0. The top card overhangs the second one by a maximum of a half of a card length, the second one a fourth over the third one, the third one the sixth over the fourth one and so on. So for $N+1$ cards the total overhang is

$$L = \frac{a}{2}\sum_{i=1}^{N}\frac{1}{i},$$

where $a = 88$ mm is the card length.

a$_0$) If you have many cards $(N \to \infty)$ you can build the bridge of infinite length $\left(\lim_{N\to\infty} L(N) \to \infty\right)$. Do you believe that it is real?

b$_0$) I give you 100 cards (two deck of cards, $(L(100) \approx 23\text{ cm})$). Can you at home on table build the bridge by 22 cm-long?

1. The field in the dielectric is reduced by $\varepsilon/\varepsilon_0$ times. So, the field in the first dielectric is

$$E_1 = \frac{q}{4\pi\varepsilon_1 r^2},$$

and in the second one

$$E_2 = \frac{q}{4\pi\varepsilon_2 r^2}.$$

The potential is respective for these fields.

a$_1$) Suppose that the interface between the two dielectric media coincides with the plane z=0. Your relations $E_i = \dfrac{q}{4\pi\varepsilon_i r^2}$ (i=1,2) implies a jump in the z component of the electric field across the interface $(E_1)_{z\to+0} \neq (E_2)_{z\to-0}$. If I understood you well, you assume that there is a bound charge sheet (z=0) on the interface between the two dielectric media. Please provide for me an expression for the bound surface charge density $\sigma = \sigma(x,y)$.

3. By introducing the new variable $t = a\sqrt{x}$ this expression simplifies to

$$\int_0^\infty \frac{tdt}{1+e^t} = \frac{\pi^2}{12}.$$

a$_3$) you have successfully carried out change of variables. Sorry but I do not see the proof here. Why $\int_0^\infty \dfrac{dt}{e^{a\cdot t}+1} = \dfrac{\pi}{12}^2$, may be $\int_0^\infty \dfrac{dt}{e^t+1} = \dfrac{1}{12}\cdot\left(\dfrac{355}{113}\right)^2$.