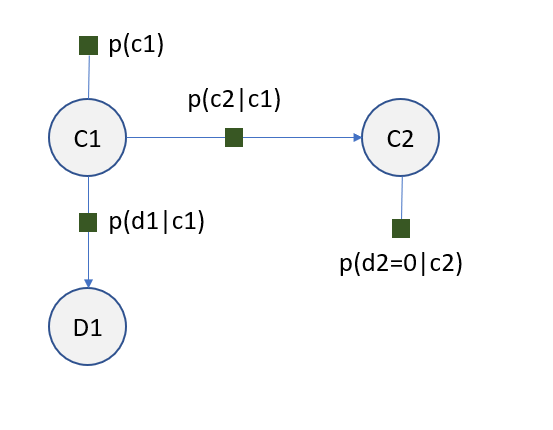
**Homework #7: Car Tracking**

# Acknowledgements

# Problem 1

## Problem 1a

After removing non-ancestors of query or evidence, converting to a factor graph, and conditioning on / removing the evidence variable , we are left with the following factor graph:



Eliminating requires defining a new factor . In turn, eliminating requires definition of a second new factor . Taking the product of the remaining new factors about the remaining variable yields the generalized posterior probability distribution with respect to :

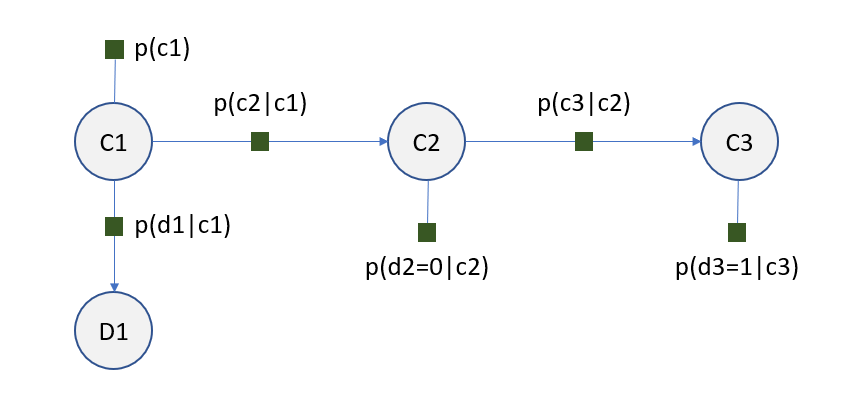
Noting that and that is a constant, we can simplify to

Therefore, by substitution and normalization we have

The above makes intuitive sense – at time step 2, the probability of the car being in position 1 given that the sensor reads position 0 is simply equal to the probability that the sensor gives an erroneous reading.

## Problem 1b

After performing the same steps as in the previous problem (removing non-ancestors, transforming into a factor graph, and conditioning on the evidence variables), we are left with the following factor graph:



In order to eliminate , we must define a new factor . Since the conditional value of is the same as in the previous problem and the two pieces of the factor graph are independent about , we can express the solution here in terms of the original solution:

So by substituting and normalizing we have

## Problem 1c

### 1c(i)

Supposing that we have , by substitution into the respective expressions we arrive at

### 1c(ii)

The above shows that the additional evidence increases the probability of the car’s position at time step 2. This makes sense when we consider the relative values of . We see that the probability of the car moving is very low, and actually less likely than the sensor having an erroneous reading. Therefore, the additional evidence about actually strengthens our inference about the car’s position at time step 2.

### 1c(iii)

Equating both expressions yields

Inspection of the above shows that  **is a solution for the above equality, i.e. the two probabilities are equal at this value.**

Recall that has the sensor at the second position as evidence, while the second one has sensors at time steps 2 and 3 as evidence. However, because the probability of staying and moving are equal at , having the additional sensor output for time step 3 as evidence provides no additional useful information about the car’s position (since it’s just as likely to stay as it is to move), and as a result the probability is unchanged.

# Problem 5

## Problem 5a

Applying Bayes’ rule, we know that

By simply looking at proportionality and noting that the two cars move independently, we can directly substitute the local conditional probabilities

In order to determine the remaining probability, we note that the conditional probability is given by the Gaussian distribution function, so that we have

## Problem 5b

Practically, being the same for all( means that the local conditional probability is the same for all cars at the first time step. Applying Bayes’ Rule, we have

## Problem 5c