Fourier spectroscopy of a spin-orbit coupled Bose gas

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Abstract.

The coupling between an electron's spin and motion lies withing the heart cool things in condensed matter systems. The high degree of control in cold atoms systems makes them ideal candidates for studying new phases of matter and simulating quantum systems. Here we generate spin-orbit coupling in a spin-1 Bose-Einstein condensate using Raman transitions. We are able to measure the system's spin and momentum dependent energy spectrum by looking at the time evolution of the three spin states. We drive transitions at different detunings from Raman resonance and extract the Fourier components of the time dependent evolution to reconstruct the spectrum. We also add a periodic modulation to one Raman field which allows us to have a fully tunable spin-orbit coupling dispersion that we can directly measure using our spectroscopy technique. (maybe switch the order of the last ant previous to last things ideas introduced) Mention this allows also to look at the Floquet quasienergies?

Contents

1. Introduction

The the relation between the dynamics/time evolution of a system is rooted in the heart of quantum mechanics.

Does driving strength exceed transition frequency? No, but it does exceed the quadratic zeeman shift.

From the floquet paper: The observed system dynamics is very well described in terms of quasienergies and quasienergy states, as predicted by Floquet theory. In particular, we observe several frequency components in the dynamics, in very good agreement with theory

Think about the coupling strength vs quadratic zeeman shift in terms of excited floquet states.

Think about fourier spectroscopy for Bloch bands.

In analogy to solid state stuff we introduce a new Fourier based spectroscopy technique that we use to measure the spin dependent energy-momentum dispersion bands of the system. With the addition of a one dimensional optical lattice, this work opens the ground for measurements of the Hofstadter butterfly spectrum.

Previous studies have shown that strong modulation in the Raman coupling strength for an effective spin 1/2 system leads to tunable spin-orbit coupling strength. In addition to this, the use of two Raman coupling frequencies in a spin one system which is equivalent to a single frequency amplitude modulated coupling leads to new magnetic phases (cite spin one papar). Here we extend the study of the effects of amplitude modulation of the Raman coupling strength/multiple frequency couplings. We will show that for a spin one system, we can independently tune the spin orbit-coupling gap and strength, and we can additionally engineer a cyclic coupling between the three m_F magnetic sub-levels.

Fixes:

- Write frequencies as ω_L and $\omega_L + \Delta\omega \pm \delta\omega$. Is there too many δ symbols, confusing?

- Call J_0 the zeroth order Bessel functions of the first kind.
- don't say modulation or multiple frequencies, just say we amplitude modulate by using
- no effective model, say the floquet Hamiltonian takes de form
- mention 2 paths

effective Hamiltonian that captures the essential characteristics of the modulated system. This strategy exploits the fact that modulation schemes can be tailored in such a way that effective Hamiltonians reproduce the Hamiltonians of interesting static systems.

Corrections of the effective Hamiltonian are of the order $1/\delta\omega$.

Order of ideas here: First spectroscopy. Describe for a single frequency spin one SOC Hamiltonian. Describe the periodic drive case, brief intro to Floquet theory.

2. Methods

2.1. Modulated Raman Coupling

Order of ideas here: Explain how we get SOC, how we modulate it. Explain pulsing procedure. Explain effective mass measurement.

We consider a spin one system in a uniform magnetic field $B\mathbf{e}_z$ that Zeeman splits the energy levels by $\omega_Z/2\pi = g_f\mu_B B = 12MHz$. A quadratic Zeeman shift ϵ breaks the $m_F = \pm 1 \leftrightarrow m_F = 0$ degeneracy. We generate spin-orbit coupling between the magnetic sub levels with a pair of intersecting, cross-polarized Raman beams, with wavelength $\lambda = 790.33nm$ propagating along $\mathbf{e}_x + \mathbf{y}$ and $\mathbf{e}_x - \mathbf{e}_y$ as shown in Fig 1a. The system can be fully described by the Hamiltonian

Here I need to introduce delta and frequency differences.

$$\begin{aligned}\hat{H} =& \frac{\hbar^2\hat{k}^2}{2m} + \alpha_0\hat{k}\hat{F}_z + 4E_L\mathbb{I} + \frac{\Omega_R}{2}\hat{F}_x] \\ &+ (\epsilon + 4E_L)(\hat{F}_z^2 - \mathbb{I}) + \Delta_0\hat{F}_z,\end{aligned} \tag{1}$$

where we have introduced the natural units of our system: the single photon recoil momentum $k_L = \frac{2\pi}{\lambda_R}\sin(\theta/2)$ and its associated recoil energy $E_L = \frac{\hbar^2 k_L^2}{2m}$, determined by the wavelength and geometry of the Raman field. We have additionally introduced the spin-orbit coupling strength $\alpha_0 = \frac{\hbar^2 k_L}{m}$ and the Raman coupling strength Ω_R which is proportional to the field intensity.

