# THEORETICAL COMPETITION

## January 16, 2008

## Please read this first:

1. The time available for the theoretical competition is 4 hours. There are three questions.
2. Use only the pen provided.
3. You can use your own calculator for numerical calculations. If you don't have one, please ask for it from Olympiad organizers.
4. You are provided with ***Writing sheet*** and additional paper. You can use the additional paper for drafts of your solutions but these papers will not be checked. Your final solutions which will be evaluated should be on the ***Writing sheets***. Please use as little text as possible. You should mostly use equations, numbers, figures and plots.
5. Use only the front side of ***Writing sheets*.** Write only inside the bordered area.
6. Begin each question on a separate sheet.
7. Fill the boxes at the top of each sheet of paper with your country (**Country),** your student code (**Student Code**), the question number (**Question Number)**, the progressive number of each sheet **(Page Number**), and the total number of ***Writing sheets*** used (**Total Number of Pages**). If you use some blank ***Writing sheets*** for notes that you do not wish to be evaluated, put a large X across the entire sheet and do not include it in your numbering.
8. At the end of the exam, arrange all sheets for each problem in the following order:
    - Used ***Writing sheets*** in order;
    - The sheets you do not wish to be evaluated
    - Unused sheets and the printed question.

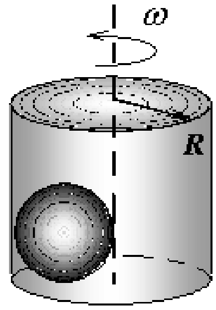
   Place the papers inside the envelope and leave everything on your desk. You are not allowed to take any paper out of the room.

## Theoretical Question 1

This problem consists of four unrelated parts

### 1A (2 points)

Uniform ball of radius *R/2* is located inside the vertical cylindrical vessel of radius *R* filled with water. The vessel is rotating near its vertical symmetry axis with angular velocity $\omega$. Determine the pressure force exerted on a ball by the side surface of the vessel. Water density equals $\rho_0$ and density of ball equals $\rho$.



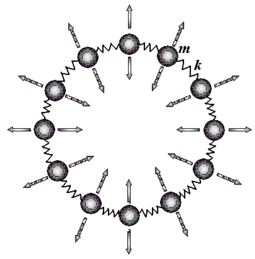

### 1B (3 points)

Heat capacities can depend on temperature (for example, at low temperatures). Two identical bodies with temperature-dependent specific heats

$$c(t) = c_0(1+\alpha t)$$

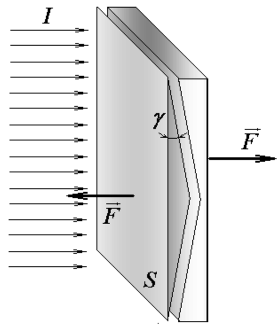
(here $c_0$ and $\alpha$ are known constants) are put into a thermal contact. Initial body temperatures are equal to $t_1$ and $t_2$. Determine the final temperature. You can neglect heat losses.

**1C (2 points)**




Ring consists of *n* identical masses connected by *n* massless springs of stiffness *k*. Find the period of small radially symmetric oscillations of such ring assuming $n >> 1$.

**1D (3 points)**




Thin glass plate with parallel surfaces of area *S = 10 sm*$^2$ and refraction index *n = 1,5* is cut into two pieces. One of them is a Fresnel biprism with a small deflection angle *γ = 0,1* Rad. The plate is irradiated by a powerful ray of intensity I = 10 kW/sm$^2$ perpendicular to the surface of the plate. Find the force *F* required to separate two plates. You can neglect intensity losses in the plate and reflection from the surfaces.

## Theoretical Question 2

## Electromagnetic cannon

Engineers suggested the model of electromagnetic cannon described below. Two highly conducting parallel wires are situated at distance *h* from each other, and their angle to horizon equals $\alpha$. Mobile conductor of mass *m* and resistance *R* can slide on the top of the wires (see Fig. 1). Lower ends of the wires are connected to a source with electromagnetic force (emf) *E* and zero resistance. The system is located in magnetic field *B* perpendicular to the plane of the wires. Length of the wires equals *L*, and initially the mobile conductor is at rest near the source.

***a)*** Determine the minimal emf $E_{\min}$ for which the mobile conductor starts moving upwards ***(1 point)***

b) After some time the current in the system reaches the constant value $I_0$. Determine it assuming $E > E_{\min}$. ***(1 point)***

c) After some time the velocity of the mobile conductor reaches the constant value $u_0$. Determine it assuming $E > E_{\min}$. ***(2 points)***

d) For large enough L one can assume that the velocity of the mobile conductor when it reaches the end of the wires is given by $u_0$. Then the total charge *q* which passes the source can be written as

$$q = C_1 L + C_2 .$$

Determine $C_1$ and $C_2$. ***(3 points)***

e) Under the conditions specified in the subsection above the total heat $Q$, dissipated in the mobile conductor can be written as

$$Q = C_3 L + C_4$$

Determine $C_3$ and $C_4$. ***(3 points)***

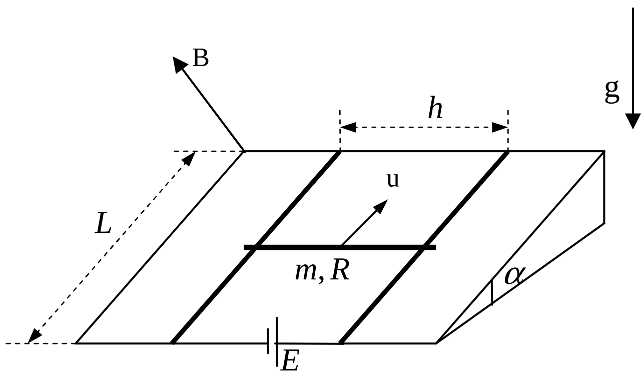



Fig 1. Electromagnetic cannon scheme.

**Note**: *In this problem you can neglect friction and self-induction.*

## Theoretical Question 3
## Helium atom

Helium atom is the simplest many-electron system. According to a planetary model it consists of a nucleus with charge *+2e* and two electrons circulating around it. Complete solution of the problem of helium atom using quantum mechanics is rather complicated, and takes into account the indistinguishability of electrons. However, many qualitative features of helium atom can be derived using Bohr quantization rules.

In this problem you are asked to qualitatively investigate helium atom. In what follows always assume that the atom has the minimal possible energy (i.e. it is in the ground state), and electrons occupy the same circular orbit of radius *r*. According to Bohr postulates, angular momentum of each electron is quantized in units of Planck constant $\hbar$ .

a) Write the relation between momentum *p* and orbit radius *r* of each electron for the ground state of helium atom ***(1 point);***
b) Write the expression for the potential energy of the system as a function of *r* ***(2 points);***
c) Write the expression for orbit radius *r* and determine its numerical value ***(2 points)***;
d) Write the expression for the total energy of the ground state of helium atom and determine its numerical value ***(1 point)***;
e) Write the expression for the energy of single ionization and determine its numerical value ***(2 point)***;

Atoms can be ionized by external pressure. Such phenomenon takes place in the cores of heavy planets.

f) Estimate the pressure of helium atom ionization ***(2 points)***;

**Numerical values:**

| | |
|---|---|
| Elementary charge | $e = 1.6022 \cdot 10^{-19} C$ |
| Mass of electron | $m_e = 0.911 \cdot 10^{-30}$ kg; |
| Electric constant | $\varepsilon_0 = 8.854 \cdot 10^{-12}$ F/m; |
| Plank constant | $\hbar = \dfrac{h}{2\pi} = 1.055 \cdot 10^{-34}$ J·s. |