Chaos in a box

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1. Abstract

For this project, we decided to research the NetLogo model Chaos in a Box. Used in the realm of chemistry and physics, Chaos in a Box is a simple system consisting of one or more balls and zero or more obstacles enclosed in a box; the obstacles are stationary while the balls move around colliding with both obstacles and the walls of the box but not with each other. Collisions alter the trajectory of the balls and while the system primarily exhibits chaotic behavior, the system can show periodic behavior where the ball bounces around in a repeating pattern when the initial conditions are appropriate. We have used this model to explore chaotic and periodic behavior in simple systems as well as how small changes in initial conditions are sufficient to create chaotic behavior. We ran multiple experiments starting from a really simple system with just one ball and no obstacles to one that involved multiple balls and multiple obstacles. We observed chaotic and periodic behavior both with and without obstacles. When there are no obstacles the collision angles are quite predictable even though the path traversed by the ball is not. However, in the presence of obstacles, especially when balls collide with obstacles, the path traversed by the ball can become highly unpredictable. As the size of the obstacle increases, the chances of collision increases making the system significantly more chaotic. Under appropriate initial conditions the system demonstrates periodic behavior. However, when those conditions change even slightly the effect becomes very chaotic.

2. Introduction

The primary goal of the Chaos in a Box model is to model the movement of a ball as it bounces off the walls and obstacles. The model was created by two Northwestern University faculty members and the model draws inspiration from an idea that was addressed in a Complex Systems in Biology course at the Humboldt University in Berlin. In addition, the model employs a multitude of advanced techniques, such as anonymous procedures to make the setup procedure flexible with respect to the placement of the ball and the obstacles and binary search to correct for the error that arises due to the chaotic nature of the system. The model assumes that the collisions are perfectly elastic, the obstacles remain stationary, and there is no friction that is being applied to the ball, in other words, the ball never changes speed and instead changes direction. In addition, the model is only modelling the movement of a single ball rather than the movement of multiple balls because in the end multiple balls only represent the change that occurs due to minute changes in the initial position. Moreover, the model is able to correct the position of the ball when it is subjected to the collisions that occur when the ball overlaps an obstacle or the edge of the room and since the collisions are elastic, the ball's new angle is calculated as the angle of reflection of its heading that's around the surface that the ball is colliding with. Some modelling approaches that have used the Chaos in a Box model are the approaches used in the GasLab models and the Kick Rotator model. In addition, the Chaos in a Box model and other similar modelling approaches are used in the branch of mathematics called Chaos Theory in order to study the chaotic behavior that exists in many systems such as fluid flow, heartbeat irregularities, weather and climate, the stock market, and road traffic just to name a few. The applications of the Chaos in a Box model in the field of nonlinear dynamics and chaos is discussed in the paper titled Modeling Nonlinear Dynamics and Chaos: A Review which was written by Luis A. Aguirre from the Federal University of Minas Gerais in Brazil and Christophe Letellier from the University of Rouen in France. In the paper, Luis and Christophe discuss the application of important developments of modeling techniques in the field of nonlinear dynamics and chaos. In addition, the authors cover the important topics that are related to the development of modeling techniques such as model representations, parameter estimation techniques, data requirements, and model validation, all of which encompass over two decades since papers on the topic of nonlinear dynamics and chaos first appeared in scientific literature.

3. Model Description

The model works by modelling the path that the ball takes as it travels around the room that it is placed in. In this model, the important components are the NUM-OBSTACLES slider, the TWO BALLS button, the SETUP-RANDOM button, the NUM-OBSTACLES button, the SETUP-PERIODIC-X and SETUP-PERIODIC-QUILT buttons, the TRACE-PATH switch and CLEAR-DRAWING button, and lastly the DRAG button. The role of the number of obstacles slider is to adjust the number of obstacles that are placed around the room. The role of the two balls button is to both create another ball that's in a different position compared to the original ball as well as show how significant of an effect slight changes in the initial conditions can have in chaotic systems. The role of the setup random button is to initialize the system in a way that will most likely be chaotic and the role of the number of obstacles button is to place obstacles of different sizes at random positions throughout the room. The setup periodic x button is responsible for predicting the kind of periodic behavior that the system can potentially exhibit and the setup periodic quilt button is responsible for creating another example of periodic behavior over a long period. Lastly, the role of the trace path switch is to outline the path of the ball, the clear drawing button clears the outlined path, and the drag button allows the user to interact with the model, in particular the balls, obstacles, and the respective direction and size. In the model, although each of the components work individually of each other and each component has a dedicated role and function, there are some components that work together. For instance, both the SETUP-PERIODIC-X and SETUP-PERIODIC-QUILT buttons work together to create periodic behavior that could potentially be exhibited by the system. In addition, the components that are required to set up the model work in a cohesive way to create a system that will most likely be chaotic.

