

Chaos in a Box Model

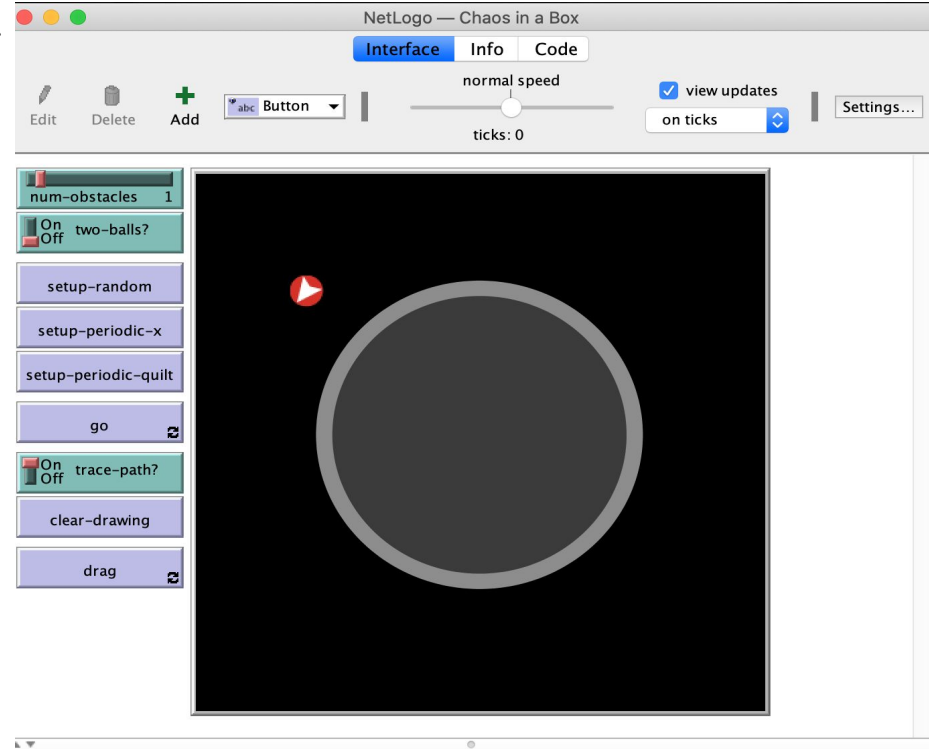
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Math 168 - Winter 2020

Professor Pancaldi

Background (Anaswar)

- Chaos in a Box is a simple system consisting of one or more balls and zero or more obstacles enclosed in a box
- Collisions alter the trajectory of the balls
- The model assumes that the collisions are perfectly elastic, the obstacles remain stationary, and there is no friction being applied to the ball.



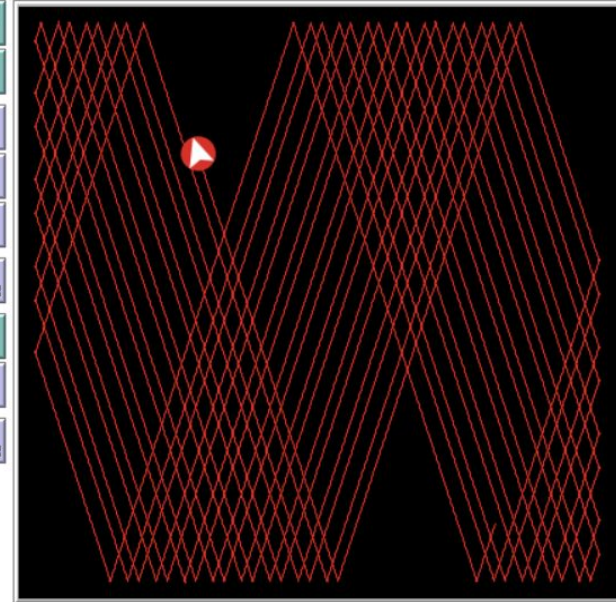
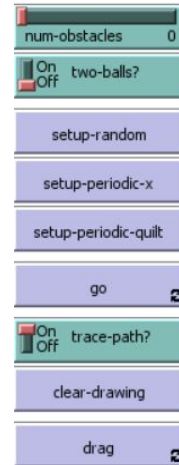
Model Description (Anaswar)

