THEORETICAL COMPETITION

January 16, 2009

Please read this first:

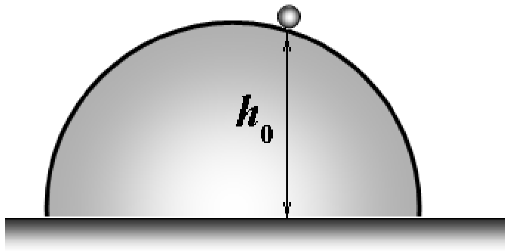
1. The time available for the theoretical competition is 4 hours. There are three questions.
2. Use only the pen provided.
3. You can use your own calculator for numerical calculations. If you don't have one, please ask for it from Olympiad organizers.
4. You are provided with ***Writing sheet*** and additional paper. You can use the additional paper for drafts of your solutions but these papers will not be checked. Your final solutions which will be evaluated should be on the ***Writing sheets***. Please use as little text as possible. You should mostly use equations, numbers, figures and plots.
5. Use only the front side of ***Writing sheets*.** Write only inside the bordered area.
6. Begin each question on a separate sheet.
7. Fill the boxes at the top of each sheet of paper with your country (**Country),** your student code (**Student Code**), the question number (**Question Number)**, the progressive number of each sheet **(Page Number**), and the total number of ***Writing sheets*** used (**Total Number of Pages**). If you use some blank ***Writing sheets*** for notes that you do not wish to be evaluated, put a large X across the entire sheet and do not include it in your numbering.
8. At the end of the exam, arrange all sheets for each problem in the following order:
    - Used ***Writing sheets*** in order;
    - The sheets you do not wish to be evaluated
    - Unused sheets and the printed question.

   Place the papers inside the envelope and leave everything on your desk. You are not allowed to take any paper out of the room.

Problem 1 (10 points)
This problem consists of four unrelated parts.

1A (2 points)

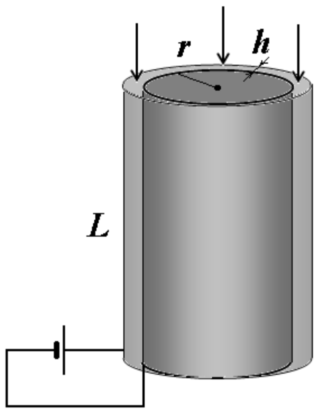
A small body is put on the surface of the frictionless fixed semisphere of radius R at the initial height of h_0 above the table. After that, the body is released without any push. Find the altitude h above the table at which the body loses contact with the sphere surface.



1B (3 points)

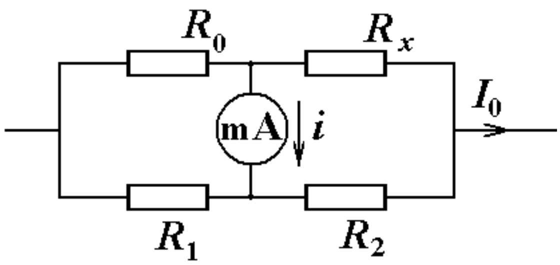
An electrical water heater consists of two well-conducting coaxial cylinders of length L. The radius of the internal cylinder equals r and the distance h between the cylinders is much smaller than their radii. Cylinders are attached to a source of constant voltage U_0. Water is slowly passing between the cylinders, and is being heated by the electric current passing through it. Calculate the velocity of water required for its temperature to rise by Δt after it comes out of the heater.

The water density is γ, its resistivity is ρ and its specific heat capacity is c. Neglect heat capacities of the cylinders and heat losses into the environment.



1C (2 points)

Bridge schemes are widely used to measure resistances. The figure on the right shows the scheme of Wheatstone bridge used to measure an unknown resistance R_x. Resistances R_1 and R_2 can be slowly varied, while resistance R_0 is fixed and exactly known. By changing R_1 and R_2 one can achieve zero total current through milliammeter and the bridge is then called balanced.



1. Express the unknown resistance R_x as a function of resistances R_1, R_2, and R_0 when the bridge is balanced.
2. In real measurements it is impossible to exactly determine the conditions under which the bridge is balanced due to finite sensitivity of the milliammeter. Assume the sensitivity of the milliammeter equal i_0 (i.e. for $i < i_0$ milliammeter shows zero). Determine the relative error of the measurement of R_x due to the finite sensitivity of the milliammeter. Assume that the total current through the bridge is fixed to I_0 with $I_0 >> i_0$.

1D (3 points)

In the morning one can see shiny spots on the road surface that look much like “puddles”. These spots are the simplest demonstration of the mirage phenomenon. In reality these “puddles” are just the reflection of the sky. In this problem you are asked to theoretically describe this phenomenon.