4. Simulation Results

To gain insight into our model, we ran a number of trials with varying parameters. For run 1, we initiated the number of obstacles present in the box to 0 (**Figure 1**). Then we selected 'setup-random' and 'trace-path' and ran our simulation. We observed that the movement of the object was periodic when no obstacles were present. It appears that the trajectory of the ball converges to a periodic trajectory if and only if there is nothing that disturbs its current path. However, when an obstacle is present in the simulation the ball ceases to exhibit periodic behavior. For run 2 we initialized an obstacle of size 1 at coordinates (0,0) (**Figure 2**). Early in the simulation the target hit the obstacle and steered off its trajectory, resulting in temporary erratic behavior. However, within a few seconds it seemed as if the target's trajectory was converging back into a periodic one. This behavior continued until the target hit the obstacle and once again exhibited chaotic behavior. For run 3, we increased the size of the obstacle to size 2 and again set its position at (0,0) **(Figure 3**). Here, we observed that increasing the size of the obstacle to size 3 prevented the target from displaying periodic behavior. As the trajectory begins to converge the target is abruptly taken off course and once again exhibits chaotic behavior. For run 4 we increased the size of the obstacle to size 3 and set its position at (0,0) (**Figure 4**). The same behavior in run 3 was present in run 4. For run 5, we initiated the 'setup-periodic-x' and the obstacle was set at size 3 at position (0,0) (**Figure 5**). The trajectory of the target displayed periodic behavior even after colliding with the obstacle. For run 6 we used the SETUP-PERIODIC-QUILT option and again noticed periodic behavior, however the trajectory of the ball followed a quilt like pattern. This option places the obstacle at position (0,8) with a size of 1 (**Figure 6**). Similar to the setup-periodic option, the setup-periodic-quilt option displayed periodic behavior even after colliding with the obstacle.

Next, we ran trials with two balls. Using the setup-periodic option we observed that the initial conditions of the the balls were very near to each other but were not exact (**Figure 7**). When running the simulation we noted that the trajectories of the balls were similar up until they collided with the obstacle. This behavior was again present when we ran the simulation using the setup-periodic-quilt option (**Figure 8**). Here we observed that the balls had initial conditions that were near each other but were not exactly the same. In addition, the two balls had similar trajectories until they came into contact with the obstacle.

Modification #1:

For our project we extended the model by increasing the number of balls in the simulation and implemented a slider into the user interface (**Figure 9**). This allowed for the inclusion of a maximum of 10 balls in the simulation. In addition, each new instance of a ball is placed in a random initial position and is represented with a different color so that its individual trajectory and behavior can be visualized. To gain a better understanding of how this modification affects our model we ran 3 simulations using the random setup option. In run 1 we placed our obstacle at position (0,0) with a size of 1 and included 3 balls for this simulation. When running the simulation we noted that the behavior of the balls coincided with the behavior that we had previously observed when only one and two balls were present. In other words, we observed that the balls exhibit chaotic behavior after colliding with the obstacle but then attempt to converge into a periodic trajectory before again colliding with the obstacle and falling into chaos. For runs 2 and 3 we placed the obstacle at position (0,0) with a size of 2 and 3 respectively. Again we found that the balls exhibit chaotic behavior after colliding with the obstacle but then attempts to converge into a periodic trajectory before again colliding with the obstacle and returning to chaotic movement.

Modification #2:

In addition to increasing the number of balls in the model, we also changed the size and shape of the box. Initially the box is at a size of 8 in height and 8 in width with values of min-pxcor, max-pxcor, min-pycor, and max-pycor of -8,8,-8, and 8 respectively. First we increased the size of the box by 50% to look at how this modification would affect our model. We observed that the model behaved similarly to previous runs and exhibited chaotic behavior only after colliding with the obstacle. However, one thing to note is that increasing the size of the box allowed for a greater time spent in a periodic trajectory before colliding and reverting to chaotic behavior. Next, we changed the shape of the box from a square to a rectangle with a width of 5 and a length of 10 (**Figure 10**). This modification to the model resulted in more chaotic behavior when compared to a simulation with a square box. We observed that this configuration gave the ball a more streamlined path to the obstacle and thus resulted in more chaotic behavior.

