

SOLUTION FOR THE EXPERIMENTAL COMPETITION

Advertising tricks

Part 1. Lens

1.1-1.2 The figure on the right shows the path of the laser beam reflected from the lens surface. It is seen that the reflected beam makes the angle 2α to the lens axis, where

$$\alpha = \frac{x}{R}, \quad (1)$$

R is the radius of the lens surface and O is the center of curvature. Consequently, the displacement of the reflected beam on the screen is given by

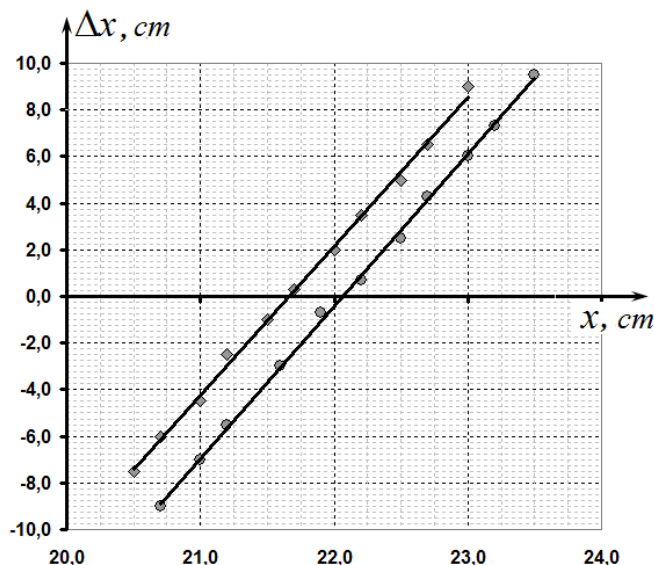
$$\Delta x = 2\alpha L = \frac{2L}{R} x. \quad (2)$$

Thus, the slope on the graph showing the laser beam displacement Δx against the lens displacement x determines the radius of the surface curvature. To find the slope it is not necessary to accurately determine the position of the surface center, i.e. the reference point can be chosen arbitrarily.

Table 1 shows the measured beam displacement for the two surfaces of the lens. The measurements have been made for $L = 22\text{ cm}$. The graphs of the obtained dependences are also presented.

Table 1

Side 1		Side 2	
$x, \text{ cm}$	$\Delta x, \text{ cm}$	$x, \text{ cm}$	$\Delta x, \text{ cm}$
23,5	9,5	23,0	9,0
23,2	7,3	22,7	6,5
23,0	6,0	22,5	5,0
22,7	4,3	22,2	3,5
22,5	2,5	22,0	2,0
22,2	0,7	21,7	0,3
21,9	-0,7	21,5	-1,0
21,6	-3,0	21,2	-2,5
21,2	-5,5	21,0	-4,5
21,0	-7,0	20,7	-6,0
20,7	-9,0	20,5	-7,5



The slopes are preferably calculated by the least square method. The radii are calculated by the formula which follows from equation (2):

$$K = \frac{2L}{R} \Rightarrow R = \frac{2L}{K}. \quad (3)$$

Numerical calculations lead to the following values:

For the first side of the lens $K_1 = 6,51 \pm 0,14$; $R_1 = (6,76 \pm 0,14)\text{ cm}$

For the second side of the lens $K_2 = 6,4 \pm 0,2$; $R_2 = (6,9 \pm 0,2)\text{ cm}$

1.3-1.4 The figure on the right shows the path of the laser beam after refraction in the lens. It follows from the similarity of the triangles that $\frac{x}{F} = \frac{\Delta x}{L}$. Therefore the dependence of the laser beam displacement on the lens displacement is given by

