Dr. Sasha

At first, I would like to thank you for the attachment containing the file answer15.pdf.

It is the citation from your text (answer on task N1)

This equation has a duplicate solution $\omega_{1,2} = \omega_0\sqrt{3}$. So, the system has an extra mode with frequency $\omega = \omega_0\sqrt{3}$. It may be proved that the amplitudes A_i of the oscillations may be arbitrary satisfying the relation

$$A_1 + A_2 + A_3 = 0,$$

i.e. no motion of the "center of mass". The arbitrarity of the amplitudes (or, in fact, of their ratios) is in fact an extra degree of freedom which corresponds to the multiplicity 2 of the root. This may be understood as two modes of oscillation degenerated into one, but with arbitrary amplitudes.

I agree with your solution. However, I have a question. For example, I select a solution $A_2 = 0,\ A_1 = -A_3$ which satisfy the relation $A_1 + A_2 + A_3 = 0$. How you think, the system can evolution in this manner: one mass is at rest and the two others move in opposite directions.

Problem specification

2) Three masses, each of mass m, are interconnected by identical massless springs of spring constant k and are placed on a smooth circular hoop as shown in figure below. The hoop is fixed in space. Neglect gravity and friction. Determine the natural frequencies of the system, and the shape of the associated modes of vibration.