

5.04.2016 – N4

Show your work for each problem using numbers, sketches, or words.

Name: \_\_\_\_\_

0) The top card in a deck may overhang the bottom card by up to  $n$  card widths (without dropping).



Find the value of  $n$ . Do you think that  $n_{Earth} = n_{Mercury}$ .

**Hint:** Custom Poker Cards (63mm x 88 mm).

1) A point charge  $q$  is placed on the planar separation boundary between two homogeneous infinite dielectrics with permittivities  $\varepsilon_1$  and  $\varepsilon_2$ . Find the potential  $\varphi$ , the field strength  $E$ , and the induction  $D$ .

2) A point charge  $q$  lies on a straight line which is the line of intersection of three planes, the angles between the planes being  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3 = 2\pi - \alpha_1 - \alpha_2$ . The space between each pair of planes is filled with homogeneous dielectrics with permittivities  $\varepsilon_1$ ,  $\varepsilon_2$  and  $\varepsilon_3$ . Find the potential  $\varphi$ , the field strength  $E$ , and the induction  $D$ .

3) It is the truth that:

$$6 \cdot \int_0^{\infty} \frac{dx}{e^{a\sqrt{x}} + 1} = \left( \frac{\pi}{a} \right)^2, \quad a > 0$$

4) The equation  $3y = z^3 + 3xz$  defines  $z(x, y)$  implicitly as a function of  $x$  and  $y$ . Verify that  $z(x, y)$  is a solution of

$$x \cdot \frac{\partial^2 z}{\partial y^2} + \frac{\partial^2 z}{\partial x^2} = 0$$

5) A cart rolls on a long table with velocity  $\beta = \frac{v_x}{c}$ . A smaller cart rolls on the first cart in the same direction with velocity  $\beta = \frac{v_x}{c}$  relative to the first cart. A third cart rolls on the second cart in the same direction with relative velocity  $\beta = \frac{v_x}{c}$ , and so on up to  $n$  carts. What is the velocity  $v_x(n)$  of the  $n$ th cart in the frame of the table? What does  $v_x(n)$  tend to as  $n \rightarrow \infty$ ?