

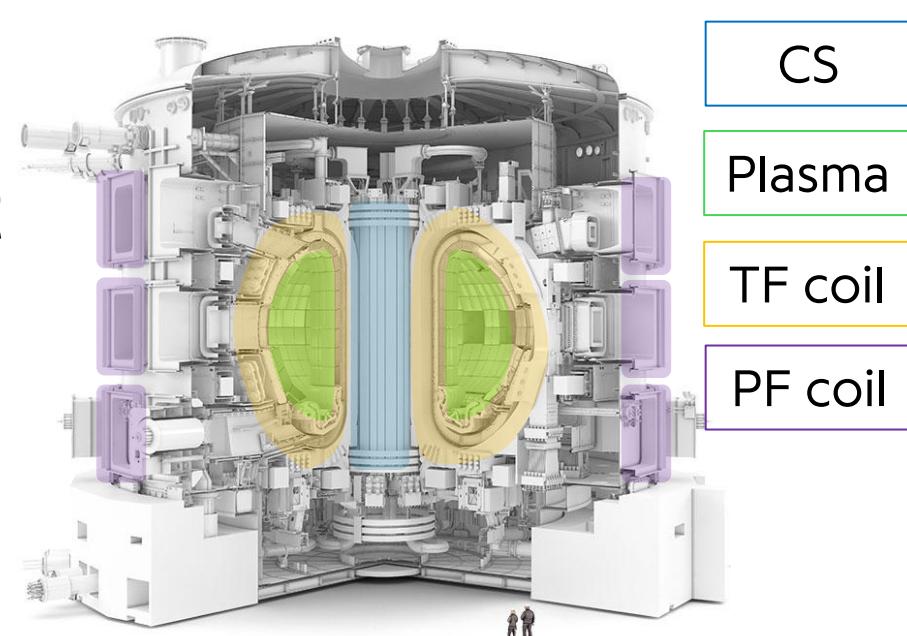
# Simulation and optimisation of tokamak plasma

## Outline.

Generating stable initial data is a crucial phase of the modelling of nuclear fusion tokamak reactors. This work achieves that by solving the free-boundary Grad-Shafranov (GS) equilibrium equation, and follows up by using evolutionary algorithms to optimise this equilibrium by adjusting the position of poloidal field (PF) coils and plasma current  $I_p$  with the aim of maximising the safety factor  $q_{95}$  and poloidal beta  $\beta_p$  to achieve high stability and efficiency, respectively.

## Background.

Tokamak reactors confine hot plasma using magnetic fields provided by the central solenoid (CS), toroidal field coils (TF) and poloidal field coils (PF).



Plasma is modelled using the magnetohydrodynamic (MHD) system, resulting from the combination of Maxwell's and Euler's equations. For static equilibrium, this system produces the GS equation:

$$\frac{\partial^2 \Psi}{\partial R^2} - \frac{1}{R} \frac{\partial \Psi}{\partial R} + \frac{\partial^2 \Psi}{\partial Z^2} = -\mu_0 R^2 \frac{dp}{d\Psi} - g \frac{dg}{d\Psi}$$

Two important metrics to maximise: safety factor  $q_{95}$ , poloidal beta  $\beta_p$ .

$$q_{95} = \frac{1}{2\pi} \oint_{\psi_n=0.95} \frac{B_t}{RB_p} dr dz, \quad \beta_p = \frac{8\pi}{\mu_0} \frac{\oint_{\partial p} p dr dz}{I_p^2}$$

## Numerical Implementation.

- Write and validate a fixed-boundary GS solver.
- Solve the free-boundary GS equation for the MAST reactor design using CAMReX's *Gssolver*.
- Optimise for  $q_{95}$  and  $\beta_p$  by adjusting (1) the plasma current  $I_p$ , (2-4) the radial position  $r$  of two central PF coils and/or the vertical distance  $d$  between the two central PF.

minimise:	$-q_{95}$	$-\beta_p$
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subject to: (1)  $1 \times 10^5 \leq I_p \leq 1 \times 10^6$