The refraction index of the air is related to its number density γ as

$$n = 1 + \frac{\alpha\gamma}{2}, \tag{1}$$

where $\alpha = 2{,}3 \cdot 10^{-29}\, m^3$ is the average polarizability of the molecules of the air. Assume that the temperature of the air is $t_0 = 20°C$ and its pressure is $P_0 = 1{,}0 \cdot 10^5\, Pa$. Due to the sun heating the surface of the road has the temperature which is higher than that of the surrounding air by $\Delta t = 2{,}0°C$.

A driver moves along the straight horizontal road and his eyes are located at the height $h = 1{,}2\, m$ above the road surface. Estimate the distance from the driver to the nearest mirage-"puddle" that the driver can clearly see on the road. The Boltzmann constant is $k_B = 1{,}38 \cdot 10^{-23}$ J/K.

Problem 2 (10 points)
Electromagnetic swings

Two very long conducting rods are connected to each other by an electromagnetic coil of self-inductance L, and they create a plane with an inclination angle α with the horizon. There is a mobile conductor of mass m that can move along that plane (see figure below). The friction coefficient between the mobile conductor and the rods equals μ, the distance between the rods is h, and the acceleration of gravity is denoted g. The whole system is located in a magnetic field B perpendicular to the plane. In this problem you can neglect resistance and inductance of the rods and of the mobile conductor. Assume that the mobile conductor is at rest at the initial moment and, then, it is released without any push. Find answers to the following questions:

1. Find the inequality for the mobile conductor to start moving downward. Write your answer in terms of α, μ. (1 point)
2. Assume that the inequality in question 1 is satisfied and the mobile conductor starts moving downward. Express the current strength I in the coil through the displacement x of the mobile conductor from its initial position. Write your answer in terms of h, B, L, x. (2 points)
3. Find the maximal velocity $u_{\max}$ of the mobile conductor while it is moving. Write your answer in terms of $h, B, L, m, \alpha, g, \mu$. (1 point)
4. Find the maximal current strength $I_{\max}$ in the coil while the mobile conductor is moving. Write your answer in terms of h, B, m, α, g, μ. (1 point)
5. Assuming that the friction coefficient is small, find the total amount of heat released due to the friction after large time. Write your answer in terms of h, B, L, m, α, g. (3 points)
6. What is the relative error of the answer of the previous item for $\mu = \dfrac{\mathrm{tg}\alpha}{2009}$? (2 points)

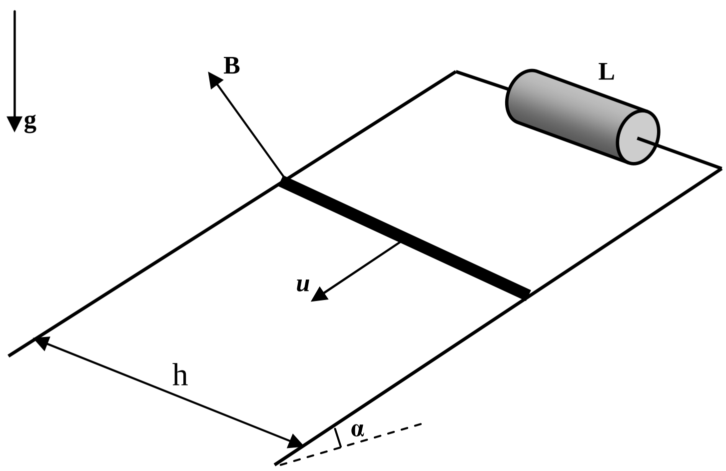


Figure. Sketch of the electromagnetic swings.

Problem 3
Thermal radiation (10 points)

Electromagnetic radiation emitted by heated bodies is called a **thermal** radiation. The thermal radiation is characterized by integral and spectral luminosities.

The integral luminosity R is the total radiation energy emitted by the unit area per unit of time. **The spectral luminosity** r_λ in the small interval of wavelength from λ till $\lambda + \Delta\lambda$ is the ratio of the luminosity ΔR in this interval to the width of this interval, $r_\lambda = \frac{\Delta R}{\Delta \lambda}$. Both the integral and spectral luminosities depend on the properties of the body and its temperature. In general, the luminosity depends on the ability of bodies to absorb radiation. The better the body absorbs the radiation, the better it emits under heating. If the body absorbs all incident radiation, it is called a **black body.** The distribution of black body radiation (i.e. its spectral luminosity) is well investigated both theoretically and experimentally. In figures 1 and 2 we show the plots of spectral luminosities of black bodies at two different temperatures, $T_1 = 2000K$ (Fig. 1) and $T_2 = 1300K$ (Fig. 2).

