Chaos in a box

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1. Abstract

For this report we investigated the NetLogo model Chaos in a Box. Applicable towards a variety of different fields, the Chaos in a Box model is a system that simulates the motion of one or more balls present with one or more obstacles within a square box. This model designates the obstacles as stationary while allowing the balls to move freely around the box. While moving freely, the balls collide with both the obstacles and the edges of the box, but do not collide with each other. The trajectory of the balls are altered by collisions, and while the system primarily exhibits chaotic behavior, it can also show periodic behavior when the initial conditions are appropriate. We utilized this model to study the underlying dynamics of chaotic and periodic behavior as well as explore how miniscule changes in initial conditions affect the behavior of a system. We ran multiple trials which included a system with only one ball and no obstacles present, one ball and one obstacle present, and multiple balls and one obstacles present. During the simulations we observed that chaotic and periodic behavior was possible when obstacles were both present and absent. We noted that when obstacles were absent all initial conditions result in periodic trajectories, but only very specific conditions resulted in periodic trajectories when an obstacle was present. We also concluded that as the size of the obstacle increased, the likelihood that a collision between the ball and the obstacle would occur increased, resulting in chaotic motion.

2. Introduction

The main goal of the Chaos in a Box model is to simulate the movement of one or more balls as they collide with the walls and obstacles. This model was created by two faculty members from Northwestern University who drew inspiration from a topic that was covered in a Complex Systems in Biology course at the Humboldt University in Berlin. This model also utilizes a number of advanced techniques, such as anonymous procedures that allow the setup procedure to be flexible with respect to the initial position of the obstacles and the ball and the binary search method which corrects for errors that occur during chaotic systems such as in the current model. The Chaos in a Box model makes a number of assumptions. First it assumes that the collisions that occur are all perfectly elastic. Second, it assumes that the obstacles are always stationary. Lastly, the model assumes that there is no friction applied to the ball, resulting in no changes in speed.

Also, this model only models the movement of a single ball as opposed to the movement of multiple balls since multiple balls are only representative of change that results due to small changes in initial position. Furthermore, this model has been programmed with the capability to correct the position of the ball when a collision occurs due to overlap between either itself and the walls of the box or itself and the obstacle. During this correction, the ball's new angle is calculated as the angle of reflection of its heading that is around the surface that the ball is colliding with. The Chaos in a Box model has been utilized in other models found on NetLogo such as GasLab's Gas in a Box and the Kick Rotator model. Moreover, the Chaos in a Box model has been used in the branch of mathematics called Chaos Theory in order to study the chaotic behavior that exists in a variety of complex systems ranging from fluid flow, the stock market, weather and climate, heartbeat irregularities, and road traffic. The application of Chaos in a Box in the field of nonlinear dynamics and chaos is discussed in the paper "Modeling Nonlinear Dynamics and Chaos: A Review", written by Christophe Letellier from the University of Rouen in France. In this paper, the authors discuss the application of important developments and modeling techniques in the field of nonlinear dynamics and chaos. In addition, the authors cover the important topics that are related to the development of modeling techniques such as model representations, parameter estimation techniques, data requirements, and model validation

3. Model Description

The model works by showing the path that the ball takes as it travels around the room that it is placed in. In this model, the important components are the NUM-OBSTACLES slider, the TWO BALLS button, the SETUP-RANDOM button, the NUM-OBSTACLES button, the SETUP-PERIODIC-X and SETUP-PERIODIC-QUILT buttons, the TRACE-PATH switch and CLEAR-DRAWING button, SIZE-BALL slider, NUMBER-BALLS slider, SIZE-OBSTACLES slider, INPUT-X-COR input, INPUT-Y-COR input, INPUT-X-DIREC input, INPUT-Y-DIREC input, SETUP-BALL button,and lastly the DRAG button. The number of obstacles slider allows the user to adjust the number of obstacles that are placed around the room. The two balls button allows for the creation of another ball that's in a different position compared to the original ball as well as show how significant of an effect slight changes in the initial conditions can have in chaotic systems. The setup random button initializes the system so that chaotic behavior is most likely shown and the number of obstacles button places obstacles of different sizes at random positions throughout the room. The setup periodic x button predicts the kind of periodic behavior that the system can potentially exhibit and the setup periodic quilt button creates another example of periodic behavior except that the periodic behavior demonstrated by setup periodic quilt is over a long period. The size of ball and obstacle easily allows for the observation of the ball's motion. The number of balls button allows for the user to enter the number of balls that appear in the box. The initial position of the ball is set up by the input values of x-coordinate and y-coordinate and the initial direction of the ball is set up in a similar way. The setup-ball button is for people who want to see the result of the input value. Lastly, the trace path switch outlines the path of the ball, the clear drawing button clears the outlined path, and the drag button allows for user interaction with the model, in particular the balls, obstacles, and the respective direction and size. In the model, although each of the components work independent of each other, each component has a dedicated role and function, there are some components that work together. For instance, both the SETUP-PERIODIC-X and SETUP-PERIODIC-QUILT buttons work together to create periodic behavior that could potentially be exhibited by the system. In addition, the components that are required to set up the model work in a cohesive way to create a system that will most likely be chaotic.

