Fourier spectroscopy of a spin-orbit coupled Bose gas

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Abstract.

We propose a time domain technique to measure the band structure of a spin-1 spin-orbit coupled Bose-Einstein condensate that relies on the Hamiltonian evolution of the system. We drive transitions at different values of detuning from Raman resonance and extract the Fourier components of the time dependent evolution to reconstruct the spin and momentum dependent energy spectrum. We add a periodic modulation to one Raman field which results in a tunable spin-orbit coupling dispersion and a spectrum of Floquet quasi-energies that we can directly measure, showing the robustness of our technique.

Contents

1. Introduction

Start with corny paragraph, about soc and how cold atoms are awesome. Describe SOC at high field and at four photon resonance. Describe Fourier spectroscopy. Describe modulated Raman.

1.1. Fourier Spectroscopy

Think transfer functions and spectroscopy. Spectroscopy is a vertical cut, it looks at response of the system when driven with frequencies within the Fourier limited bandwith of the pulse.

In order to explicitly measure the modified energy-momentum dispersion relation, we will use a Fourier based spectroscopy technique, which relies on the time evolution of an atomic state after a dressing field is suddenly turned on, and the initially bare states become superpositions of dressed states undergoing Rabi oscillations in time with spectral components related to the relative energies of the dressed states.

For the case of a spin-orbit coupled atomic system, the dressed state energies are explicitly dependent on both the particle's spin and momentum. Therefore, in order to fully characterize the energy-momentum dispersion we must prepare an atomic state at a given spin and momentum $|k_i, m_{f_i}\rangle$, pulse on the Raman field, and measure the time evolution.

In practice it may not be as straightforward to reliably prepare an arbitrary momentum state. The measurement however can be simplified by noticing that a non-moving atom cloud in the laboratory reference frame dressed by a field with non-zero detuning is equivalent to a moving cloud with a resonant field in a suitable moving reference frame. This can be explicitly seen in the Hamiltonian 1 where the detuning term δ/Er and the momentum term $4k/k_R$ have the same effect in the relative energies. There is an additional Doppler shift associated with the transformation between reference frames, which gets canceled when we look at the energy differences.

Therefore, for the purpose of our experiments, momentum and detuning are equivalent up to a numerical pre factor.

The method described above only allows us to measure relative energies and we must add a known energy reference if we want to recover the dispersion relation. We can do so by measuring the effective mass $m^\star = \hbar^2[\frac{d^2E(k_x)}{dk_x}]^{-1}$ of the nearly quadratic lowest branch of the dispersion, and then shifting the measured frequencies accordingly.

Here an image of dispersion, time evolution, FFT, spectrum of energy differences and reconstructed spectrum.

In order to maximize our signal to noise ratio (SNR) and minimize the required number of data points we use some fancy algorithm that I'm still not sure which one will work best. We also choose the spacing and the total number of pulses for each spectra so that the bandwith and resolution of the Fourier transform allow us to resolve the frequencies of interest.

1.2. Modulated Raman Coupling

Order of ideas here: Explain how we get SOC, how we modulate it. Explain pulsing procedure. Explain effective mass measurement.

We consider a spin one system in a uniform magnetic field $B\mathbf{e}_z$ that Zeeman splits the energy levels by $\omega_Z/2\pi = g_f\mu_B B = 12MHz$. A quadratic Zeeman shift ϵ breaks the $m_F = \pm 1 \leftrightarrow m_F = 0$ degeneracy. The system can be fully described by the Hamiltonian

Here I need to introduce delta and frequency differences.

$$\begin{aligned}\hat{H} =& \frac{\hbar^2\hat{k}^2}{2m} + \alpha_0\hat{k}\hat{F}_z + 4E_L\mathbb{I} + \frac{\Omega_R}{2}\hat{F}_x] \\ &+ (\epsilon + 4E_L)(\hat{F}_z^2 - \mathbb{I}) + \Delta_0\hat{F}_z,\end{aligned} \tag{1}$$

where we have introduced the natural units of our system: the single photon recoil momentum $k_L = \frac{2\pi}{\lambda_R}\sin(\theta/2)$ and its associated recoil energy $E_L = \frac{\hbar^2k_L^2}{2m}$, determined by the wavelength and geometry of the Raman field. We have additionally introduced the spin-orbit coupling strength $\alpha_0 = \frac{\hbar^2k_L}{m}$ and the Raman coupling strength Ω_R which is proportional to the field intensity.

