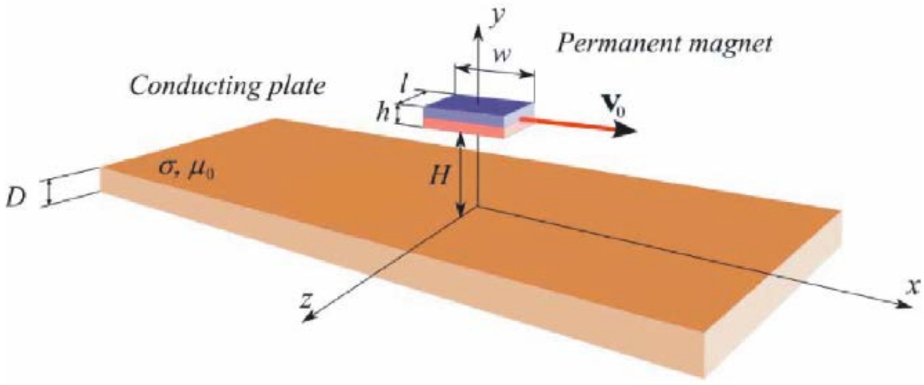
24.04.2016 – N5

Show your work for each problem using numbers, sketches, or words.

Name:__________________

0)



A rectangular permanent magnet (h×w×l) is moved with a constant speed $\vec{\mathrm{v}} = \mathrm{v}_0 \cdot \vec{e}_x + 0 \cdot \vec{e}_y + 0 \cdot \vec{e}_z$ on a height H above a nonmagnetic conducting plane of thickness D and conductivity σ (see figure above).

a) Assuming that the permanent magnet could be regarded as a vertical magnetic dipole $\left(\vec{\mathrm{M}} = -\mathrm{M}_0 \cdot \vec{e}_y\right)$ calculate the components of a lift force ($F_{lift}\left(\mathrm{v}_0\right) = F_y \cdot \vec{e}_y$) and a drag force ($F_{drag}\left(\mathrm{v}_0\right) = F_x \cdot \vec{e}_x$) on the dipole.

b) The magnetic field distribution $\vec{B}\left(\vec{r}\right)$ of the permanent magnet (Fig.1) at point P (x, y, z), in the space out of the permanent magnet can be written as follows [1]:

$$B_x = -\frac{K}{2}\Big[f\left(w - x, \mathrm{y}, z - \mathrm{z}_0\right) + f\left(w - x, \mathrm{h} - \mathrm{y}, z - \mathrm{z}_0\right)\Big]\Big|_0^l + \\ + \frac{K}{2}\Big[f\left(x, \mathrm{y}, z - \mathrm{z}_0\right) + f\left(x, \mathrm{h} - \mathrm{y}, z - \mathrm{z}_0\right)\Big]\Big|_0^l \quad (1.1)$$

$$B_y\left(\vec{r}\right) = -\frac{K}{2}\Big[f\left(h - y, \mathrm{x}, z - \mathrm{z}_0\right) + f\left(h - y, \mathrm{w} - x, z - \mathrm{z}_0\right)\Big]\Big|_0^l + \\ + \frac{K}{2}\Big[f\left(\mathrm{y}, \mathrm{x}, z - \mathrm{z}_0\right) + f\left(\mathrm{y}, \mathrm{w} - x, z - \mathrm{z}_0\right)\Big]\Big|_0^l \quad (1.2)$$

$$
\begin{aligned}
B_z = -K\left[g\left(y, \mathrm{w}-\mathrm{x}, z-z_0\right) + g\left(h-y, \mathrm{w}-\mathrm{x}, z-z_0\right)\right]\Big|_0^l \\
-K\left[g\left(\mathrm{x}, h-y, z-z_0\right) + g\left(w-x, h-y, z-z_0\right)\right]\Big|_0^l \\
-K\left[g\left(\mathrm{h}-\mathrm{y}, \mathrm{x}, z-z_0\right) + g\left(\mathrm{y}, \mathrm{x}, z-z_0\right)\right]\Big|_0^l \\
-K\left[g\left(w-x, \mathrm{y}, z-z_0\right) + g\left(\mathrm{x}, \mathrm{y}, z-z_0\right)\right]\Big|_0^l
\end{aligned}
\qquad (1.3)
$$

