# Efficient evaluation of hypergeometric functions

Asked 15 years, 11 months ago Modified 15 years, 10 months ago

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Does anyone have experience with algorithms for evaluating hypergeometric functions? I would be interested in general references, but I'll describe my particular problem in case someone has dealt with it.

My specific problem is evaluating a function of the form 3F2(a, b, 1; c, d; 1) where a, b, c, and d are all positive reals and c+d > a+b+1. There are many special cases that have a closed-form formula, but as far as I know there are no such formulas in general. The power series centered at zero converges at 1, but very slowly; the ratio of consecutive coefficients goes to 1 in the limit. Maybe something like Aitken acceleration would help?

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asked Jan 25, 2009 at 21:16

John D. Cook

30k 10 69 94

## 2 Answers

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I tested Aitken acceleration and it does not seem to help for this problem (nor does Richardson extrapolation). This probably means Pade approximation doesn't work either. I might have done something wrong though, so by all means try it for yourself.

I can think of two approaches.

One is to evaluate the series at some point such as z = 0.5 where convergence is rapid to get an initial value and then step forward to z = 1 by plugging the hypergeometric differential equation into an ODE solver. I don't know how well this works in practice; it might not, due to z = 1 being a singularity (if I recall correctly).

The second is to use the definition of 3F2 in terms of the Meijer G-function. The contour integral defining the Meijer G-function can be evaluated numerically by applying Gaussian or doubly-exponential quadrature to segments of the contour. This is not terribly efficient, but it should work, and it should scale to relatively high precision.

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answered Jan 29, 2009 at 8:14

Fredrik Johansson
27k ● 3 ● 28 ● 17

Fantastic question, and an answer that's up to the challenge. Well done. – duffymo Jan 29, 2009 at 12:54

I use a Burlisch-Stoer stepper to successfully solve the ODE for 2F1. The method works well, provided you take some care when choosing your branch cuts. For 2F3, this seems more involved since it is fourth order and has more singular points, but this should be feasible. – Alexandre C. Oct 28, 2011 at 14:55

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Is it correct that you want to sum a series where you know the ratio of successive terms and it is a rational function?

I think Gosper's algorithm and the rest of the tools for proving hypergeometric identities (and finding them) do exactly this, right? (See Wilf and Zielberger's A=B book online.)

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answered Jan 25, 2009 at 23:07

ShreevatsaR
39k • 17 • 106 • 127

Yes, the ratio of the series coefficients is a rational function of the index. But I have not found a useful hypergeometric identity. functions.wolfram.com lists thousands of identities, but none of them help. – John D. Cook Jan 25, 2009 at 23:39

I don't know much -- don't these algorithms *find* an identity as well? I haven't read the A=B book in detail, but the Maple packages it mentions might have better implementations... – ShreevatsaR Jan 26, 2009 at 1:34