1. Due to Newton law for rotation about the axis of the pendulum

$$ml^2\ddot{\theta} = m\omega^2 l^2 \sin\theta\cos\theta - mgl\sin\theta,$$

which implies

$$\ddot{\theta} = \omega^2 \sin\theta\cos\theta - \frac{g}{l}\sin\theta.$$

For a small perturbation $\delta\theta$

$$\delta\ddot{\theta} = \left(\omega^2 - \frac{g}{l}\right)\delta\theta.$$

So, the equilibrium at $\theta = 0$ becomes unstable if $\ddot{\theta} > 0$, or $\omega > \sqrt{g/l}$. The new equilibrium corresponds to

$$\cos\theta = \frac{g}{\omega^2 l}.$$

For $\omega \leq \sqrt{g/l}$ and $\theta = 0$ the frequency of oscillations

$$\Omega = \sqrt{\frac{g}{l} - \omega^2}.$$

For $\omega > \sqrt{g/l}$

$$\delta\ddot{\theta} = \delta\theta\left(\omega^2\cos 2\theta - \frac{g}{l}\cos\theta\right),$$

which implies

$$\Omega = \sqrt{-\left(\omega^2\cos 2\theta - \frac{g}{l}\cos\theta\right)} = \sqrt{\omega^2 - \frac{g^2}{\omega^2 l^2}}.$$

2. The described system has a zero-frequency mode corresponding to one-directional rotation. As the system has 3 bonds, other 2 modes exist. Let x_i be the deviations of the balls. Then

$$\begin{aligned}
\ddot{x}_1 &= \omega_0^2\left(x_2 + x_3 - 2x_1\right),\\
\ddot{x}_2 &= \omega_0^2\left(x_1 + x_3 - 2x_2\right),\\
\ddot{x}_3 &= \omega_0^2\left(x_1 + x_2 - 2x_3\right),
\end{aligned}$$

where $\omega_0 = \sqrt{k/m}$. Let's look for a solution of the form $x_1 = A_1\sin\omega t$, $x_2 = A_2\sin\omega t$, $x_3 = A_3\sin\omega t$. Substituting x_i into the motion equations we get

$$\begin{aligned}
-\omega^2 A_1 &= \omega_0^2\left(A_2 + A_3 - 2A_1\right),\\
-\omega^2 A_2 &= \omega_0^2\left(A_1 + A_3 - 2A_2\right),\\
-\omega^2 A_3 &= \omega_0^2\left(A_1 + A_2 - 2A_3\right).
\end{aligned}$$

This is a system of linear equations with variable determinants $\Delta_x = \Delta_y = \Delta_z = 0$. According to the general theory of linear algebra, this system has a non-zero solution iff the main determinant $\Delta = 0$. Thus we get

$$\begin{vmatrix} 2\omega_0^2 - \omega^2 & -\omega_0^2 & -\omega_0^2 \\ -\omega_0^2 & 2\omega_0^2 - \omega^2 & -\omega_0^2 \\ -\omega_0^2 & -\omega_0^2 & 2\omega_0^2 - \omega^2 \end{vmatrix} = 0 \Leftrightarrow \begin{vmatrix} 3\omega_0^2 - \omega^2 & 0 & 0 \\ 0 & 3\omega_0^2 - \omega^2 & 0 \\ 0 & 0 & 3\omega_0^2 - \omega^2 \end{vmatrix} = 0 \Leftrightarrow \omega^2 = 3\omega_0^2.$$

This equation has a duplicate solution $\omega_{1,2} = \omega_0\sqrt{3}$. So, the system has an extra mode with frequency $\omega = \omega_0\sqrt{3}$. It may be proved that the amplitudes A_i of the oscillations may be arbitrary satisfying the relation

$$A_1 + A_2 + A_3 = 0,$$

i.e. no motion of the "center of mass". The arbitrarity of the amplitudes (or, in fact, of their ratios) is in fact an extra degree of freedom which corresponds to the multiplicity 2 of the root. This may be understood as two modes of oscillation degenerated into one, but with arbitrary amplitudes.