Modification #3:

Lastly we changed the y coordinate of the ball's initial position from 7 to 6.99 in the setup-periodic-quilt option (**Figure 11**). When we made this change, we observed that the behavior of the model became chaotic and the path of the ball was no longer a mirror image of the ball's original path. This correlates strongly with the fact that periodic behavior can only be demonstrated under appropriate conditions and when those conditions are subjected to even minute changes, the system instantly becomes chaotic rather than periodic. Moreover, the fact that even minute changes to the system results in instant chaotic behavior also shows that chaotic behavior makes it difficult for systems to be precisely predicted and doing so would require one to have exact measurements of the systems' initial conditions and therefore measuring a system to that degree of accuracy is almost impossible.

5. Conclusion and Future Work

Currently, this model does not consider friction and for this report we did not look at the dynamics of the ball when the obstacle is moving. Under these conditions the speed of the ball does not change, only its direction. Also, because the collisions that occur when the ball collides with either the obstacle or wall are perfectly elastic there is no change in speed, only the direction of the angle is changed. This observation is present throughout all of our simulations.

When running a simulation with no obstacles we noted that the trajectory of the ball was periodic and continued to be periodic throughout the entirety of the run. In contrast, when an obstacle is present the trajectory of the ball is periodic up until it collides with the obstacle. Therefore, we conclude that it is the continual disturbance in trajectory that prevents the ball from converging into periodicity. This is observed in all of the setup-random runs where at least one obstacle is present. However, when selecting the setup-periodic and setup-periodic-quilt options we observed that even with an obstacle present in the box, the ball still exhibited a periodic trajectory. Upon closer examination we noted that the ball seemed to hit the obstacle at about a 90 degree angle. When changing the position of the ball we found that the trajectory of the ball will no longer be periodic. As a result, we hypothesize that in order for the ball to remain periodic it must collide with the obstacle at a 90 degree angle.

During our runs we also noted that the initial position and direction of the ball directly influences its trajectory throughout the simulation. This is seen during runs using two balls. Selecting this option places the balls near each other and facing approximately the same direction, but not exactly the same. This results in entirely different trajectories once they collide with the obstacle. Therefore, we conclude that differing initial conditions, despite how miniscule the differences may be, will result in completely different outcomes.

While this model does an exceptional job at providing insight into the behavior of chaotic systems it can be improved and extended. For future work we plan on changing the shape of the box itself and identifying initial conditions that will result in a periodic trajectory. In addition we will also add hit detection for the balls themselves. This will result in the trajectory of one ball being dependent on the trajectory of others in addition to the walls and any obstacles present. Lastly we will add the option to include balls of varying sizes. Including all of these additional parameters will allow for the modelling of various real-world cases such as the physical interaction between multiple different gases within an enclosed space.

6. References/Figures

References

1. Head, B. and Wilensky, U. (2017). NetLogo Chaos in a Box model. http://ccl.northwestern.edu/netlogo/models/ChaosinaBox. Center for Connected Learning and Computer-Based Modeling, Northwestern University, Evanston, IL.
2. Wilensky, U. (1999). NetLogo. http://ccl.northwestern.edu/netlogo/. Center for Connected Learning and Computer-Based Modeling, Northwestern University, Evanston, IL.
3. Aguirre, Luis A., and Christophe Letellier2. "Modeling Nonlinear Dynamics and Chaos: A Review." *Mathematical Problems in Engineering*, vol. 2009, 2 June 2009, pp. 1-35. *https://scholar.google.com/*, doi:10.1155. Accessed 9 Feb. 2020.

