Solution of the experimental task

a) Since the resistance of the voltmeter is much larger than the resistance of the solar cell, one can directly measure emf of the solar cell using voltmeter as shown in Fig.1(below on the left).Numerical value is $E \approx 0{,}5$ V.

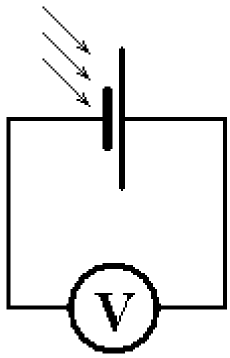


Рис. 1

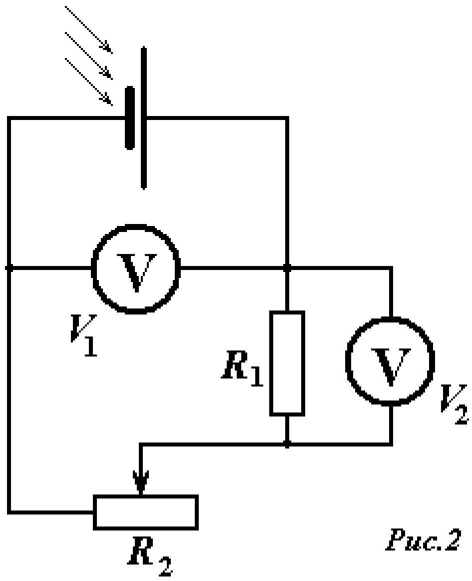


Рис.2

b) To measure the dependence of the current through the solar cell on the voltage drop on the external resistance,one can use the circuit shown in Fig. 2 (above, on the right). Here $R_1 = 1{,}00 \pm 0.01$ Ohm is the fixed resistance.Readings of the voltmeter V2 can be used to measure the current. One can change the current by varying the resitance R_2 Results of the measurements are shown in Table 1. The plot is shown in Fig. 3 (below).

Dependence of the current on the external resistance

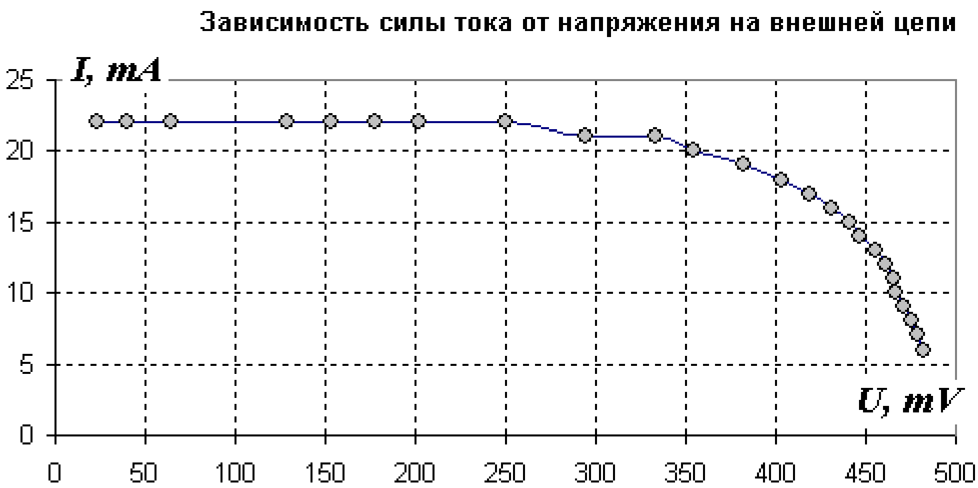


Рис.3

Table with the results of the measurements.

V1, mV	*V2, mV*	*I, mA*	*R=* $R_1 + R_2$ *, Ohm*	*P, mW*	*1/I, (1/A)*
23	22	22	1,05	0,51	45,45
40	22	22	1,82	0,88	45,45
64	22	22	2,91	1,41	45,45
129	22	22	5,86	2,84	45,45
153	22	22	6,95	3,37	45,45
178	22	22	8,09	3,92	45,45
202	22	22	9,18	4,44	45,45
250	22	22	11,36	5,50	45,45
294	21	21	14,00	6,17	47,62
333	21	21	15,86	6,99	47,62
354	20	20	17,70	7,08	50,00
382	19	19	20,11	7,26	52,63
403	18	18	22,39	7,25	55,56
419	17	17	24,65	7,12	58,82
431	16	16	26,94	6,90	62,50
441	15	15	29,40	6,62	66,67
447	14	14	31,93	6,26	71,43
456	13	13	35,08	5,93	76,92
461	12	12	38,42	5,53	83,33
465	11	11	42,27	5,12	90,91
467	10	10	46,70	4,67	100,00
471	9	9	52,33	4,24	111,11
476	8	8	59,50	3,81	125,00
479	7	7	68,43	3,35	142,86
482	6	6	80,33	2,89	166,67

