Dr. Sasha

Result for the task N12

1) – rightly

2) – I doing verification

3) – I doing verification

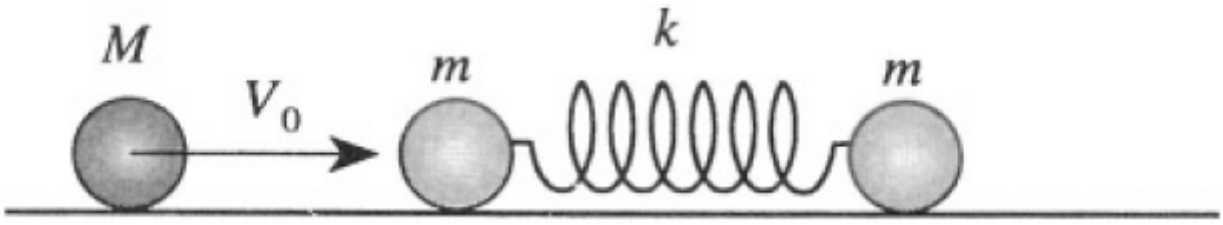
4) – I doing verification

5) – rightly

Please, solve this problem and also show the steps to solve it.

1.06.2015 – N13

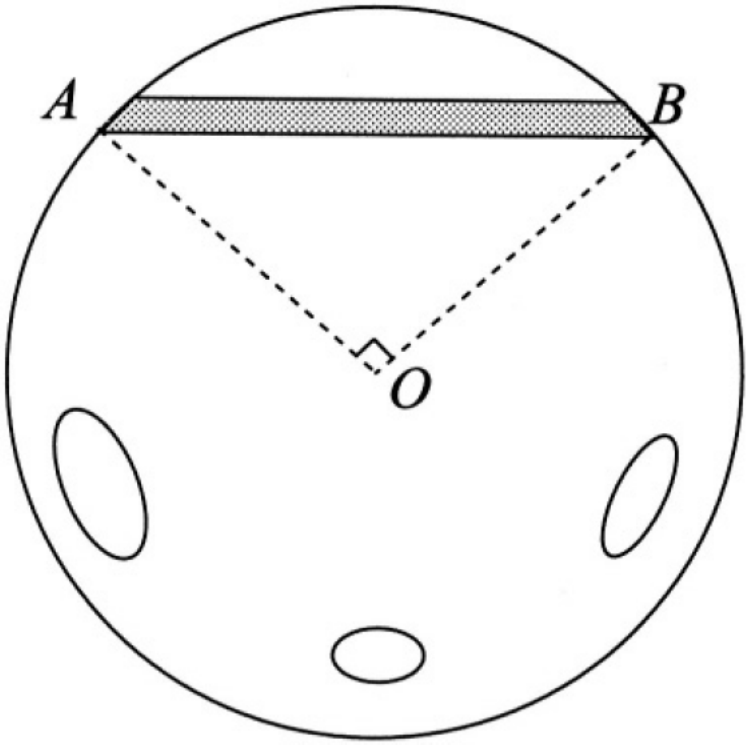
1) A ball of mass M moving with velocity V_0 on a frictionless plane strikes the first of two identical balls, each of mass $m = 2$ kg, connected by a massless spring with spring constant $k = 1$ kg/s^2 (see figure below). Consider the collision to be central and elastic and essentially instantaneous.



a) Find the minimum value of the mass M for the incident ball to strike the system of two balls again.

b) How much time will elapse between the two collisions?

2) In one of his novels, H. G. Wells describes an encounter of amateur earthling astronauts with a lunar civilization living in very deep caverns beneath the surface of the Moon. The caverns are connected to the surface by long channels filled with air. The channel is dug between points A and B on the surface of the Moon so that the angle $AOB = 90^0$ (see figure below). Assume that the air pressure in the middle of a channel is $P_0 = 1$ atm. Estimate the air pressure in the channel near the surface of the Moon. The radius of the Moon $R_{Moon} \approx 1750$ km. The acceleration due to gravity on the surface of the Moon $g_{\text{Moom}} \approx g_{\text{Earth}}/6$, where g_{Earth} is the acceleration due to gravity on the surface of the Earth.



3) A small amount of water of mass $m = 50\,\text{g}$ in a container at temperature $T = 273$ K is placed inside a vacuum chamber which is evacuated rapidly. As a result, part of the water freezes and becomes ice and the rest becomes vapor.

a) What amount of water initially transforms into ice? The latent heat of fusion (ice/water) $q_{i/w} = 80\dfrac{\text{cal}}{\text{g}}$ and the latent heat of vaporization (water/vapor) $q_{\text{w/v}} = 600\dfrac{\text{cal}}{\text{g}}$.

4) Pauli matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\ \sigma_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Truth or false:

a) $\sigma_1^{\,2} = \sigma_2^{\,2} = \sigma_3^{\,2} = I$, where I – identity matrix.

b) $\sigma_i \cdot \sigma_j = -\sigma_j \cdot \sigma_i$, for $i \neq j$ $\left(i = 1,\ 2,\ 3;\ j = 1,\ 2,\ 3\right)$

c) For any complex numbers c_1, c_2, c_3

$$\left(c_1 \cdot \sigma_1 + c_2 \cdot \sigma_2 + c_3 \cdot \sigma_3\right)^2 = \left(c_1^{\,2} + c_2^{\,2} + c_3^{\,2}\right) \cdot I$$

d) $\exp(\sigma_1)\cdot\exp(\sigma_2)=\exp(\sigma_1+\sigma_2)$

Define exponentiation of matrices via: $\exp(A)=\sum_{n=0}^{\infty}\frac{A^n}{n!}$.