500 handwritten zip code digits, 466 were read correctly by an optical character recognition (OCR) system operated by the U.S. Postal Service (USPS). USPS would like to know whether the rate is at least 90% correct. Do the data provide evidence that the rate is at least 90% at $\alpha = 0.01$?

Problem 6 (10 pts + bonus 5 pts)

What is the difference between commuting patterns for students and professors? A study compares mean commuting distances (in miles) for students and professors. Summary statistics:

	n	$\bar{x}$	s
students	38	6.8	4.8
professors	40	11.2	7.2

a) Determine whether there is any difference between mean commuting distances at $\alpha = 5\%$ level of significance?

b) (bonus) Construct a 95% confidence interval for the difference between commuting patterns for students and professors. How the result you have observed is connected to (a)?

Problem 7 (Bonus 10 pts)

Let $X_1, X_2, ..., X_n$ be a random sample of size from a distribution with probability density function

$$f(x) = \lambda x^{\lambda-1},\ 0 < x < 1,\ \lambda > 0$$

a) (bonus) Show that $T = X^\lambda$ is a pivotal quantity. (*Hint:* You need to find cdf and then pdf of T and show that the distribution doesn't depend on λ. The last slide in the lecture 17 might help.)

b) (bonus) Construct 95% confidence interval for λ using the pivotal quantity in (a). You don't need to find optimal q_1, q_2. Just write down how you can approach the problem.