Tunable Spin-Orbit coupling: spectroscopy and cyclic coupling

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SOC rocks, pero latex apesta

Aqui hablo sobre espectroscopia y tunable soc

A. Modulated/tripple frequency coupling

I. TRIPLE FREQUENCY COUPLING

$$\hat{H}_R(t) = \{\Omega_{21}\cos(2k_Rx - \omega_{21}t + \frac{\phi_1}{2}) + \Omega_{31}\cos(2k_Rx - \omega_{31}t - \frac{\phi_1}{2}) + \Omega_{41}\cos(2k_Rx - \omega_{41}t + \phi_2)\}\hat{F}_x \tag{1}$$

where $\Omega_{ij} \propto \vec{E}_i \times \vec{E}_j^\star$ represents the coupling strenght associated to each pair of Raman beams and $\omega_{ij} = \omega_i - \omega_j$. The frequencies are chosen so that $\omega_{31} + \omega 21$ is at 4 photon resonance with the $m_f = +1 \rightarrow m_f = -1$. Under a rotation about the z axis $\hat{U} = e^{i\omega t\hat{F}_z}$ at frequency $\bar{\omega} = \frac{\omega_{21}+\omega_{31}}{2}$, and after applying the rotating wave approximation (RWA), the Hamiltonian transforms to

$$\begin{aligned}\hat{\tilde{H}}_R(t) =& \hat{U}^\dagger \hat{H}_R(t)\hat{U}\\ =& \frac{1}{2}\{\Omega_{21}\cos(2k_Rx + (\bar{\omega} - \omega_{21})t + \frac{\phi_1}{2}) + \Omega_{31}\cos(2k_Rx + (\bar{\omega} - \omega_{31})t - \frac{\phi_1}{2})\\ &+ \Omega_{41}\cos(2k_Rx + (\bar{\omega} - \omega_{41})t + \phi_2)\}\hat{F}_x\\ &- \frac{1}{2}\{\Omega_{21}\sin(2k_Rx + (\bar{\omega} - \omega_{21})t + \frac{\phi_1}{2}) + \Omega_{31}\sin(2k_Rx + (\bar{\omega} - \omega_{31})t - \frac{\phi_1}{2})\\ &+ \Omega_{41}\sin(2k_Rx + (\bar{\omega} - \omega_{41})t + \phi_2)\}\hat{F}_y\end{aligned} \tag{2}$$

where we have used

$$e^{-i\theta\hat{F}_z}\hat{F}_x e^{i\theta\hat{F}_z} = \cos\theta\hat{F}_x + \sin\theta\hat{F}_y \tag{3}$$

and applied the rotating wave approximation (RWA)to get rid of the fast terms.

I want the time dependent Hamiltonian to look like it has a constant offset and a frequency modulated term. This can be done by choosing $\Omega_{21} = \Omega_{31} = \Omega$, $\Omega_{41} = \Omega_0$, and $\omega_{41} = \frac{\omega_{21}+\omega_{31}}{2}$. The Hamiltonian is now reduced to

$$\hat{\tilde{H}}_R(t) = \frac{\Omega}{2}\cos(\delta\omega t)[\cos(2k_Rx)\hat{F}_x - \sin(2k_Rx)\hat{F}_y] + \frac{\Omega_0}{2}[\cos(2k_Rx + \phi_2)\hat{F}_x - \sin(2k_Rx + \phi_2)\hat{F}_y] \tag{4}$$

where $\delta\omega = \frac{\omega_{31}-\omega_{21}}{2}$. Also notice that by redefining the time origin I can get rid of the first phase.

Things are looking good so far!

Now I can apply the usual x dependent rotation $\hat{U} = e^{i2k_Rx\hat{F}_z}$. I won't work out all the algebra explicitly here, but the complete system's Hamiltonian (in the interaction picture) should be something like

$$\begin{aligned}\hat{H}(t) =& \frac{\hbar^2}{2m}(\hat{k} - 2k_R\hat{F}_z)^2 + \frac{1}{2}[\Omega_0\cos\phi_2 + \Omega\cos(\delta\omega t)]\hat{F}_x - \frac{1}{2}\Omega_0\sin\phi_2\\ =& \frac{\hbar^2\hat{k}^2}{2m} - \frac{\hbar^2}{m}2k_R\hat{k}\hat{F}_z + 4E_r\hat{F}_z^2 + \frac{1}{2}[\Omega_0\cos\phi_2 + \Omega(t)]\hat{F}_x - \frac{1}{2}\Omega_0\,\mathrm{s}\end{aligned} \tag{5}$$

with $\Omega(t) = \Omega\cos(\delta\omega t)$.

To get rid of the time dependence in the Hamiltonian and ultimately getting the 'tunable' spin-orbit coupling we can choose a transoformation of the Hamiltonian such that $\hat{U}^\dagger\frac{\partial\hat{U}}{\partial t} = -i\frac{\Omega(t)}{2}\hat{F}_x$. This will be satisfied for

$$\hat{U} = e^{-i\frac{\Omega}{2}\int_0^t\cos(\delta\omega t')dt'} = e^{-i\frac{\Omega}{2\delta\omega}\sin(\delta\omega t)}. \tag{6}$$

Under this time dependent transformation, the time evolution of the system will be given by

$$\begin{aligned}\hat{\tilde{H}} =& \hat{U}^\dagger\hat{H}(t)\hat{U} + \hat{U}^\dagger\frac{\partial\hat{U}}{\partial t}\\ =& \frac{\hbar^2\hat{k}^2}{2m} - \frac{\hbar^2}{m}2k_R\hat{k}\hat{U}^\dagger\hat{F}_z\hat{U} + \frac{1}{2}\Omega_0\cos\phi_2\hat{F}_x - \frac{1}{2}\Omega_0\sin\phi_2\hat{U}^\dagger\hat{F}_y\hat{U} + \Delta\end{aligned} \tag{7}$$