4. Simulation Results

In order to elucidate the underlying dynamics of our model we ran multiple trials with varying parameters. As done in our previous paper, here we report our observations when running the "Gas in a Box" model under different conditions. For run 1, we set the number of obstacles inside of the world to 0 (**Figure 1**). Next we clicked the 'setup-random' option and began the simulation. Throughout the model we noted that the trajectory of the ball is periodic. For run 2 we set an obstacle present at coordinate (0,0) and initialized it at a size of 1 (**Figure 2**). In this simulation we observed that when the ball does not make contact with the obstacle, the trajectory is periodic. However, once contact is made the trajectory exhibits chaotic behavior. After a few seconds the trajectory converges to a new periodic path until it once again comes into contact with the obstacle. For run 3, we initialized the position of the obstacle at coordinate (0,0) and assigned it a size of 2 (**Figure 3**). We observed that increasing the size of the obstacle resulted in a greater likelihood that a collision with the ball would occur, leading to more chaotic behavior. For run 4 we again increased the size of the obstacle and assigned it a size of 4. We positioned it at (0,0) and ran the simulation using the 'setup-random' option (**Figure 4**). Here, we observed similar behavior to run 3. Increasing the size of the obstacle reduced the amount of time that the ball spent in a periodic trajectory and increased the amount of time spent in a chaotic trajectory. For run 5 we ran the simulation using the 'setup-periodic-x' option and initialized the obstacle at size 3 and position (0,0) (**Figure 5**). We noted that despite there being an obstacle present in the box the ball still exhibited a periodic trajectory. For run 6 we ran the simulation using the 'setup-periodic quilt' option, which placed the obstacle at position (0,8) with a size of 1 (**Figure 6**). The behavior of the ball in run 6 was similar to that of run 5 with regards to both displaying periodic behavior, however the trajectory of the ball in run 6 followed a quilt like pattern.

We then ran trials of the 'Gas in a Box' model using the 'two-balls?' option. When selecting the 'setup-periodic', we noted that the initial directions and positions of the balls were very near each other but were not exactly the same (**Figure 7**). When running the simulation we observed that the two balls had trajectories that were similar when they were periodic, but completely different after colliding with the obstacle. This behavior is again observed when we ran the simulation using the 'setup-periodic-quilt option (**Figure 8**). We noted that the balls' directions and positions were near each other but not equal, which resulted in completely different trajectories after colliding with the obstacle.

Modification #1:

For this project we looked to extend the model to allow for a more realistic interpretation of how different gases in an enclosed space would interact. Given the fact that gases are of varying sizes, masses and speeds, we opted to include the following parameters into the ‘Chaos in a Box’ model: ‘ball-size’, ‘num-balls’, and ‘ball-speed’ (**Figure 9**). We implemented these parameters using sliders so that it would allow for the user to set the number of balls present, the ball size, and ball speed. For run 1 of our modification, we initialized 4 balls with different colors and set their size to 1 (**Figure 10**). We set their parameters to ball-size 1, and ‘ball-speed’ 0.25. We observed that the trajectory of all the balls was periodic when no obstacles were present in the box, which agrees with our previous findings. For run 2 we kept all parameters constant and initialized an obstacle at coordinate (0,0) (**Figure 11**). When running the simulation, we noted that the balls’ trajectories were no longer periodic after colliding with the obstacle. Upon colliding the balls adopt a chaotic trajectory. For run three we investigated how increasing the speed would affect the balls’ trajectories. When increasing the speed of the balls to 0.5 we observed that this led to more chaotic behavior throughout the simulation (**Figure 12**). Lastly, for run 4 we increased the size of the ball to 4 (**Figure 13**). This change resulted in more chaotic behavior through the simulation as the balls had a smaller distance to travel before colliding with the obstacle.

Modification #2:
In addition to these modifications, we’ve also included the option of entering the exact position and direction that a ball is initialized at. For run 1 of modification 2 we set 4 balls with a direction vector of (2,2) and a position at (0,0) (**Figure 14**). Running the simulation with no obstacles, we observed that the balls followed a periodic trajectory, which further confirmed our previous results.

Modification #3:

Similar to the modification we made in our previous report, we increased the size of the box. Initially,the size of the world is at a size of 8 with x and y maximums at 8, and x and y minimums at -8. For the first run we increased the size of the box by 100% to look at how this would affect the dynamics of our model (**Figure 15**). We observed that the model behaved similarly to previous runs and exhibited chaotic behavior only after colliding with the obstacle. However, one thing to note is that increasing the size of the box allowed for a greater time spent in a periodic trajectory before colliding and reverting to chaotic behavior.

5. Conclusion and Future Work

As stated in the model description for ‘Chaos in a box, this model does not consider friction and for this report we did not take into account the condition where the obstacle is moving. In addition, the speed of the ball does not change as the collisions are perfectly elastic.

When running a simulation with no obstacles we noted that the trajectory of the ball was periodic and continued to be periodic throughout the entirety of the run. On the other hand, when an object is present the ball exhibits chaotic motion. From these findings we conclude that it is the constant perturbation from a periodic trajectory that results in chaotic motion. This is observed in all of the setup-random runs where at least one obstacle is present. However, we noted that there are specific cases where even when an obstacle is placed within the box the ball still displays periodic motion, as seen in the case of ‘setup-periodic-x’ and ‘setup-periodic-quilt’. Taking a closer look, we noted that when the ball collided with the obstacle at a 90 degree angle the trajectory remained periodic. This was confirmed when we adjusted the direction of the ball slightly, resulting in the trajectory no longer being periodic. Thus, we hypothesize that the ball must collide with the obstacle at a 90 degree angle in order for the ball’s trajectory to remain periodic.