Previous studies have shown that driven systems such as cold atoms in time dependent optical fields[karina, optical lattices, germany group] exhibit effective coupling terms in the Hamiltonian that arise from averaging the dynamics of the system. Here we will show that we can get tunable spin-orbit coupling using a multiple frequency Raman field which is equivalent to periodically modulating the Raman coupling strength.

With the addition of these multiple frequency couplings, the Hamiltonian in Eq.1 remains unchanged, except for the coupling strength that takes the form $\Omega_R(t) = \Omega_0 + \Omega\cos(\delta\omega t)$. Introduce a little bit of Floquet theory?

If the driving frequency is chosen so that $\delta\omega \gg \epsilon$ and $\delta\omega \gg 4E_L$ the effective Floquet Hamiltonian retains the form of 1 with renormalized coefficients and quadratic Zeeman shift, and an additional term that explicitly couples the $m_f = -1$ and $m_f = +1$ states:

$$\begin{aligned}\hat{H} =&\frac{\hbar^2\hat{k}^2}{2m} + \alpha\hat{k}\hat{F}_z + 4E_L\mathbb{I} + \frac{\Omega_0}{2}\hat{F}_x \\ &+ \frac{\tilde{\Omega}}{2}\hat{F}_{xz} + (\tilde{\epsilon} + 4E_L)(\hat{F}_z^2 - \mathbb{I}) + \tilde{\Delta}\hat{F}_z,\end{aligned} \tag{2}$$

with $\alpha = J_0(\Omega/2\delta\omega)\alpha_0$, $\tilde{\Omega} = 1/4(\epsilon + 4E_L)(J_0(\Omega/\delta\omega) - 1)$, $\Delta = J_0(\Omega/2\delta\omega)\Delta_0$, and $\tilde{\epsilon} = 1/4(4E_L - \epsilon) - 1/4(4E_L + 3\epsilon)J_0(\Omega/\delta\omega)$

There are two limiting cases of this effective Hamiltonian 2 which will be of interest: (1) for large quadratic Zeeman shift the system can be described as an effective spin 1/2 (cite Lindsay) system where the spin orbit coupling strength and the Raman coupling can be independently tuned and (2) for small quadratic Zeeman shift we can tune the $m_f = +1 \leftrightarrow m_f = -1$ and $m_f = 0 \leftrightarrow m_f = \pm 1$ coupling strength as well as the spin-orbit coupling strength and energy of the $m_f = 0$ state relative to the other two states. In this work we will only focus on the high field regime, and measure the spin-orbit dispersion for three different configurations: No modulation, pure modulation, and modulation plus dc offset.

In the high field regime, when $\epsilon > 4E_R$, the $m_f = -1 \leftrightarrow m_f = 0$ and $m_f = 0 \leftrightarrow m_f = +1$ transition cannot be resonantly addressed with the same frequency. By adiabatically eliminating the $m_f = 0$ state we can describe the system in terms of an effective spin 1/2 with an effective Hamiltonian

$$\hat{H}_{eff} = \frac{\hbar^2}{2m}(\hat{k} + 2k_R\hat{\sigma}_z)^2 + \frac{\hbar\Omega'}{2}\hat{\sigma}_x + \Delta\hat{\sigma}_z \tag{3}$$

where we have defined an effective coupling between the $m_f = -1$ and $m_f = +1$ states $\Omega' = \tilde{\Omega} + \hbar\Omega_0^2/2(\tilde{\epsilon})$.

Fig1b shows the hight field dispersion relation, both for the modulated and unmodulated cases (here goes an image of bands). The minima, originally locted at $\pm 2k_L$, are shifted and the size of the spin-orbit gap is changed for different choices of Ω_0, Ω, and $\delta\omega$.

2. Experiment

2.1. Spectroscopy experimental sequence

We start our experiments with a Rb^{87} Bose-Einstein condensate (BEC) with $N \approx 4 \times 10^4$ (measure) atoms in the $|F = 1, m_F = 0\rangle$ state, confined in a 1064 nm crossed optical dipole trap, with trapping frequencies $(\omega_x, \omega_y, \omega_z) = 2\pi(42(3), 34(2), 133(3))$ Hz. We break the degeneracy between the m_F magnetic sub-levels by applying a 17.0556 G bias field along the z axis, which produces a Zeeman splitting of 12 MHz and a quadratic Zeeman shift that lowers the energy of the $|F = 1, m_F = 0\rangle$ state by 20.9851 kHz. We apply a pair of microwave pulses to monitor and stabilize the bias field and. We generate spin-orbit coupling between the magnetic sub levels with a pair of intersecting, cross-polarized Raman beams, with wavelength $\lambda = 790.33nm$ propagating along $\mathbf{e}_x + \mathbf{y}$ and $\mathbf{e}_x - \mathbf{e}_y$ as shown in Fig 1a. We offset the frequency of the beams using two acusto optic modulators (AOMs), one of them driven a superposition of up to three different frequencies. We set the

