1. Let μ be the mass of unit length of the rope. Then the net force (in the rotating frame) acting on a small piece dr is

$$dF = \left(-\frac{GM}{r^2} + \omega^2 r\right) \mu dr,$$

where $G = 6.67 \cdot 10^{-11}$ m^3/(kg $\cdot$ s^2) is the gravitational constant, $M = 5.97 \cdot 10^{24}$ kg is the mass of Earth, $\omega = 7.27 \cdot 10^{-5}$ rad/s is the frequency of rotation of Earth. Then the net force on the whole rope is

$$\frac{F}{\mu} = -GM\left(\frac{1}{R} - \frac{1}{R+L}\right) + \frac{\omega^2}{2}\left((R+L)^2 - R^2\right) = \frac{\omega^2 L(L+2R)}{2} - \frac{GML}{R(R+L)},$$

where $R = 6.378 \cdot 10^6$ m is the radius of Earth. As the rope hangs free, $F = 0$. So,

$$\frac{2GM}{\omega^2} = R(R+L)(2R+L),$$

or, in terms of the geostationary radius $R_0 = (GM/\omega^2)^{1/3}$,

$$RL^2 + 3R^2L + 2R^3 - 2R_0^3 = 0.$$

The solution of this equation is

$$L = \frac{1}{2R}\left(-3R^2 + \sqrt{R_4 + 8R_0^3 R}\right) = \frac{R}{2}\left(-3 + \sqrt{1 + \frac{8R_0^3}{R^3}}\right) = 1.44 \cdot 10^8 \text{ m}.$$

2. The angular velocity appears to be $\omega = 1$, and the radius is a. So the linear velocity is $v = \omega a = a$, and the arc length is $l = 2\pi a$.

3. Let x_i $(i = 1, 2, 3)$ be the displacements of the bodies. Then due to Newton's law

$$\begin{aligned} x_1'' &= \omega_0^2(x_2 - x_1) + \omega^2\delta\cos\omega t, \\ x_2'' &= \omega_0^2(x_1 + x_3 - x_2), \\ x_3'' &= \omega_0^2(x_2 - x_3), \end{aligned}$$

where $\omega_0 = \sqrt{k/m}$ and $\delta = f/(m\omega^2)$. The solution of this system is (obtained by *Mathematica*):

$$\xi_1 = \frac{x_1}{\delta} = \frac{3\omega^2\left(\omega^2 - 3\omega_0^2\right)\cos\omega_0 t - 6\left(\omega^4 - 3\omega_0^2\omega^2 + \omega_0^4\right)\cos\omega t + \left(\omega^2 - \omega_0^2\right)\left(2\omega^2 - 6\omega_0^2 + \omega^2\cos\omega_0 t\sqrt{3}\right)}{6\left(\omega^4 - 4\omega_0^2\omega^2 + 3\omega_0^4\right)},$$

$$\xi_2 = \frac{x_2}{\delta} = \frac{\omega^2\left(1 - \cos\omega_0 t\sqrt{3}\right) - 3\omega_0^2\left(1 - \cos\omega t\right)}{3\left(\omega^2 - 3\omega_0^2\right)},$$

$$\xi_3 = \frac{x_3}{\delta} = \frac{6\left(\omega^4 - 3\omega_0^2\omega^2 + \omega_0^4\right) + \left(\omega^2 - \omega_0^2\right)\left(2\omega^2 - 6\omega_0^2 + \omega^2\cos\omega_0 t\sqrt{3}\right)}{6\left(\omega^4 - 4\omega_0^2\omega^2 + 3\omega_0^4\right)},$$

or, in a dimensionless form with $\varphi = \omega_0 t$ and $\gamma = \omega/\omega_0$,

$$
\begin{aligned}
\xi_1 &= \frac{3\gamma^2\left(\gamma^2-3\right)\cos\varphi - 6\left(\gamma^4-3\gamma^2+1\right)\cos\gamma\varphi + \left(\gamma^2-1\right)\left(2\gamma^2-6+\gamma^2\cos\varphi\sqrt{3}\right)}{6\left(\gamma^4-4\gamma^2+3\right)}, \\
\xi_2 &= \frac{\gamma^2\left(1-\cos\varphi\sqrt{3}\right) - 3\left(1-\cos\gamma\varphi\right)}{3\left(\gamma^2-3\right)}, \\
\xi_3 &= \frac{6\left(\gamma^4-3\gamma^2+1\right) + \left(\gamma^2-1\right)\left(2\gamma^2-6+\gamma^2\cos\varphi\sqrt{3}\right)}{6\left(\gamma^4-4\gamma^2+3\right)}.
\end{aligned}
$$

