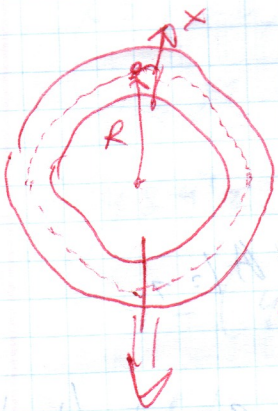
Задача 1

$$v_{13} = \frac{v_{12} + v_{23}}{1 + \frac{v_{12} v_{23}}{c^2}}$$

$$v = c \operatorname{th} \frac{f}{c}$$

$$f = c \operatorname{arcth} \frac{v}{c}$$

Задача 2



$$R = \frac{e}{2\pi}$$

$$\varepsilon = \frac{x}{R}$$

$$w = \frac{1}{2} E \varepsilon^2 = \frac{E}{2} \left(\frac{x}{R}\right)^2$$

$$dW = w \cdot e \, dS$$

$$dW = \frac{2E\pi^2}{e} e \int x^2 dS$$

↑ мом. ин. кольца отн. оси y

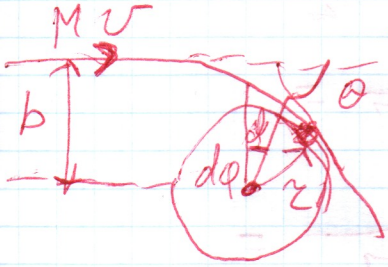
$$\int x^2 dS = \frac{1}{2} \int (x^2 + y^2) dS = \frac{1}{2} \int r^2 dS$$

↑ мом. ин. отн. оси z

$$\int r^2 dS = \int_{r_1}^{r_2} r^2 \cdot 2\pi r \, dr = \frac{\pi}{2} (r_2^4 - r_1^4)$$

$$W = \frac{2\pi^2 E}{l} \cdot \frac{1}{2} \cdot \frac{8}{2} \left(r_2^4 - r_1^4\right)$$

Задача 3

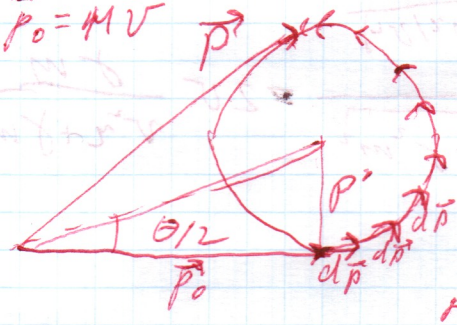


$$Mvb = Mr^2 \frac{d\varphi}{dt} \qquad M \ll m$$

$$\vec{F} = -\frac{\gamma m M}{r^2}\vec{n} \qquad \vec{n} = \frac{\vec{r}}{r}$$

$$d\vec{p} = \vec{F}\,dt = -\frac{\gamma m M}{r^2}\vec{n}\,dt = -\underbrace{\frac{\gamma m M}{v b}}_{p'}\vec{n}\,d\varphi$$

$\vec{p}_0 = M\vec{v}$



$$\operatorname{tg}\frac{\theta}{2} = \frac{p'}{p_0} = \frac{\gamma m}{b v^2}$$

нужен макс θ - мин b,

$r_{min} = r$; $\quad \frac{Mv^2}{2} = \frac{Mu^2}{2} - \gamma\frac{mM}{r}$

u - скорость в r_{min} $\quad Mvb = Mur$

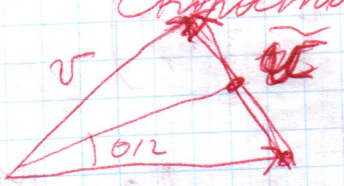
$$\frac{Mv^2}{2} = \frac{Mv^2 b^2}{2r^2} - \gamma\frac{mM}{r}$$

$$\frac{v^2}{r^2}b^2 = v^2 + 2\gamma\frac{m}{r}$$

$$b^2 = r^2 + 2\gamma\frac{mr}{v^2}$$ — мин. b

$$\operatorname{tg}\frac{\theta}{2} = \frac{\gamma m}{v^2\sqrt{r^2 + 2\gamma\frac{mr}{v^2}}}$$

Скорость в начальной СО



$$v' = 2v \sin\frac{\theta}{2} =$$

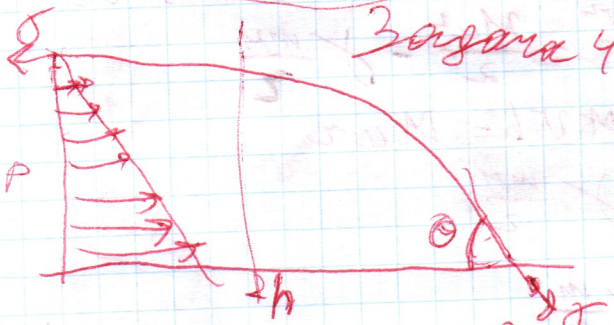
$$= 2v \frac{\operatorname{tg}\frac{\theta}{2}}{\sqrt{1+\operatorname{tg}^2\frac{\theta}{2}}} =$$

$$= 2v \frac{\frac{\gamma m}{v^2\sqrt{r^2+2\gamma m r/v^2}}}{\sqrt{1+\frac{\gamma^2 m^2}{v^4(r^2+2\gamma m r/v^2)}}} =$$

$$= 2v \frac{\gamma m}{\sqrt{v^4 r^2 + 2\gamma m r v^2 + \gamma^2 m^2}} = 2v \frac{\gamma m}{v^2 r + \gamma m}$$

$$= \frac{2v}{1+\gamma m/v^2 r}$$

Задача 4



$$\sigma = \sigma\cos\theta + \frac{1}{2}\rho g h^2$$

$$h = \sqrt{\frac{2\sigma}{\rho g}(1-\cos\theta)}$$