Raman detuning conditions by keeping the magnetic field constant and changing the value of the frequency ω_A. On resonance, the laser frequencies satisfy the condition $\omega_A - \omega_B = \omega_A - \frac{\omega_{B+}+\omega_{B-}}{2} = \omega_Z$.

For a given detuning value, we pulse our Raman beams for time intervals between 5 μs and up to 900 μs. We then release the atoms from the optical dipole trap and let them fall for a 21 ms time of flight (TOF) time before we image them using resonant absorption imaging. Our images reveal the atoms spin and momentum distribution, from which we can extract the full dynamics of the system.

In order to account for missing data points we use a not-uniform fast Fourier transform algorithm (NUFFT) which allows us to get the power spectral density for data points that are not necessarily evenly spaced in time as required by regular FFT algorithms.

2.2. Effective mass measurement

We measure the atom's ground state effective mass by inducing dipole oscillations in our BECs for both the bare and Raman dressed atoms. The effective mass $m^\star$ of the dressed atoms is related to the bare mass m and the bare and dressed trapping frequencies ω and ω^star by the ratio $m^\star/m = \sqrt{\omega^\star/\omega}$

To measure the trapping frequencies, we prepare our system in $|F = 1, m_F = 0\rangle$ and adiabatically turn on the Raman in ≈ 10 ms while also ramping the detuning to a non-zero value, around $0.5E_R$. Our system does not have the capability to dynamically change the laser frequency while maintaining phase stability, so unlike the pulsing experiments, we ramped the magnetic field to change the resonance conditions. This magnetic field induced detuning shifts the minima in the ground state energy away from zero quasi-momentum. We then suddenly snap the field back to resonance which changes the equilibrium conditions of the system and excites the dipole mode of our optical dipole trap. For the bare state frequency, we repeat the same procedure but we completely snap off the Raman after the initial ramp.

For this set of measurements we modified our trapping frequencies to $(\omega_x, \omega_y, \omega_z) = 2\pi(35.9, 32.5, xx)$ Hz so that they were nominally symmetric along the $x - y$ plane. Our Raman beams are at a 45° (can probably measure with more accuracy) angle with respect to the optical dipole trap beams, so a rotation $\mathbf{e}_x + \mathbf{e}_y$ $x \to \frac{x+y}{\sqrt{2}}$, $y \to \frac{x-y}{\sqrt{2}}$ leads to a trapping frequency along the Raman recoil momentum direction $\omega = \sqrt{\omega_x^2 + \omega_y^2}$ for axially symmetric traps.

2.3. Magnetic field stabilization

We stabilized the magnetic field and measured fluctuations about the desired set point by applying a pair of microwave pulses with frequencies close to resonance from the $5^2S_{1/2}$ $F = 2$ state, and imaging the in-situ the population transfered by each pulse.

We first prepare our BEC in the $|F = 1, m_f = 0\rangle$ state and apply a 17.0556 G bias filed along the z axis. We then apply a pair of $250\mu s$ microwave pulses, each one blue (red) detuned from the $|F = 1, m_F = 0\rangle \leftrightarrow |F = 2, m_F = 1\rangle$ transition by $+(-)2$ kHz. We can separately and non-destructively image the atoms transfered into $F = 2$ and extract the transferred population imbalance, which gives us a 4 kHz wide error signal. We use the error signal both to feed forward our bias coils and actively stabilize the bias field, and also to post select that is 0.5 mG of the desired magnetic field set-point. We additionally trigger our sequence to the line and both the

microwave and Raman pulses are timed at integer periods of 60 Hz and performed at the zero-derivative point of the 60 Hz curve in order to minimize additional magnetic field fluctuations

3. Results

3.1. Dispersion relations

Here put some pretty plots.