Figures

Figure 1: No obstacles in box.(Periodic)

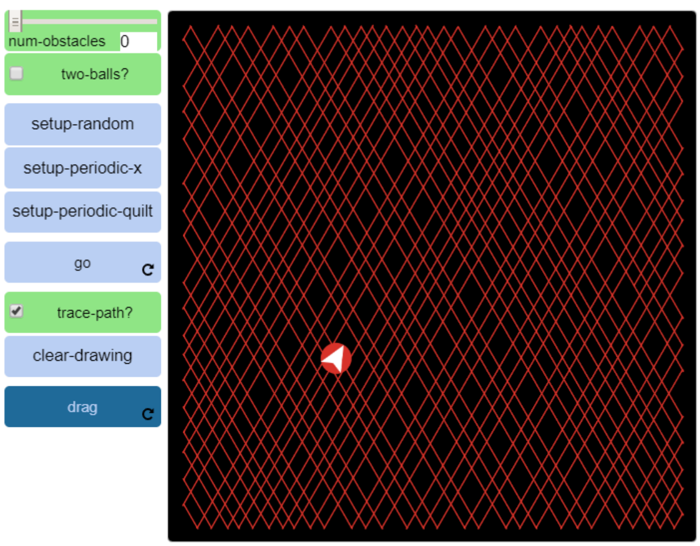

Figure 2: Obstacle of size 1 at (0,0). (Chaotic)

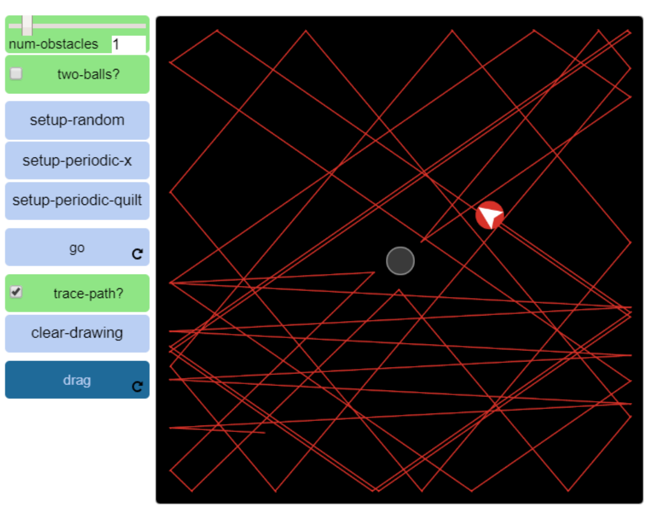

Figure 3: Obstacle of size 2 at (0,0). (Chaotic)

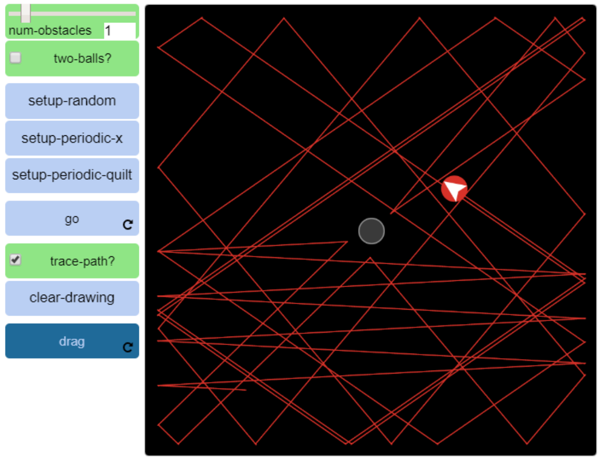

Figure 4: Obstacle of size 3 at (0,0). (Chaotic)

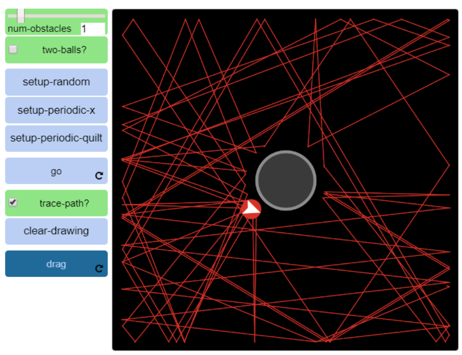

Figure 5: Set-up-periodic option with 1 ball. (Periodic)

