

1. Integration gives

$$B = \frac{\mu_0 I}{2\pi} \left(1 - \frac{a+d}{\sqrt{a^2+d^2}} \right),$$

the positive direction is by default downwards (away from the picture).

2. Consider a loop current i_1 in the bigger loop. It creates a field

$$B_0 = \frac{\mu_0 i_1}{2R_1}$$

in its center. In case of $R_1 \gg R_2$ we can consider B_0 as a uniform field containing the smaller coil. Then the magnetic flux is

$$\Phi_2 = \pi R_2^2 B_0 = \pi R_2^2 \cdot \frac{\mu_0 i_1}{2R_1},$$

then the mutual inductance is

$$M = \frac{\mu_0 \pi R_2^2}{2R_1}.$$

3. The general formula is

$$1 - \beta^2 = \left(\frac{mc^2}{E} \right)^2.$$

As $E \gg mc^2$ (for an electron $mc^2 = 512 \text{ keV}$), then a small quantity

$$\varepsilon = 1 - \beta = \frac{1}{2} \left(\frac{mc^2}{E} \right)^2 = 3.6 \cdot 10^{-9},$$

and the electron's velocity is

$$\beta = 1 - \varepsilon = 0.9999999964.$$

4. The rockets' relative velocity is

$$\beta = \frac{\beta_1 + \beta_2}{1 + \beta_1 \beta_2} = 0.8.$$

Then the contraction factor is $l_0/l = \gamma = (1 - \beta^2)^{-1/2} = 5/3$.

5. The momentum of the system is conserved and is equal to

$$p = \frac{m_0 v}{\sqrt{1 - v^2/c^2}}.$$

At the end the velocity of the system is

$$\beta_1 = \left(1 + \left(\frac{2m_0 c}{p} \right)^2 \right)^{-1/2} = \left(\frac{4c^2}{v^2} - 3 \right)^{-1/2},$$

so the mass of the system is

$$m_1 = \frac{p}{\beta_1 c} = m_0 \sqrt{\frac{4 - 3\beta_0^2}{1 - \beta_0^2}},$$

where β_0 corresponds to v/c .

6. Consider a photon with momentum \vec{p} creating an electron-positron pair with momenta \vec{p}_1 and \vec{p}_2 respectively. Then due to conservation laws

$$\vec{p} = \vec{p}_1 + \vec{p}_2,$$

$$pc = \sqrt{p_1^2 c^2 + m^2 c^4} + \sqrt{p_2^2 c^2 + m^2 c^4},$$

where m corresponds to the electron's (positron's) mass. Squaring both these equations and eliminating $p^2 c^2$, we get

$$\vec{p}_1 \cdot \vec{p}_2 = m^2 c^2 + \sqrt{(p_1^2 + m^2 c^2)(p_2^2 + m^2 c^2)}.$$

This can't correspond to the reality as $\vec{p}_1 \cdot \vec{p}_2 \leq p_1 p_2 < \sqrt{(p_1^2 + m^2 c^2)(p_2^2 + m^2 c^2)}$.