- ‘Chaos in a Box’ works by modelling the path that the ball takes as it travels around the room that it is placed in.
- In this model, the important components are:
 - NUM-OBSTACLES
 - TWO BALLS
 - SETUP-RANDOM
 - SETUP-PERIODIC-X
 - SETUP-PERIODIC-QUILT
 - TRACE-PATH
 - CLEAR-DRAWING
 - DRAG
 - GO



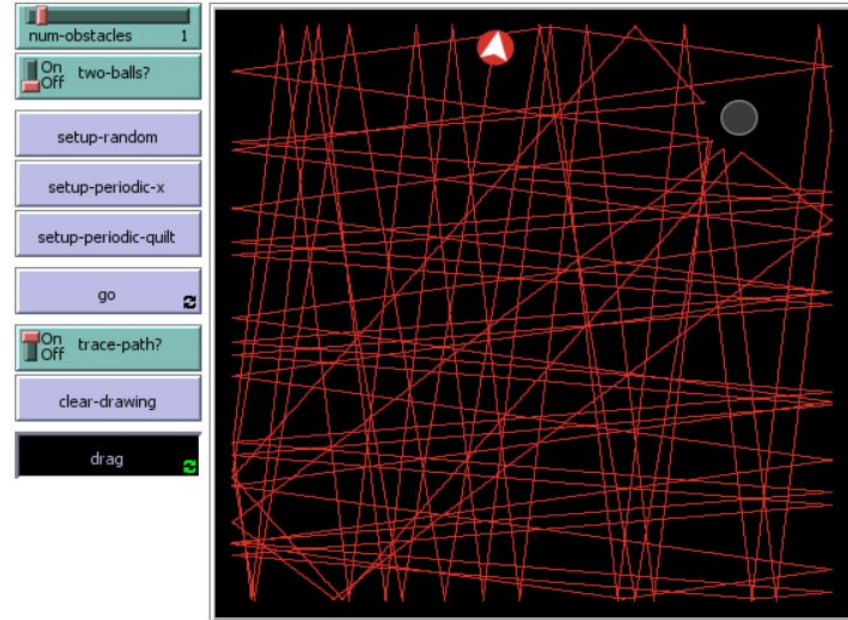
Results -No obstacle present (Anaswar)

- In order to determine the underlying dynamics of our model we ran multiple trials with varying parameters
- When no obstacles are present we observed that the ball's trajectory converges towards a periodic trajectory



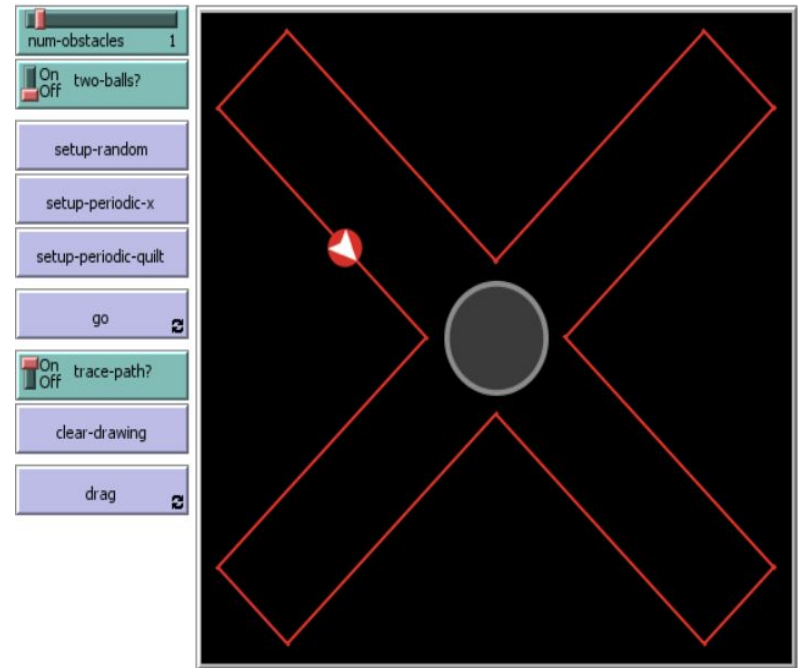
Results-One obstacle present (Anaswar)

