Theoretical Problem No. 1 (10 points)

Fred & Barney's car

Fred and Barney built a car having as "wheels" two identical square prisms (figure 1). The wheels are perfectly suited to the road's shape (that is a periodical repetition of identical bumps), so that the center of mass of the car does not move vertically during the trip. The wheels never slip on the road. During the car's motion, the wheel's long edge always touches a „valley" of the road; road „heights" are reached by a line passing through the half of a rectangular face of the "wheels". The vehicle's movement starts in the tops of two road bumps. Initially, the horizontal translational velocity is $\vec{v}_0$.

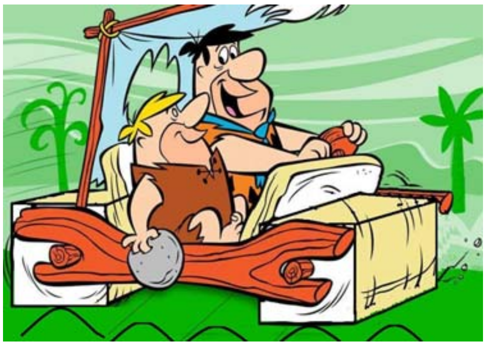

Figure 1

The total weight of the vehicle excluding its „wheels" is $M \cdot \vec{g}$, distributed equally on the two wheel axles. Further, assume that the only forces acting on car are gravity and normal force.

Task No. 1 - Kinetic energy of the car

1.a. Determine the expression of J - the moment of inertia of wheel reported at its own axles if its mass is m and the length of its square side is $2a$.

1.b. Determine the expression of kinetic energy of the car and the expression of the angular velocity of its wheels:

i. when the car pass on the tops of the bumps;

ii. when the car pass with each wheel through a „valley".

Task No. 2 - Road's shape

In the figure 2 is shown the motion of the lower side of the square cross-section of the wheel on a bump (the curve passing through the points x_s, T, x_d).

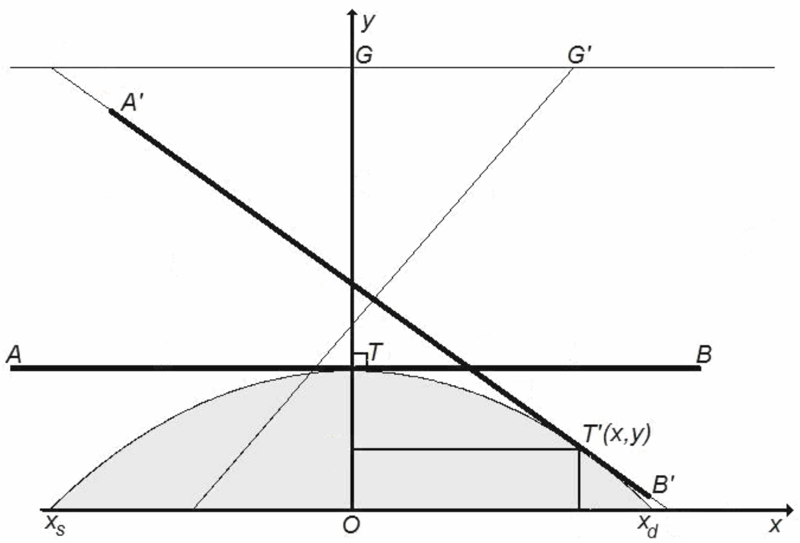

Figure 2

The road is a periodic repetition of such a bump.

The Ox axis of the coordinate system passes through valleys of the relief and the Oy axis passes through bump tip. *AB is wheels side, T is the contact point between wheel and road and G is wheel axis position – in initial position. $A'B'$, T', G' have the same meaning – but for a certain position.* In cross-section the wheel axis is always on the same vertical line with the contact point between wheel and road.

2.a. Give an explanation for this finding.

2.b. Determine the analytical form $y = y(x)$ of cross-section of the road.

Hint: The derivative $y'(x)$ of the function $y = y(x)$ is the slope of the tangent to the graph of function in x. Consider known that $\int \frac{dx}{\sqrt{x^2 - a^2}} = ln\left|x + \sqrt{x^2 - a^2}\right| + C$. We recommend to use the notation $ch(x) = \frac{e^x + e^{-x}}{2}$; $sh(x) = \frac{e^x - e^{-x}}{2}$ with property $ch^2(x) - sh^2(x) = 1$

2.c. Determine the expression of the length (on horizontal) of a bump of road.

2.d. Determine the expression of minimum distance between wheel axles Fred's car.

