

## SOLUTIONS FOR EXPERIMENTAL COMPETITION

### Part 1

The measurement results are shown in Table 1. The last column gives the calculated values of the strain  $\varepsilon = (l - l_0)/l_0$ . The plot of the rubber cord length relative extension against the weights is shown in Fig. 1; 1 weight = 100 g.

Number of weights, N	L, cm	$\varepsilon$
0	20,8	0,00
1	23,1	0,11
2	28,1	0,35
3	37,2	0,79
4	49,7	1,39
5	61,3	1,95
6	71,2	2,42
5	65,2	2,13
4	55,3	1,66
3	42,1	1,02
2	31,5	0,51
1	24,6	0,18
0	20,8	0,00

Table 1

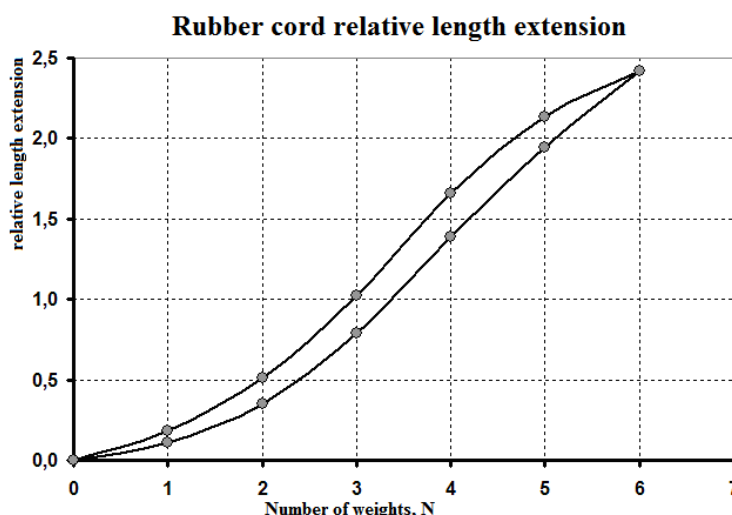


Figure 1

### Part 2

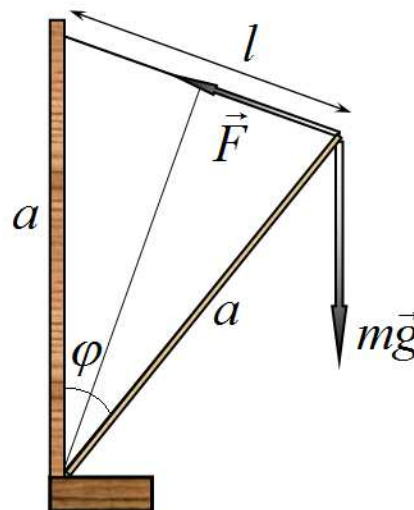
2.1 The equilibrium condition follows from equating the torques of the elastic and the gravity forces written as

$$F(l)a \cos \frac{\varphi}{2} = mga \sin \varphi,$$

where  $\varphi$  is the angle of deviation from the vertical. Taking into account that  $\sin \varphi = 2 \sin \frac{\varphi}{2} \cos \frac{\varphi}{2}$  and  $2a \sin \frac{\varphi}{2} = l$ , we obtain the condition (1).

2.2 When plotting these graphs it is necessary to renormalize the length of the rubber cord for each value of the stretching force:  $l = \frac{l_0}{L_0} L$ . The curves of the functions  $f(l) = mg \frac{l}{a}$  are

straight lines passing through the origin. Required drawing is carried out in Fig. 2.



