Worst-Case Robust Approximation (RMPC) Using YALMIP Toolbox

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Motivation

- Model Predictive Control (MPC) is an advanced method of control that combines open-loop optimal control with feedback to close the loop
- A dynamics model for our state transitions is assumed to be known and time-invariant
- Noise in the model dynamics can compromise controller performance

Problem Definition

- Extension of linear MPC for a problem with bounded noise with known distribution
- Introduction of Robust-MPC (RMPC) to account for noise
- Our system dynamics is simulated with noise, i.e.:

$$x(t+1) = Ax(t)+Bu(t)+Ew(t), w \sim unif(-1,1)$$
$$y(t) = Cx(t)$$

$$A = \begin{bmatrix} 2.9 & -0.73 & 0.25 \\ 4 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0.25 \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} -0.2 \\ 0.04 \\ 0.07 \end{bmatrix} \quad E = \begin{bmatrix} 0.0625 \\ 0.0625 \\ 0.0625 \end{bmatrix}$$

- For simplicity, we had our system track a constant target of y = 0.5

Theory

For the case of MPC with no noise, we solve the problem of the following form:

$$J_t^*(\mathbf{x}(t)) = \min_{\mathbf{u}_{t|t},\ldots,\mathbf{u}_{t+N-1|t}} p(\mathbf{x}_{t+N|t}) + \sum_{k=0}^{N-1} c(\mathbf{x}_{t+k|t}, \mathbf{u}_{t+k|t})$$

$$\begin{aligned} \text{subject to} \quad & \mathbf{x}_{t+k+1|t} = A\mathbf{x}_{t+k|t} + B\mathbf{u}_{t+k|t}, \quad k = 0, \ldots, N-1 \\ & \mathbf{x}_{t+k|t} \in X, \quad \mathbf{u}_{t+k|t} \in U, \quad k = 0, \ldots, N-1 \\ & \mathbf{x}_{t+N|t} \in X_f \\ & \mathbf{x}_{t|t} = \mathbf{x}(t) \end{aligned}$$

(Pavone 2019)

With the introduction of a noise term in the dynamics, the constraints no longer make sense.

$\Longrightarrow$ We need to formulate a new problem.

Introduction of noise is akin to adding a secondary player (the adversary) that is trying to maximize the stage costs, trying to push the state outside of its constraining set

We want to counteract this and remain within our constraints. Our cost function becomes a minimax problem of the following form:

$$J_t^*(x(t)) = \min_{u,y} \max_{w} ||y - y_{ref}|| + \delta ||u - u_{max}||$$
$$|w| \leq w_{max}$$
$$|u| \leq u_{max}$$
$$x(t+1) = Ax(t) + Bu(t)$$
$$x(t+N) \in X_f$$

Method

- To solve the minimax problem, we used the MATLAB toolbox YALMIP
- The toolbox has formulations that solve open-loop RMPC as well as an approximate closed-loop RMPC

