Tunable Spin-Orbit Coupling using a 'Molmer-Sorensen' coupling scheme

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1 Introduction

Aqui hablo sobre espectroscopia y tunable soc

1.1 Modulated/tripple frequency coupling

2 Triple frequency coupling

Consider 3 pairs of Raman beamsg coupling the three m_f states in the ^{87}Rb $F = 1$ manifold as shown in fig no... . The Hamiltonian describing the interaction is:

$$\hat{H}_R(t) = \{\Omega_{21}\cos(2k_R x - \omega_{21}t + \frac{\phi_1}{2}) + \Omega_{31}\cos(2k_R x - \omega_{31}t - \frac{\phi_1}{2}) + \Omega_{41}\cos(2k_R x - \omega_{41}t + \phi_2)\}\hat{F}_x \tag{1}$$

where $\Omega_{ij} \propto \vec{E}_i \times \vec{E}_j^\star$ represents the coupling strenght associated to each pair of Raman beams and $\omega_{ij} = \omega_i - \omega_j$. The frequencies are chosen so that $\omega_{31} + \omega 21$ is at 4 photon resonance with the $m_f = +1 \rightarrow m_f = -1$. Under a rotation about the z axis $\hat{U} = e^{i\omega t\hat{F}_z}$ at frequency $\bar{\omega} = \frac{\omega_{21}+\omega_{31}}{2}$, and after applying the rotating wave approximation (RWA), the Hamiltonian transforms to

$$\begin{aligned}
\hat{\tilde{H}}_R(t) =& \hat{U}^\dagger \hat{H}_R(t)\hat{U} \\
=& \frac{1}{2}\{\Omega_{21}\cos(2k_R x + (\bar{\omega} - \omega_{21})t + \frac{\phi_1}{2}) + \Omega_{31}\cos(2k_R x + (\bar{\omega} - \omega_{31})t - \frac{\phi_1}{2}) \\
&+ \Omega_{41}\cos(2k_R x + (\bar{\omega} - \omega_{41})t + \phi_2)\}\hat{F}_x \\
&- \frac{1}{2}\{\Omega_{21}\sin(2k_R x + (\bar{\omega} - \omega_{21})t + \frac{\phi_1}{2}) + \Omega_{31}\sin(2k_R x + (\bar{\omega} - \omega_{31})t - \frac{\phi_1}{2}) \\
&+ \Omega_{41}\sin(2k_R x + (\bar{\omega} - \omega_{41})t + \phi_2)\}\hat{F}_y
\end{aligned} \tag{2}$$

where we have used

$$e^{-i\theta\hat{F}_z}\hat{F}_x e^{i\theta\hat{F}_z} = \cos\theta\hat{F}_x + \sin\theta\hat{F}_y \tag{3}$$

and applied the rotating wave approximation (RWA)to get rid of the fast terms.

I want the time dependent Hamiltonian to look like it has a constant offset and a frequency modulated term. This can be done by choosing $\Omega_{21} = \Omega_{31} = \Omega$, $\Omega_{41} = \Omega_0$, and $\omega_{41} = \frac{\omega_{21}+\omega_{31}}{2}$. The Hamiltonian is now reduced to

$$\hat{\tilde{H}}_R(t) = \frac{\Omega}{2}\cos(\delta\omega t)[\cos(2k_R x)\hat{F}_x - \sin(2k_R x)\hat{F}_y] + \frac{\Omega_0}{2}[\cos(2k_R x + \phi_2)\hat{F}_x - \sin(2k_R x + \phi_2)\hat{F}_y] \tag{4}$$

where $\delta\omega = \frac{\omega_{31}-\omega_{21}}{2}$. Also notice that by redefining the time origin I can get rid of the first phase.

Things are looking good so far!

Now I can apply the usual x dependent rotation $\hat{U} = e^{i2k_R x\hat{F}_z}$. I won't work out all the algebra explicitly here, but the complete system's Hamiltonian (in the interaction picture) should be something like

$$\begin{aligned}\hat{H}(t) =&\frac{\hbar^2}{2m}(\hat{k} - 2k_R\hat{F}_z)^2 + \frac{1}{2}[\Omega_0\cos\phi_2 + \Omega\cos(\delta\omega t)]\hat{F}_x - \frac{1}{2}\Omega_0\sin\phi_2\hat{F}_y + \Delta\hat{F}_z + \epsilon(\hat{F}_z^2 - \mathbb{I})\\ =&\frac{\hbar^2\hat{k}^2}{2m} - \frac{\hbar^2}{m}2k_R\hat{k}\hat{F}_z + 4E_r\hat{F}_z^2 + \frac{1}{2}[\Omega_0\cos\phi_2 + \Omega(t)]\hat{F}_x - \frac{1}{2}\Omega_0\sin\phi_2\hat{F}_y + \Delta\hat{F}_z + \epsilon(\hat{F}_z^2 - \mathbb{I}),\end{aligned} \tag{5}$$

with $\Omega(t) = \Omega\cos(\delta\omega t)$.