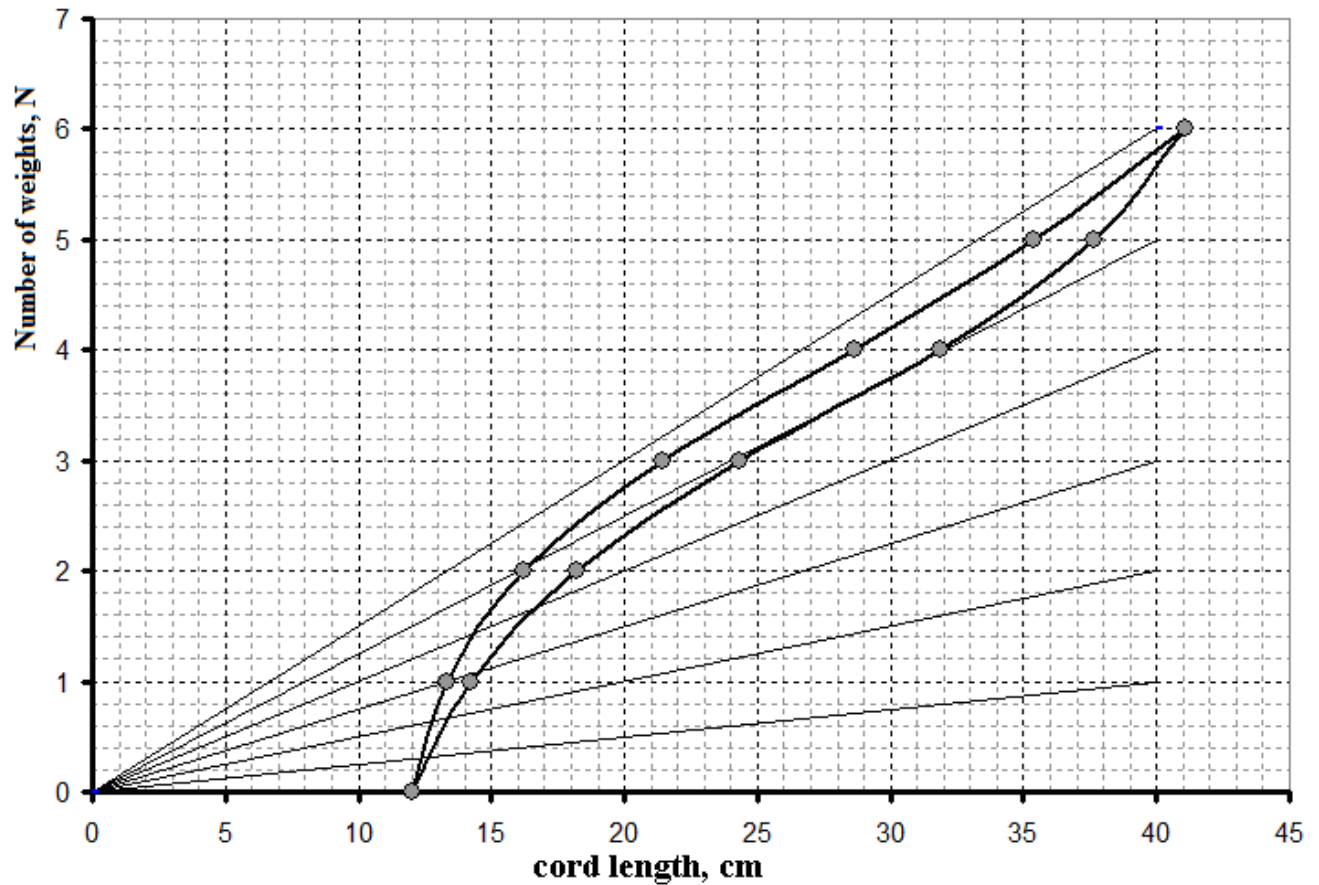


Figure 2

Equilibrium positions correspond to the intersection points of the graphs.

Table 2 represents the equilibrium positions found both with the help of Fig. 2 and obtained experimentally. Fig. 3 shows the corresponding curves, a fairly good agreement is achieved.

F, N	L, exp.	L, theory
0	11,6	12,0
1	11,7	12,5
2	12,0	12,7
3	13,1	13,2
4	14,6	14,0
5	18,4	16,5
6	51,0	47,0
5	48,7	40,0
4	18,2	17,0
3	14,6	14,0
2	13,0	13,5
1	11,8	12,5
0	11,6	12,0