Results

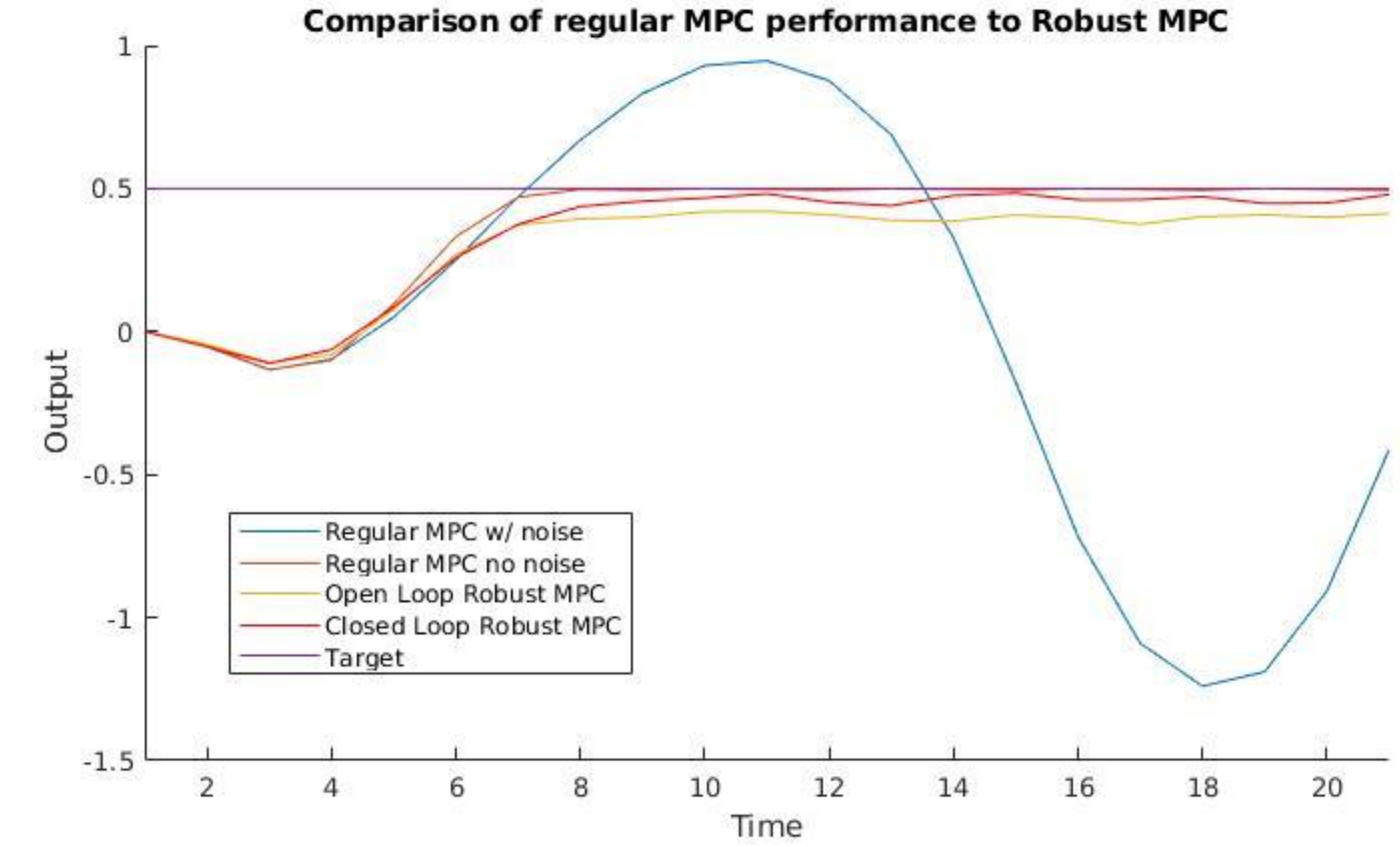

Figure 1: Output y vs time for 4 different schemes: 1. MPC w/ noise in dynamic model, 2. MPC without noise, 3. Open loop RMPC, 4. Closed loop RMPC. All MPC's were optimized with time horizon of N=10

Figure 1 shows the resulting output y values from noisy simulation for controllers:

1. Simulated MPC with and without noise included in the dynamic model as a baseline
2. Any RMPC controllers (both OL/ approx. CL) are effective in bringing the system to convergence without overshooting reference
3. OL solution is more conservative than the CL
4. As anticipated, tradeoff between final cost and performance:

MPC w/o noise	MPC w noise	OL RMPC	CL RMPC
6.64	108.29	102.89	16.85

Table 1: Control effort of MPC with different strategies

References

- Dimitri P. Bertsekas, "Dynamic Programming and Optimal Control, 2nd Edition" (2000). Athena Scientific
- https://yalmip.github.io/