(2a)  $1.5 \leq r \leq 2.0$

(2b)  $1.0 \leq d \leq 2.0$

(3)  $1.5 \leq r \leq 2.0$  AND  $1.0 \leq d \leq 2.0$

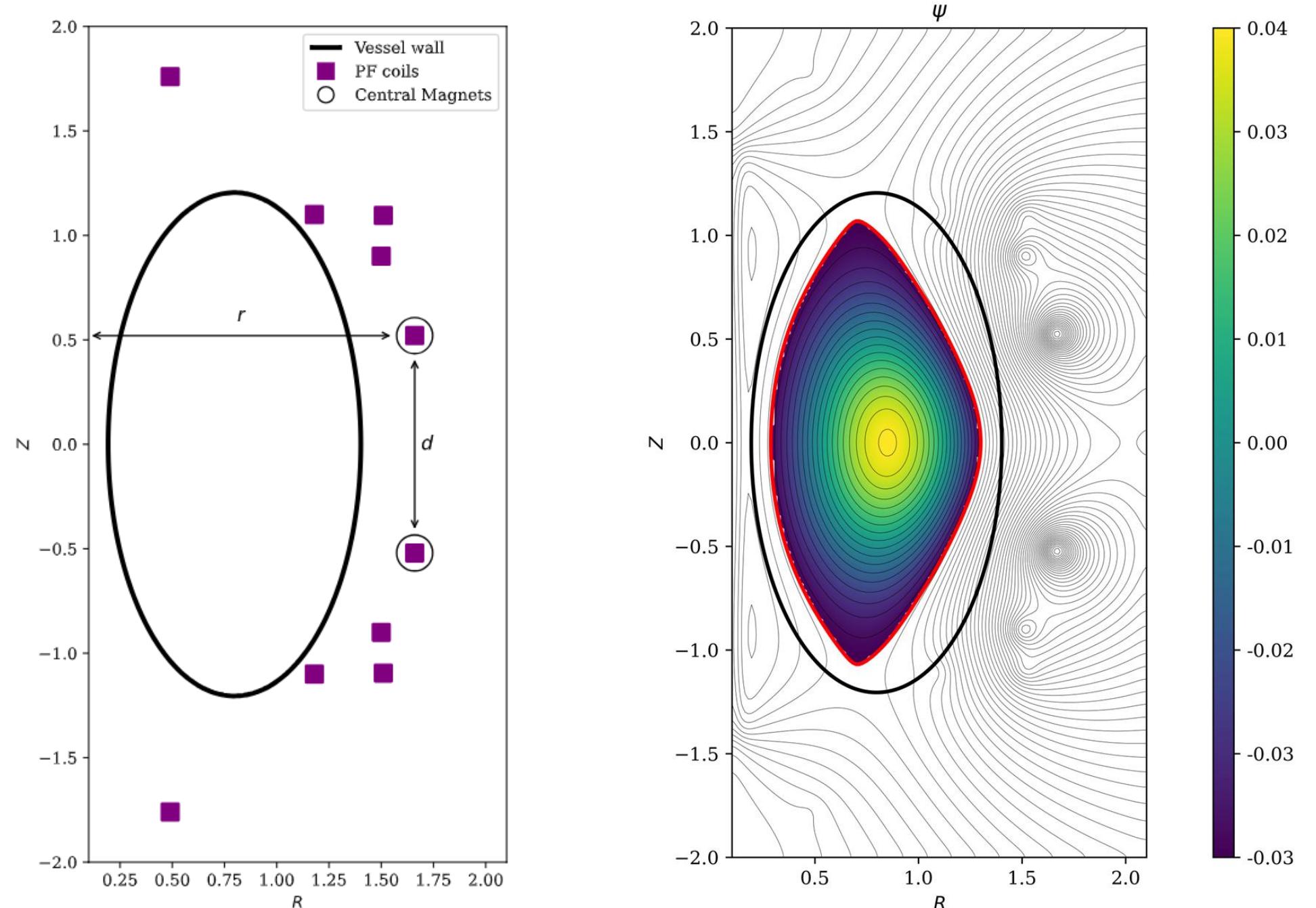
minimise:	$\{-q_{95}, -\beta_p\}$
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subject to: (4)  $1.5 \leq r \leq 2.0$  AND  $1.0 \leq d \leq 2.0$