While analyzing our runs for the model, we noted that the initial conditions of the simulation had a significant impact on the trajectory of the ball. A slight change in either direction or position will result in a completely new trajectory. As a result, we conclude that small changes in initial conditions will result in large changes in the dynamics of the system.

The modifications that we included for this report also allowed us to determine that any change in parameters that facilitate contact with the obstacle, whether it be increased ball size or speed, will result in more chaotic behavior displayed by the system.

This model provides insight into the behavior of chaotic systems. In this report we have attempted to extend the model in order to better understand the kinetic interactions that gases undergo within an enclosed space. However, this can still be improved. The most effective way to create a more robust model would be to implement collision detection for the balls themselves. This would allow for a more accurate representation of what occurs when gases collide. In addition it would be beneficial to include non-elastic collision behavior in order to properly simulate a system with high entropy. Since gases behave in a highly disordered manner, an improved model would allow for deeper insight into how different gases differ in their dynamics.

6. References/Figures

References

1. Head, B. and Wilensky, U. (2017). NetLogo Chaos in a Box model. http://ccl.northwestern.edu/netlogo/models/ChaosinaBox. Center for Connected Learning and Computer-Based Modeling, Northwestern University, Evanston, IL.
2. Wilensky, U. (1999). NetLogo. http://ccl.northwestern.edu/netlogo/. Center for Connected Learning and Computer-Based Modeling, Northwestern University, Evanston, IL.
3. Aguirre, Luis A., and Christophe Letellier2. "Modeling Nonlinear Dynamics and Chaos: A Review." *Mathematical Problems in Engineering*, vol. 2009, 2 June 2009, pp. 1-35. *https://scholar.google.com/*, doi:10.1155. Accessed 9 Feb. 2020.
4. Wilensky, U. (1997). NetLogo GasLab Gas in a Box model. http://ccl.northwestern.edu/netlogo/models/GasLabGasinaBox. Center for Connected Learning and Computer-Based Modeling, Northwestern University, Evanston, IL.
5. Math 168 Chaos in a box, Project 1. 2020 Spring

Figures

Figure 1: Zero obstacles in the box (Depicts periodic motion)

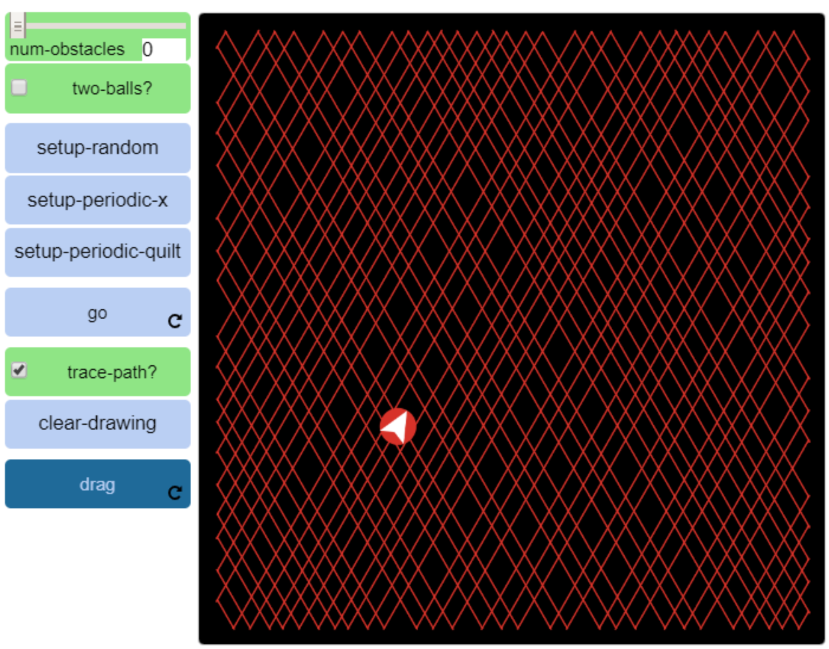

Figure 2: Obstacle of size 1 at (0,0). (Depicts chaotic motion)

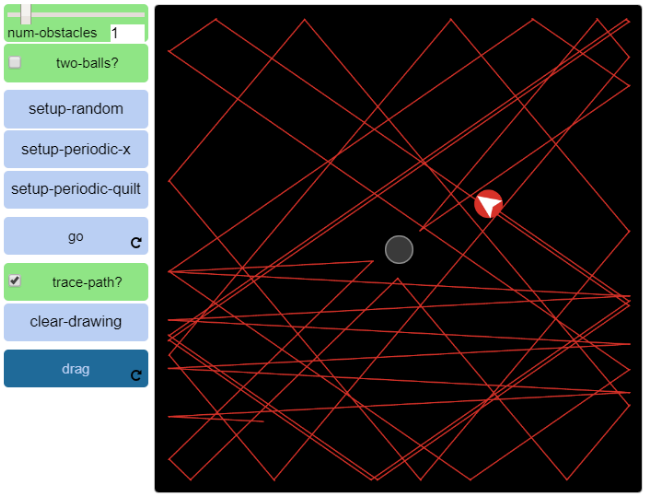

Figure 3: Obstacle of size 2 at (0,0). (Chaotic)