2.e. Decide whether a machine wheeled regular hexagonal prism conveniently chosen could run on the road whose form was determined in 2.b., without vertical displacement of the center of mass of the wheel.

Task No. 3 - Accident

Assuming that analytical expression of road's bumps is $y = k - h \cdot ch(x/a)$ where k and h are two constants.

3.a. Determine the domain of possible values of the horizontal velocity of the car, under the circumstances established in problem's statement.

3.b. Determines how the amount of heat that can be released by plastic collision of Fred's car with an obstacle depends on position of obstacle on road. Immediately after the collision the car stops.

3.c. Determine the expression of heat released by collision.

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ANSWER SHEET

Theoretical Problem No. 1 (10 points)

Fred & Barney's car

Task No. 1 - Kinetic energy of the car

1.a. Determine the expression of J - the moment of inertia of wheel reported at its own axles.		1,00p
1.b.i. Determine the expression of kinetic energy of the car and the expression of the angular velocity of its wheels, when the car passes on the tops of the bumps.		1,00p
1.b.ii. Determine the expression of kinetic energy of the car and the expression of the angular velocity of its wheels: when the car passes with each wheel through a „valley”.		1,00p

Task No. 2 - Road's shape

2.a. In cross-section the wheel axis is always on the same vertical line with the contact point between wheel and road. Give an explanation for this finding.

	0,50p

2.b. Determine the analytical form $y = y(x)$ of cross-section of the road.		2,00p
2.c. Determine the expression of the length (on horizontal) of a bump of road.		0,50p
2.d. Determine the expression of minimum distance between wheel axles Fred's car.		0,50p

2.e. Decide whether a machine wheeled regular hexagonal prism conveniently chosen could run on the road whose form was determined in 2.b., without vertical displacement of the center of mass of the wheel. Give a short explanation.

	0,50p

Task No. 3 - Accident

3.a. Determine the domain of possible values of the horizontal velocity of the car, under the circumstances established in problem's statement.		2,00p

3.b. Determines how the amount of heat that can be released by plastic collision of Fred's car with an obstacle depends on position of obstacle on road. Give a short explanation.

0,50p

3.c. Determine the expression of heat released by collision.

0,50p

Theoretical Problem No. 2 (10 points)

Compton scattering

An electron storage ring contains circulating electrons with high energy produced by an accelerator and rotating in an appropriate magnetic field. X - ray photons are directed so as to collide with electrons stored in storage ring. Phenomenon that occurs is known as Inverse Compton Scattering.

A photon of wavelength λ_i is scattered by a moving, free electron. As a result the electron stops and the resulting photon of wavelength λ_0 is scattered at an angle $\theta = 60°$ with respect to the direction of the incident photon; this photon is again scattered by a second free electron at rest. In this second scattering process a photon with wavelength of $\lambda_f = 1{,}25 \times 10^{-10} m$ emerges at an angle $\theta = 60°$ with respect to the direction of the photon of wavelength λ_0.

To characterize the photons and the electrons during the processes use the following notation:

	initial photon	photon – after the first scattering	final photon		first electron before collision	first electron after collision	second electron before collision	Second electron after collision
moment	$\vec{p}_i$	$\vec{p}_0$	$\vec{p}_f$	moment	$\vec{p}_{1e}$	0	0	$\vec{p}_{2e}$
energy	E_i	E_0	E_f	energy	E_{1e}	E_{0e}	E_{0e}	E_{2e}
wavelength	λ_i	λ_0	λ_f	speed	$\vec{v}_{1e}$	0	0	$\vec{v}_{2e}$

The following constants are known:

$h = 6{,}6 \times 10^{-34} J \cdot s$ - Planck's constant

$m_0 = 9{,}1 \times 10^{-31} kg$ - rest mass of the electron

$c = 3{,}0 \times 10^{8} m/s$ - speed of light in vacuum

Task No. 1 - First collision

1.a. Draw simple sketches marking the moments of the electron and photon before and after the first collision. Clearly specify the coordinate system used.

1.b. Express the energy and moment of the electron implied in the first collision as a function of the initial speed of the electron $\vec{v}_{1e}$ and its rest mass m_0.

1.c. Express the energy and wavelength of the photon after the first collision as a function of the wavelength of the initial photon λ_i, the scattering angle θ and $\Lambda = h/(m_0 \cdot c)$.

Task No. 2 - Second collision

2.a. Draw simple sketches marking moment of the electron and photon before and after the second collision. Clearly specify the coordinate system used.

2.b. Express the energy and wavelength of the photon after the second collision as a function of the wavelength of the photon before the collision λ_0, the scattering angle θ and $\Lambda = h/(m_0 \cdot c)$.

