Instrumentation provided:

A Platform with 6 banana jacks

B Pickup coil embedded into the platform

C Ferrite U-core with two coils marked ``A" and ``B"

D Ferrite U-core without coils

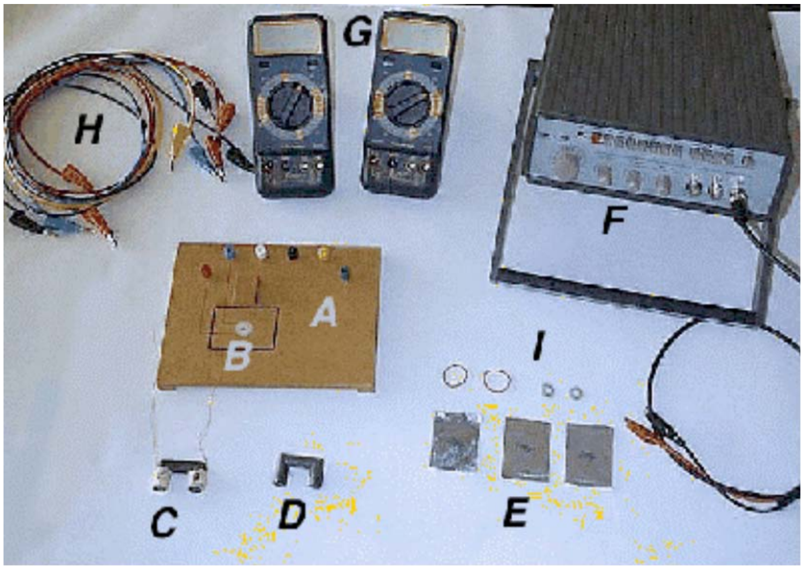
E Aluminium foils of thicknesses: 50 µm, 100 µm and 200 µm

F Function generator with output leads

G Two multimeters

H Six leads with banana plugs

I Two rubber bands and two plastic spacers



Multimeters
The multimeters are two-terminal devices that in this experiment are used for measuring AC voltages, AC currents, frequency and resistance. In all cases one of the terminals is the one marked **COM**. For the voltage, frequency and resistance measurements the other terminal is the red one marked **V-Ω**. For current measurements the other terminal is the yellow one marked **mA**. With the central dial you select the meter function (V~ for AC voltage, A~ for AC current, Hz for frequency and Ω for resistance) and the measurement range. For the AC modes the measurement uncertainty is ± (4% of reading + 10 units of the last digit). **To get accurate current measurements a change of range is recommended if the reading is less than 10% of full scale**.

Function generator
To turn on the generator you press in the red button marked **PWR**. Select the 10 kHz range by pressing the button marked **10k**, and select the sine waveform by pressing the second button from the right marked with a wave symbol. No other buttons should be selected. You can safely turn the amplitude knob fully clockwise. The frequency is selected with the large dial on the left. The dial reading multiplied by the range selection gives the output frequency. The frequency can be verified at any time with one of the multimeters. Use the output marked **MAIN**, which has 50 Ω internal resistance.

Ferrite cores
Handle the ferrite cores gently, they are brittle!! Ferrite is a ceramic magnetic material, with low electrical conductivity. Eddy current losses in the cores are therefore low.

Banana jacks
To connect a coil lead to a banana jack, you loosen the colored plastic nut, place the tinned end between the metal nut and plastic nut, and tighten it again.

