1. Consider a current i flowing through the straight wire. It creates a magnetic field

$$B(r) = \frac{\mu_0 i}{2\pi r}.$$

Then the flux through the circular loop is

$$\Phi = \int_0^{2a} B(r)\sqrt{a^2 - (a-r)^2}dr = \frac{\mu_0 i}{2\pi}\int_0^{2a}\sqrt{\frac{2a-r}{r}}dr = \frac{\mu_0 i a}{2}.$$

This yields the mutual inductance

$$M = \frac{\Phi}{i} = \frac{\mu_0 a}{2}.$$

2. The magnetic field separates the charges on the sphere by the Lorentz force, as a consequence the charges create a force field equivalent to an electric field $E = Bv$ in the $-z$-direction. Then the surface charge density depends on the angle φ between the z axis and the radius-vector by law

$$\sigma(\varphi) = -3\varepsilon_0 E\cos\varphi = -3\varepsilon_0 Bv\cos\varphi.$$

3. The distribution of the magnetic field induces a current in the loop which dissipates the gravitational energy. The total resistance of the loop is

$$R = \frac{4\rho D}{d^2},$$

and the mass is

$$m = \frac{\pi^2 d^2 D\rho_m}{4}.$$

The induced emf is

$$\mathcal{E} = \frac{d\Phi}{dt} = \frac{\pi D^2\kappa v}{4},$$

where v is the velocity of the loop. Then the power of dissipation is

$$P = \frac{\mathcal{E}^2}{R} = \frac{\pi^2 d^2 D^3\kappa^2 v^2}{64\rho}.$$

For a steady velocity $P = mgv$, which yields

$$v = \frac{16\rho\rho_m g}{\kappa^2 D^2}.$$

4. Integrating the wave function gives

$$A = 2\alpha^{3/4}\left(\frac{2}{\pi}\right)^{1/4}.$$

Substituting the given functions $\psi(x)$ and $U(x)$ into the differential equation, we get

$$\alpha = \frac{m\omega}{2\hbar}$$

and

$$\varepsilon = \frac{3\hbar\omega}{2}.$$

Several integrations yield

$$\langle x \rangle = \int\limits_{-\infty}^{\infty} x\psi^2(x)dx = 0,$$

$$\langle x^2 \rangle = \int\limits_{-\infty}^{\infty} x^2\psi^2(x)dx = \frac{3A^2}{16\alpha^{5/2}}\sqrt{\frac{\pi}{2}} = \frac{3}{4\alpha},$$

$$\langle p \rangle = -i\hbar \int\limits_{-\infty}^{\infty} \psi(x)\psi'(x)dx = 0,$$

$$\langle p^2 \rangle = \hbar^2 \int\limits_{-\infty}^{\infty} \psi(x)\psi''(x)dx = \frac{3A^2}{4}\sqrt{\frac{\pi}{2\alpha}} = 3\hbar^2\alpha.$$

Then

$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \frac{1}{2}\sqrt{3}\alpha,$$

$$\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2} = \hbar\sqrt{3\alpha}.$$

In this case

$$\Delta p \cdot \Delta x = \frac{3\hbar}{2} \geq \frac{\hbar}{2}.$$