c) To determine the characteristics of the solar cell more precisely, one should linearize Ohm's law for small currents. This can be done in several ways..

Approach 1. From Ohm's law one has

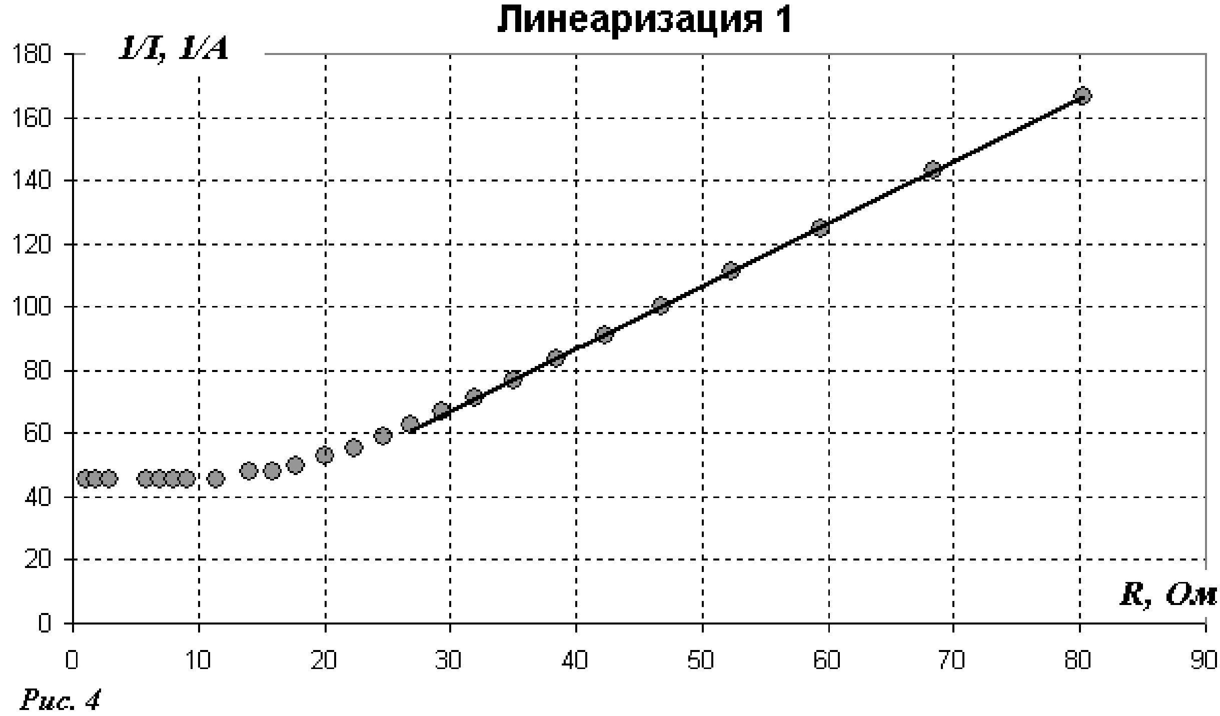
$$I = \frac{E}{R+r}. \tag{1}$$

It follows then that $1/I$ should depend linearly on R for constant emf of the cell and constant resistance of the cell:

$$\frac{1}{I} = \frac{1}{E}R + \frac{r}{E}. \tag{2}$$

This dependence is shown in Fig. 4 below.