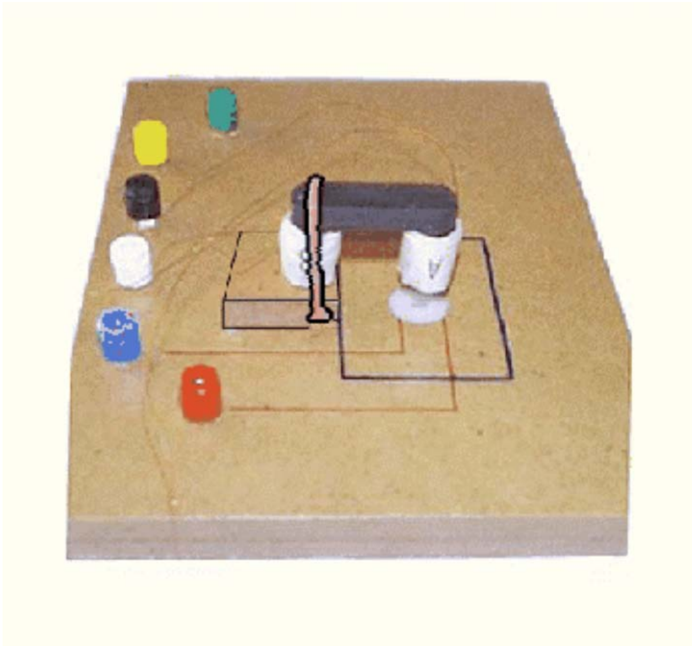


Figure 1: *Experimental arrangement for part I.*

Part I. Magnetic shielding with eddy currents

Time-dependent magnetic fields induce eddy currents in conductors. The eddy currents in turn produce a counteracting magnetic field. In superconductors the induced eddy currents will expel the magnetic field completely from the interior of the conductor. Due to the finite conductivity of normal metals they are not as effective in shielding magnetic fields.

To describe the shielding effect of aluminium foils we will apply a phenomenological model

$$B = B_0 e^{-\alpha d} \qquad (1)$$

where B_0 is the magnetic field in the absence of foils. B is the magnetic field beneath the foils, α an attenuation constant, and d the foil thickness.

Experiment

Put the ferrite core with the coils, with legs down, on the raised block such that coil A is directly above the pickup coil embedded in the platform, as shown in Figure 1. Secure the core on the block by stretching the rubber bands over the core and under the block recess.

1. Connect the leads for coils A and B to the jacks. Measure the resistance of all coils to make sure you have good connections. You should expect values of less than 10 Ω. Write your values in field 1 on the answer sheet.
2. Collect data to validate the model above and evaluate the attenuation constant α for the aluminum foils (50 - 300 μm), for frequencies in the range of 5 - 20 kHz. Place the foils inside the square, above the pickup coil, and apply a sinusoidal voltage to coil A. Write your results in field 2 on the answer sheet.
3. Plot α versus frequency, and write in field 3 on the answer sheet, an expression describing the function $\alpha(f)$.

Part II. Magnetic flux linkage

The response of two coils on a closed ferrite core to an external alternating voltage (V_g) from a sinusoidal signal generator is studied.

Theory

In the following basic theoretical discussion, and in the treatment of the data, it is assumed that the ohmic resistance in the two coils and hysteresis losses in the core have insignificant influence on the measured currents and voltages. Because of these simplifications in the treatment below, some deviations will occur between measured and calculated values.

Single coil

Let us first look at a core with a single coil, carrying a current *I*. The magnetic flux Φ, that the current creates in the ferrite core inside the coil, is proportional to the current *I* and to the number of windings *N*. The flux depends furthermore on a geometrical factor *g*, which is determined by the size and shape of the core, and the magnetic permeability $\mu = \mu_r \mu_0$, which describes the magnetic properties of the core material. The relative permeability is denoted μ_r and μ_0 is the permeability of free space.

The magnetic flux Φ is thus given by

$$\Phi = \mu g N I = c N I \qquad (2)$$

where $c = \mu g$. The induced voltage is given by Faraday's law of induction,

$$\varepsilon(t) = -N \frac{d\Phi(t)}{dt} = -cN^2 \frac{dI(t)}{dt} \qquad (3)$$

The conventional way to describe the relationship between current and voltage for a coil is through the self inductance of the coil *L*, defined by,

$$\varepsilon(t) = -L \frac{dI(t)}{dt} \qquad (4)$$

A sinusoidal signal generator connected to the coil will drive a current through it given by

$$I(t) = I_0 \sin \omega t \qquad (5)$$

where ω is the angular frequency and I_0 is the amplitude of the current. As follows from equation (3) this alternating current will induce a voltage across the coil given by

$$\varepsilon(t) = -\omega c N^2 I_0 \cos \omega t \qquad (6)$$

The current will be such that the induced voltage is equal to the signal generator voltage V_g. There is a 90 degree phase difference between the current and the voltage. If we only look at the magnitudes of the alternating voltage and current, allowing for this phase difference, we have

$$\varepsilon = \omega c N^2 I \qquad (7)$$

Two coils

Let us now assume that we have two coils on one core. Ferrite cores can be used to link magnetic flux between coils. In an ideal core the flux will be the same for all cross sections of the core. Due to flux leakage in real cores a second coil on the core will in general experience a reduced flux compared to the flux-generating coil. The flux Φ_B in the secondary coil B is therefore related to the flux Φ_A in the primary coil A through