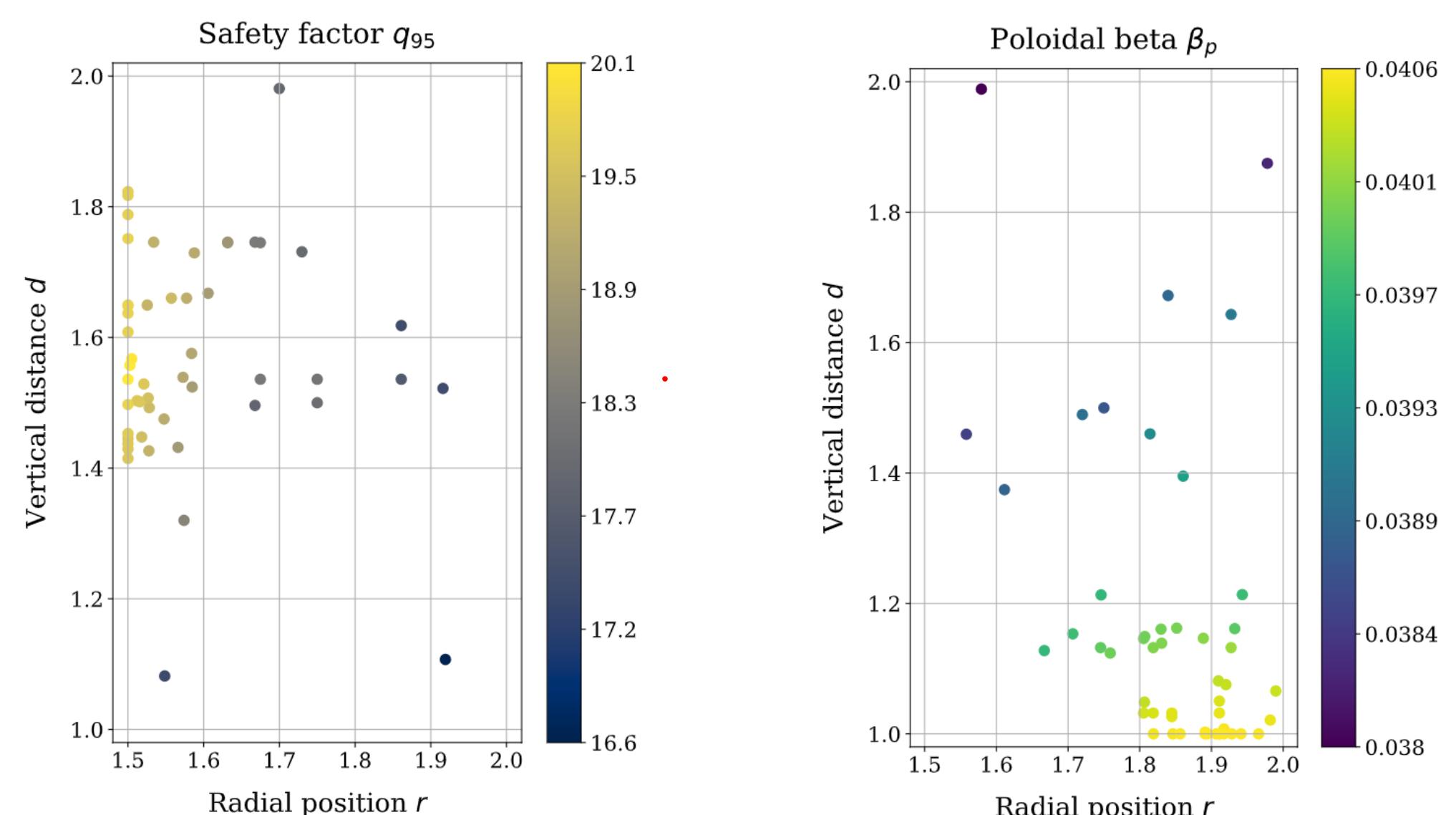
## Results.

Some selected results are presented here.