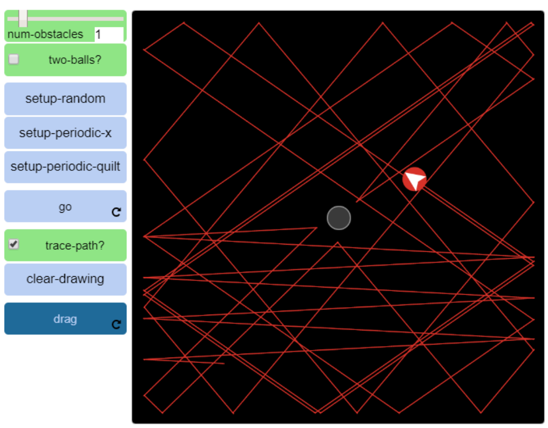

Figure 4: Obstacle of size 3 at (0,0). (Depicts chaotic)

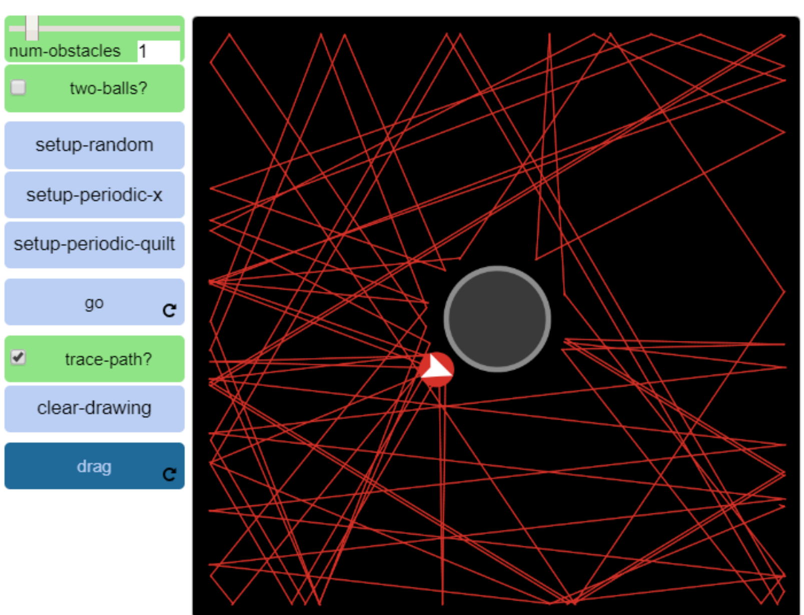

Figure 5: Set-up-periodic option with 1 ball. (Depicts periodic)

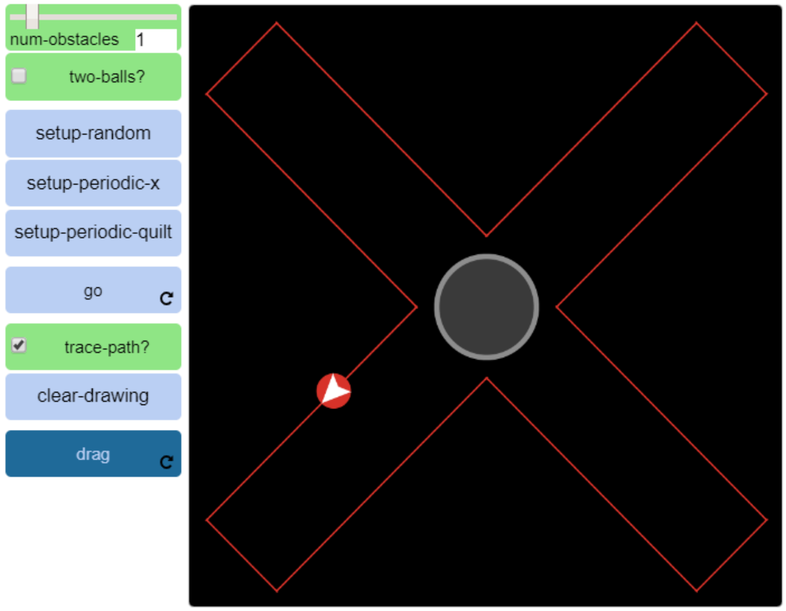

Figure 6: Set-up-periodic-quilt option with 1 ball. (Depicts periodic motion)