$$\Delta x = \frac{L}{F} x. \quad (4)$$

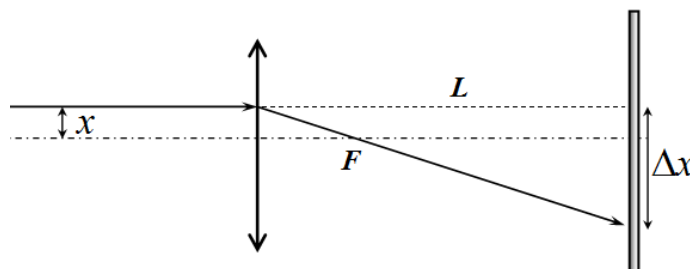
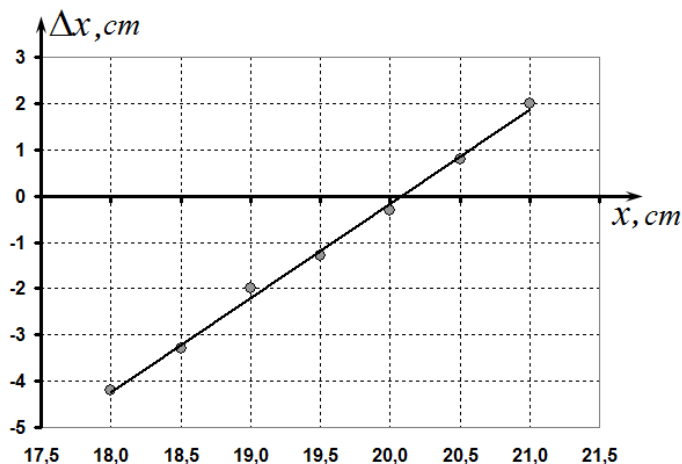


Table 2 shows the results of experimental measurements carried out for $L = 15 \text{ cm}$. The corresponding graph is plotted in the figure on the right.

Table 2

$x, \text{ cm}$	$\Delta x, \text{ cm}$
21,0	2,0
20,5	0,8
20,0	-0,3
19,5	-1,3
19,0	-2,0
18,5	-3,3
18,0	-4,2



The slope of the dependence is calculated by the least square method $K_3 = 2,03 \pm 0,10$. Accordingly, the focal length of the lens is found as

$$F = \frac{L}{K_3} = (7,4 \pm 0,4) \text{ cm}. \quad (5)$$

1.5 The refractive index of the lens can be calculated from the formula for the focal length

$$n = 1 + \frac{\frac{1}{F}}{\frac{1}{R_1} + \frac{1}{R_2}} = 1 + \frac{\frac{1}{7,4}}{\frac{1}{6,76} + \frac{1}{6,9}} \approx 1,5.$$

Experimental error turns out to be $\Delta n \approx 0,2$.

Part 2. Fresnel mirror

Fresnel mirror can be considered as a diffraction grating with the varying period. If the radii of the rings depends on the consecutive number from the center as

$$r_k = r_0 k^\gamma, \quad (6)$$

then, at large k the distance between the rings is found as

$$\Delta r = r_0 \left((k+1)^\gamma - k^\gamma \right) = r_0 k^\gamma \left(\left(1 + \frac{1}{k} \right)^\gamma - 1 \right) \approx \gamma r_0 k^{\gamma-1}. \quad (7)$$

This quantity can be treated as the grating period d . To process the experimental data it is necessary to express the grating period through the distance to the mirror center. It is derived from equation (6) that

$$k = \left(\frac{r}{r_0} \right)^{\frac{1}{\gamma}}. \quad (8)$$

Therefore, the grating period varies with the distance as

$$d = \gamma r_0 \left(\frac{r}{r_0} \right)^{\frac{\gamma-1}{\gamma}}. \quad (9)$$

Experimentally, this dependence can be obtained using the formula of the diffraction grating

$$d \sin \varphi = \lambda. \quad (10)$$

Therefore, the sine of the deflection angle depends on the mirror displacement as:

$$\sin \varphi = \frac{\lambda}{d} = Cr^{\frac{1-\gamma}{\gamma}}. \quad (11)$$

In the experiment it is possible to measure the dependence of the reflected beam displacement Δx on the distance r measured from the point of laser beam to the mirror center. Since the deflection angles in this case can not be considered small, the sine of the deflection angle φ should be calculated according to the formula

