USING PORTFOLIO OPTIMIZATION TECHNIQUES TO DETERMINE THE OPTIMAL PORTFOLIO

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Background

- This project focuses on optimizing a portfolio consisting of S&P 500 stocks
- The objective is to maximize the return on investment (ROI) while minimizing financial risk associated with the portfolio
- The eventual result of the model should give the best portfolio or distribution of assets out of the set of all possible portfolios
- Low risk investment = means a small chance to lose some or all of the investment money
- Generally a high investment return will have more risk associated with it

Source - https://www.bnpparibasmf.in/learn-invest/understanding-risk-return

Modern Portfolio Theory

- Our model uses Markowitz Mean-Variance Portfolio Analysis otherwise known as Modern Portfolio Theory
- What is Modern Portfolio Theory (MPT)?
  - MPT is a mathematical framework that determines the minimum level of risk for an expected return.
  - Specifically, MPT provides an explanation for maximizing the ROI of a portfolio given a particular risk level
  - MPT is centered around the Markowitz model which was developed by Harry Markowitz in 1952
  - The Markowitz model assumes that lower risk portfolios will be favored over those with higher risk
    - This assumption forms a central part of MPT: that is how the return and risk of a given portfolio is affected by a certain asset
- Mean-Variance Analysis is a normative theory
  - A normative theory is one that outlines the behavior that should be followed when constructing a portfolio rather than a prediction concerning actual behavior

Modern Portfolio Theory

- The goal of MPT is to reduce Idiosyncratic Risk
  - Idiosyncratic Risk is a type of investment risk that is unique to an individual asset or a class of assets
- MPT can be used to find best portfolio based on certain risk constraints or certain ROI constraints
  - For example, an investment portfolio that focuses on one asset category can will be greatly affected if the category is going down.
- The “Don’t put all of your eggs in one basket” approach best summarizes MPT

Modern Portfolio Theory Example

- Example - Hospitality stocks impacted by Covid-19
  - On the top, American Airlines Group Stock - you can see that the stock value dropped around March 2020
  - On the bottom Starwood Property Trust - you can see that the stock value also dropped around March 2020
  - If an investment is only focused on hospitality stock and someone invested few months before March 2020, the investment value will drop around September.
- Using MPT on the investment, will result with more diversification and will minimize the risk of the portfolio.

In general:

- Expected return:

  $$\mathrm{E}(R_p) = \sum_i w_i \, \mathrm{E}(R_i)$$

  where R_p is the return on the portfolio, R_i is the return on asset *i* and w_i is the weighting of component asset i (that is, the proportion of asset "i" in the portfolio).

- Portfolio return variance:

  $$\sigma_p^2 = \sum_i w_i^2 \sigma_i^2 + \sum_i \sum_{j \neq i} w_i w_j \sigma_i \sigma_j \rho_{ij},$$

  where σ is the (sample) standard deviation of the periodic returns on an asset, and ρ_{ij} is the correlation coefficient between the returns on assets *i* and *j*. Alternatively the expression can be written as:

  $$\sigma_p^2 = \sum_i \sum_j w_i w_j \sigma_i \sigma_j \rho_{ij},$$

  where $\rho_{ij} = 1$ for $i = j$, or

  $$\sigma_p^2 = \sum_i \sum_j w_i w_j \sigma_{ij},$$

  where $\sigma_{ij} = \sigma_i \sigma_j \rho_{ij}$ is the (sample) covariance of the periodic returns on the two assets, or alternatively denoted as $\sigma(i,j)$, cov_{ij} or $\mathrm{cov}(i,j)$.

- Portfolio return volatility (standard deviation):

  $$\sigma_p = \sqrt{\sigma_p^2}$$

Mathematical formulation for MPT

Determining the asset allocation that minimizes the portfolio risk for two assets

- To determine the asset allocation that minimizes portfolio risk, we differentiate the portfolio variance equation with respect to the weight of that specific asset.
- We then equate the first derivative to zero and solve for the minimum point.
    - For example, if we are interested in finding the value $W_{A.}$ that minimizes portfolio risk, we will differentiate the portfolio variance below with respect to W_A.
    - By differentiating, equating to zero, and solving for W_A we will get the formula for asset allocation that minimizes the portfolio risks for two assets.

$$W_A = \frac{\sigma_B^2 - Cov0V(A,B)}{(\sigma_A^2 + \sigma_B^2 - 2C_{AB})}$$

Objectives/Challenges

- The objective of our model is maximizing the Sharpe Ratio
  - The Sharpe ratio is simply a measure for how well an investment performs relative to its risk.
  - The Sharpe ratio combines maximizing the return and minimizing the risk into a single unit measure, therefore making it the ideal objective to maximize
  - Maximizing the Sharpe ratio is critical in the decision problem since it is a key metric in describing the overall performance of the portfolio
- Some of the challenges that we faced included the following:
  - Finding an appropriate LP model due to the plethora of models that have been developed for portfolio optimization
  - Determining the optimal objective for the LP model as different models optimize for different objectives
  - Formulating the LP model such that it adheres to the general assumptions that summarize the relationship between risk, return, and diversification