$$\Phi_B = k\Phi_A \tag{8}$$

Similarly a flux component Φ_B created by a current in coil B will create a flux $\Phi_A = k\Phi_B$ in coil A. The factor *k*, which is called the coupling factor, has a value less than one.
The ferrite core under study has two coils A and B in a transformer arrangement. Let us assume that coil A is the primary coil (connected to the signal generator). If no current flows in coil B ($I_B=0$), the induced voltage ε_A due to I_A is equal and opposite to V_g. The flux created by I_A inside the secondary coil is determined by equation (8) and the induced voltage in coil B is

$$\varepsilon_B = \omega k c N_A N_B I_A \tag{9}$$

If a current I_B flows in coil B, it will induce a voltage in coil A which is described by a similar expression. The total voltage across the coil A will then be given by

$$V_g = \varepsilon_A = \omega c N_A^2 I_A - \omega k c N_A N_B I_B \tag{10}$$

The current in the secondary coil thus induces an opposing voltage in the primary coil, leading to an increase in I_A. A similar equation can be written for ε_B. As can be verified by measurements, *k* is independent of which coil is taken as the primary coil.

Experiment
Place the two U-cores together as shown in Figure 2, and fasten them with the rubber bands. Set the function generator to produce a 10 kHz, sine wave. Remember to set the multimeters to the most sensitive range suitable for each measurement. The numbers of the windings of the two coils, A and B, are: N_A = 150 turns and N_B = 100 turns (±1 turn on each coil).

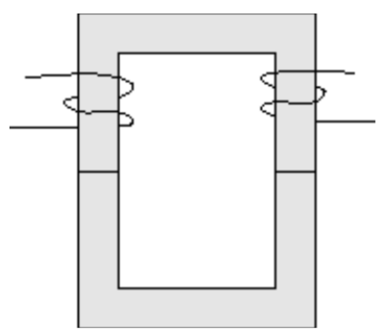


Figure 2: *A transformer with a closed magnetic circuit.*

1. Derive algebraic expressions for the self inductances L_A and L_B, and the coupling factor *k*, in terms of measured and given quantities and write your results in field 1.a on the answer sheet. Draw circuit diagrams in field 1.b on the answer sheet, showing how these quantities are determined. Calculate the numerical values of L_A, L_B and *k* and write their values in field 1.c on the answer sheet.
2. When the secondary coil is short-circuited, the current I_P in the primary coil will increase. Use the equations above to derive an expression giving I_P explicitly and write your result in field 2.a on the answer sheet. Measure I_P and write your value in field 2.b on the answer sheet.
3. Coils A and B can be connected in series in two different ways such that the two flux contributions are either added to or subtracted from each other.
   3.1. Find the self inductance of the serially connected coils, L_{A+B}, from measured quantities in the case where the flux contributions produced by the current *I* in the two coils add to (strengthen) each other and write your answer in field 3.1 on the answer sheet.

3.2. Measure the voltages V_A and V_B when the flux contributions of the two coils oppose each other. Write your values in field 3.2.a on the answer sheet and the ratio of the voltages in field 3.2.b. Derive an expression for the ratio of the voltages across the two coils and write it in field 3.2.c on the answer sheet.

4. Use the results obtained to verify that the self inductance of a coil is proportional to the square of the number of its windings and write your result in field 4 on the answer sheet.
5. Verify that it was justified to neglect the resistances of the coils and write your arguments as mathematical expressions in field 5 on the answer sheet.
6. Thin plastic spacers inserted between the two half cores (as shown in Figure 3) reduce the coil inductances drastically. Use this reduction to determine the relative permeability μ_r of the ferrite material, given Ampere's law and continuity of the magnetic field **B** across the ferrite - plastic interface.

Assume $\mu = \mu_0 = 4\pi\times10^{-7}$ Ns^2/C^2 for the plastic spacers and a spacer thickness of 1.6 mm. The geometrical factor can be determined from Ampere's law

$$\oint \frac{1}{\mu} B dl = I_{total} \tag{11}$$

where I_{total} is the total current flowing through a surface bounded by the integration path. Write your algebraic expression for μ_r in field 6.a on the answer sheet and your numerical value in field 6.b.

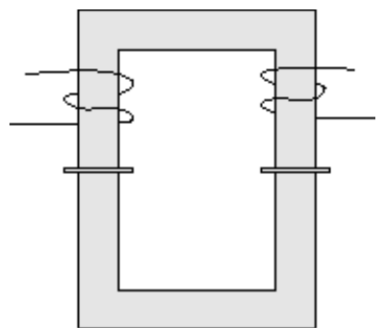


Figure 3: *The ferrite cores with the two spacers in place.*