1. Using figure 1 of the spectral luminosity at $T_1 = 2000K$, determine the ratio of energies emitted in spectral intervals $(\lambda_1, \lambda_1 + \Delta\lambda)$ and $(\lambda_2, \lambda_2 + \Delta\lambda)$ for $\lambda_1 = 2{,}0 \cdot 10^{-6} m$, $\lambda_2 = 4{,}0 \cdot 10^{-6} m$, $\Delta\lambda = 0{,}5 \cdot 10^{-6} m$. (1 point)

2. Wien established that the wavelength λ_m which corresponds to maximal spectral luminosity is related to the body temperature as
$$\lambda_m = bT^n \qquad (1)$$
where b is called Wien's constant, and n is an integer number. Determine the values of b and n using the plots given. (2 points)

3. Stefan and Boltzmann established that the integral luminosity of black bodies is related to their temperature as
$$R = \sigma T^m,$$
where σ is the Stefan-Boltzmann constant and m is an integer number. Determine the values of σ and m using the plots given. (3 points)

4. The spectral luminosity of the Sun is close to the black body radiation with $\lambda_m = 0{,}48 \cdot 10^{-6} m$. Estimate the time after which the mass of the Sun will decrease by 1%. The mass of the Sun is $2 \cdot 10^{30} kg$, and its radius is $7 \cdot 10^8 m$. (4 points)

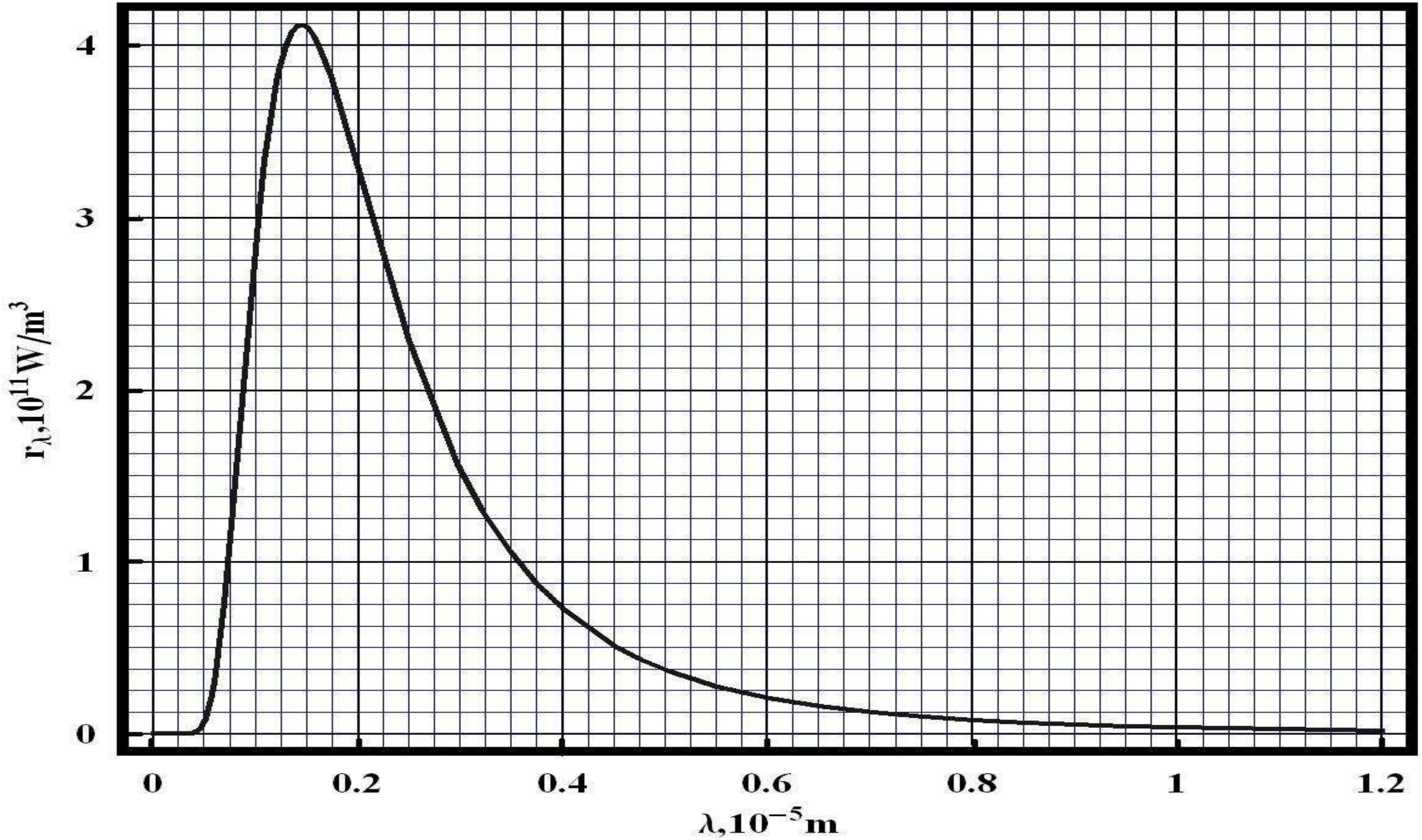


Figure 1. Spectral luminosities of a black body for the temperature $T_1 = 2000K$.