Previous studies have shown that driven systems such as cold atoms in time dependent optical fields[karina, optical lattices, germany group] exhibit effective coupling terms in the Hamiltonian that arise from averaging the dynamics of the system. Here we will show that we can get tunable spin-orbit coupling using a multiple frequency Raman field which is equivalent to periodically modulating the Raman coupling strength.

We choose t With the addition of these multiple frequency couplings, the Hamiltonian in Eq.1 remains unchanged, except for the coupling strength that takes the form $\Omega_R(t) = \Omega_0 + \Omega\cos(\delta\omega t)$. In order to describe the full quasi-energy spectrum If the driving frequency is chosen so that $\delta\omega \gg \epsilon$ and $\delta\omega \gg 4E_L$ the effective Hamiltonian retains the form of 1 with renormalized coefficients and quadratic Zeeman shift, and an additional term that explicitly couples the $m_f = -1$ and $m_f = +1$ states:

In order to describe this time periodic system we can find the quasi-energies using Floquet theory change here , or we can find an effective Hamiltonian applying an appropriate time dependent rotation and averaging out fast oscillating terms.

$$\begin{aligned}\hat{H} =&\frac{\hbar^2\hat{k}^2}{2m} + \alpha\hat{k}\hat{F}_z + 4E_L\mathbb{I} + \frac{\Omega_0}{2}\hat{F}_x \\ &+ \frac{\tilde{\Omega}}{2}\hat{F}_{xz} + (\tilde{\epsilon} + 4E_L)(\hat{F}_z^2 - \mathbb{I}) + \tilde{\Delta}\hat{F}_z,\end{aligned} \tag{2}$$

with $\alpha = J_0(\Omega/2\delta\omega)\alpha_0$, $\tilde{\Omega} = 1/4(\epsilon + 4E_L)(J_0(\Omega/\delta\omega) - 1)$, $\Delta = J_0(\Omega/2\delta\omega)\Delta_0$, and $\tilde{\epsilon} = 1/4(4E_L - \epsilon) - 1/4(4E_L + 3\epsilon)J_0(\Omega/\delta\omega)$

There are two limiting cases of this effective Hamiltonian 2 which will be of interest: (1) for large quadratic Zeeman shift the system can be described as an effective spin 1/2 (cite Lindsay) system where the spin orbit coupling strength and the Raman coupling can be independently tuned and (2) for small quadratic Zeeman shift we can tune the $m_f = +1 \leftrightarrow m_f = -1$ and $m_f = 0 \leftrightarrow m_f = \pm 1$ coupling strength as well as the spin-orbit coupling strength and the quadratic Zeeman shift, which with the addition of an optical lattice can lead to measurements of Hofstadter butterfly spectrum(cite synthetic dimensions). In this work we will only focus in the first case.

In the high field regime, when $\epsilon > 4E_R$, the $m_f = -1 \leftrightarrow m_f = 0$ and $m_f = 0 \leftrightarrow m_f = +1$ transition cannot be resonantly addressed with the same frequency. By adiabatically eliminating the $m_f = 0$ state we can describe the system in terms of an effective spin 1/2 with an effective Hamiltonian

$$\hat{H}_{eff} = \frac{\hbar^2}{2m}(\hat{k} + 2k_R\hat{\sigma}_z)^2 + \frac{\hbar\Omega'}{2}\hat{\sigma}_x + \Delta\hat{\sigma}_z \tag{3}$$

where we have defined an effective coupling between the $m_f = -1$ and $m_f = +1$ states $\Omega' = \tilde{\Omega} + \hbar\Omega_0^2/2(\tilde{\epsilon})$.

Fig1b shows the hight field dispersion relation, both for the modulated and unmodulated cases (here goes an image of bands). The minima, originally locted at $\pm 2k_L$, are shifted and the size of the spin-orbit gap is changed for different choices of Ω_0, Ω, and $\delta\omega$.

2.2. Fourier Spectroscopy

Think transfer functions and spectroscopy. Spectroscopy is a vertical cut, it looks at response of the system when driven with frequencies within the Fourier limited bandwith of the pulse.

In order to explicitly measure the modified energy-momentum dispersion relation, we will use a Fourier based spectroscopy technique, which relies on the time evolution of an atomic state after a dressing field is suddenly turned on, and the initially bare states become superpositions of dressed states undergoing Rabi oscillations in time with spectral components related to the relative energies of the dressed states.