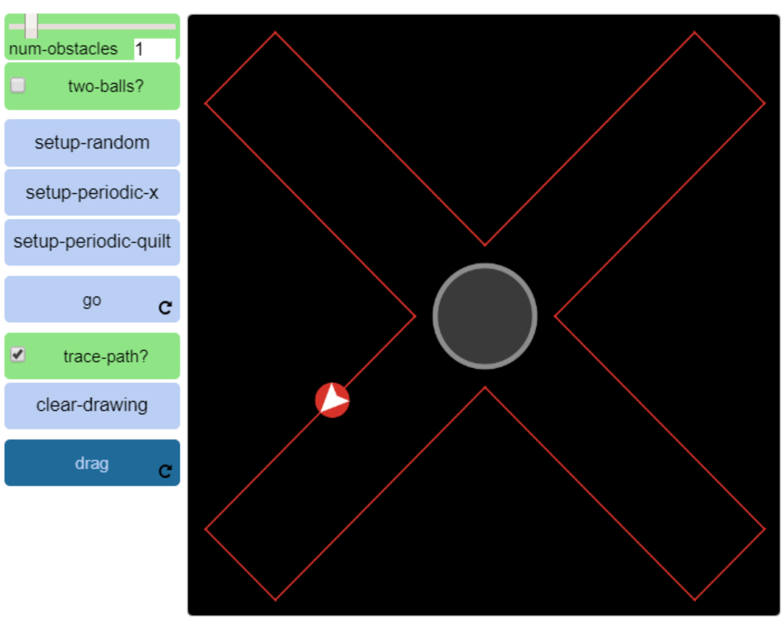

Figure 6: Set-up-periodic-quilt option with 1 ball. (Periodic)

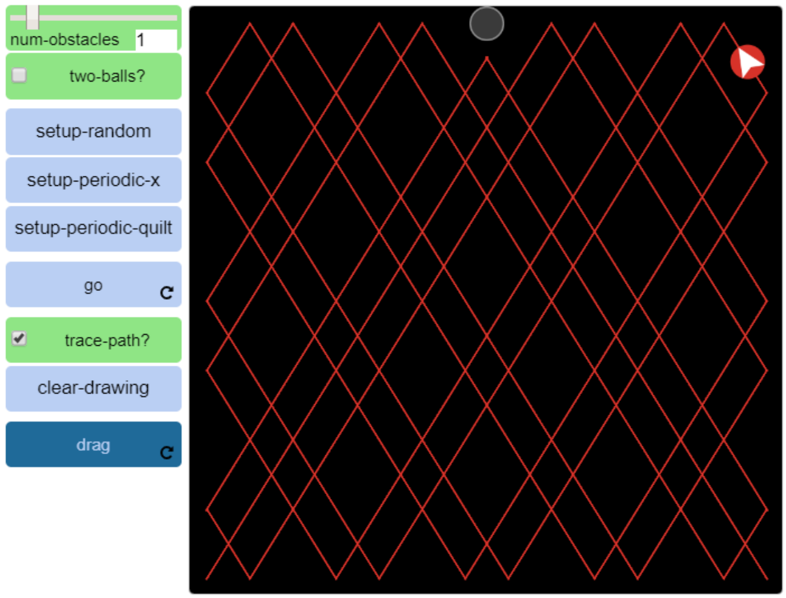

Figure 7: Setup-periodic option with 2 balls. (No collision, one Periodic, one Chaotic)

