11.03.2016 – N3

Show your work for each problem using numbers, sketches, or words.

Name:__________________

1) Derive the formula for the electric potential generated by a charge distributed along a spatial curve L with continuous line density $\mu(x,y,z)$ where $x=x(t)$, $y=y(t)$, $z=z(t)$, $t_0 \le t \le t_1$ are parametric equations of the curve L.

2) The moon orbits about the Earth with an average speed of just over 1000 m/s; yet its acceleration is less than 0.003 m/s^2 . It is the truth that the moon is a fast-moving object with a low acceleration.

3) Prove that:

$$\cos(\theta)+\cos(\theta+\alpha)+\ldots+\cos(\theta+n\cdot\alpha)=\frac{\sin\left(\dfrac{\alpha\cdot(n+1)}{2}\right)\cdot\cos\left(\theta+\dfrac{\alpha\cdot n}{2}\right)}{\sin\left(\dfrac{\alpha}{2}\right)}$$

4) Find the function $\phi(|\vec{r}|)$ which satisfies the condition $\operatorname{div}\left(\phi(|\vec{r}|)\vec{r}\right)=0$, by using the expressions for the divergence in terms of spherical coordinates.

4) Consider the nonlinear system of ordinary differential equations:

$$\frac{du_1}{dt}=(\lambda_3-\lambda_2)\cdot u_2\cdot u_3$$
$$\frac{du_2}{dt}=(\lambda_1-\lambda_3)\cdot u_1\cdot u_3$$
$$\frac{du_3}{dt}=(\lambda_2-\lambda_1)\cdot u_2\cdot u_1$$

where $\lambda_i \in R$. Show that the first integrals are given by:

$I_1=(u_1)^2+(u_2)^2+(u_3)^2$, $I_2=\lambda_1\cdot(u_1)^2+\lambda_2\cdot(u_2)^2+\lambda_3\cdot(u_3)^2$

Hint: the first integral - is a quantity that is conserved throughout the motion.