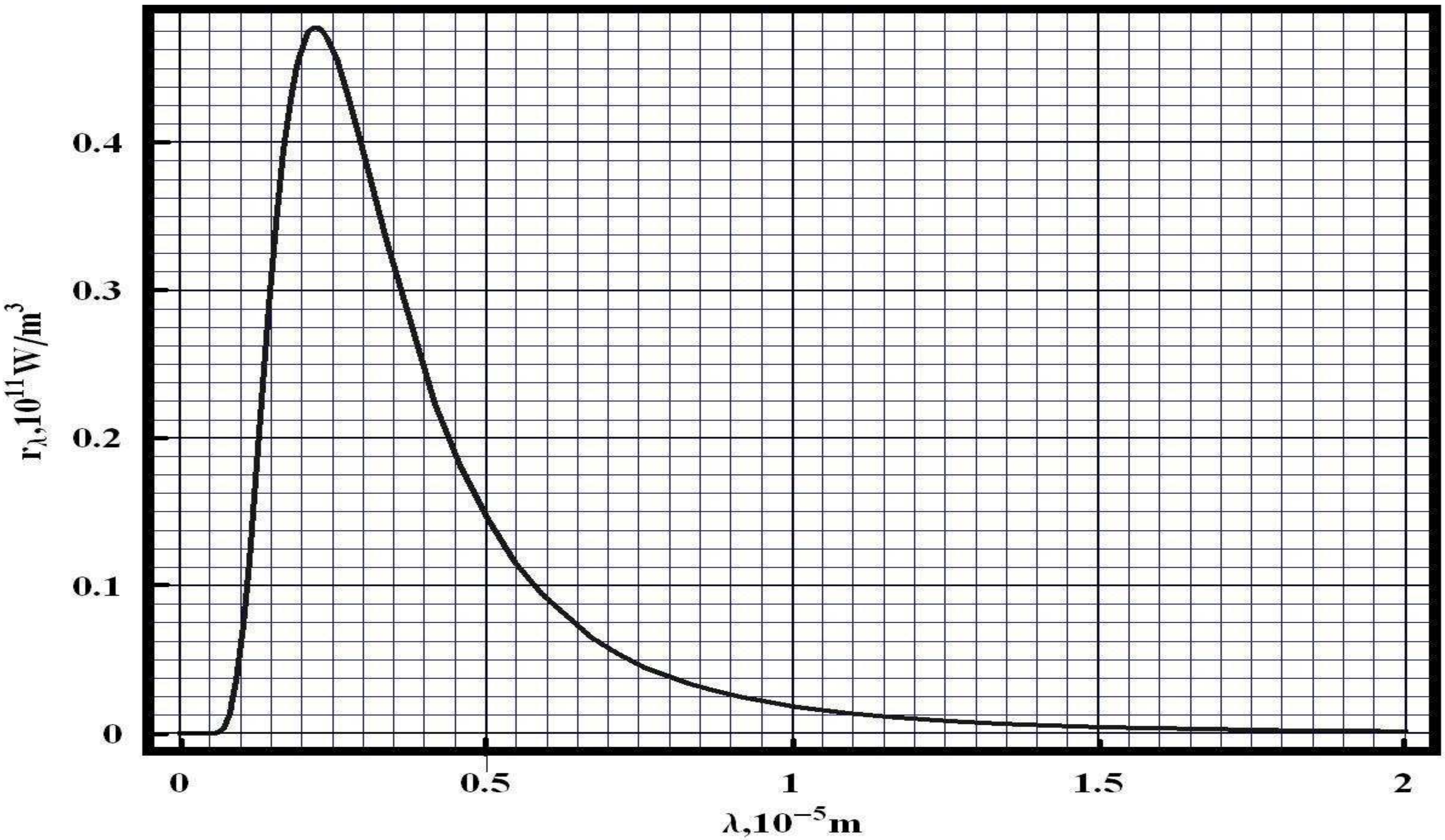


Figure 2. Spectral luminosities of a black body for the temperature $T_2 = 1300K$