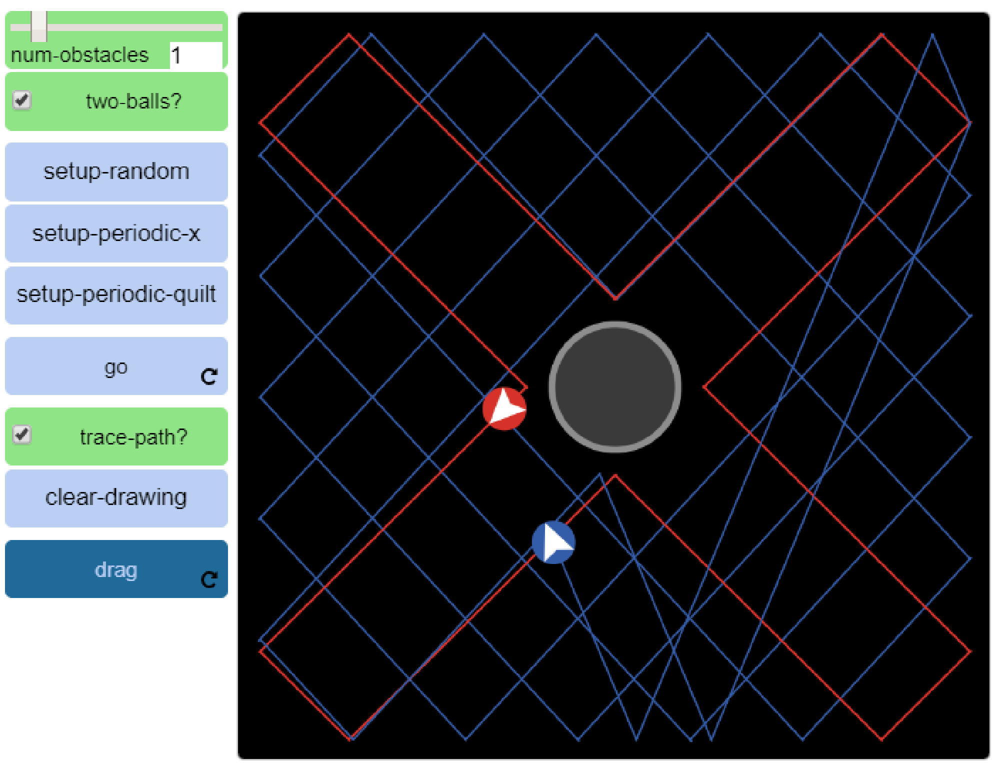

Figure 8: Set-up-periodic-quilt option with 2 balls.(No collision, one Periodic, one Chaotic)

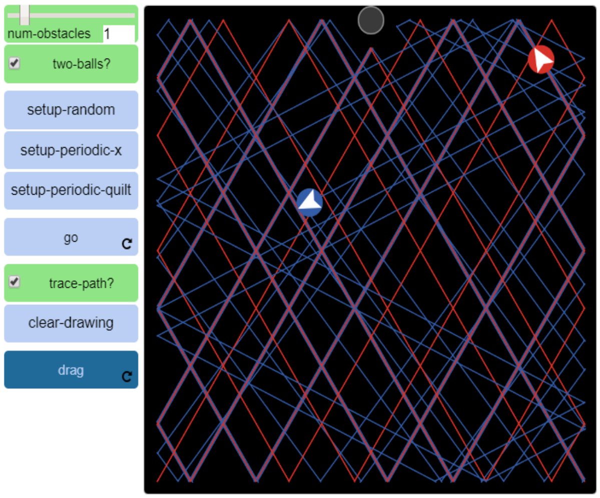

Figure 9: Setup-random option with 3 balls. (Chaotic)