- Initialized an obstacle at coordinate (5,5) with a size of 1
- The trajectory of the ball was periodic until it collided with the obstacle, resulting in chaotic motion



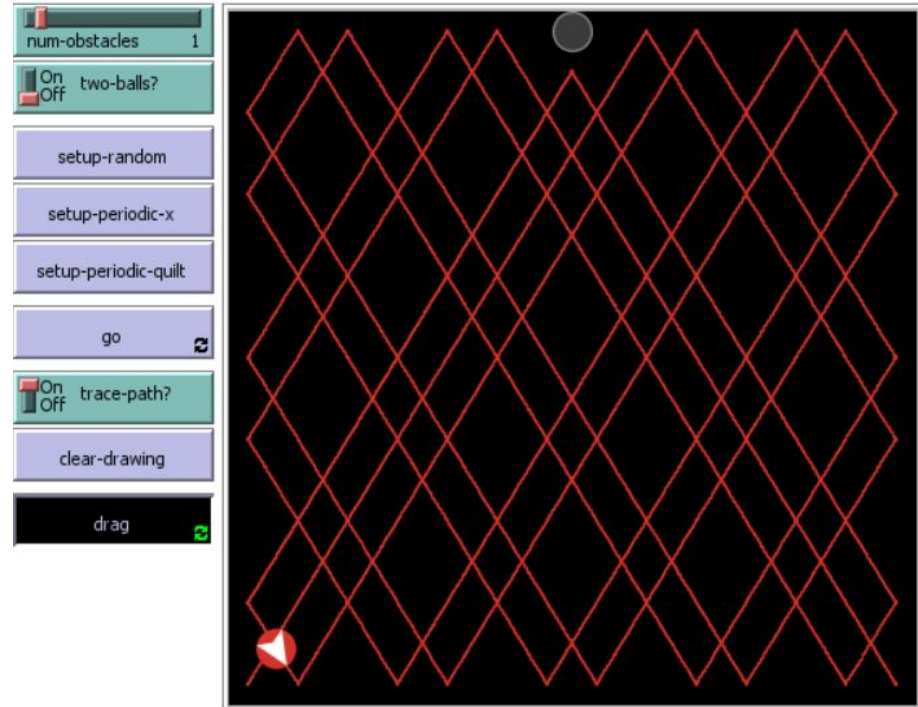
Results-Setup-periodic x (Tianyang)

- The setup-periodic-x option places the obstacle in the center at (0,0) with a size of 3
- Upon colliding the ball changes its direction 90 degrees
- Ball remains in a periodic trajectory despite there being an obstacle present