Linearization 1



From the plot above it is clear that for currents smaller than ~15 mA assumptions of constant emf and internal resistance hold, since in this regime the plot presented above is linear

One can determine the parameters of dependence $\frac{1}{I} = aR + b$, using the least squares method as

$$a = (1{,}98 \pm 0{,}02)\, V^{-1}$$
$$b = (7{,}7 \pm 0{,}9)\, A^{-1}.$$

From equation (2) one obtains

$$E = \frac{1}{a} \approx 0{,}506 \text{ V}, \qquad \Delta E = E\frac{\Delta a}{a} \approx 0{,}005 \text{ V},$$

$$r = \frac{b}{a} \approx 3{,}9 \text{ Ohm}, \qquad \Delta r = r\sqrt{\left(\frac{\Delta a}{a}\right)^2 + \left(\frac{\Delta b}{b}\right)^2} \approx 0{,}5 \text{ Ohm}$$

Approach 2 For constant internal resistance the dependence of the voltage drop on the external resistance on the current through the solar cell is given by a linear function

$$U = E - Ir. \tag{3}$$

Linearization 2

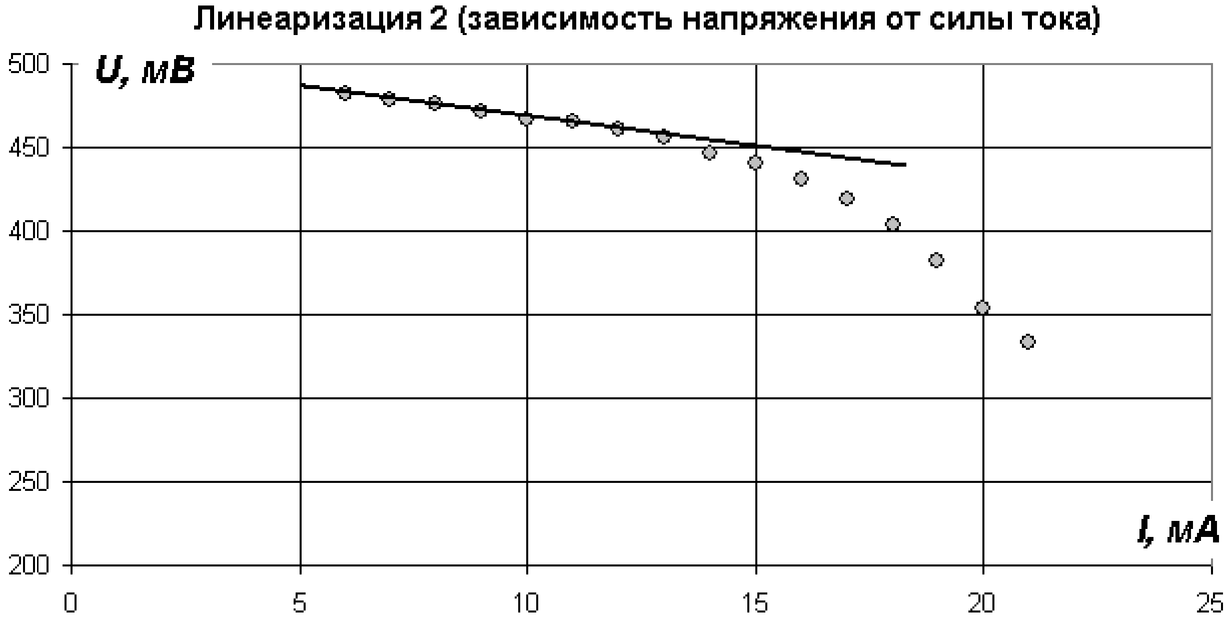


Рис. 5

This dependence is shown in Fig. 5(above). Drawing a line through several initial points with small current, one can obtain internal resistance and emf as

$$E = (508 \pm 7)\ \text{mV}, \quad r = (4{,}1 \pm 0{,}6)\ \text{Ohm}$$

d) The heat power dissipated in the external resistance is given by

$$P = I^2 R = \left(\frac{E}{R+r}\right)^2 R\,.$$

The power is maximal when the derivative of this expression vanishes:

$$\left(\frac{R}{(R+r)^2}\right)' = \frac{(R+r)^2 - 2R(R+r)}{(R+r)^4} = \frac{r^2 - R^2}{(R+r)^4} = 0\,.$$

Thus the power is maximal for $r = R$

e) One can easily calculate the external resistance and dissipated power as

$$R = \frac{U}{I}, \quad P = UI\,.$$

Dependence of the power dissipated on the external resistance is shown in Fig. 6 (below).