$$\sin \varphi = \frac{\Delta x}{\sqrt{(\Delta x)^2 + L^2}}. \quad (12)$$

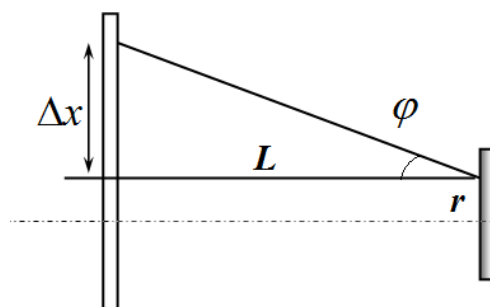
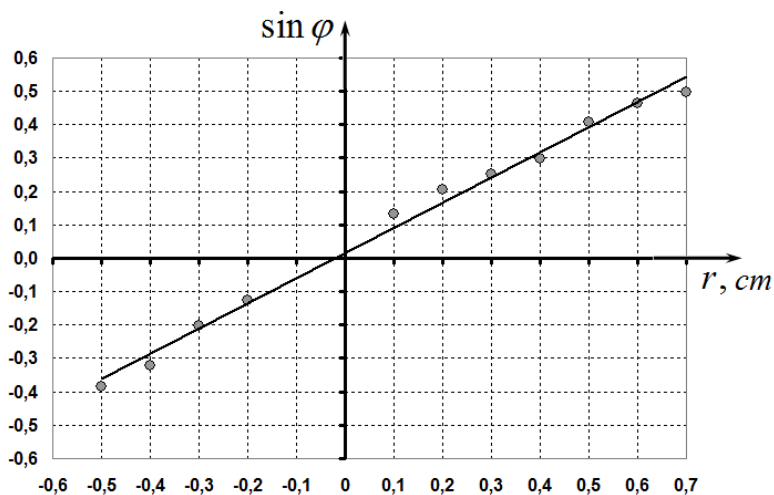


Table 3 shows the results of the measurements of the laser beam displacement Δx the distance r . The same table shows the calculated values of the sine of the deflection angles. The graph on the right represents a plot of the sine of the deflection angle as a function of r .

Table 3

r, cm	$\Delta x, cm$	$\sin \varphi$
0,7	11,0	0,497
0,6	10,0	0,463
0,5	8,5	0,406
0,4	6,0	0,297
0,3	5,0	0,253
0,2	4,0	0,207
0,1	3,0	0,159
-0,2	-2,5	-0,127
-0,3	-4,0	-0,201
-0,4	-6,5	-0,321
-0,5	-8,0	-0,385



Since the obtained dependence is close to linear, in accordance with the general formula (11) the coefficient γ is found from the figure to be 0,5.

Part 3. Glossy reflective stripe

In the reflected light at least five of the reflected beams are visible. One is at the center and four are at the corners of a square. Such a pattern can be explained by the two-dimensional diffraction grating. The diffraction angle is approximately measured to be equal 45 degrees. Hence, the grating period is found to be

$$d = \frac{\lambda}{\sin 45^\circ} = \frac{680 \cdot 10^{-9} m}{\frac{\sqrt{2}}{2}} \approx 9,6 \cdot 10^{-7} m.$$

Part 4. Plastic refractive element

When the laser beam passes through the plastic element six spots are clearly seen on the screen. They are located at the vertices of a regular hexagon. Such a pattern is explained by the refraction of laser beams on the facets of the regular pyramids. Base of the pyramids fill the plane forming a hexagonal structure.

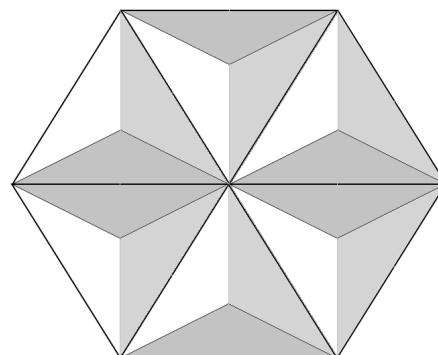
If the screen is placed at a distance $L = 6,5 \text{ cm}$, then the side length of the hexagon is equal $a = 4,5 \text{ cm}$. The same distance is measured from the center of the regular hexagon. These data allow us to calculate the sine of the deflection angle of laser beam

$$\sin \alpha = \frac{a}{\sqrt{a^2 + L^2}} = 0,57.$$

In the problems of the theoretical competition the following formula for the deflection angle was derived

$$\sin \gamma = \sqrt{\frac{2}{3}} \sqrt{n^2 - \frac{2}{3}} - \frac{\sqrt{2}}{3},$$

which can be used to calculate the refraction index to be $n = 1,51$.