3. Let $\overrightarrow{p}_\gamma$ be the momentum of the incident photon, $\overrightarrow{p}_p$ and $\overrightarrow{p}_\pi$ be the momentum of the proton and the pion respectively after the reaction. Then due to conservation laws

$$\overrightarrow{p}_\gamma = \overrightarrow{p}_p + \overrightarrow{p}_\pi,$$
$$p_\gamma c + m_p c^2 = \sqrt{m_p^2 c^4 + p_p^2 c^2} + \sqrt{m_\pi^2 c^4 + p_\pi^2 c^2}.$$

It may be proved that in case of constant momentum the energy of the products is minimal when $p_\pi = 0$. Then the equations may be simplified:

$$p_\gamma c + m_p c^2 = \sqrt{m_p^2 c^4 + p_\gamma^2 c^2} + m_\pi c^2,$$

which implies

$$p_\gamma = m_\pi c \frac{2m_p - m_\pi}{2\left(m_p - m_\pi\right)}.$$

Then the threshold energy of the photon is

$$\varepsilon_{\min} = p_\gamma c = m_\pi c^2 \frac{2m_p - m_\pi}{2\left(m_p - m_\pi\right)} = 146.3 \text{ MeV}.$$

4. Let Ox axis be parallel to the plane, Oy axis be perpendicular to it, both axes are in the plane of the dipole. The conducting plane acts as as a mirror, creating an image dipole. As the electric field exists only in half of the space, the energy is half the energy of the dipole-dipole interaction. The original dipole is $\overrightarrow{p} = (p\sin\theta, p\cos\theta)$, the image is $\overrightarrow{p_1} = (-p\sin\theta, p\cos\theta)$. The first dipole creates a field

$$\overrightarrow{E} = \frac{4\pi\varepsilon_0}{r^3}\left(-\overrightarrow{p} + \frac{(\overrightarrow{p}, \overrightarrow{r})\overrightarrow{r}}{r^2}\right),$$

where $\overrightarrow{r}$ is the radius-vector measured from the center of the dipole, $\overrightarrow{r} = (0, -2h)$ for the location of the second dipole. Then the energy of their interaction is

$$W = -(\overrightarrow{E}, \overrightarrow{p_1}) = -\frac{\pi\varepsilon_0 p^2}{2h^3}\left(1 + \cos^2\theta\right).$$

The work to be done to remove the dipole to infinity is half of that:

$$A = -\frac{W}{2} = \frac{\pi\varepsilon_0 p^2}{4h^3}\left(1 + \cos^2\theta\right).$$

5. Consider $d_1, d_2 \ll L_1, L_2$ so that the plates form a type of a capacitor. Let Ox axis be along the L_2-side, with $x = 0$ at the left pair of sides (see picture). Then the electric field inside is

$$E(x) = \frac{V}{d_1\left(1 - \frac{x}{L_2}\right) + d_2\frac{x}{L_2}}.$$

The energy density is $w = \varepsilon_0 E^2/2$, so the energy is

$$W = \frac{\varepsilon_0 L_1}{2}\int_0^{L_2}\left(d_1\left(1 - \frac{x}{L_2}\right) + d_2\frac{x}{L_2}\right)E^2(x)dx = \frac{\varepsilon_0 V^2 L_1}{2}\int_0^{L_2}\frac{dx}{d_1 + (d_2 - d_1)\frac{x}{L_2}} = \frac{\varepsilon_0 V^2 L_1 L_2}{2\,(d_2 - d_1)}\ln\frac{d_2}{d_1}.$$

By definition of capacitance the energy is $W = CV^2/2$, then the capacitance is

$$C = \frac{\varepsilon_0 L_1 L_2}{d_2 - d_1}\ln\frac{d_2}{d_1}.$$

In the limit of $d_1 \to d_2 = d$ the expression becomes

$$C = \frac{\varepsilon_0 L_1 L_2}{d},$$

as for an ordinary capacitor.