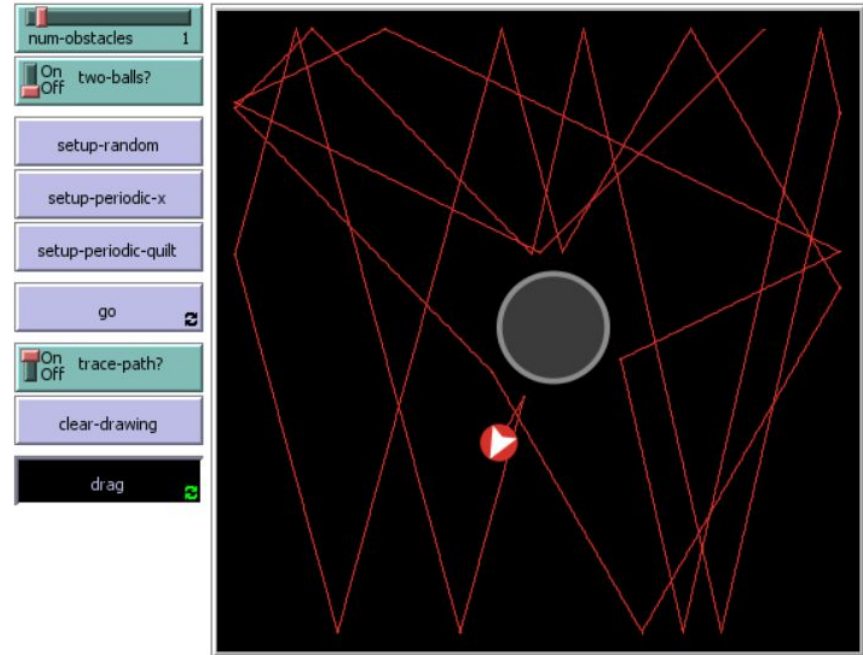
Results-Setup periodic quilt (Tianyang)

- The ‘setup-periodic-quilt’ option initializes the obstacle at coordinate (0,8) with a size of 1
- The ball displays a periodic trajectory that resembles a quilt
- Result is similar to ‘setup-periodic-x’ with regards to remaining in a periodic trajectory



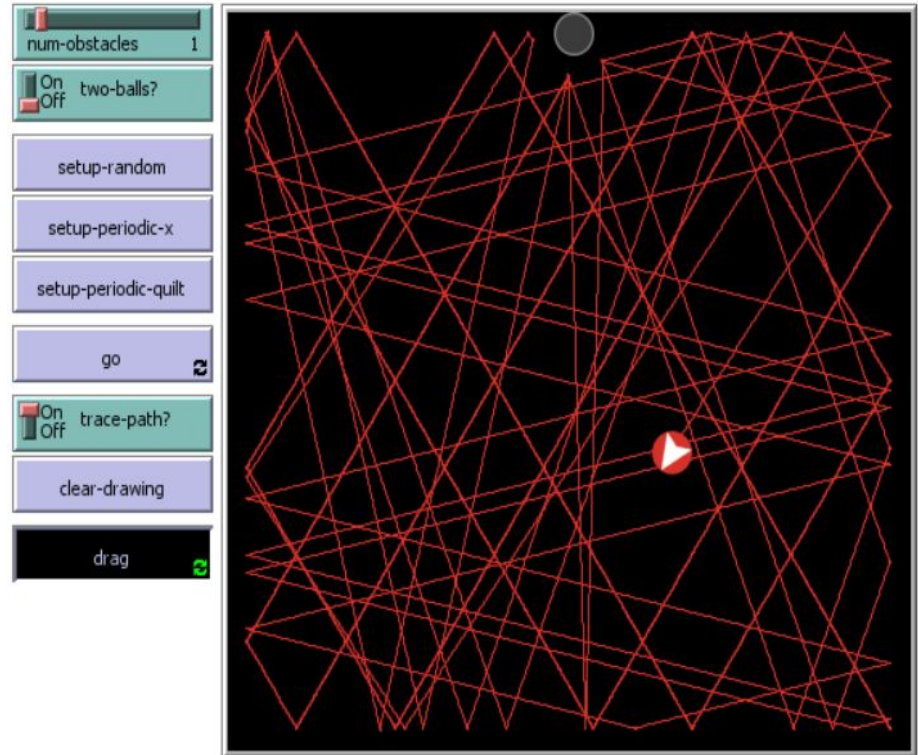
Results- Setup Periodic x shifted 0.01 to the right (Tianyang)

