

Experimental competition

Solution

Part 1.

1. It is necessary to obtain the motion law of the rod. In this experiment it is preferable to measure time intervals at which the rod passes preliminary fixed paths, say 1 sm. Table 1 shows the corresponding measurements and Fig.1 graphically demonstrates the same dependence.

Table 1.

x, sm	t, s
0	0
1	6,9
2	14,3
3	21,8
4	29,2
5	36,8
6	44,1
7	51,0
8	58,4
9	65,7
10	72,2
11	79,0
12	85,9
13	92,9
14	100,3
15	107,7
16	114,6

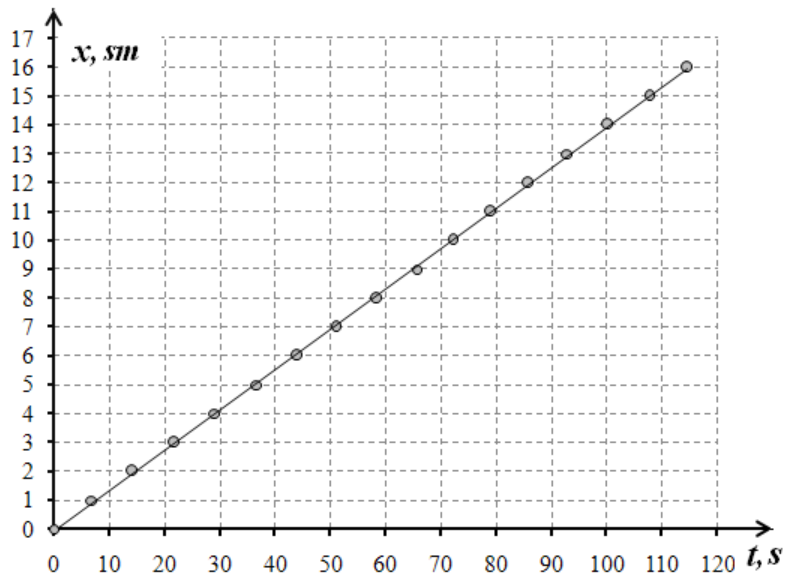


Fig.1 The motion law of the rod

The experimental line in Fig. 1 is linear with high accuracy which proves that the rod really moves uniformly. The most accurate technique of determining average velocity of the rod is least-squares method. The slope of the straight line in Fig.1 gives an average velocity which in this particular case is

$$V = (0,140 \pm 0,001) sm/s$$

2. In this part of the experiment it is necessary to measure the rod velocity several times and to guarantee maximal accuracy the fixed path for the rod to pass should be over 10 sm. Table 2 presents the measurement results for the time intervals t for the rods of different diameters and the calculated velocities V .

Table 2.

d, mm	x, sm	t_1, s	t_2, s	t_3, s	$\langle t \rangle, s$	$\Delta t, s$	$V, sm/s$	$\Delta V, sm/s$
4,0	10	77,5	76,6	81,7	78,60	5,4	0,13	0,01
3,5	10	11,7	11,6	12,0	11,76	0,48	0,85	0,03
3,0	10	3,4	3,3	3,3	3,35	0,12	2,99	0,11
2,5	20	3,3	3,5	3,2	3,33	0,30	6,01	0,54

To calculate the average velocity the following formula is used

$$V = \frac{x}{\langle t \rangle}, \quad (1)$$

where $\langle t \rangle$ is the average of the measured time intervals.

The experimental error can be evaluated as

$$\Delta t = 2 \sqrt{\frac{\sum (t_i - \langle t \rangle)^2}{n}}, \quad \Delta V = V \frac{\Delta t}{\langle t \rangle} \quad (2)$$

The corresponding dependence is plotted in Fig.2.

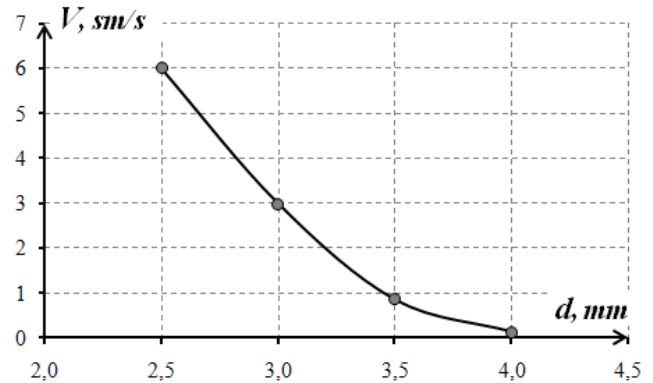


Fig.2 The dependence rod velocity on the rod diameter

3. To check the validity of the formula written in the experimental task it is necessary to plot in logarithmic scale the average velocity dependence on the difference between the diameters of the test tube and the rod. It is done in Fig.3.