To get rid of the time dependence in the Hamiltonian and ultimately getting the 'tunable' spin-orbit coupling we can choose a transoformation of the Hamiltonian such that $\hat{U}^\dagger\frac{\partial\hat{U}}{\partial t} = -i\frac{\Omega(t)}{2}\hat{F}_x$. This will be satisfied for

$$\hat{U} = e^{-i\frac{\Omega}{2}\int_0^t\cos(\delta\omega t')dt'} = e^{-i\frac{\Omega}{2\delta\omega}\sin(\delta\omega t)}. \tag{6}$$

Under this time dependent transformation, the time evolution of the system will be given by

$$\begin{aligned}\hat{\tilde{H}} =&\hat{U}^\dagger\hat{H}(t)\hat{U} + i\hat{U}^\dagger\frac{\partial\hat{U}}{\partial t}\\ =&\frac{\hbar^2\hat{k}^2}{2m} - \frac{\hbar^2}{m}2k_R\hat{k}\hat{U}^\dagger\hat{F}_z\hat{U} + \frac{1}{2}\Omega_0\cos\phi_2\hat{F}_x - \frac{1}{2}\Omega_0\sin\phi_2\hat{U}^\dagger\hat{F}_y\hat{U} + \Delta\hat{U}^\dagger\hat{F}_z\hat{U} + \epsilon(\hat{U}^\dagger\hat{F}_z^2\hat{U} - \mathbb{I}) + 4E_r\hat{U}^\dagger\hat{F}_z^2\hat{U}.\end{aligned} \tag{7}$$

The transformations for the angular momentum operators that don't commute with $\hat{F}_x$ are given by:

$$\begin{aligned}e^{i\theta\hat{F}_x}\hat{F}_z e^{-i\theta\hat{F}_x} &= \cos\theta\hat{F}_z + \sin\theta\hat{F}_y\\ e^{i\theta\hat{F}_x}\hat{F}_y e^{-i\theta\hat{F}_x} &= -\sin\theta\hat{F}_z + \cos\theta\hat{F}_y\\ e^{i\theta\hat{F}_x}\hat{F}_z^2 e^{-i\theta\hat{F}_x} &= \cos^2\theta\hat{F}_z^2 + \sin^2\theta\hat{F}_y^2 + \sin\theta\cos\theta(\hat{F}_z\hat{F}_y + \hat{F}_y\hat{F}_z).\end{aligned} \tag{8}$$

The JacobiAnger expansion will be useful to write an (almost) exact expression for the transformed Hamiltonian:

$$\cos(z\sin\theta) = J_0(z) + 2\sum_{n=1}^{\infty}J_{2n}(z)\cos(2n\theta) \approx J_0(z) \tag{9}$$

$$\sin(z\sin\theta) = 2\sum_{n=0}^{\infty}J_{2n+1}(z)\sin((2n+1)\theta) \approx 0, \tag{10}$$

so we can write

$$
\begin{aligned}
\hat{\tilde{H}} =&\frac{\hbar^2\hat{k}^2}{2m} - \frac{\hbar^2}{m}2k_R J_0(\Omega/2\delta\omega)\hat{k}\hat{F}_z + \frac{\Omega_0}{2}(\cos\phi_2\hat{F}_x - \sin\phi_2 J_0(\Omega/2\delta\omega)\hat{F}_y) + \Delta J_0(\Omega/2\delta\omega)\hat{F}_z \\
&+ \epsilon(J_0^2(\Omega/2\delta\omega)\hat{F}_z^2 + \frac{1}{2}(1 - J_0(\Omega/\delta))\hat{F}_y^2 - \mathbb{I}) + 4E_r(J_0^2(\Omega/\delta)\hat{F}_z^2 + \frac{1}{2}(1 - J_0(\Omega/\delta\omega))\hat{F}_y^2 \\
=&\frac{\hbar^2}{2m}(\hat{k} - 2k_R J_0(\Omega/2\delta\omega)\hat{F}_z)^2 + \frac{\Omega_0}{2}(\cos\phi_2\hat{F}_x - \sin\phi_2 J_0(\Omega/2\delta\omega)\hat{F}_y) + \Delta J_0(\Omega/2\delta\omega)\hat{F}_z \\
&+ \epsilon[J_0^2(\Omega/2\delta\omega)\hat{F}_z^2 + \frac{1}{2}(1 - J_0(\Omega/\delta\omega))\hat{F}_y^2 - \mathbb{I}] + 2E_r(1 - J_0(\Omega/\delta\omega))\hat{F}_y^2
\end{aligned} \tag{11}
$$

The terms on the last line can be simplified:

$$
\epsilon[J_0^2(\Omega/\delta)\hat{F}_z^2 + \frac{1}{2}(1 - J_0(2\Omega/\delta))\hat{F}_y^2 - \mathbb{I}] + 2E_r(1 - J_0(2\Omega/\delta))\hat{F}_y^2) = \tag{12}
$$

This looks almost like a spin one spin-orbit coupled system with some extra weird terms and couplings between $m_f = 1$ and $m_f = -1$. Adiabatic elimination of the $m_f = 0$ (ground) state leads to an effective Hamiltonian

$$
\hat{H}_{eff} = \frac{\hbar^2}{2m}(\hat{k} - 2k_R J_0(\Omega/\delta)\hat{F}_{2z})^2 + \Delta J_0(\Omega/\delta)\hat{F}_{2z} \tag{13}
$$

2.1 Spectroscopy

3 Methods

4 Results

5 Discussion

6 Conclusion