, where $K = \dfrac{\mu_0 \cdot M_0}{4\pi}$, $f\left(x, \mathrm{y}, z\right) = \ln\left(\dfrac{\sqrt{x^2+y^2+z^2}-y}{\sqrt{x^2+y^2+z^2}+y}\right)$,

$g\left(x, \mathrm{y}, z\right) = \begin{cases} ArcTan\left(\dfrac{x \cdot z}{y \cdot \sqrt{x^2+y^2+z^2}}\right), \; if \;\; y \neq 0 \\ 0, \; if \;\; y = 0 \end{cases}$, "$[\sim]\big|_0^l$" denote the subtraction between the value of the function [~] at $z_0 = l$ and at $z_0 = 0$.

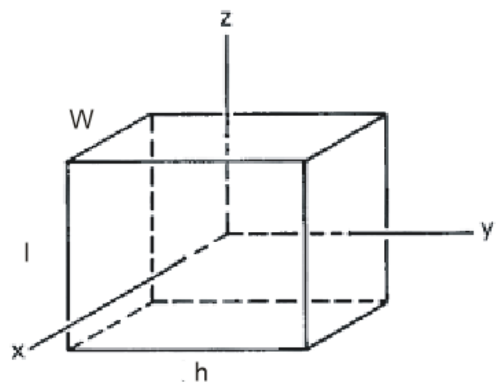


Figure 1. The rectangular permanent magnet h×w×l, which is magnetized sufficiently in one direction (z-axis) and saturated.

Assuming D=10cm, $\mathrm{H}=1\mathrm{mm}$, $\mathrm{l}=5\mathrm{cm}$, $\mathrm{h}=1\mathrm{cm}$, $\mathrm{w}=5\mathrm{cm}$, $\sigma_{Al} = 3.8 \cdot 10^8 \; S/m$ and expressions (1.1) – (1.3) calculate the components of the force $\dfrac{F_{lift}\left(\mathrm{v}_0\right)}{M_0^2}$ and $\dfrac{F_{drag}\left(\mathrm{v}_0\right)}{M_0^2}$ on the permanent magnets versus the speed v_0.

1) Find the general solution for each of the following equations:

a) $2 \cdot u_{xx} - 5 \cdot u_{xy} + 3 \cdot u_{yy} = 0$

b) $2 \cdot u_{xx} + 6 \cdot u_{xy} + 4 \cdot u_{yy} + u_x + u_y = 0$

c) $u_{yy} - 2 \cdot u_{xy} + 2 \cdot u_x - u_y = 4 \cdot e^x$

<u>Hint:</u> The general solution of equation $u_{xx} - u_{tt} = 0$ has the form $u(x,t) = f(x+t) + g(x-t)$ were f and g are arbitrary twice continuously differentiable functions.

2) Show that if $u(x,t)$ is a solution of equation $u_{xx} - u_{tt} = 0$ then the function $v(x,t) = u\left(\frac{x}{x^2 - t^2}, \frac{t}{x^2 - t^2}\right)$ is also a solution of that equation at each point where it is defined.

3) Evaluate the integral $\oint_C \left(x^2 \cdot e^{5z} dx + x \cdot Cos(y) dy + 3y dz\right)$ where C is the curve parameterized by $\vec{r}(t) = 2 \cdot (1 + Cos(t)) \cdot \vec{e}_y + 2 \cdot (1 + Sin(t)) \cdot \vec{e}_z$.

Reference

[1] X. F. Gou, Y. Yang, and X. J. Zheng, Appl. Math. Mech. 25, 297 (2004).