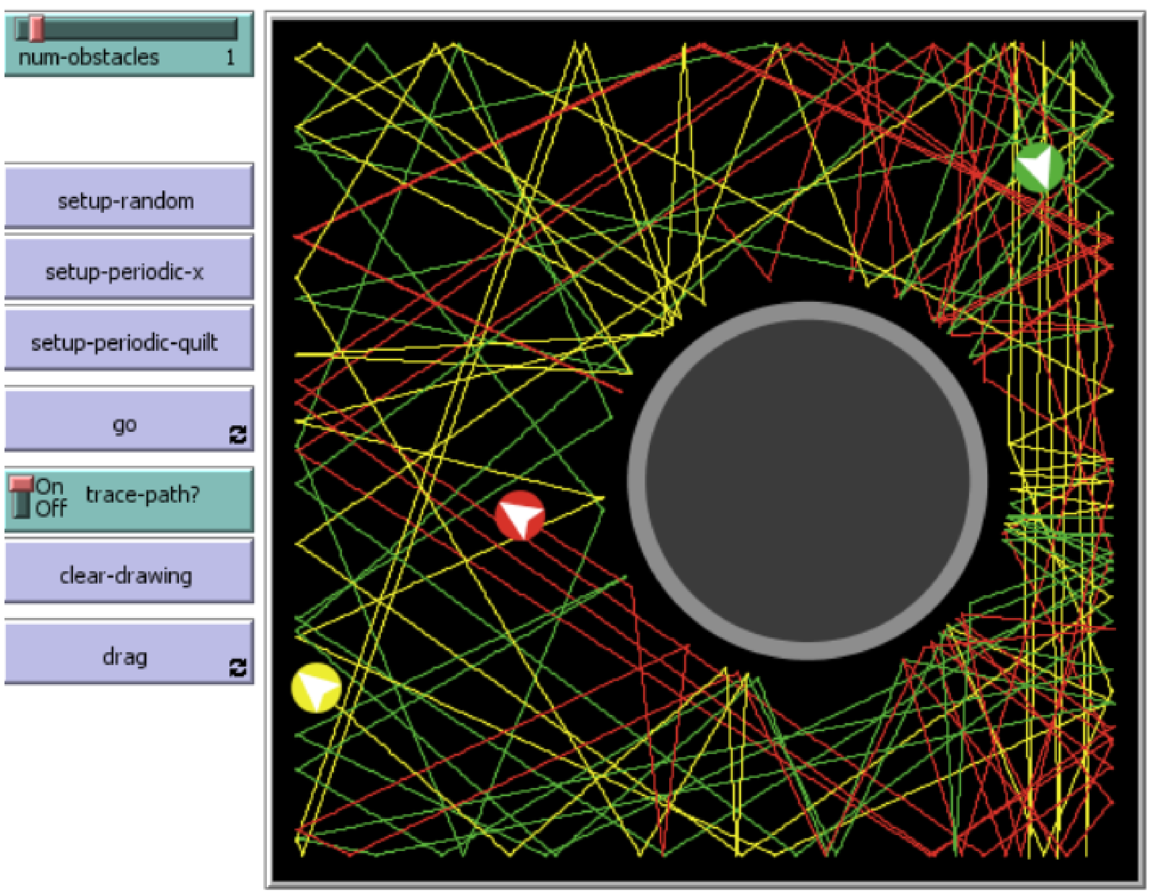

Figure 10: Setup-random option with box with width of 5 and length of 10

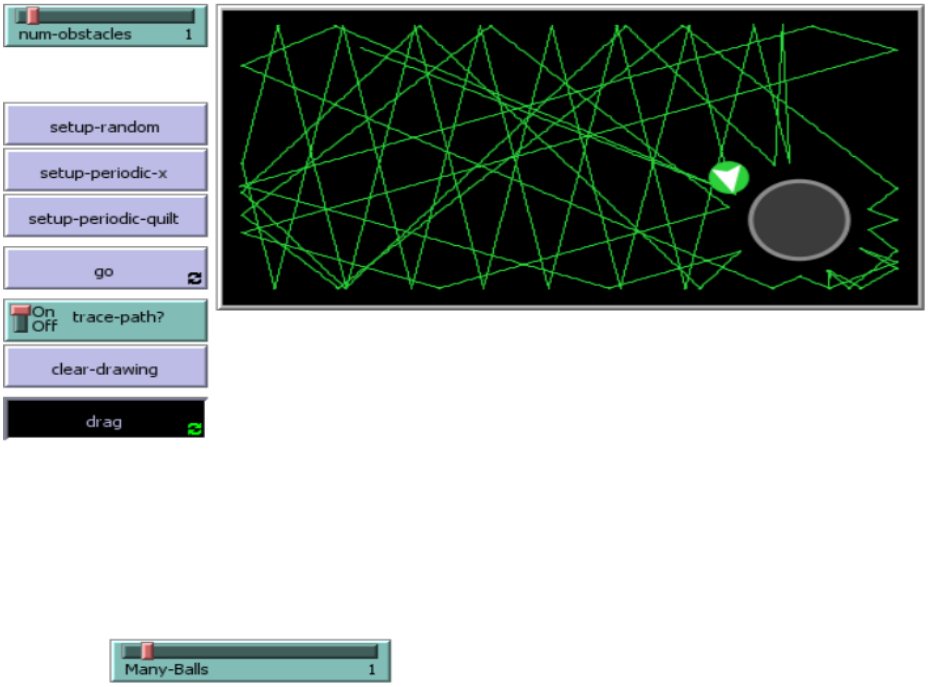

Figure 11: In setup-periodic-quilt, 1 ball and 1 obstacle, however the y coordinate of the ball's initial position was changed slightly thus resulting in chaotic behavior in the system