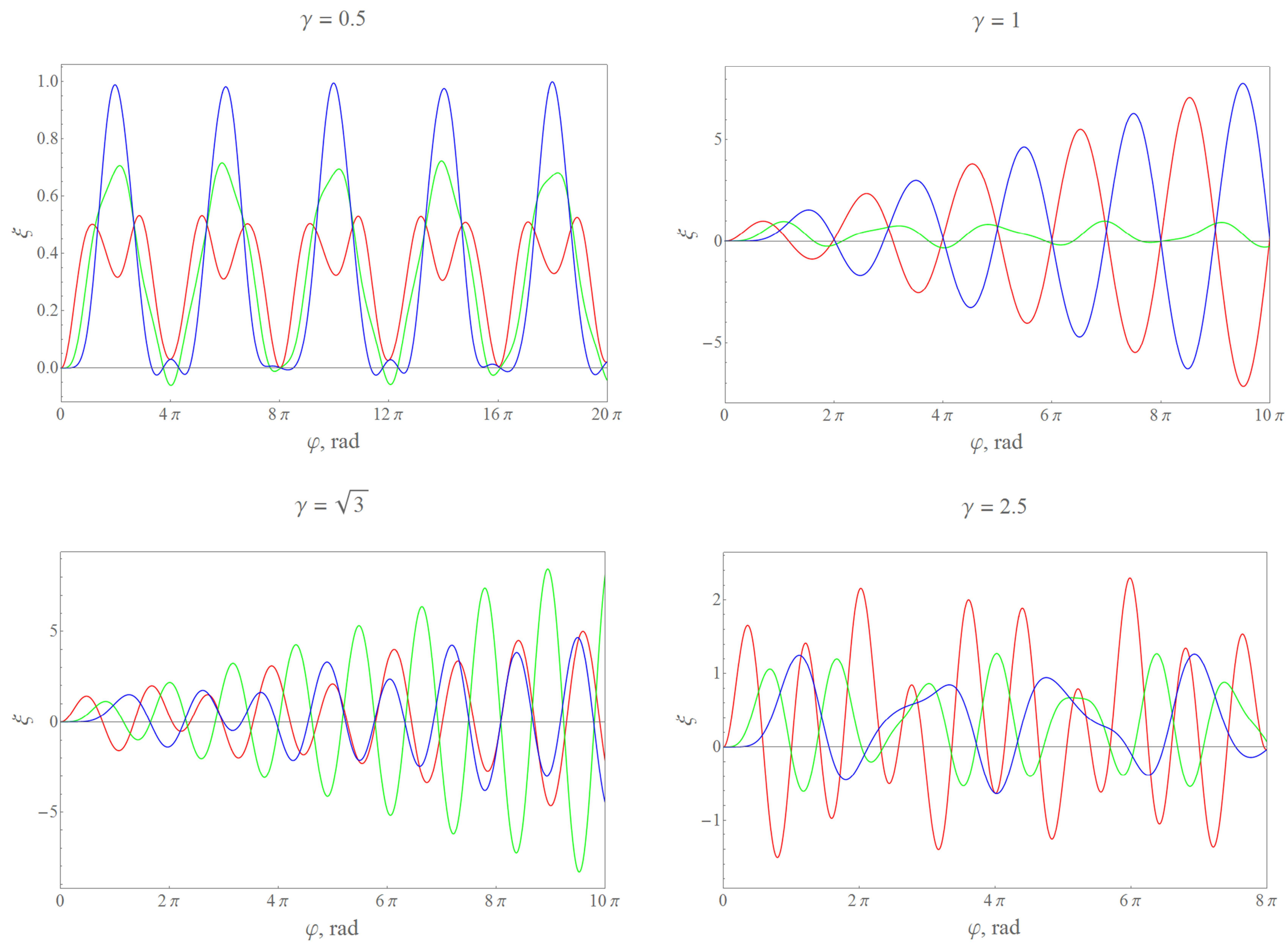
In case of a resonance $\gamma = 1$ this solution is degenerated to

$$
\begin{aligned}
\xi_1 &= \frac{1}{12}\left(4 - 3\cos\varphi - \cos\varphi\sqrt{3} + 3\varphi\sin\varphi\right), \\
\xi_2 &= \frac{1}{6}\left(2 - 3\cos\varphi + \cos\varphi\sqrt{3}\right), \\
\xi_3 &= \frac{1}{12}\left(4 - 3\cos\varphi - \cos\varphi\sqrt{3} - 3\varphi\sin\varphi\right).
\end{aligned}
$$

There's also a resonance $\gamma = \sqrt{3}$, in that case

$$
\begin{aligned}
\xi_1 &= \frac{1}{12}\left(4 + 9\cos\varphi - 13\cos\varphi\sqrt{3} + \varphi\sqrt{3}\sin\varphi\sqrt{3}\right), \\
\xi_2 &= \frac{1}{6}\left(2 - 2\cos\varphi\sqrt{3} - \varphi\sqrt{3}\sin\varphi\sqrt{3}\right), \\
\xi_3 &= \frac{1}{12}\left(4 - 9\cos\varphi + 5\cos\varphi\sqrt{3} + \varphi\sqrt{3}\sin\varphi\sqrt{3}\right).
\end{aligned}
$$

The plots of these dependencies are shown below.



The plots of the $\xi(\varphi)$ dependencies for the bodies, body 1 corresponds to the red curve, body 2 — to the green, body 3 — to the blue one.