3.2. Coupling strength and detuning calibrations

Both Raman lasers were generated by diode laser with frequency set at 790.024 nm, the magic wavelength of Rb^{87}. We offset their frequency using two acousto-optic modulators, one driven at a single frequency near 80 MHz, and the second one driven with a superposition of up to 3 frequencies. We detune the Raman lasers by changing the driving frequency of the AOMs while keeping the bias magnetic field constant. We calibrated the Raman coupling strength Ω and the detuning from Raman resonance Δ by fitting the three-level Rabi oscillations of the $m_F = 0$ and $m_f = \pm 1$ states to the time evolution given by the Hamiltonian in Eq. 1.

Put a plot with time evolution and fit. One for simple soc case, one for Molmer-Sorensen case. Fourier trasform and spectra. One image that shows how it is done and then several other spectra. It would be great if I show both tunability and and Floquet bands. Effective mass measurement data? Show higher harmonics. Spin resolved spectroscopy.

3.3. Dark states at $q = 0$

4. Discussion

Heating problems? Talk about Mention the possibility of measuring the Floquet bands. Possibility to extend this to the regime where cyclic coupling are not negligible and do butterfly physics.

Higher coupling strength compared to quadratic zeeman shift means more coupling within different Floquet manifolds. So interpreting the 'tuned' bands becomes more challenging. Think about the coupling strength vs quadratic zeeman shift in terms of excited floquet states. Spectroscopy of bloch bands? Does driving strength exceed transition frequency? No, but it does exceed the quadratic zeeman shift. From the floquet paper: The observed system dynamics is very well described in terms of quasienergies and quasienergy states, as predicted by Floquet theory. In particular, we observe several frequency components in the dynamics, in very good agreement with theory

Heating due to scattering of spontaneously emitted photons is always present in our system. The time scales of our pulsing experiments never exceeded 1 ms which is small compared to the lifetime of our system (measure lifetime with and without Raman?). Heating is also present in periodically driven systems, and while it can be minimized by increasing the driving frequency one in exchange requires more power to achieve the same tunability in the system.

5. Conclusion

In conclusion, we can measure the spin and momentum dependent dispersion relation for a spin-1 spin-orbit coupled BEC using our Fourier spectroscopy thechnique. This method is good for any (effective) three level system with a quadratic Zeeman shift $\epsilon > 4E_L$ and does not require any additional hardware as it relies only on the Hamiltonian evolution of the system. Our technique can also be useful to measure the Floquet quasi energy spectrum and the coupling within different Floquet manifolds for driven systems.

The relation between the dynamics/time evolution of a system is rooted in the heart of quantum mechanics.

Does driving strength exceed transition frequency? No, but it does exceed the quadratic zeeman shift. From the floquet paper: The observed system dynamics is very well described in terms of quasienergies and quasienergy states, as predicted by Floquet theory. In particular, we observe several frequency components in the dynamics, in very good agreement with theory Think about the coupling strength vs quadratic zeeman shift in terms of excited floquet states. Think about fourier spectroscopy for Bloch bands. In analogy to solid state stuff we introduce a new Fourier based spectroscopy technique that we use to measure the spin dependent energy-momentum dispersion bands of the system. With the addition of a one dimensional optical lattice, this work opens the ground for measurements of the Hofstadter butterfly spectrum.

Previous studies have shown that strong modulation in the Raman coupling strength for an effective spin 1/2 system leads to tunable spin-orbit coupling strength. In addition to this, the use of two Raman coupling frequencies in a spin one system which is equivalent to a single frequency amplitude modulated coupling leads to new magnetic phases (cite spin one papar). Here we extend the study of the effects of amplitude modulation of the Raman coupling strength/multiple frequency couplings. We will show that for a spin one system, we can independently tune the spin orbit-coupling gap and strength, and we can additionally engineer a cyclic coupling between the three m_F magnetic sub-levels.

Fixes:

- Write frequencies as ω_L and $\omega_L+\Delta\omega\pm\delta\omega$. Is there too many δ symbols, confusing?
- Call J_0 the zeroth order Bessel functions of the first kind.
- don't say modulation or multiple frequencies, just say we amplitude modulate by using
- no effective model, say the floquet Hamiltonian takes de form
- mention 2 paths

effective Hamiltonian that captures the essential characteristics of the modulated system. This strategy exploits the fact that modulation schemes can be tailored in such a way that effective Hamiltonians reproduce the Hamiltonians of interesting static systems.

Corrections of the effective Hamiltonian are of the order $1/\delta\omega$.