- Initialized the simulation using 'setup-periodic-x' and moving the obstacle to the right 0.01
- Trajectory no longer periodic

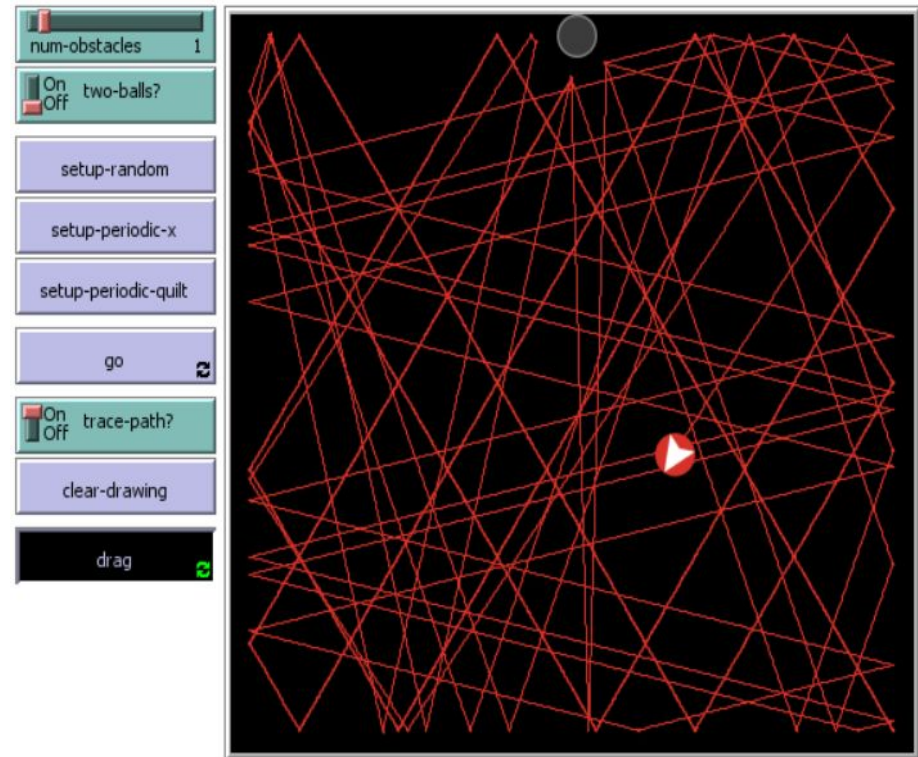
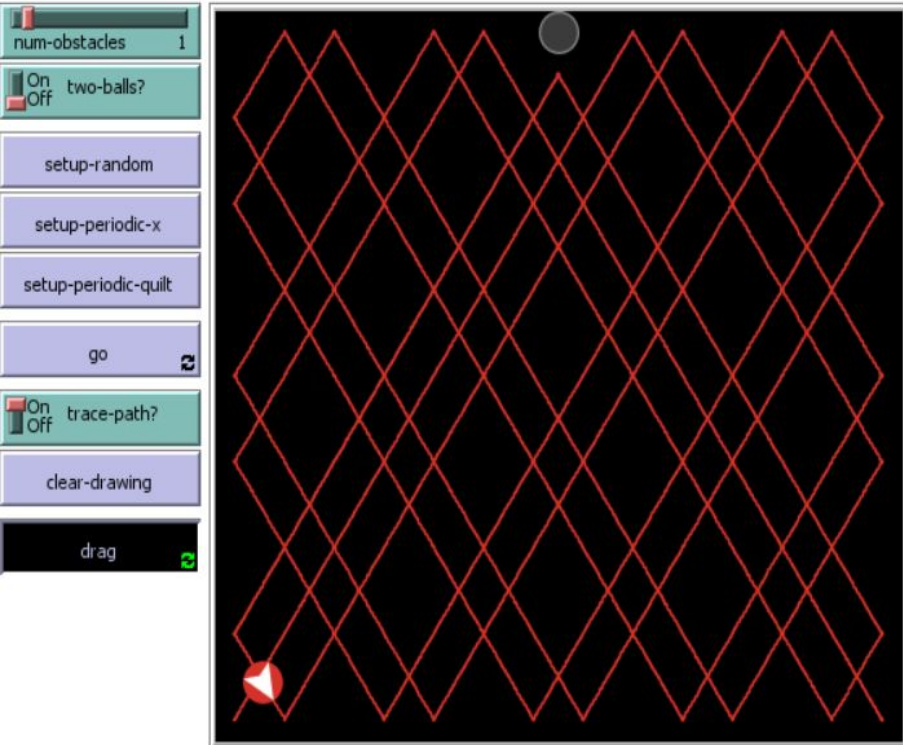