The transformations for the angular momentum operators that don't commute with $\hat{F}_x$ are given by:

$$\begin{aligned}e^{i\theta\hat{F}_x}\hat{F}_z e^{-i\theta\hat{F}_x} =& \cos\theta\hat{F}_z + \sin\theta\hat{F}_y\\ e^{i\theta\hat{F}_x}\hat{F}_y e^{-i\theta\hat{F}_x} =& -\sin\theta\hat{F}_z + \cos\theta\hat{F}_y\\ e^{i\theta\hat{F}_x}\hat{F}_z^2 e^{-i\theta\hat{F}_x} =& \cos^2\theta\hat{F}_z^2 + \sin^2\theta\hat{F}_y^2 + \sin\theta\cos\theta(\hat{F}_z\hat{F}_y + \hat{F}_y\hat{F}_z).\end{aligned} \tag{8}$$

The JacobiAnger expansion will be useful to write an (almost) exact expression for the transformed Hamiltonian:

$$\cos(z\sin\theta) = J_0(z) + 2\sum_{n=1}^{\infty}J_{2n}(z)\cos(2n\theta) \approx J_0(z) \tag{9}$$

$$\sin(z\sin\theta) = 2\sum_{n=0}^{\infty}J_{2n+1}(z)\sin((2n+1)\theta) \approx 0, \tag{10}$$

so we can write

$$\hat{\hat{H}} =\frac{\hbar^2\hat{k}^2}{2m}-\frac{\hbar^2}{m}2k_R J_0(\Omega/2\delta\omega)\hat{k}\hat{F}_z+\frac{\Omega_0}{2}(\cos\phi_2\hat{F}_x-\sin\phi_2 J_0(\Omega/2\delta\omega)\hat{F}_y)+\Delta J_0(\Omega/2\delta\omega)\hat{F}_z$$
$$+\epsilon(J_0^2(\Omega/2\delta\omega)\hat{F}_z^2+\frac{1}{2}(1-J_0(\Omega/\delta))\hat{F}_y^2-\mathbb{I})+4E_r(J_0^2(\Omega/\delta)\hat{F}_z^2+\frac{1}{2}(1-J_0(\Omega/\delta\omega))\hat{F}_y^2$$
$$=\frac{\hbar^2}{2m}(\hat{k}-2k_R J_0(\Omega/2\delta\omega)\hat{F}_z)^2+\frac{\Omega_0}{2}(\cos\phi_2\hat{F}_x-\sin\phi_2 J_0(\Omega/2\delta\omega)\hat{F}_y)+\Delta J_0(\Omega/2\delta\omega)\hat{F}_z$$
$$+\epsilon[J_0^2(\Omega/2\delta\omega)\hat{F}_z^2+\frac{1}{2}(1-J_0(\Omega/\delta\omega))\hat{F}_y^2-\mathbb{I}]+2E_r(1-J_0(\Omega/\delta\omega))\hat{F}_y^2 \tag{11}$$

The terms on the last line can be simplified:

$$\epsilon[J_0^2(\Omega/\delta)\hat{F}_z^2+\frac{1}{2}(1-J_0(2\Omega/\delta))\hat{F}_y^2-\mathbb{I}]+2E_r(1-J_0(2\Omega/\delta))\hat{F}_y^2)= \tag{12}$$

This looks almost like a spin one spin-orbit coupled system with some extra weird terms and couplings between $m_f = 1$ and $m_f = -1$. Adiabatic elimination of the $m_f = 0$ (ground) state leads to an effective Hamiltonian

$$\hat{H}_{eff} = \frac{\hbar^2}{2m}(\hat{k}-2k_R J_0(\Omega/\delta)\hat{F}_{2z})^2+\Delta J_0(\Omega/\delta)\hat{F}_{2z} \tag{13}$$

A. Spectroscopy

The time evolution of the system is given by the time dependent Schrödinger equation.

$$i\hbar\frac{d|Psi\rangle}{dt} = \hat{H}|\Psi\rangle \tag{14}$$

Under the presence of a Raman field,

$$|m_f\rangle \xrightarrow{t} \sum_n c_n e^{-iE_n t/\hbar}|\psi_n\rangle|\langle m_{f'}|m_f(t)\rangle|^2 = |\sum_{n,k} c_{nk} e^{-iE_n t/\hbar}\langle\psi_k|\psi_n\rangle|^2 \tag{15}$$

$$\tag{16}$$

II. METHODS

A. Fourier Spectroscopy

The Fourier Spectroscopy technique takes advantage of the time evolution of a bare atomic state after a dressing field is suddenly turned. The initial state becomes a superposition of dressed states and it undergoes Rabi oscillations in time. The spectral components of these oscillations contain information about the energies of the dressed states.

Aqu algo sobre el caso particular de las energias SOC.

The measurement can be simplified by noticing that a non-moving atom cloud in the laboratory reference frame dressed by a field with non-zero detuning is equivalent to a moving cloud with a resonant field in a suitable moving reference frame. As can also be seen in the Hamiltonian (citarlo aqui) the detuning term δ/Er and the momentum term $4k/k_R$ have the same effect in the energy differences.

For the case of our spin-orbit coupled BECs, the bare state The system is let to evolve for a finite time T and afterwards the field is snapped off. A Stern-Gerlach pulse applied at our 21 ms time of flight (TOF) allows us to project the state of the condensate back into the bare m_f basis.

For the experimental sequence we start

III. RESULTS

IV. DISCUSSION

V. CONCLUSION