For the case of a spin-orbit coupled atomic system, the dressed state energies are explicitly dependent on both the particle's spin and momentum. Therefore, in order to fully characterize the energy-momentum dispersion we must prepare an atomic state

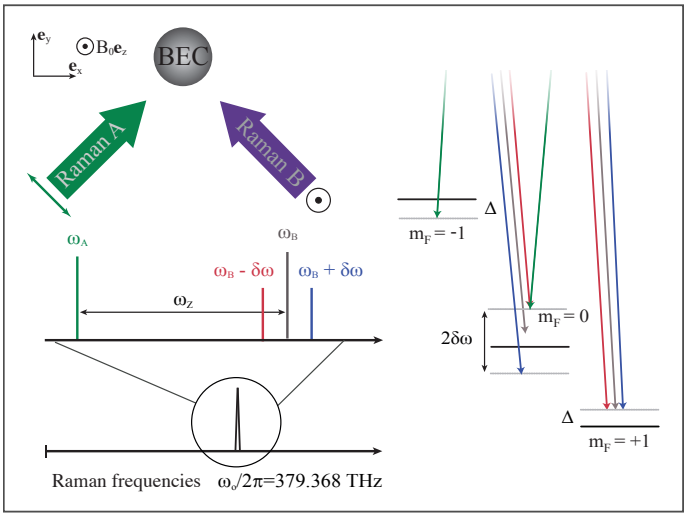

Figure 1. Experimental setup A pair of Raman laser bla bla

at a given spin and momentum $|k_i, m_{f_i}\rangle$, pulse on the Raman field, and measure the time evolution.

In practice it may not be as straightforward to reliably prepare an arbitrary momentum state. The measurement however can be simplified by noticing that a non-moving atom cloud in the laboratory reference frame dressed by a field with non-zero detuning is equivalent to a moving cloud with a resonant field in a suitable moving reference frame. This can be explicitly seen in the Hamiltonian 1 where the detuning term δ/Er and the momentum term $4k/k_R$ have the same effect in the relative energies. There is an additional Doppler shift associated with the transformation between reference frames, which gets canceled when we look at the energy differences. Therefore, for the purpose of our experiments, momentum and detuning are equivalent up to a numerical pre factor.

The method described above only allows us to measure relative energies and we must add a known energy reference if we want to recover the dispersion relation. We can do so by measuring the effective mass $m^\star = \hbar^2[\frac{d^2E(k_x)}{dk_x}]^{-1}$ of the nearly quadratic lowest branch of the dispersion, and then shifting the measured frequencies accordingly.

Here an image of dispersion, time evolution, FFT, spectrum of energy differences and reconstructed spectrum.

In order to maximize our signal to noise ratio (SNR) and minimize the required number of data points we use some fancy algorithm that I'm still not sure which one will work best. We also choose the spacing and the total number of pulses for each spectra so that the bandwith and resolution of the Fourier transform allow us to resolve the frequencies of interest.

2.3. Experimental sequence

We start our experimental sequence with a Rb^{87} Bose-Einstein condensate (BEC) with $N \approx 4\times10^4$ atoms in the $|F = 1, m_F = 0\rangle$ state, confined in a 1064 nm crossed optical

dipole trap, with trapping frequencies $(\omega_x, \omega_y, \omega_z) = 2\pi(42(3), 34(2), 133(3))$ Hz. We break the degeneracy between the m_F magnetic sub-levels by applying a 17.0556 G bias field along the z axis, which produces a Zeeman splitting of 12 MHz and a quadratic Zeeman shift that lowers the energy of the $|F = 1, m_F = 0\rangle$ state by 20.9851 kHz. We use a microwave pulsing protocol (see Supplementary Information) to monitor the bias field and lock it to the desired value. Once the bias field is locked, the sequence we used to study the time evolution of our spin-orbit coupled system was morally the same for all the data we present. For the single frequency case we choose a Raman coupling strength and set the detuning by changing the frequency of our Raman lasers away from four-photon resonance. For the multiple frequency case, when the detuning is set to zero, both ω_0 and $\frac{\omega_+ + \omega_-}{2}$ are at four photon resonance. Otherwise we detune the frequency of ω_0 and $\frac{\omega_+ + \omega_-}{2}$ by the same amount. We suddenly turn on the lasers, let the system evolve for a time t, snap off the lasers and let the atoms fall for a 21 ms time of flight (TOF) time before we image them using resonant absorption imaging. Our images reveal the atoms spin and momentum distribution, from which we can extract the full dynamics of the system.