2.c. Express the kinetic energy $(T_2 = E_{2e} - E_{0e})$ and momentum p_{2e} of the electron after the second collision as a function of the photon wavelengths after the collision λ_f , m_0 , c and h .

Task No. 3 - Quantitative description of processes

Using the values of the given physical constants and numerical values of λ_f and θ determine the expressions and numerical values for:

3.a. the De Broglie wavelength of the initial electron;

3.b. the energy and frequency of the initial photon;

3.c. the speed of the second electron after collision;

3.d. the variation of the photon wavelength after each collision process.

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Theoretical Problem No. 2 (10 points)

Compton scattering

Task No. 1 - First collision

1.a. Draw simple sketches marking the moments of the electron and photon before and after the first collision. Clearly specify the coordinate system used.

	0,50p

1.b. Express the energy and moment of the electron implied in the first collision as a function of the initial speed of the electron $\vec{v}_{1e}$ and its rest mass m_0.		0,80p
1.c. Express the energy and wavelength of the photon after the first collision as a function of the wavelength of the initial photon λ_i, the scattering angle θ and $\Lambda = h/(m_0 \cdot c)$.		1,20p

Task No. 2 - Second collision

2.a. Draw simple sketches marking moment of the electron and photon before and after the second collision. Clearly specify the coordinate system used.

0,50p

2.b. Express the energy and wavelength of the photon after the second collision as a function of the wavelength of the photon before the collision λ_0, the scattering angle θ and $\Lambda = h/(m_0 \cdot c)$.

1,20p

2.c. Express the kinetic energy $(T_2 = E_{2e} - E_{0e})$ and momentum p_{2e} of the electron after the second collision as a function of the photon wavelengths after the collision λ_f, m_0, c and h.

0,80p

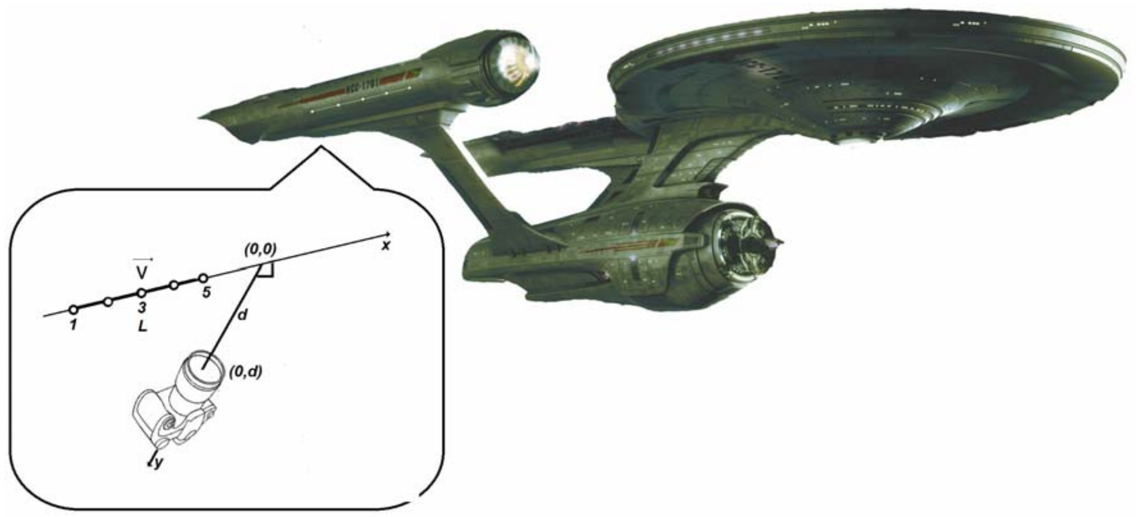
Task No. 3 - Quantitative description of processes

3.a. Determine the expression for the de Broglie wavelength of the initial electron		1,00p
Determine the numerical value for the de Broglie wavelength of the initial electron		1,00p
3.b. Determine the expressions for the energy and for the frequency of the initial photon		0,50p
Determine the numerical values for the energy and for the frequency of the initial photon		0,50p
3.c. Determine the expression for the speed of the second electron after collision		0,50p
Determine the numerical value for the speed of the second electron after collision		0,50p

3.d. Determine the expression for the variation of the photon wavelength after each collision process		0,50p
Determine the numerical value for the variation of the photon wavelength after each collision process		0,50p

Theoretical Problem No. 3 - Star Trek Watching (10 points)

Description of the situation



Star Ship Enterprise has on each of the two warp engines a succession of five collinear, equidistant, numbered, beacons as shown in the figure (for the propeller on right side). When traveling, SS Enterprise moves with constant speed $\vec{v}$ so that, for the questions that follow, beacons located on a propeller are always on the same line that is Ox axis of the coordinate system. In the situation room of another spacecraft which is at rest and which oversees the SS Enterprise, are analyzed the images of line of beacons. The images are taken by a camera with the aperture located at the point $(0,d)$ - in the orthogonal coordinate system shown in the inset in figure above. An image of line of beacons is produced by light rays that arrive simultaneously at camera aperture that opens for a short time. On a picture of SS Enterprise at rest, the length of line of beacons is L. In the problem, the beacons can be considered as points.