Results- Setup-Periodic quilt shifted 0.1 to the right (Tianyang)

- Initialized the simulation using 'setup-periodic-quilt' and moving the obstacle to the right 0.1
- Trajectory no longer periodic

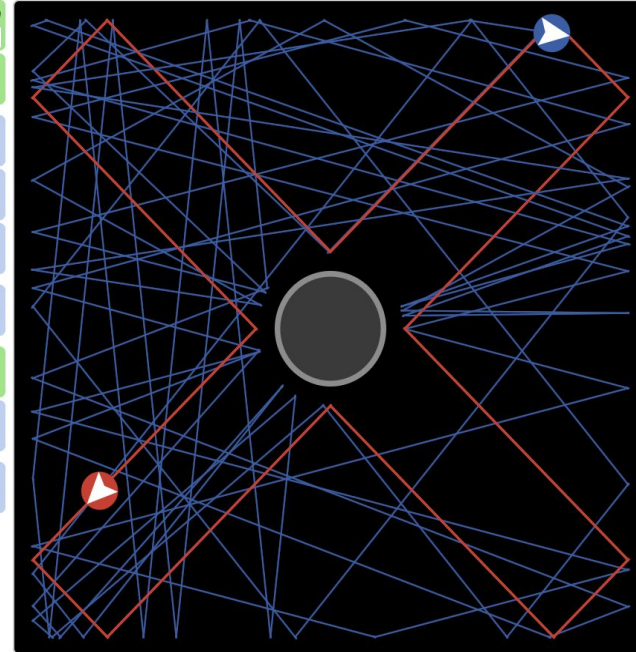
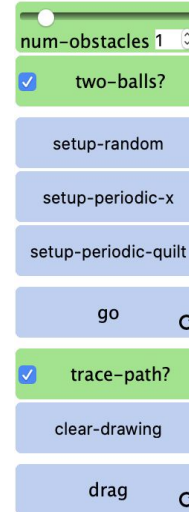


Marco



Results-Setup-periodic-quilt with two balls (Marco)

- Selecting the two balls option places another ball in the box with a slightly different position than the first
- This results in the trajectory no longer being periodic



Setup-periodic-quilt with two balls (Marco)

- Selecting the two balls option places another ball in the box with a slightly different position than the first
- This results in the trajectory no longer being periodic

num-obstacles 1

☒ two-balls?

