How do you find a point at a given perpendicular distance from a line?

Asked 16 years, 3 months ago Modified 5 years, 1 month ago

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55

I have a line that I draw in a window and I let the user drag it around. So, my line is defined by two points: (x1,y1) and (x2,y2). But now I would like to draw "caps" at the end of my line, that is, short perpendicular lines at each of my end points. The caps should be N pixels in length.

Thus, to draw my "cap" line at end point (x1,y1), I need to find two points that form a perpendicular line and where each of its points are N/2 pixels away from the point (x1,y1).

So how do you calculate a point (x3,y3) given it needs to be at a perpendicular distance N/2 away from the end point (x1,y1) of a known line, i.e. the line defined by (x1,y1) and (x2,y2)?

algorithm math graphics geometry coordinates

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edited Nov 16, 2019 at 18:33

David Nehme

21.6k • 8 • 81 • 121

asked Sep 25, 2008 at 15:12

AZDean

1,804 2 18 24

For a detailed worked out solution, [see here]. – legends2k Oct 19, 2013 at 5:58

4 Answers

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96

You need to compute a unit vector that's perpendicular to the line segment. Avoid computing the slope because that can lead to divide by zero errors.

```
dx = x1-x2
dy = y1-y2
dist = sqrt(dx*dx + dy*dy)
dx /= dist
dy /= dist
x3 = x1 + (N/2)*dy
y3 = y1 - (N/2)*dx
x4 = x1 - (N/2)*dy
y4 = y1 + (N/2)*dx
```

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edited Sep 25, 2008 at 15:28

answered Sep 25, 2008 at 15:22

David Nehme

21.6k 8 81 121

1 I keep thinking there must be a way to avoid that nasty sqrt in there, possibly by using Breshenham's Line, but I can't think of it off hand. – Paul Tomblin Sep 25, 2008 at 15:29

2 I think you have a sign error in your computations or points 3 and 4. Use (+ - - +) or (- + + -) in the last four lines, no? – dmckee --- ex-moderator kitten Sep 25, 2008 at 15:29

5 Put together a visualization of the formula on JSFiddle: jsfiddle.net/n2gqw8of You can change the values of ax,ay and bx,by to modify the line and see the perpendicular lines adjust accordingly... – Kevin Jurkowski Apr 8, 2015 at 21:48

1 @PaulTomblin an approximate solution would be to flip the line itself by 90 degrees, w/ `(dx,dy)->(-dy,dx)` ; then scaling it *approximately* by `N/L_aprx` where `L_aprx = (|dx|+|dy|)*0.83` . might just be good enough. The lengths of the two ticks will be `(10% (+-10%))` off. – Will Ness Feb 17, 2016 at 10:31

1 Can any one explain what this pseudo code does, I do get it that it works but my question is how? – Santhosh Dec 13, 2017 at 9:30

7

You just evaluate the orthogonal versor and multiply by N/2

```
vx = x2-x1
vy = y2-y1
len = sqrt( vx*vx + vy*vy )
ux = -vy/len
uy = vx/len

x3 = x1 + N/2 * ux
Y3 = y1 + N/2 * uy
```

```
x4 = x1 - N/2 * ux
Y4 = y1 - N/2 * uy
```

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edited Sep 25, 2008 at 15:31

answered Sep 25, 2008 at 15:25

Giacomo Degli Esposti

2,442 ● 1 ● 16 ● 21

4

Since the vectors from 2 to 1 and 1 to 3 are perpendicular, their dot product is 0.

This leaves you with two unknowns: x from 1 to 3 (x13), and y from 1 to 3 (y13)

Use the Pythagorean theorem to get another equation for those unknowns.

Solve for each unknown by substitution...

This requires squaring and unsquaring, so you lose the sign associated with your equations.

To determine the sign, consider:

```
while x21 is negative, y13 will be positive
while x21 is positive, y13 will be negative
while y21 is positive, x13 will be positive
while y21 is negative, x13 will be negative
```

Known: point 1 : x1 , y1

Known: point 2 : x2 , y2

```
x21 = x1 - x2
y21 = y1 - y2
```

Known: distance |1->3| : N/2

equation a: Pythagorean theorem

```
x13^2 + y13^2 = |1->3|^2
x13^2 + y13^2 = (N/2)^2
```

Known: angle 2-1-3 : right angle

vectors 2->1 and 1->3 are perpendicular

2->1 dot 1->3 is 0

equation b: dot product = 0

```
x21*x13 + y21*y13 = 2->1 dot 1->3
x21*x13 + y21*y13 = 0
```

ratio b/w x13 and y13:

```
x21*x13 = -y21*y13
x13 = -(y21/x21)y13

x13 = -phi*y13
```

equation a: solved for y13 with ratio

```
  plug x13 into a
phi^2*y13^2 + y13^2 = |1->3|^2

  factor out y13
y13^2 * (phi^2 + 1) =

  plug in phi
y13^2 * (y21^2/x21^2 + 1) =

  multiply both sides by x21^2
y13^2 * (y21^2 + x21^2) = |1->3|^2 * x21^2

  plug in Pythagorean theorem of 2->1
y13^2 * |2->1|^2 = |1->3|^2 * x21^2

  take square root of both sides
y13 * |2->1| = |1->3| * x21

  divide both sides by the length of 1->2
y13 = (|1->3|/|2->1|) *x21

  lets call the ratio of 1->3 to 2->1 lengths psi
y13 = psi * x21

  check the signs
    when x21 is negative, y13 will be positive
    when x21 is positive, y13 will be negative

y13 = -psi * x21
```

equation a: solved for x13 with ratio

```
  plug y13 into a
x13^2 + x13^2/phi^2 = |1->3|^2

  factor out x13
x13^2 * (1 + 1/phi^2) =

  plug in phi
x13^2 * (1 + x21^2/y21^2) =
```

```
  multiply both sides by y21^2
x13^2 * (y21^2 + x21^2) = |1->3|^2 * y21^2

  plug in Pythagorean theorem of 2->1
x13^2 * |2->1|^2 = |1->3|^2 * y21^2

  take square root of both sides
x13 * |2->1| = |1->3| * y21

  divide both sides by the length of 2->1
x13 = (|1->3|/|2->1|) *y21

  lets call the ratio of |1->3| to |2->1| psi
x13 = psi * y21

  check the signs
    when y21 is negative, x13 will be negative
    when y21 is positive, x13 will be negative

x13 = psi * y21
```

to condense

```
x21 = x1 - x2
y21 = y1 - y2

|2->1| = sqrt( x21^2 + y^21^2 )
|1->3| = N/2

psi = |1->3|/|2->1|

y13 = -psi * x21
x13 =  psi * y21
```

I normally wouldn't do this, but I solved it at work and thought that explaining it thoroughly would help me solidify my knowledge.

Share Improve this answer edited Mar 26, 2011 at 1:31

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answered Mar 26, 2011 at 1:25

user677616

41 ● 2

1

If you want to avoid a sqrt, do the following:

```
in: line_length, cap_length, rotation, position of
line centre

define points:
  tl (-line_length/2, cap_length)
  tr (line_length/2, cap_length)
  bl (-line_length/2, -cap_length)
  br (line_length/2, -cap_length)

rotate the four points by 'rotation'
offset four points by 'position'

drawline (midpoint tl,bl to midpoint tr,br)
drawline (tl to bl)
drawline (tr to br)
```

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edited Jul 3, 2012 at 14:07

user142162

answered Sep 25, 2008 at 15:39

Skizz

71k ● 10 ● 74 ● 109