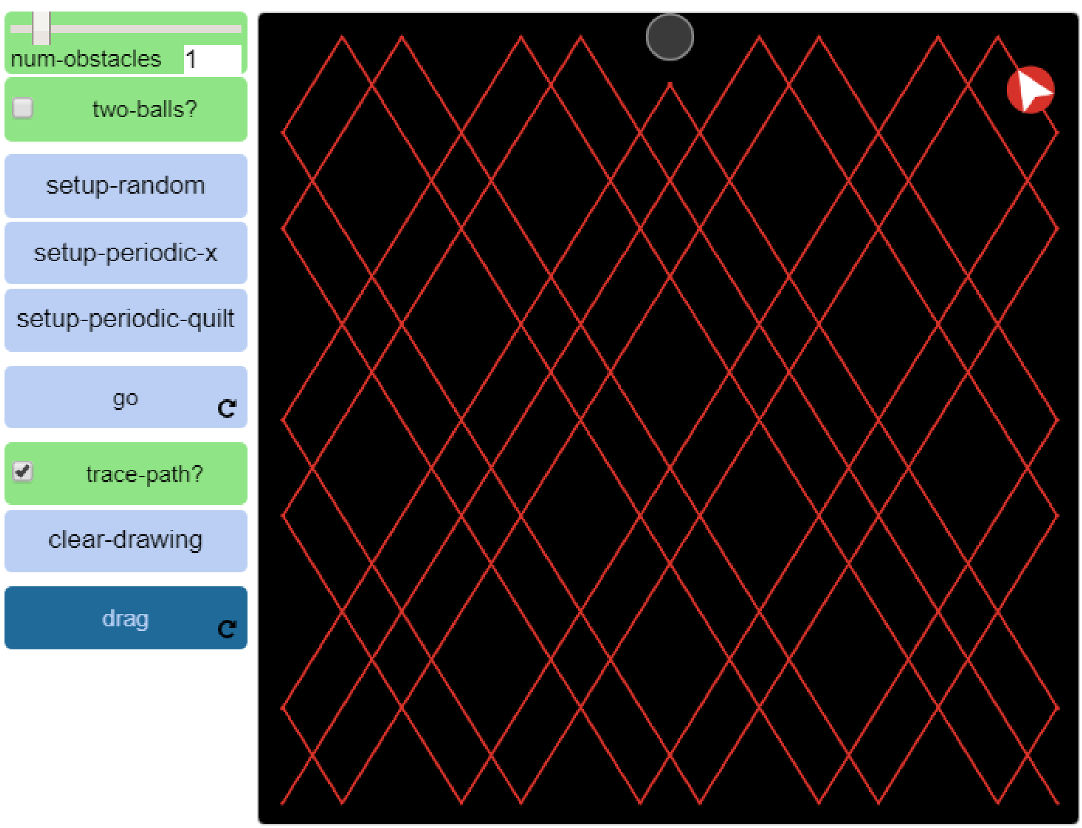

Figure 7: Setup-periodic option with 2 balls. (Depicts periodic and chaotic motion)

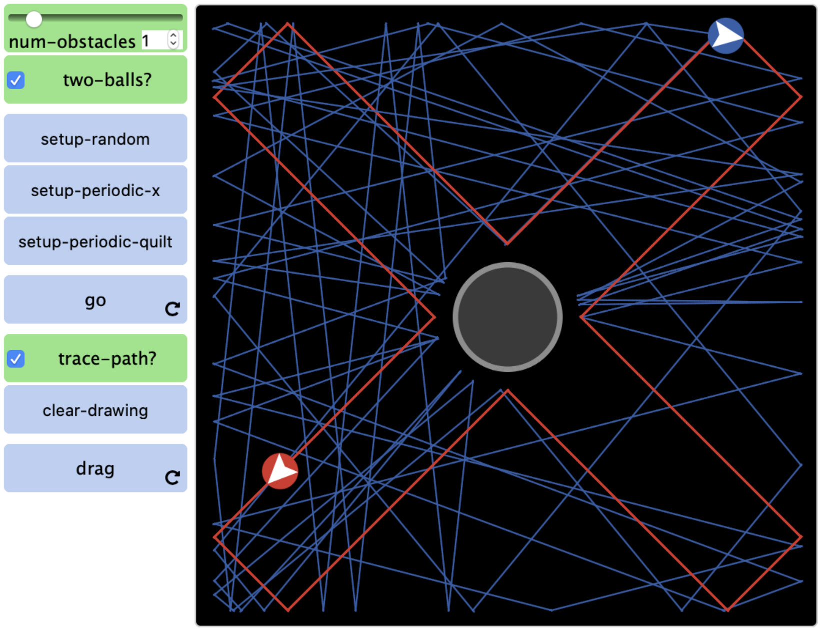

Figure 8: Set-up-periodic-quilt option with 2 balls.(Depicts chaotic and periodic motion)