setup-random

setup-periodic-x

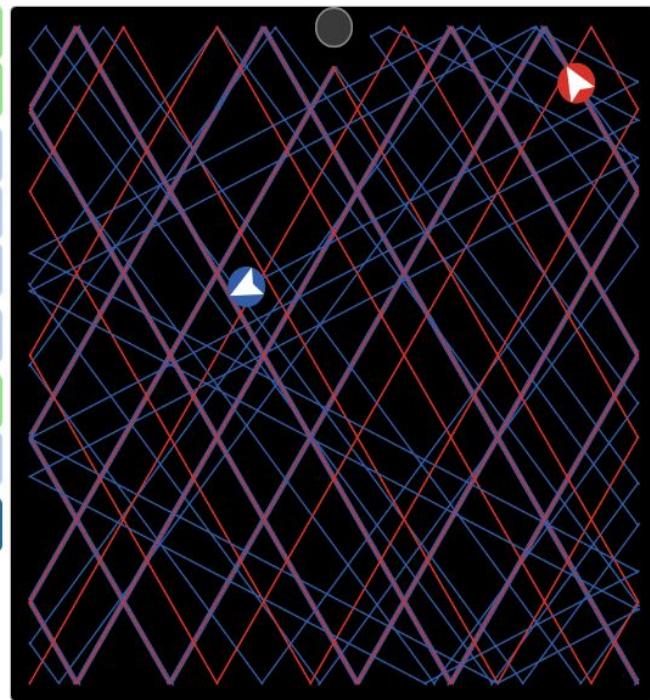
setup-periodic-quilt

go

☒ trace-path?

clear-drawing

drag



Modifications (Marco)

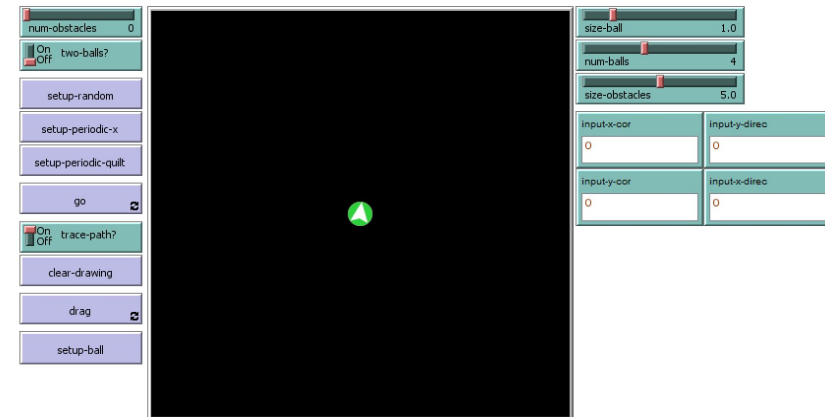
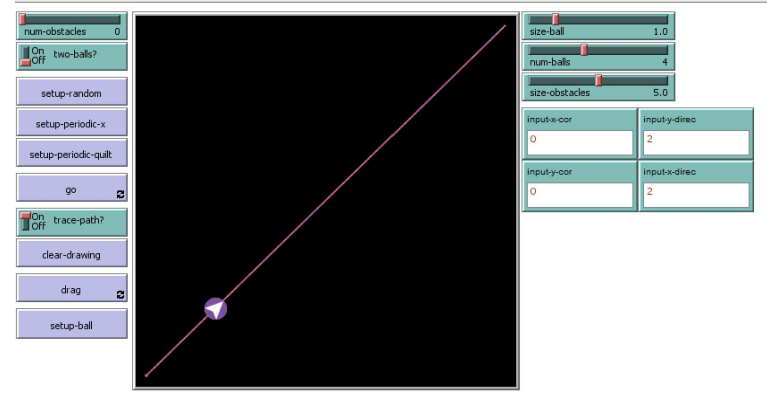
- Increased the number of parameters used in the model:
 - Position of Ball
 - Direction of Ball
 - Size of Ball
 - Number of Balls
 - Size of Obstacles