The speed of light is $c = 3{,}00 \times 10^8 m \cdot s^{-1}$.

Employ the quantities $\beta = v/c$ and $\gamma = 1\big/\sqrt{1-\beta^2}$, if they help to simplify your result.

Task No. 1 - The link between image and the actual position

The actual position is the position in the frame in which the camera is at rest.

Next, the camera remains stationary in the point $(0,d)$ and the SS Enterprise is moving at velocity $\vec{v}$ on Ox direction. In an image provided by the camera described above is observed that one of the bright beacons is placed in the position x_i.

1.a. Determine the expression for the actual position of the bright beacon when the image is formed in the camera.

1.b. Find also the corresponding inverse relation, that is the expression of x_i in terms of x, d, L, v and c.

Task No. 2 - Apparent length of the beacons line

The camera takes a picture of the instant when the actual position of the center of beacons line is at some x_0 point.

2.a. Determine the expression for the apparent length of the beacons line on this picture.

2.b. Indicate – using a mathematical expression – how the apparent length of beacons line changes with time, when SS Enterprise moves along the Ox axe. The space ship comes from far away, passes through the origin of the system and then receding far away.

Task No. 3 - Symmetrical image

One of the pictures taken by the camera shows both ends of beacons line at the same distance from the aperture of camera.

3.a. Determine the expression for the apparent length of beacons line on this picture.

3.b. Determine the expression for the actual position of the middle beacons at the time when this picture is taken.

3.c. Determine where will be situated on the picture the image of the middle beacons.

Task No. 4 - Pictures of SS Enterprise, being far away and approaching or being far away and receding

Camera takes a picture when SS Enterprise is approaching from far away and another image when the ship (and the beacons line) is far away and receding. On one of these images the apparent length of the line of beacons is $160m$ and the other is $600m$.

4.a. Decide which of the following statements is correct.

i. Apparent length is $200m$ on the image of „approaching” ship and $600m$ on the image of „receding” ship.

ii. Apparent length is $600m$ on the image of „approaching” ship and $200m$ on the image of „receding” ship.

Write in the appropriate box in the answer sheet the letter corresponding to the answer you think is correct. Briefly justify your choice.

4.b. Determine the speed v of the ship.

4.c. Calculate the value of length of the beacons line at rest.

4.d. Calculate the value of length of the beacons line on the symmetric picture, described at task no. 3.a.

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ANSWER SHEET

Theoretical Problem No. 3 - Star Trek Watching (10 points)

Task No. 1 - The link between image and the actual position

1.a. The expression for the actual position of the bright beacon when the image is formed in the camera — 1,00p

1.b. Corresponding inverse relation, that is the expression of x_i in terms of x, d, L, v and c — 1,00p

Task No. 2 - Apparent length of the beacons line

2.a. The expression for the apparent length of the beacons line on this picture — 1,00p

2.b. Indicate – using a mathematical expression – how the apparent length of beacons line changes with time, when SS Enterprise moves along the Ox axe.

1,00p

Task No. 3 - Symmetrical image

3.a. The expression for the apparent length of beacons line on this picture — 1,00p

3.b. The expression for the actual position of the middle beacons at the time when this picture is taken		1,00p
3.c. Determine where will be situated on the picture the image of the middle beacons		1,00p

Task No. 4 - Pictures of SS Enterprise, being far away and approaching or being far away and receding

4.a. Decide which of the following statements is correct.

i. Apparent length is $200m$ on the image of „approaching" ship and $600m$ on the image of „receding" ship.

ii. Apparent length is $600m$ on the image of „approaching" ship and $200m$ on the image of „receding" ship.

Write in the appropriate box in the answer sheet the letter corresponding to the answer you think is correct. Briefly justify your choice.

1,00p

4.b. The speed v of the ship		1,00p
4.c. The value of length of the beacons line at rest		0,50p
4.d. The value of length of the beacons line on the symmetric picture		0,50p