The system of differential equations in ρ and φ corresponding to the pursuit is:

$$v-\rho'(t)\cdot\cos(\varphi(t))+\rho(t)\cdot\varphi'(t)\cdot\sin(\varphi(t))=v\cdot\cos(\varphi(t)),$$
$$-\rho'(t)\cdot\sin(\varphi(t))-\rho(t)\cdot\varphi'(t)\cdot\cos(\varphi(t))=v\cdot\sin(\varphi(t)),$$
$$\rho(0)=h/2,\ \varphi(0)=\frac{\pi}{2};$$

or shortened:

$$v-\dot\rho\cos\varphi+\rho\dot\varphi\sin\varphi=v\cos\varphi,\ \|-\sin\varphi\,\|\cos\varphi \qquad (1)$$
$$-\dot\rho\sin\varphi-\rho\dot\varphi\cos\varphi=v\sin\varphi;\|\cos\varphi\,\|\sin\varphi \qquad (2)$$

Let's multiplicate (1) by –sin $\varphi$ and (2) by cos $\varphi$ and then add:

$$\rho\dot\varphi=-v\sin\varphi; \qquad (3)$$

analogically (1) by cos $\varphi$; (2) by sin $\varphi$; (1) + (2):

$$\dot\rho=v\,(\cos\varphi-1); \qquad (4)$$

So,

$$\rho=-\frac{v\sin\varphi}{\dot\varphi}; \qquad (5)$$

Differentiating (5) by $t$, we get:

$$\dot\rho=-v\cos\varphi+\frac{v\ddot\varphi\,\sin\varphi}{\dot\varphi^2}; \qquad (6)$$

(4) = (6):

$$\frac{\dot\varphi\ (2\cos\varphi-1)}{\sin\varphi}=\frac{\ddot\varphi}{\dot\varphi}; \qquad (7)$$

Now it's time to integrate:

$$\int\frac{\dot\varphi(2\cos\varphi-1)}{\sin\varphi}dt=\int\frac{2\cos\varphi-1}{\sin\varphi}d\varphi=\ln\left(4\sin\frac{\varphi}{2}\cos^3\frac{\varphi}{2}\right);$$

$$\int\frac{\ddot\varphi}{\dot\varphi}dt=\ln\dot\varphi-\ln C,$$

where $C$ is a constant which will be found from the initial conditions.

$$\dot\varphi=C_1\cdot4\sin\frac{\varphi}{2}\cos^3\frac{\varphi}{2}. \qquad (8)$$

Now let's find $C_1$.

$\rho(0)=h,\varphi(0)=\pi/2$, so $\dot\varphi(0)=-\dfrac{v}{h}$ (from (3)). As $\sin\dfrac{\pi}{4}\cos^3\dfrac{\pi}{4}=0.25, C_1=1.$

Integrating once more by $t$, we get the equation for $\varphi$:

$$1-\frac{4vt}{h}=2\ln \operatorname{tg}\frac{\varphi}{2}+\operatorname{tg}^2\frac{\varphi}{2}; \tag{9}$$

The equation of the trajectory in polar coordinates (in the coordinate system of the hare) is:

$$\rho=\frac{h}{2\cos^2\varphi/2}; \tag{10}$$

As $\varphi(\infty)=0,\ \cos^2\frac{\varphi}{2}=1;\ \rho(\infty)=\frac{h}{2}.$

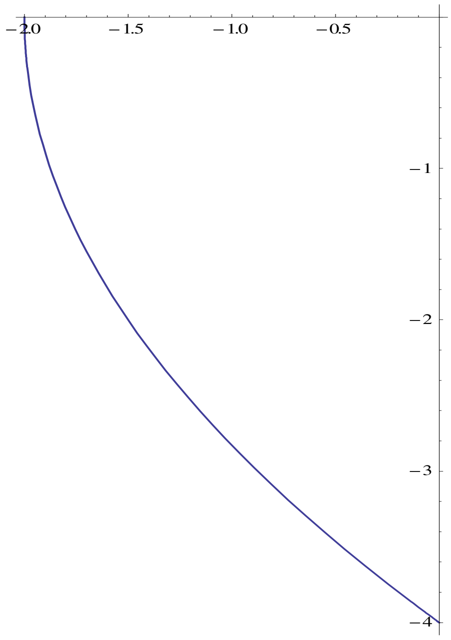
The conversion into normal coordinates looks like:

$$x=vt-\rho\cos\varphi, \tag{11}$$

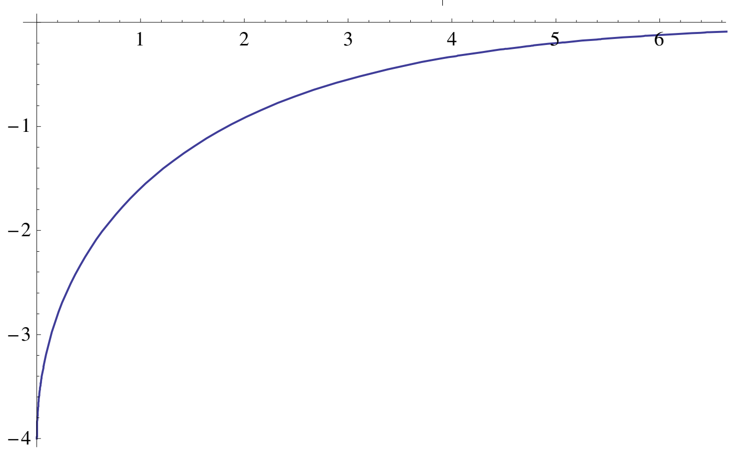
$$y=-\rho\sin\varphi; \tag{12}$$

$$\dot{x}=v\cos\varphi,\ \dot{y}=v\sin\varphi.$$

Differentiating (11) and (12) by $t$, we get (1) and (2).




This is the graph of (10) while h = 4.




This is the path of the fox in the fixed coordinate system while h = 4.