The linearity of the obtained line clearly demonstrates the power dependence between the chosen values, i.e. proves the validity of formula (1) written in the experimental task// It is possible that the last point in Fig.3 deviates slightly from the straight line due to the big gap between the rod and the test tube walls.

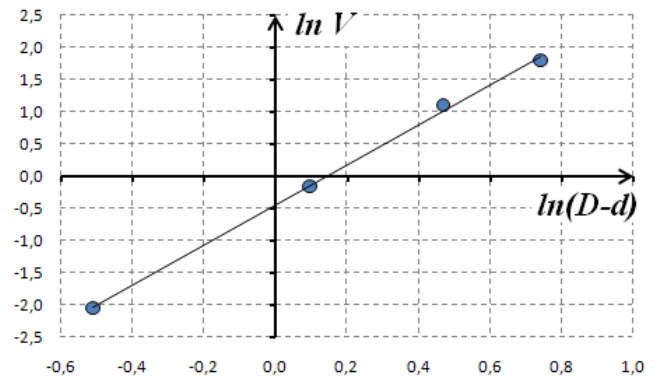


Fig. 3 The rod velocity versus the size of the gap (log-log scale)

4. The power index in formula (1) is numerically equal to the slope of the straight line in Fig.3. Calculation with the least-squares method gives rise to the following value

$$\gamma = 3,11 \pm 0,16$$

5. When the rod sinks the liquid has to flow between the lateral surface of the rod and the test tube walls (Fig.4). To overcome the viscous friction of the liquid, the pressure difference $\Delta P = P_0 - P_1$ should appear between the lower and upper ends of the rod. It is this difference in the pressure that produces an additional force acting on the sinking rod. Besides, the rod is slowed down by the above mentioned viscous friction acting on its lateral surface.

It is reasonable to assume that the viscous friction force acting on the liquid in the gap from the tube walls is proportional to the average velocity of the flowing liquid and inversely proportional to the gap size.

$$F_v = \beta \frac{v}{h} \quad (3)$$

At uniform flowing of the liquid this force is compensated by the pressure difference (the weight of the liquid in the gap is negligible due to the small size of the gap)

$$\beta \frac{v}{h} \approx S_1 \Delta P, \quad (4)$$

where $S_1 = 2\pi R h$ is the area of the transversal section of the gap. Since the rod moves slowly, one can neglect the viscous friction and write

$$mg = S_0 \Delta P, \quad (5)$$

where $S_0 = \pi R^2$ is the area of the rod transversal section. It follows from formulas (4)-(5) that the average velocity of the liquid in the gap is found as

$$v = \frac{mg}{R\beta} h^2 \quad (6)$$

The rod velocity u is related to the average velocity of the liquid in the gap as $S_0 u = S_1 v$, thus

$$u = \frac{2h}{R} v \sim h^3. \quad (7)$$

The results obtained above verify formula (7) within the experimental error.

Part 2.

1. The experimental technique used in this task is quite analogous to the same task in Part 1. In Table 3 and Fig.5 the results are presented for the wood stick.

Table 3.

$x, \text{ cm}$	$t, \text{ c}$
0	0,0
1	13,1
2	25,6
3	37,0
4	48,8
5	61,3
6	73,9
7	86,1
8	98,2
9	109,6
10	121,7
11	134,0
12	146,0

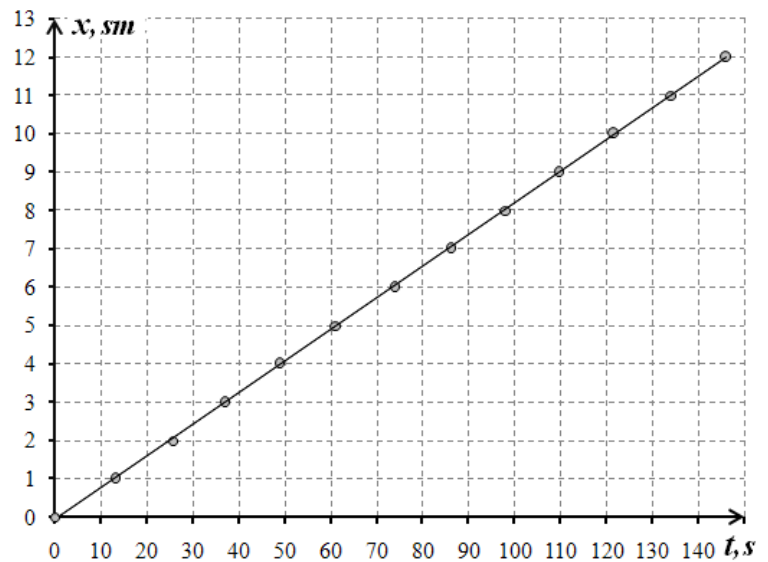
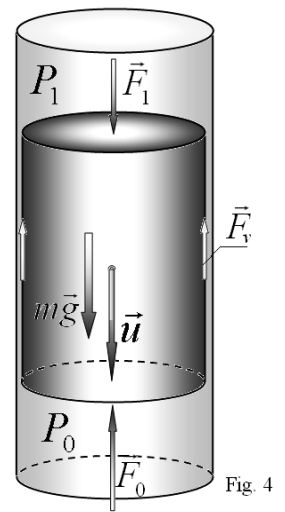