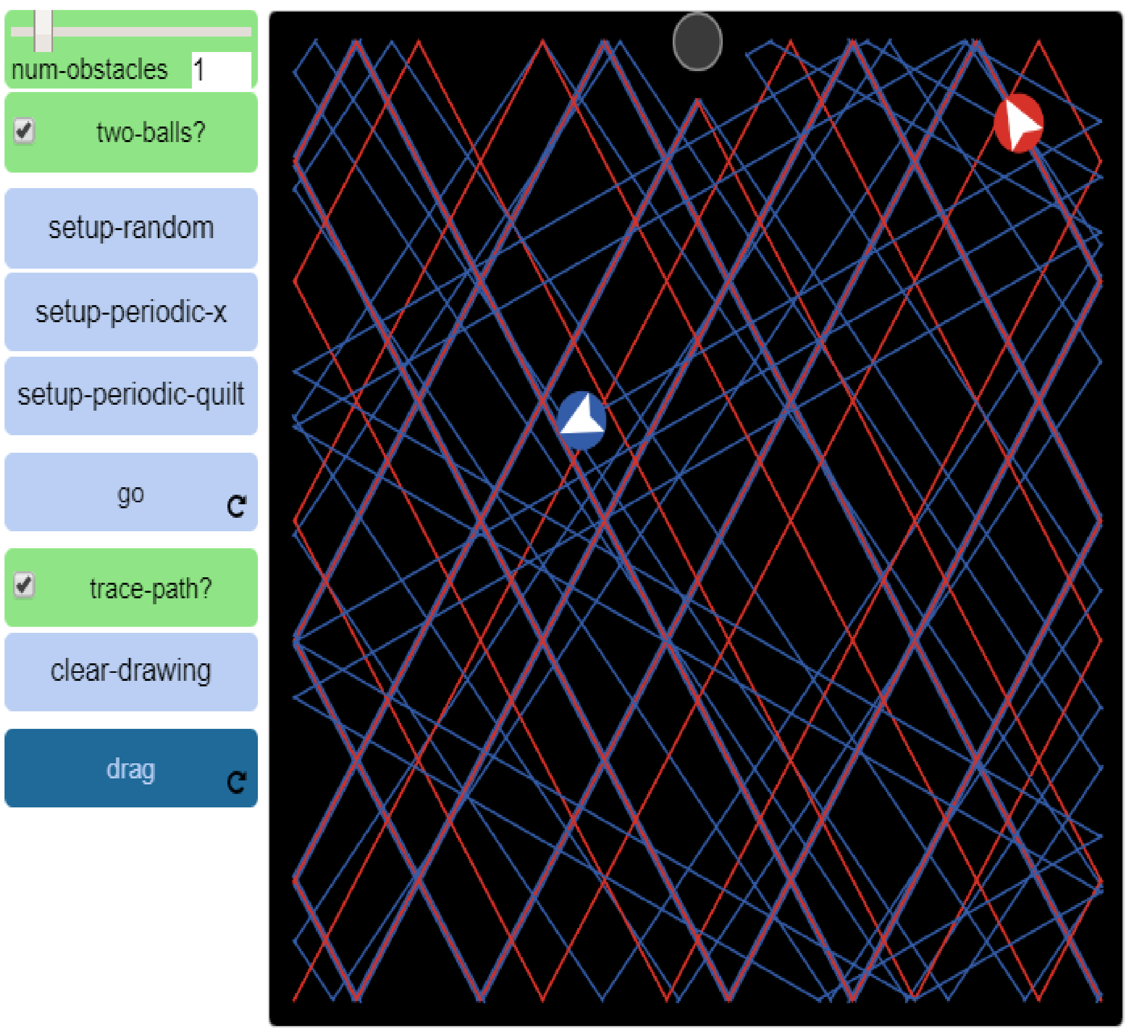

Figure 9: Parameters added as part of modification to model

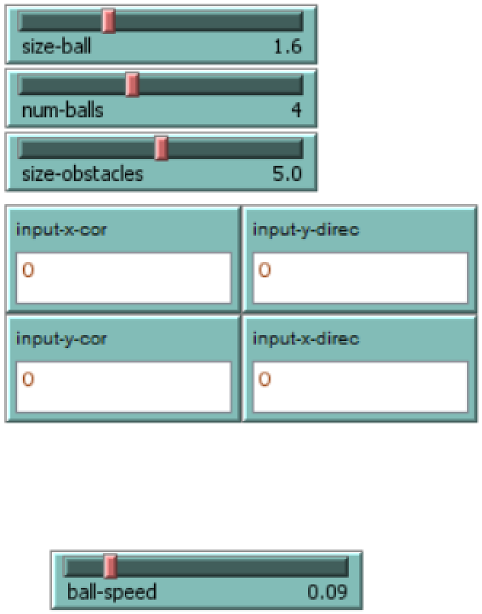

Figure 10: 4 balls initialized with no obstacles (Depicts periodic motion

