Dr. Sasha

It is the information about the task N19

4) I used you expression for speed of the spacecraft $v(t)=\dfrac{v_0}{\sqrt{1+\dfrac{2\rho\cdot v_0\cdot A\cdot t}{m_0}}}$. Then, the length $L[a,b]$ (total distance traveled between time a and time b) of the path $x(t)$ for $t\in(0,\infty)$ is

$$L = x(t)\Big|_{t=0}^{t=\infty} = \int_0^\infty v(t)dt = \frac{m_0}{\rho\cdot A}\sqrt{1+\frac{2\rho\cdot v_0\cdot A\cdot t}{m_0}}\Bigg|_{t=0}^{t=\infty} = \infty \qquad (1)$$

How it is possible, that the path length of the spacecraft is infinity? Do you think the spacecraft will not stop?

It is the information about the task N20

1) - correctly solved problem.

2) It is the citation from your text (file answer20.pdf)

2. The angular velocity appears to be $\omega = 1$, and the radius is a. So the linear velocity is $v = \omega a = a$, and the arc length is $l = 2\pi a$.

It is the plot of one arch of the cycloid $x(t)=a\cdot t-a\sin(t)$, $y(t)=a-a\cos(t)$ ($a=1$).

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ParametricPlot[{u - Sin[u], 1 - Cos[u]}, {u, 0, 2 Pi}]
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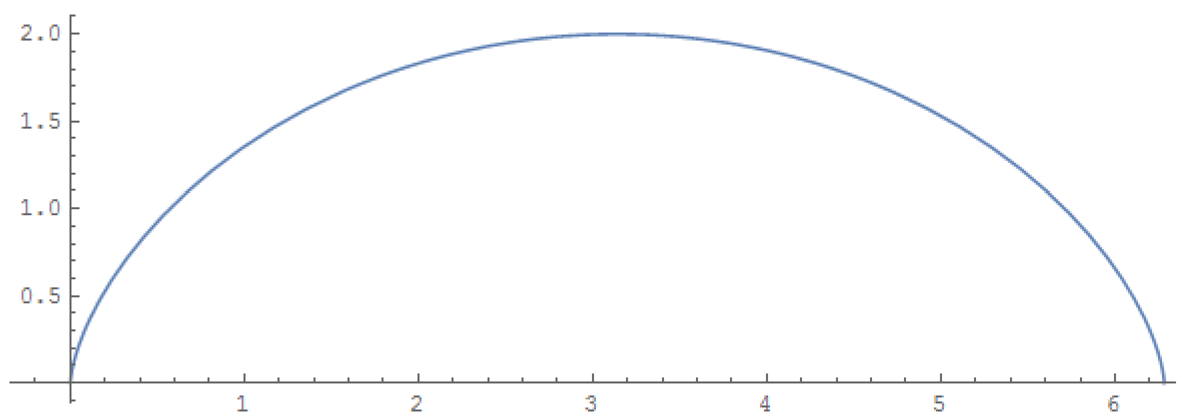


Do you really believe that, the one arc length is 2π for $a=1$?

3) - correctly solved problem.

Please, solve this problem and also show the steps to solve it.

22.06.2015 – N21

1) Superconductors are only superconducting below a certain temperature, called the critical temperature T_c, which varies from material to material. Consider a sphere of radius a initially above its particular T_c. You hold it in a region of constant magnetic field $B_0\hat{z}$ and cool it below its Tc. Calculate the induced surface current $\vec{J}(\theta)$ as a function of the polar angle θ.

<u>Hint:</u> A superconductor is a material that has zero electrical resistance. Any net charge resides on its surface. Superconductors also have the fascinating property that $\vec{B}=0$ inside them. Superconductivity is a distinctly quantum mechanical effect. However, Maxwell's equations still allow us to make sensible statements about the $\vec{E}$ and $\vec{B}$ fields in them.

2) Consider two superconducting wires: one tin (type I) and one Nb_3Sn (type II), each 1 mm in diameter.

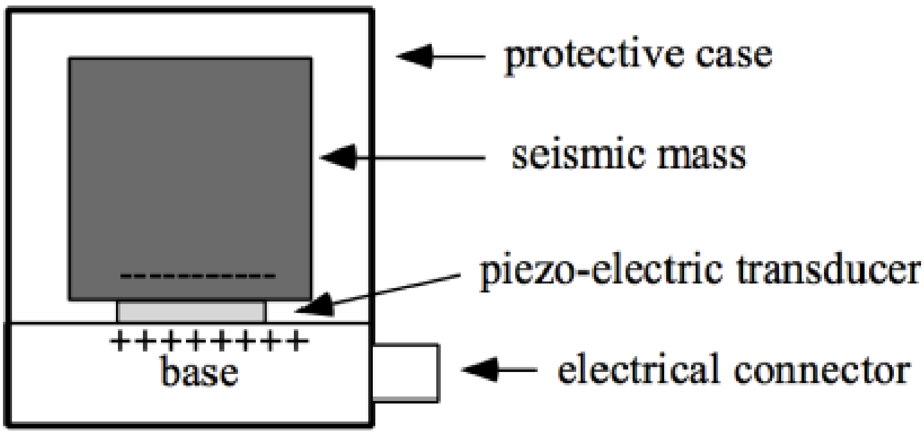
a) Assuming that Sn loses superconductivity when the magnetic field at its surface reaches the critical field ($H_c = 0.2$ T). Calculate the maximum current and hence the critical current density for the Sn wire.

b) Calculate the maximum current and critical current density for the Nb_3Sn wire using the same assumption as in (a), but using its upper critical field ($H_{c1} = 24.5$ T). Compare the results with the actual critical current density of 10^7 A/cm^2 for Nb_3Sn.

3) A conductor at potential $V=0$ has the shape of an infinite plane except for a hemispherical bulge of radius a (see figure below).

A charge q is placed above the center of the bulge, a distance p from the plane (or $p-a$ from the top of the bulge). What is the force on the charge?

4) A piezoelectric accelerometer is done by sandwiching a ceramic piezo disk between a seismic mass m (see figure below).



The sensors base is fastened to a surface you are measuring, so that when an acceleration is applied the seismic mass will move up and down, causing the piezo disk to generate a charge. For this simplified analysis, you will assume that the piezo disk is only generating charge if a force is applied in the vertical direction (due to acceleration), proportional to the piezo charge coefficient d_{yy}. The circular piezoelectric disk is 0.5 cm in diameter and 1 mm thick (denote thickness by the variable l). The seismic mass is a steel cylinder, 1 cm diameter, 1 cm long. You also know the following physical properties:

Parameter	Value	
E	71GPa	Young's modulus of the piezo material
d_{yy}	0.559 Coulombs/meter	Piezo charge coefficient
ε_r	450	dielectric coefficient of the piezo material
ε_0		dielectric coefficient of vacuum

Derive a symbolic expression for the vertical acceleration $\left(\frac{d^2y}{dt^2}\right)$ of the mass m that is only a function of the known material properties and dimensions, as well as the output voltage V of the piezoelectric accelerometer.

Hint: The mechanical spring constant k can be determined from the measured value of Young's modulus E and the dimensions of the element. The capacitance C can be determined from the measured values of ε_0, ε_r and the dimensions of the element.