input-x-cor 0	input-y-direc 0
input-y-cor 0	input-x-direc 0

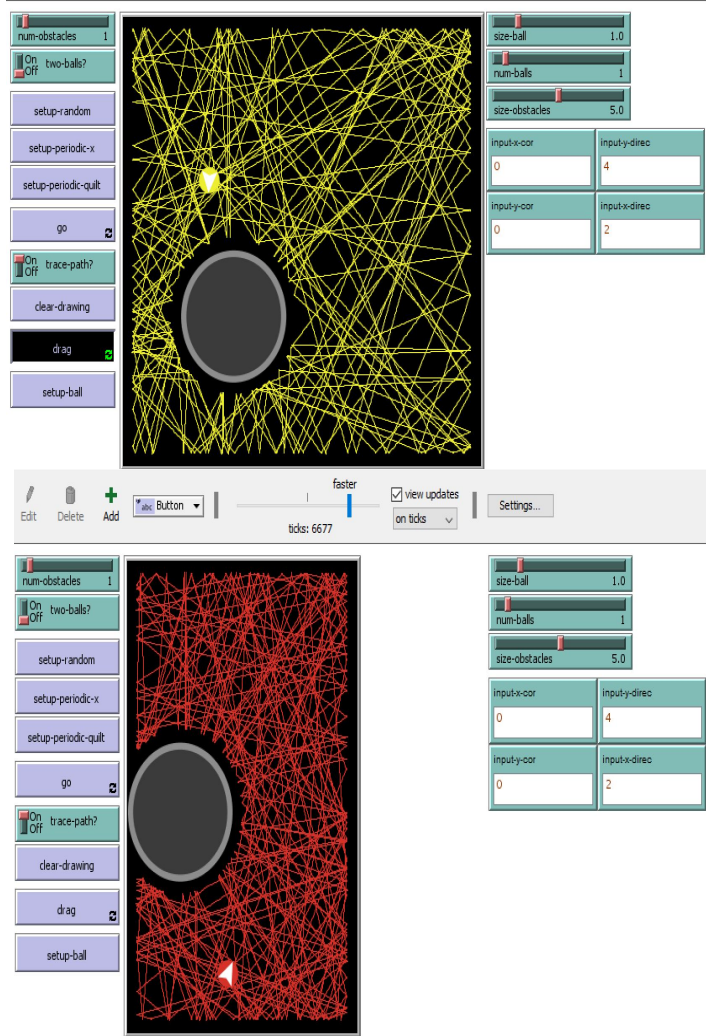
Modifications (Yan)

- The modification allowed for the exact positioning of initialized balls
- Allowed the user to specify the direction that the balls face



Modifications (Yan)

- For this modification we changed the shape of the box
- Increasing the box by 100% results in a decrease in chaotic motion
- Decreasing the shape of the box from square to rectangle with dimensions of 10Wx16L. This resulted in increased chaotic motion



Conclusions (Yan)

1. A ball's trajectory converges towards a periodic path when there no obstacles present
2. When obstacles are present, collisions result in a change of trajectory that results in chaotic behavior
3. Small changes in initial condition result in the ball's trajectory being significantly different than before
4. Under certain conditions (ie. setup-periodic-x and setup-periodic-quilt), the ball still exhibits periodic trajectory despite there still being a obstacle present in the box
5. The change in parameters that facilitate contact with the obstacle will result in more chaotic behavior (i.e. increasing/decreasing balls size and speed, number of balls, the box size and shape etc...)

Future work (Yan)

1. Include non-elastic behavior for collisions in order to properly simulate a system with high entropy
2. Implement collision between the balls themselves
3. Extend the model to include mass as a parameter
4. Potentially create a 3-dimensional version of the model

References

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- Wilensky, U. (1999). NetLogo. <http://ccl.northwestern.edu/netlogo/>. Center for Connected Learning and Computer-Based Modeling, Northwestern University, Evanston, IL.
- Aguirre, Luis A., and Christophe Letellier2. "Modeling Nonlinear Dynamics and Chaos: A Review." *Mathematical Problems in Engineering*, vol. 2009, 2 June 2009, pp. 1-35. <https://scholar.google.com/>, doi:10.1155. Accessed 9 Feb. 2020.
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- Math 168 Chaos in a box, Project 1. 2020 Winter
- Math 168 Gas in a box, Project 2. 2020 Winter