1. In the given formulation of the problem it's only possible to write the answer in a symbolic form:

$$\varphi\left(x_0, y_0, z_0\right) = \int\limits_{t_0}^{t_1} \frac{\mu}{4\pi\varepsilon_0} \sqrt{\frac{x'(t)^2 + y'(t)^2 + z'(t)^2}{\left(x - x_0\right)^2 + \left(y - y_0\right)^2 + \left(z - z_0\right)^2}} dt.$$

2. For a periodic motion the acceleration a is related with the velocity v as

$$a = \omega v,$$

where ω is the characteristic frequency. The characteristic frequency of the Moon rotation is $\omega = 4.3 \cdot 10^{-7}\ \mathrm{s}^{-1}$, so the Moon is subjected to a small acceleration while moving at a great velocity. The galaxies move with velocities of order 10^3 km/s and with acceleration which is not more than $10^{-5}\ \mathrm{m/s^2}$.

3. Let

$$A = \sum_{k=0}^{n} \cos(\theta + k\alpha).$$

This expression is easier manipulated in the complex plane. Let

$$B = \sum_{k=0}^{n} e^{i(\theta + k\alpha)},$$

then $A = \mathrm{Re}\,(B)$. Using the formula of the sum of a geometric progression

$$B = e^{i\theta} \cdot \frac{1 - e^{i(n+1)\alpha}}{1 - e^{i\alpha}} = e^{i\theta} \cdot \frac{i\left(1 - e^{i(n+1)\alpha}\right)\left(1 - e^{-i\alpha}\right)}{2\sin\alpha} = (\cos\theta + i\sin\theta) \cdot \frac{\left(1 - e^{i(n+1)\alpha}\right)\left(1 + e^{-i\alpha}\right)}{2 - 2\cos\alpha}.$$

The real part of this is

$$\begin{aligned} A = {} & \frac{1}{2\left(1 - \cos\alpha\right)} \left(\left(1 - \cos\left(n+1\right)\alpha\right)\left(\cos\theta\left(1 - \cos\alpha\right) - \sin\alpha\sin\theta\right) + \sin(n+1)\alpha\left(\sin\theta\left(1 - \cos\alpha\right) + \sin\alpha\sin\theta\right)\right) = \\ & = \frac{1}{2}\left(\cos\theta - \frac{\sin\alpha}{1 - \cos\alpha}\sin\theta - \cos\theta\cos(n+1)\alpha + \sin\theta\sin(n+1)\alpha + \frac{\sin\alpha}{1 - \cos\alpha}\sin\theta\cos(n+1)\alpha + \right. \\ & \left. + \frac{\sin\alpha}{1 - \cos\alpha}\cos\theta\sin(n+1)\alpha\right) = \frac{1}{2\sin\dfrac{\alpha}{2}}\left(\cos\theta\cos\frac{\alpha}{2} - \sin\theta\cos\frac{\alpha}{2} - \cos\left(\theta - (n+1)\alpha\right)\sin\frac{\alpha}{2} + \right. \\ & \left. + \sin\left(\theta - (n+1)\alpha\right)\cos\frac{\alpha}{2}\right) = \frac{1}{2\sin\dfrac{\alpha}{2}}\left(\sin\left(\frac{\alpha}{2} - \theta\right) + \sin\left(\theta + (n+1)\alpha - \frac{\alpha}{2}\right)\right) = \frac{\sin\dfrac{(n+1)\alpha}{2}\cos\left(\theta + \dfrac{\alpha n}{2}\right)}{\sin\dfrac{\alpha}{2}}, \end{aligned}$$

q.e.d.

4. As the vector $\vec{A} = \phi(r)\vec{r}$ has only a radial component, its divergence is

$$\operatorname{div}\vec{A} = \frac{1}{r^2}\frac{d}{dr}\left(r^3\phi\right) = 3\phi + r\frac{d\phi}{dr} = 0,$$

which implies

$$\phi \sim r^{-3}.$$

5. First multiply the first equation by u_1, the second by u_2, the third by u_3, and add:

$$u_1\frac{du_1}{dt} + u_2\frac{du_2}{dt} + u_3\frac{du_3}{dt} = 0.$$

This proves the first integral. For the second integral multiply the first equation by $\lambda_1 u_1$, the second by $\lambda_2 u_2$, the third by $\lambda_3 u_3$ and add. Again,

$$\lambda_1 u_1 \frac{du_1}{dt} + \lambda_2 u_2 \frac{du_2}{dt} + \lambda_3 u_3 \frac{du_3}{dt} = 0.$$

This proves the second integral.