0. The top card overhangs the second one by a maximum of a half of a card length, the second one a fourth over the third one, the third one the sixth over the fourth one and so on. So for $N+1$ cards the total overhang is

$$L = \frac{a}{2}\sum_{i=1}^{N}\frac{1}{i},$$

where $a = 88$ mm is the card length.

1. The field in the dielectric is reduced by $\varepsilon/\varepsilon_0$ times. So, the field in the first dielectric is

$$E_1 = \frac{q}{4\pi\varepsilon_1 r^2},$$

and in the second one

$$E_2 = \frac{q}{4\pi\varepsilon_2 r^2}.$$

The potential is respective for these fields.

2. Same as previous problem, the fields are

$$E_i = \frac{q}{4\pi\varepsilon_i r^2},$$

the potentials are

$$\varphi_i = \frac{q}{4\pi\varepsilon_i r}.$$

3. By introducing the new variable $t = a\sqrt{x}$ this expression simplifies to

$$\int_0^\infty \frac{t dt}{1+e^t} = \frac{\pi^2}{12}.$$

4. By differentiation with respect to x and y we get

$$(x+z^2)\frac{\partial z}{\partial x} + z = 0,$$
$$(x+z^2)\frac{\partial^2 x}{\partial z^2} + 2\frac{\partial z}{\partial x} + 2z\left(\frac{\partial z}{\partial x}\right)^2 = 0,$$
$$(x+z^2)\frac{\partial z}{\partial y} = 1,$$
$$(x+z^2)\frac{\partial^2 z}{\partial y^2} + 2z\left(\frac{\partial z}{\partial y}\right)^2 = 0.$$

Eliminate the first derivatives:

$$\frac{\partial^2 z}{\partial x^2} = \frac{2xz}{(x+z^2)^3},$$
$$\frac{\partial^2 z}{\partial y^2} = -\frac{2z}{(x+z^2)^3}.$$

These satisfy the required equation.

5. The equation for β_n is

$$\beta_{k+1} = \frac{\beta_k + \beta}{1+\beta_k\beta}$$

with $\beta_0 = 0$. The solution is

$$\beta_n = \frac{1-a^n}{1+a^n},$$

where

$$a = \frac{1-\beta}{1+\beta}.$$

The infinity asymptotic is

$$\beta_n \approx 1 - 2a^n.$$

There is another approach to the solution. The velocity summation law of special relativity states that the velocities β_i are not additive but the quantities $\tanh^{-1}\beta_i$ are. So,

$$\tanh^{-1}\beta_{k+1} = \tanh^{-1}\beta_k + \tanh^{-1}\beta.$$

This easily yields

$$\beta_n = \tanh\left(n \tanh^{-1}\beta\right).$$