1. Consider the center-of-mass frame of reference, let the positive Ox direction be antiparallel to v_0. In this frame the velocities of the balls before the collision are $u_0 = -2mv_0/(M+2m)$ and $u_1 = u_2 = Mv_0/(M+2m)$ respectively. Then right after the collision the velocities will be

$$u_0' = \frac{(M-m)u_0 + 2mu_1}{M+m} = \frac{2m^2 v_0}{(M+m)(M+2m)},$$
$$u_1' = \frac{2Mu_0 - (M-m)u_1}{M+m} = -\frac{M(M+3m)v_0}{(M+m)(M+2m)}.$$

Consider the event of the collision to be the point $(x,t) = (0,0)$. Then the originally incident ball will be moving by law

$$x_0(t) = u_0' t = \frac{2m^2 v_0}{(M+m)(M+2m)} t = \frac{2v_0}{(\mu+1)(\mu+2)} t,$$

and the other collided ball — by law

$$x_1(t) = \frac{u_1' + u_2}{2} t + \frac{u_1' - u_2}{2\omega} \sin \omega t = -\frac{\mu v_0}{(\mu+1)(\mu+2)} t - \frac{\mu v_0}{(\mu+1)\omega} \sin \omega t,$$

where $\omega = \sqrt{2k/m} = 1$ rad/s and $\mu = M/m$.

The second collision will happen if $x_0(t_1) = x_1(t_1)$ at some $t_1 > 0$. This condition may be simplified to

$$t_1 = -\frac{\mu}{\omega} \sin \omega t_1.$$

Obviously, this equation has a solution if μ is not less than some μ_0 which can be derived from the condition $v_0(t_1) = v_1(t_1)$ (the $x - t$ curves touch each other in a single point). So, we have a system of equations

$$\varphi = -\mu_0 \sin \varphi,$$
$$1 = -\mu_0 \cos \varphi,$$

where $\varphi = \omega t_1$. It can be reduced to a transcendental equation $\varphi = \tan \varphi$, which numeric solution is $\varphi_0 = 4.493$ rad, giving $\mu_0 = 4.603$. So, the second collision will happen if $M \geq 4.60m$ with a maximum interval $\Delta t_{\max} = \varphi_0/\omega = 4.49$ s. For an arbitrary $\mu > \mu_0$ the time interval $\Delta t = \varphi/\omega$, where φ is found from $\varphi = -\mu \sin \varphi$.

2. For an estimation let's consider the temperature of the air T inside the channel constant. Then the density of the air ρ linearly depends on p:

$$\rho(p) = \frac{p\mu}{RT},$$

where $\mu = 0.029$ kg/mol is the molar mass of air, $R = 8.314$ J/(mol · K) is the gas constant. So, the pressure gradient at a distance x from the center is

$$\frac{dp}{dx} = -\rho g(x) \frac{x}{\sqrt{x^2 + R_0^2/2}} = -\frac{p\mu g_0}{RT} \frac{x}{R_0},$$

where R_0 is the radius of the Moon, g_0 is the acceleration due to gravity on its surface. The solution of this equation is

$$p(x) = p_0 \exp\left(-\frac{x^2}{2x_0^2}\right),$$

where

$$x_0 = \sqrt{\frac{R_0 RT}{\mu g_0}} = 303.6 \text{ km}$$

at $T = 300$ K. So, the pressure near the surface is

$$p\left(\frac{R_0}{\sqrt{2}}\right) = 2.46 \cdot 10^{-4} \text{ atm} = 25 \text{ Pa}.$$

3. Let the amount of water $m_{\mathrm{i}} = \alpha m$ freeze. Then the heat balance yields

$$\alpha\lambda = (1-\alpha)(c\Delta t + r),$$

where $c = 4200$ J/(kg · K) is the heat capacity of water, $\Delta t = 100$ K, $\lambda = 330$ kJ/kg is the specific heat of melting, $r = 2.3$ MJ/kg is the specific heat of evaporation. From this equation we get

$$\alpha = \frac{r + c\Delta t}{r + c\Delta t + \lambda} = 0.89.$$

So, $m_{\mathrm{i}} = 44.5$ g of water freezes, other $m_{\mathrm{v}} = 5.5$ g vaporizes.

4. The first three statements are true. For the fourth statement we need to find $\exp(\sigma_i)$. For example,

$$\exp(\sigma_1) = I \sum_{i=0}^{\infty} \frac{1}{(2i)!} + \sigma_1 \sum_{i=0}^{\infty} \frac{1}{(2i+1)!} = I \cosh 1 + \sigma_1 \sinh 1.$$

Similarly,

$$\begin{aligned}\exp(\sigma_2) &= I \cosh 1 + \sigma_2 \sinh 1,\\ \exp(\sigma_3) &= I \cosh 1 + \sigma_3 \sinh 1.\end{aligned}$$

This means that the relation $\exp(\sigma_i \sigma_j) = \exp(\sigma_i)\exp(\sigma_j)$ is false. This relation would be true if σ_i were pairwise commutative.