

At first, I would like to thank you for the attachment containing the file answer15.pdf.

It is the citation from your text (answer on task N1)

which implies

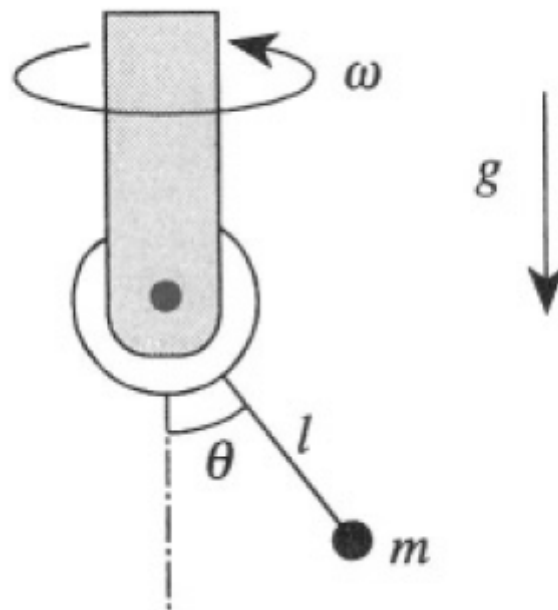
$$\Omega = \sqrt{\left(\omega^2 \cos 2\theta - \frac{g}{l} \cos \theta\right)} = \sqrt{\frac{g^2}{\omega^2 l^2} - 1}.$$

From the expression  $\Omega = \sqrt{\frac{g^2}{\omega^2 l^2} - 1}$ , the frequency  $\Omega$  is dimensionless quantity.

Please, give for me a coefficient in front of the square root.

### Problem specification

1) The bearing of a rigid pendulum of mass  $m$  is forced to rotate uniformly with angular velocity  $\omega$  (see figure below). The angle between the rotation



axis and the pendulum is called  $\theta$ . Neglect the inertia of the bearing and of the rod connecting it to the mass. Neglect friction. Include the effects of the uniform force of gravity.

a) Find the differential equation for  $\theta(t)$ .

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This equation has a duplicate solution  $\omega_{1,2} = \omega_0\sqrt{3}$ . So, the system has an extra mode with frequency  $\omega = \omega_0\sqrt{3}$ . It may be proved that the amplitudes  $A_i$  of the oscillations may be arbitrary satisfying the relation

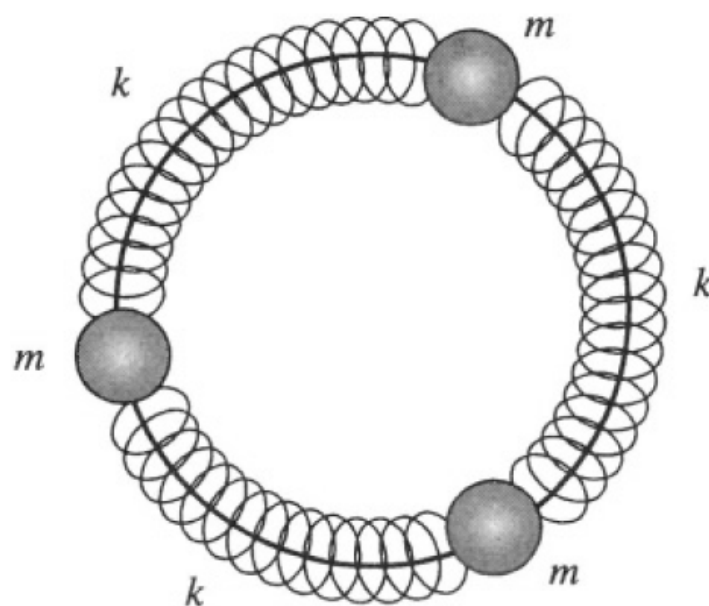
$$A_1 + A_2 + A_3 = 0,$$

i.e. no motion of the “center of mass”. The arbitrariness of the amplitudes (or, in fact, of their ratios) is in fact an extra degree of freedom which corresponds to the multiplicity 2 of the root. This may be understood as two modes of oscillation degenerated into one, but with arbitrary amplitudes.

I agree with your solution. However, I have a question. For example, I select a solution  $A_2 = 0$ ,  $A_1 = -A_3$  which satisfy the relation  $A_1 + A_2 + A_3 = 0$ . How you think, the system can evolve in this manner: one mass is at rest and the two others move in opposite directions.

## Problem specification

2) Three masses, each of mass  $m$ , are interconnected by identical massless springs of spring constant  $k$  and are placed on a smooth circular hoop as shown in figure below. The hoop is fixed in space. Neglect gravity and friction. Determine the natural frequencies of the system, and the shape of the associated modes of vibration.



1. Sorry, there was a misspelling.. The correct answer is

$$\Omega = \sqrt{\frac{g^2}{\omega^2 l^2} - \omega^2},$$

with a correct dimension.

2. Yes, that type of oscillations can exist and its frequency is  $\omega_0\sqrt{3}$ . I don't see any obstacles to its existence.