Dependence of the dissipated power on the external resistance

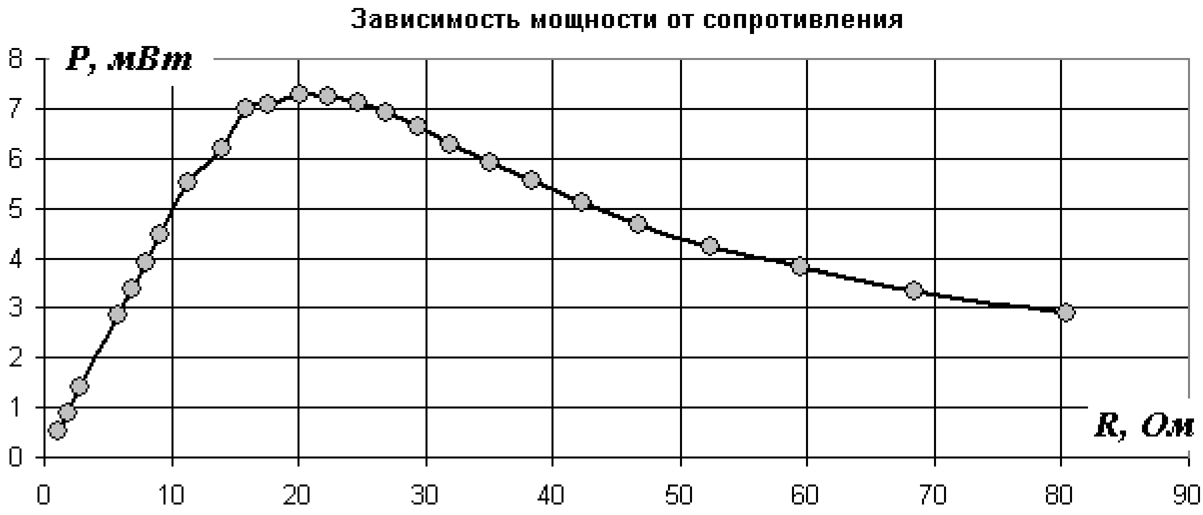


Рис.6

Maximal power is achieved for $R \approx 20\,Ohm$ and equals $P \approx 7{,}3$ mW. This resistance exceeds the theoretical prediction based on linear approximation

$$\eta \approx \frac{20}{4} \approx 5 \text{ times.}$$

f) Maximal value of current equals $I_{\max} = 22$ mA.Thus the filling coefficient equals

$$K = \frac{P_{\max}}{I_{\max} \cdot E} = \frac{7{,}3}{22 \cdot 0{,}50} \approx 0{,}66\,.$$

Experimental competition marking scheme.

Part	Content	Subdivisions	Points	Subtotal
a	Emf measurement ***(1 point)***	*Circuit*	0,5	1,0
		Numerical value (approximately 0,5 V – 0,9V)	0,5	
b	Measurements ***(5 points)***	*Circuit* *(if multimeters used as amperemeters, or if voltage on 1Ohm is measured as difference of 2 voltages)*	1,0 (-0,5)	1,0
		Measurement results: *- up to 10 points;* *- 10 and more points;* *- Two regions are present, one with approximately constant current, and linear regime for small currents* *-Minimal current should be in the region of constant resistance (about 10mA)*	 (0,5) 1,0 1,0 0.5	2,5
		Plot: *- reasonable scale;* *- axes marked and labeled;* *- points presented and connected;*	 0,5 0,5 0,5	1,5
c	Emf and resistance for small currents ***(3 points)***	*Linearization* *(only1-2 points are used)*	1 (0,5)	1
		Value of Emf obtained(no more than 20% difference from a)); *Errors;*	0,5 0,5	1
		Internal resistance (1 to 10 Ohm); *Errors;*	0,5 0,5	1
d	Theoretical question ***(1 point)***	*- Expression for power;* *-extremal conditions;*	0,5 0,5	1
e	Power as a function of external resistance ***(3 points)***	*- Expression for dissipated power (through current and voltage);* *- Evaluation of power for all experimental points;* *- Plot of power as a function of resistance(axes, etc);* *-Value of resistance corresponding to maximal power, ratio to theoretical (about 4 - 5)*	0,5 0,5 1,0 1,0	3
f	Filling coefficient ***(2 points)***	*- Emf from part (c);* *- maximal current from part (b);* *- Numerical value from 0,5 to 0,7*	0,5 0,5 1	2