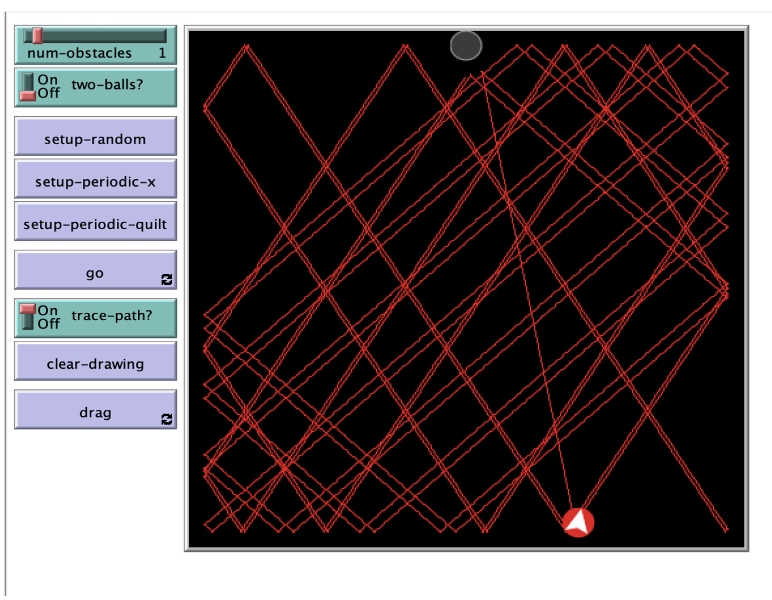

Figure 12: No obstacle location (0, 0); Direction (0, 1) no Obstacles

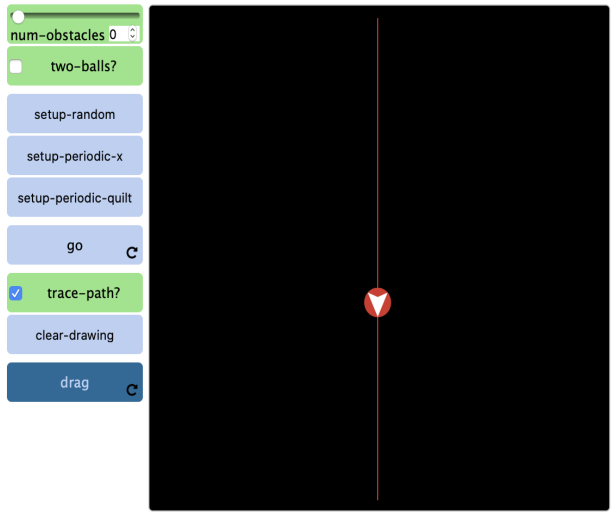

Figure 13: No obstacle, Random setup and it finally reaches all the points in the box

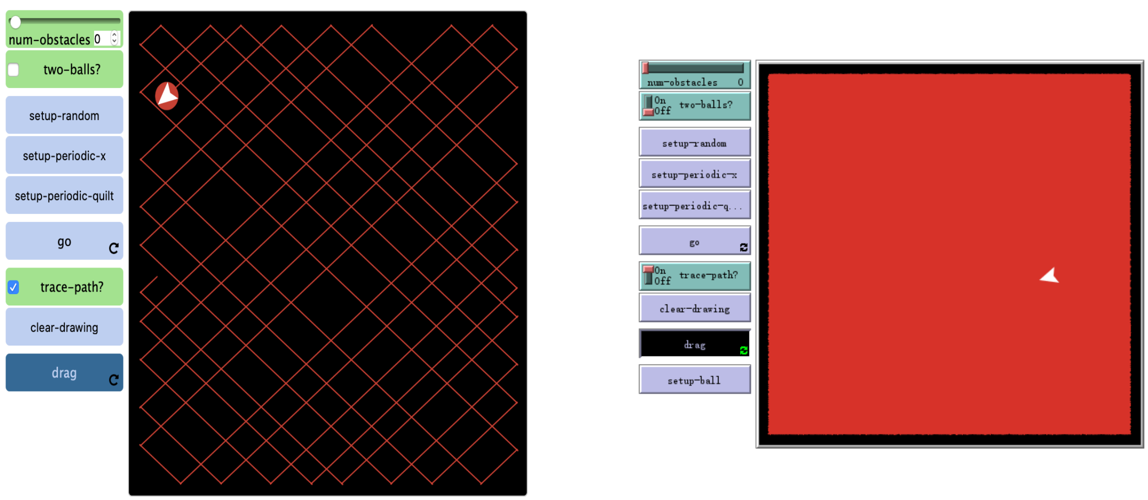

Figure 14: 1 obstacle without collision between the ball and the obstacle