Fig.5 The motion law of the wood stick

It is seen that the wood stick also emerges uniformly and its average velocity, calculated by the least-squares method from Table 3, is

$$V = (0,0825 \pm 0,0004) \text{ cm/c}$$



2. To obtain velocities it is enough to measure time intervals for a wood stick to pass a fixed path several times. Results of the measurements shows that within the experimental error the emerging velocity does not depend on the stick length and equals to

$$V = (0,08 \pm 0,01) \text{ m/s}$$

Thus, the plot is just a horizontal straight line.

3. In this case the diameters of sticks are all equal. It is obvious that the viscous friction is proportional to the stick length. $\beta \sim l$. The mass and the pressure difference is also proportional to the length l . Consequently, it follows from formula (6) that the velocity of the flowing liquid and, thus, the velocity of the stick does not depend on the length.

Experimental task

Marking scheme

No.	Items	Subtotal	Points
1.1	Uniform motion of metal rods	3,5	
	Study of law of motion		0,25
	Results of measurement: - Range of measured coordinate is not less than 10cm; - Measurements with 1 cm period; - Uniform motion is established; - Average velocity is within (0,12 – 0,16) cm/sec; within (0,05 – 0,20) cm/sec;		0,25 0,25 0,25 0,5 (0,25)
	Plot: - Plot is made; - Axes have labels (units are shown); - Data points are indicated (in accordance with the data table); - Fitting curve is plotted;		0,25 0,25 0,25 0,25
	Calculation of velocity: - Least mean square is used (or equivalent method for all the data points); - Velocity is calculated numerically; - Measurement errors for velocity are estimated; - The result is rounded in a correct way;		0,25 0,25 0,25 0,25
1.2	Study of the dependence of velocity on diameter	2,25	
	Measurements: - Velocities for 4 rods are made; - Length of the motion measured is not less than 10 cm; - Measurements of time of motion (or distance covered) for each rod is not less than 3; - Measurement errors for velocity are estimated; - Obtained results are within 25% deviation from the sample results; - <i>within 50% deviation from the sample results</i>		0,25 0,25 0,25 0,25 0,5 (0,25)
	Plot: - Axes have labels (units are shown); - Data points are indicated (in accordance with the data table); - Fitting curve is plotted;		0,25 0,25 0,25
1.3	Exploring the dependence (1)	1	
	- Linearization is made in log-log scale; - Linearization is made in some other correct way; - Calculation is made; - Linearized plot is made;		0,5 (0,25) 0,25 0,25
1.4	Determination of the exponent value	1,25	
	- Least mean square is used; - <i>Visual fit is used</i> ; - The value of exponent is within the range (2,8-3,2); - <i>The value of exponent is within the range (2,5-3,5)</i> ; - Measurement errors are estimated;		0,5 (0,25) 0,5 (0,25) 0,25
1.5	Theoretical explanation	1	
	- Dependence of viscous force on velocity and gap width; - Estimation of pressure difference; - Estimation of viscous force; - It is shown that $\gamma=3$		0,25 0,25 0,25 0,25

2.1	Uniform motion of wooden rods	3	
	Results of measurement: - Range of measured coordinate is not less than 10cm; - Measurements with 1 cm period; - Uniform motion is established; - Average velocity is within (0,06 – 0,10) cm/c; <i>within (0,04 – 0,12) cm/c;</i>		0,25 0,25 0,25 0,5 (0,25)
	Plot: - Plot is made; - Axes have labels (units are shown); - Data points are indicated (in accordance with the data table); - Fitting curve is plotted;		0,25 0,25 0,25
	Расчет скорости: - использован МНК (или равносильный метод по всем точкам); - рассчитано значение скорости; - оценена погрешность измерения скорости; - результат правильно округлен;		0,25 0,25 0,25 0,25
2.2	Study of the dependence of velocity on length of rod	2,5	
	Measurements: - Velocities for 4 rods are made; - Length of the motion measured is not less than 5 cm; - Measurements of time of motion (or distance covered) for each rod is not less than 3; - Measurement errors for velocity are estimated; - Obtained results are within 25% deviation from the sample results; <i>- within 50% deviation from the sample results</i>		0,25 0,25 0,25 0,25 0,5 (0,25)
	Plot: - Axes have labels (units are shown); - Data points are indicated (in accordance with the data table); - Error bars are shown; - Fitting curve is plotted;		0,25 0,25 0,25 0,25
2.3	Theoretical explanation	0,5	
	- Dependence of viscous force on length of rod; - Pressure difference is proportional to length of rod;		0,25 0,25
	Total for Experimental task	15	