Grading scheme

Part 1. Lens		(6)
1.1	Measurements: - L is stated; - 7 or more data points; - (4-6 data points); - (less than 4 data points);	0,2 0,7x2 (0,3x2) 0
	Graphs: - axis ticked and digitized; - all the experimental points are plotted; - line smoothing is carried out;	0,1 0,1x2 0,1x2
1.2	Calculation of the radii: - dependence (2); - Eq. (3); - slopes are calculated : - by using Least Square Method; - (by using the graph); - (by using two points on the graph); Calculation of radii: - range 6,5 -7,5 cm; - (range 5-9 cm); - (out of range 5-9 cm);	0,1 0,1 0,3x2 (0,2x2) (0,1x2) 0,2x2 (0,1x2) (0x2)

1.3	Measurements: - L is stated; - 5 or more data points; - (3-4 data points); - (less than 3 data points);	0,2 0,5 (0,3) (0)
	Graphs: - axis ticked and digitized; - all the experimental points are plotted; - line smoothing is carried out;	0,1 0,1 0,1
1.4	Measurement of the focal length: - Eq. (4); slopes are calculated : - by using Least Square Method; - (by using the graph); - (by using two points on the graph); Measurement of the focal length: - range 7 -8 cm; - (range 6-9 cm); - (out of range 6-9 cm);	0,2 0,3 (0,2) (0,1) 0,4 (0,2) (0)
1.5	Calculation of the refraction index: - Eq. to find index of refraction; - numerical value is in the range of 1,45-1,55; - (range 1,35 – 1,65); - (out of the range 1,35 - 1,65); - error estimation;	0,1 0,5 (0,2) (0) 0,2
Part 2. Fresnel Mirror		(4)
2.1	Measurements: - L is stated; - 7 or more data points; - (4-6 data points); - (less than 4 data points);	0,2 0,8 (0,4) (0)
	Eq for the sine of the angle; Values of sines are calculated ;	0,1 0,2
	Approximately linear relationship;	0,5
	Graphs: - axis ticked and digitized; - all the experimental points are plotted; - line smoothing is carried out;	0,1 0,1 0,1
	Method to find γ : - the idea of a diffraction grating ; - dependence of the grating period on the distance to the center (9); - dependence of the sine of the deflection angle on the distance to the center (11);	0,3 0,4 0,4
	The value obtained for γ : - by using linear dependence on the sine of the angle; - by using other reasonable way;	0,3 (0,2) 0,5

	$\gamma = \frac{1}{2}$; - range 0,3-0,7; - out of range 0,3 – 0,7;	(0,2) (0)
Part 3. Glossy reflective stripe.		2
	- obtained more than 5 spots on the screen; - (obtained 5 spots on the screen); - (obtained 3 spots on the screen);	0,4 (0,3) (0,2)
	Structure of the diffraction grating: - two-dimensional; - (onde-dimensional);	0,5 (0,3)
	Deflection angles of the beams are measured; The formula to calculate grating period; Calculated grating period (1 μm); - error less than 10%; - (in the range 30%); - (out of the range 30%);	0,4 0,2 0,5 (0,2) (0)
Part 4. Plastic refractive element.		3
	- 6 spots on the screen are obtained;	0,5
	Structure - pyramid;	0,5
	Filling the plane (with a hexagonal structure);	0,5
	Eq to find the refraction index;	0,5
	Measured deflection angles of the beams are measured;	0,5
	The numerical value of the refraction index:	
	- numerical value in the range of 1,45-1,55;	0,5
	- (in the range 1,35 – 1,65);	(0,3)
	- (out of range 1,35 - 1,65);	(0)