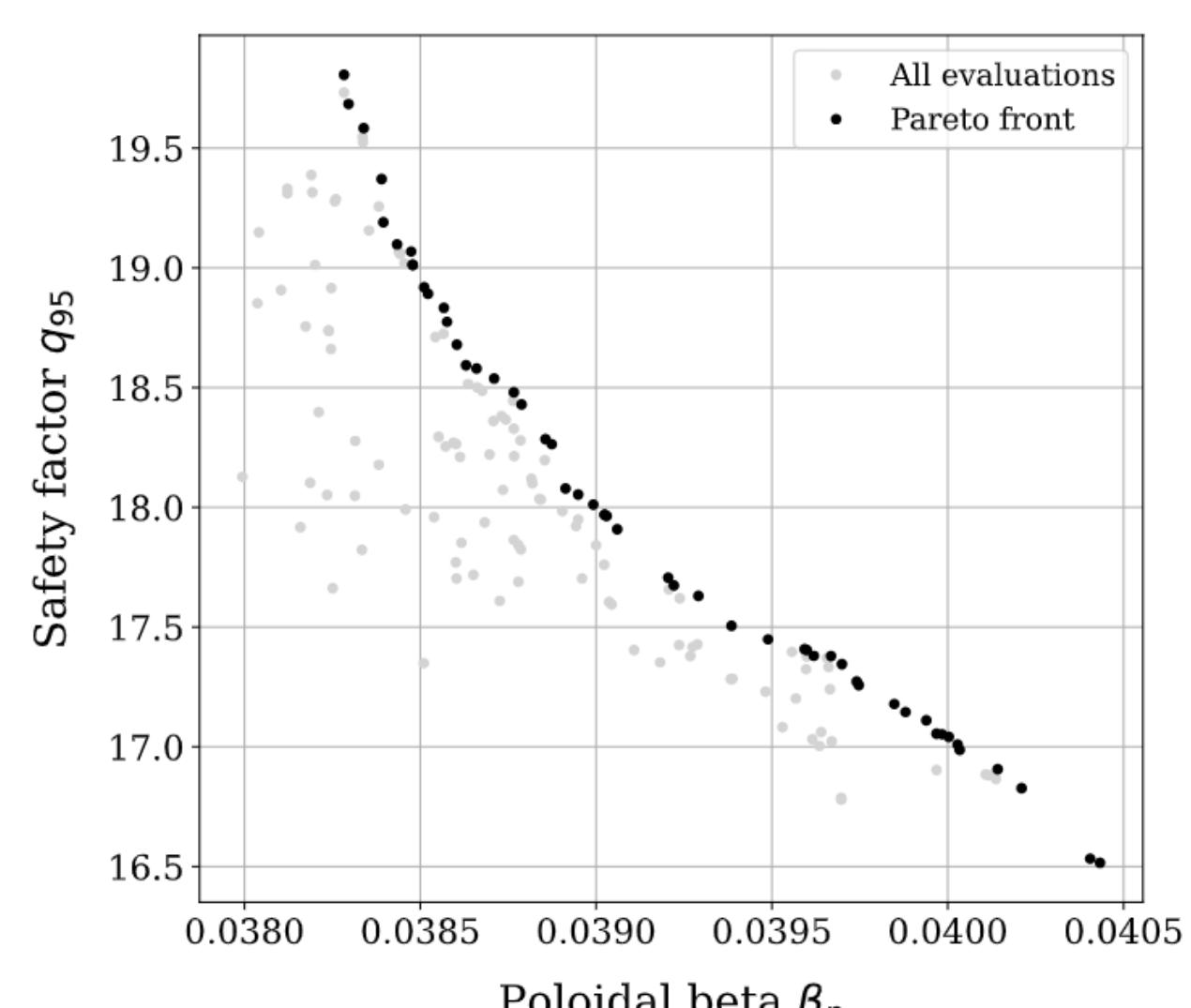
Free-boundary GS solution for MAST reference



Optimisation test 3: Optimisation in 2D



Optimisation test 4: Multiobjective optimisation



## Future work.

These optimised results can be used as initial data in the MHD system under perturbation to examine the evolution of the instability in different configurations as related to their initial stability and efficiency.