Proposed Methodologies/Assumptions

- Many of the existing models are centered around the core principles of MPT, most importantly the Markowitz Mean-Variance (MV) Model, while alternative approaches to the MV model have been devised as well
- The assumptions that explain the relationship between risk, return, and diversification are essential to modern portfolio theory and therefore formulating the LP model as such was important
- Some of these assumptions include:
  - Investors attempt to maximize returns given their unique situation
  - Asset returns follow a normal distribution
  - Investors are rational and avoid unnecessary risk
  - Unlimited amounts of capital can be borrowed at the risk-free rate
- The methodology for modeling the decision problem primarily involved formulating the appropriate LP model.

Approach/Model Formulation

- The Data
  - We picked 50 stocks out of the S&P 500
  - Each stock used historical data from 2015-2021
  - We calculated the mean, and the standard deviation of each stock that we picked
  - Using Excel “PsiNormal” function we generated random number for expected return
- Objective - Maximize the Sharpe Ratio using different weight for each stock over one year
- Variables - Weight for each stock
- Our model is based on the assumptions that summarize the relationship between, return, risk, and diversification.
- Some of the assumptions that are specific to our model include:
  - Hard constraint: .1% <= weights <= 20%
  - Soft constraint: overall risk of portfolio
- Covariance matrix is used to calculate the standard deviation of the portfolio of stocks used in the denominator of the Sharpe Ratio

Sharpe Ratio Formula

- Highly used method that calculates the the risk-adjusted return
- In our formula we assumed that risk is equal to volatility,
- Greater Sharpe ratio value points out better risk-adjusted-performance
- R_p= return of portfolio
- R_f = risk-free rate
- σ_p= standard deviation of the portfolio’s excess return

Formula and Calculation of Sharpe Ratio

$$Sharpe\ Ratio = \frac{R_p - R_f}{\sigma_p}$$

where:

R_p = return of portfolio

R_f = risk-free rate

σ_p = standard deviation of the portfolio’s excess return

Stochastic Simulation

- Expected returns of stocks is the most critical part of portfolio optimization
- Stochastic simulation finds the max/min of an objective function when randomness is present
- Our model used randomness in the expected returns of the individual stocks
- The uncertainty is modeled through sampling the normal distribution based on the historical daily returns and standard deviation of each stock
- The model uses simulation optimization to find the weights to maximize the Sharpe Ratio
- Our model use the simulation optimization feature of analytical solver
- The simulations were set to 1 and the trials set to 1000

Results

- A Sharpe Ratio between 1 and 2 is considered to be good by industry standards, between 2 and 3 considered very good, and anything over 3 is considered to be excellent
- For our model our target was a Sharpe Ratio of at least 1
- Our model ended with a average Sharpe Ratio of 1.71
- An important concept in portfolio management is Beta
    - Beta measures how a stock moves with the overall market i.e., volatility
- To balance risk a portfolio should have a good mix of high and low betas
- Our top 5 stocks have a good mix which indicates good portfolio fundamentals

- Top 6 stocks by weight

Stock	Weight	Beta
AMZN	20.00%	1.15
NVDA	20.00%	1.37
LLY	12.59%	0.25
COST	8.80%	0.64
DHR	8.10%	0.64
TSLA	8.00%	1.98

Achievements/Limitations

- Achievements :
  - A Sharpe Ratio of 1.71 was achieved
  - Creating a portfolio that had a good mix of high and low betas was achieved
    - Specifically, we were able to create a portfolio that had a good mix of small and large beta values which is important in balancing the volatility of the portfolio
  - Creating a well diversified portfolio was achieved
- Limitations:
  - Since we are using standard deviation of returns in the denominator we assume the returns are normally distributed
  - This is not the case as many stocks has spikes or downfall,
    - Example - COVID-19 impact on the ROI of technology companies
  - In the real world, this approach can be somewhat biased if the model is using skewed data
  - If we are adjusting the weights every year or so, high spikes can impact the model grealy

Future work

- Make sure that we are picking neutral look-back period for the stocks
- Make the model more Dynamic, for example adjusting the portfolio every quarter or a year
- Increase the number of stocks to pick from, we only picked 50 stock, playing with the constraints can give different values.
- Adding boolean values that define how many stocks the model should pick
- The portfolio could also include other investment vehicles such as bonds, ETFs, crypto currencies, etc