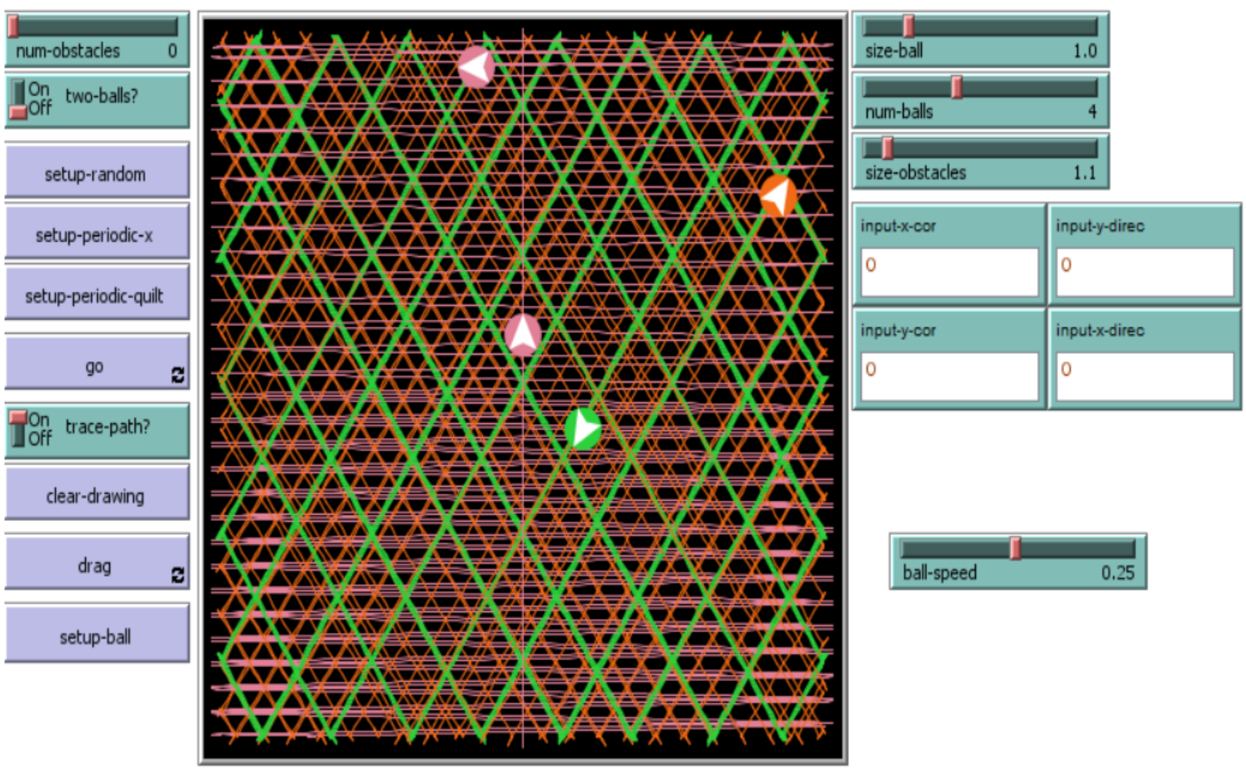

Figure 11: 4 balls 1 obstacle (Depicts chaotic motion)

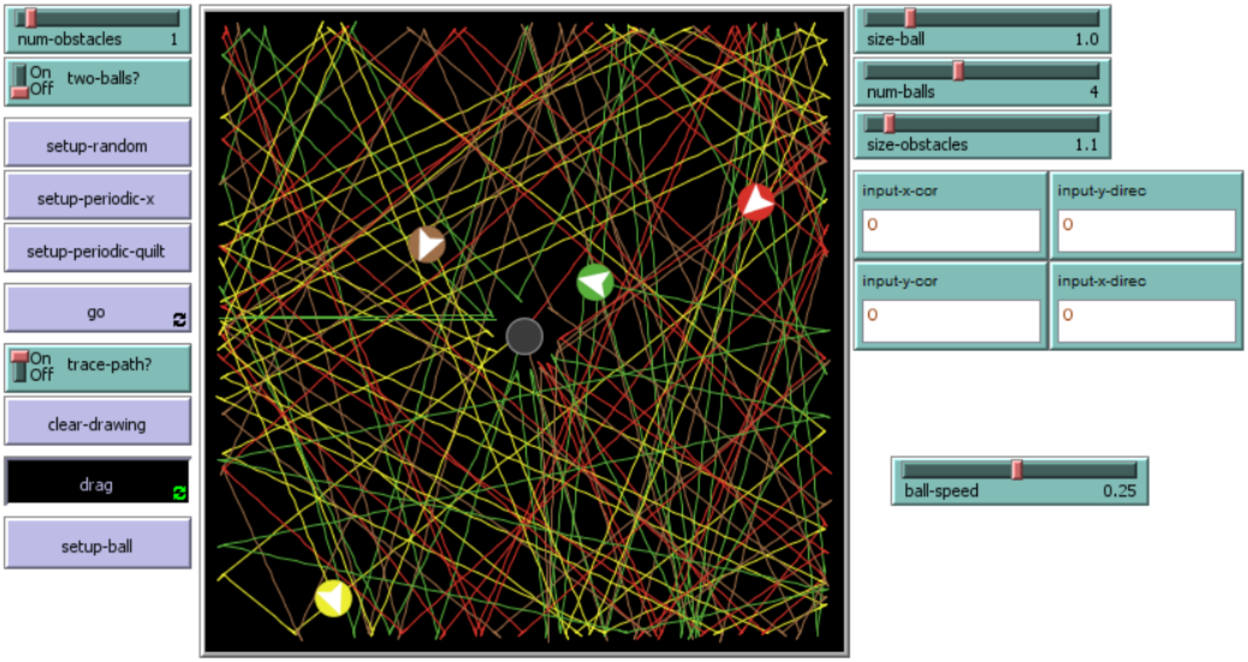

Figure 12: Increased speed of balls (Results in an increase of chaotic motion)

