

9.03.2016 – N2

Show your work for each problem using numbers, sketches, or words.

Name: \_\_\_\_\_

**1) Prove the identities:**

$$\text{a) } \left( (\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) \right) = (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{d}) - (\vec{a} \cdot \vec{d})(\vec{b} \cdot \vec{c})$$

$$\text{b) } \left( (\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) \right) = \left( \vec{a} \cdot (\vec{b} \times \vec{d}) \right) \vec{c} - \left( \vec{a} \cdot (\vec{b} \times \vec{c}) \right) \vec{d}$$

**2) Find the divergence and curl of the following vectors:**

$$(\vec{a} \cdot \vec{r}) \vec{b}, (\vec{a} \cdot \vec{r}) \vec{r}, (\vec{a} \times \vec{r}), \phi(\vec{r})(\vec{a} \times \vec{r}), (\vec{r} \times (\vec{a} \times \vec{r}))$$

where  $\vec{a}$  and  $\vec{b}$  are constant vectors.

**3) The plane  $z = 0$  is charged to a density which varies in accordance with the periodic law  $\sigma = \sigma_0 \sin(\alpha x) \cdot \sin(\beta y)$ , where  $\sigma_0$ ,  $\alpha$ ,  $\beta$  are constants. Find the potential  $\varphi$  due to this charge distribution.**