Table 2

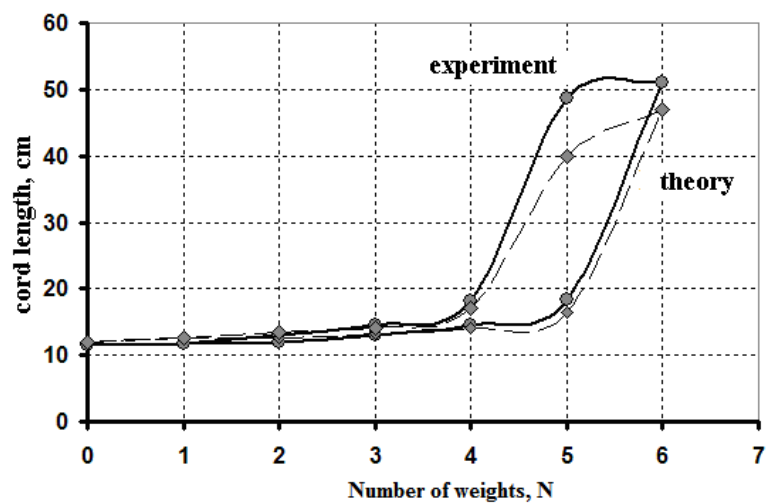


Figure 3

### Part 3. Bistability.

The bistability is possible when the straight line  $f(l) = mg \frac{l - \delta}{a}$  has three intersection points with the plot of the elastic force. A possible graph is shown in Fig. 4. Note that the intermediate position of equilibrium is unstable!

With the equipment provide the bistability is clearly observed at 5 or 6 weights (500 or 600 g).

The equilibrium positions correspond to the rubber cord lengths 13 - 14 cm and 36 - 44 cm.

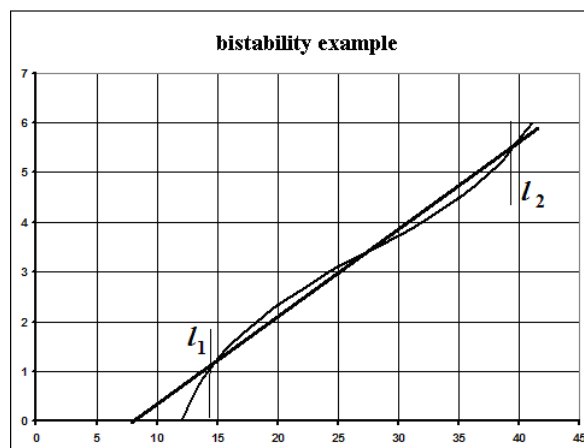


Figure 4

### Marking scheme

	Content	Points
<b>Часть 1. Растяжение.</b>		<b>4,5</b>
1.1	Measurements: - measurements made (12 points); - the hysteresis loop obtained; - measurement made with the accuracy less than 20% - measurement made with the accuracy less than 30%	1 0,5 1,0 (0,5)
1.2	The plot of the unit elongation against the gravity of hanging weights: - calculation of the unit elongation for all experimental points; - the graph plotted (the axes are named and ticked; the points are placed according to the table; the approximate curve is drawn)	1  1
<b>Part 2. Equilibrium</b>		<b>7,5</b>
2.1	The proof for the equilibrium position: - the torque of the gravity force; the torque of the elastic force; - geometrical relations and trigonometric transformations;	0,5  0,5
2.2	The theoretical calculation of the cord length: - formula for calculation of the cord length from the unit elongation; - calculation for all values of the elastic force; - the graph of the elastic force against the cord length (the axes are named and ticked; the points are placed according to the table; the approximate curve is drawn); - 6 straight lines drawn;	0,5 0,5  0,5 0,5
2.3	-the values are taken from the graph (12 points); - the theoretical curve of the cord length against the gravity force (the axes are named and ticked; the points are placed according to the table; the approximate curve is drawn);	0,5  0,5

2.4	Measurements: - measurements conducted (10 points); - the hysteresis loop is obtained (differences at 4 and 5 weights); - measurement conducted with the accuracy less than 20%; - measurement conducted with the accuracy less than 50%;	1 0,5 1,0 (0,5)
2.5	The experimental graph drawn: - the points are placed according to the table; - the approximate curve is drawn.	0,5 0,5
<b>Part 3. Bistability</b>		<b>3</b>
3.1	The qualitative explanation of the bistability: - the approximate curve of the gravity force against the cord length; - the shifted straight line is drawn to have three intersection points; - the stable equilibrium positions are determined;	0,5 0,5 0,5
3.2	the stable equilibrium positions are found experimentally: - at the weight 500 g (or 600 g is possible); - the cord lengths at equilibrium positions are obtained in the intervals stated above	1 0,5