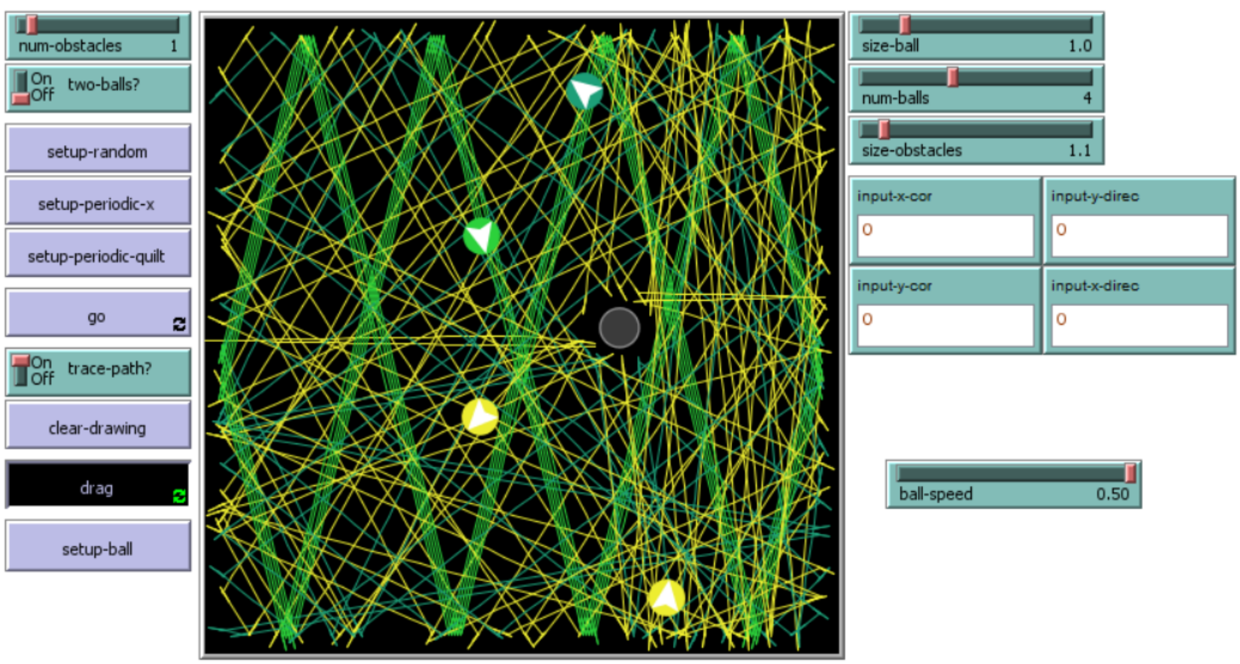

Figure 13: Increased size of balls (Results in increased chaotic motion)

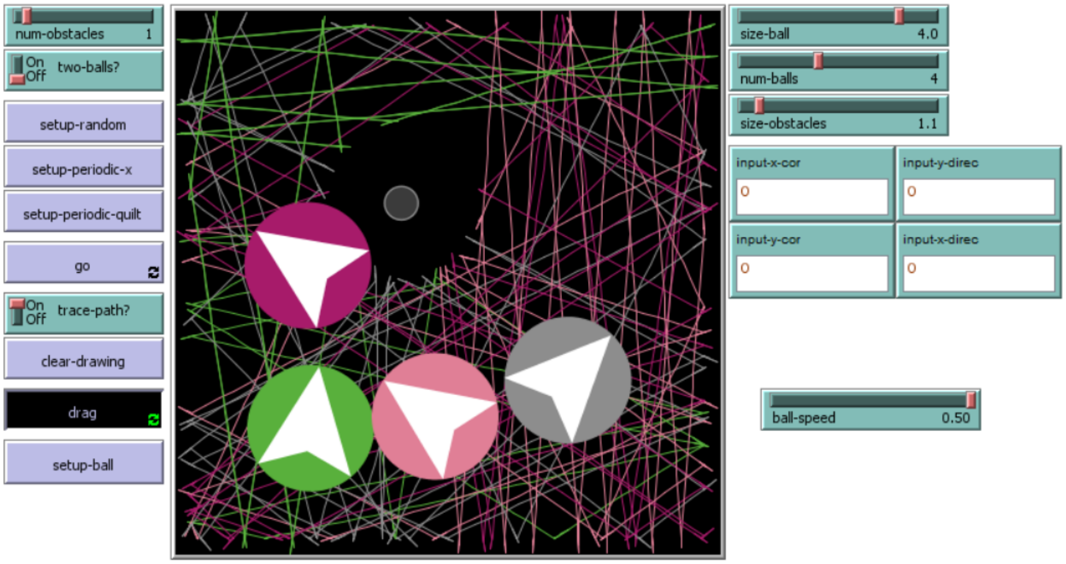

Figure 14: 4 balls initialized with a direction vector of (2,2)

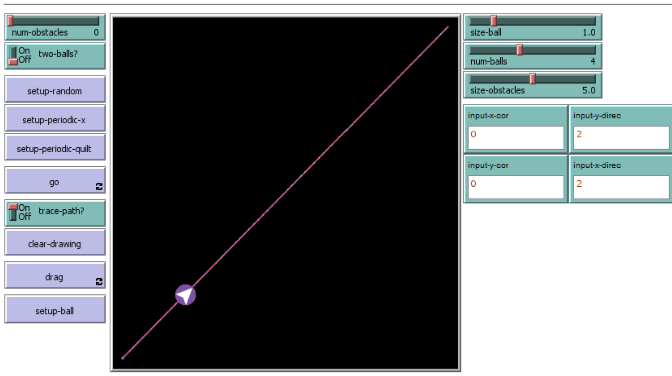

Figure 15: Increased size of the box by 100% (Resulted in a decrease of chaotic motion)