We measure the atom's effective mass by inducing dipole oscillations in our BECs along $\mathbf{e}_x$ and comparing the frequency of oscillation for the bare and dressed atoms case. We prepare our system in $|F = 1, m_F = 0\rangle$ and adiabatically turn on the Raman in xx ms while also ramping the detuning to a non-zero value, around $0.5E_R$. This detuning gives a small shift in the energy-momentum minima which causes a small displacement of our BECs. We then suddenly bring the frequency back to resonance (for the dressed state measurement) or turn off the Raman (for the bare state measurement), which excites the dipole mode of our optical dipole trap. We can extract the effective mass from the ratio $m^\star/m = \omega^\star/\omega$

3. Results

Put a plot with time evolution and fit. One for simple soc case, one for Molmer-Sorensen case. Fourier trasform and spectra. One image that shows how it is done and then several other spectra. It would be great if I show both tunability and and Floquet bands. Effective mass measurement data? Show higher harmonics. Spin resolved spectroscopy.

4. Discussion

Heating problems? Talk about Mention the possibility of measuring the Floquet bands. Possibility to extend this to the regime where cyclic coupling are not negligible and do butterfly physics.

5. Conclusion

We

6. Supplementary Information

6.1. Effective SOC Hamiltonian

The effective spin-orbit coupling Hamiltonian can be derived from the electric dipole Hamiltonian describing the interaction between our Rb atoms and the multiple frequency Raman lasers

$$\hat{H}_{AL}(t) = \left[\Omega_{21}\cos(2k_R x - \omega_{21}t + \phi_1) + \Omega_{31}\cos(2k_R x - \omega_{31}t + \phi_2) + \Omega_{41}\cos(2k_R x - \omega_{41}t + \phi_3)\right]\hat{F}_x \tag{4}$$

where $\Omega_{ij} \propto \vec{E}_i \times \vec{E}_j^{\star}$ represents the coupling strength associated to each pair of Raman beams and $\omega_{ij} = \omega_i - \omega_j$. We choose the frequencies so that $\omega_{31} + \omega 21$ is at 4 photon resonance with the $m_f = +1 \rightarrow m_f = -1$ transition. We then apply a rotation about the z axis $\hat{U} = e^{i\bar{\omega}t\hat{F}_z}$ where $\bar{\omega} = \frac{\omega_{21}+\omega_{31}}{2}$. For the choice of parameters $\Omega_{21} = \Omega_{31} = \Omega$, $\Omega_{41} = \Omega_0$, and $\omega_{41} = \frac{\omega_{21}+\omega_{31}}{2}$,and after applying the rotating wave approximation (RWA), the Hamiltonian transforms to

$$\begin{aligned}\hat{\tilde{H}}_{AL}(t) = &\frac{1}{2}\{\Omega\cos(2k_R x + \delta\omega t + \phi_1) + \Omega\cos(2k_R x - \delta\omega t + \phi_2) + \Omega_0\cos(2k_R x + \phi_3)\}\hat{F}_x \\ &- \frac{1}{2}\{\Omega\sin(2k_R x + \delta\omega t + \phi_1) + \Omega\sin(2k_R x - \delta\omega t + \phi_2) + \Omega_0\sin(2k_R x + \phi_3)\}\hat{F}_y\end{aligned} \tag{5}$$

we have the freedom of defining the time origin so we can get rid of one phase, lets make it ϕ_3 for convenience. Can I get rid of a second phase? Make $\phi_1 = -\phi_2$

$$\hat{\tilde{H}}_{AL}(t) = [\frac{\Omega_0}{2} + \Omega\cos(\delta\omega t + \phi_1)][\cos(2k_R x)\hat{F}_x - \sin(2k_R x)\hat{F}_y], \tag{6}$$

where we have defined $\delta\omega = \frac{\omega_{31}-\omega_{21}}{2}$. This Hamiltonian term describes a helically precessing effective Zeeman field with amplitude oscillating periodically in time.

The complete Hamiltonian can therefore be written as

$$\hat{H}(t) = \frac{\hbar^2}{2m}\hat{k}^2 + \mu\mathbf{B}_{eff}\cdot\hat{\mathbf{F}} \tag{7}$$

(check units here) with $\mu\mathbf{B}_{eff} = (\frac{\Omega_0}{2} + \Omega\cos(\delta\omega t + \phi_1))(\cos(2k_R x)\mathbf{e}_x - \sin(2k_R x)\mathbf{e}_y) + \Delta_0\mathbf{e}_z$

We can apply a position dependent rotation $\hat{U} = e^{i2k_R x\hat{F}_z}$ transforms our Hamiltonian into the form of Eq. 1 with a time dependent Raman coupling.

$$\begin{aligned}\hat{H}(t) =&\frac{\hbar^2}{2m}(\hat{k} - 2k_R\hat{F}_z)^2 + (\frac{\Omega_0}{2} + \Omega\cos(\delta\omega t))\hat{F}_x \\ &+ \epsilon(\hat{F}_z^2 - \mathbb{I}) + \Delta\hat{F}_z \\ =&\frac{\hbar^2\hat{k}^2}{2m} + \alpha_0\hat{k}\hat{F}_z + 4E_L\mathbb{I} + \frac{\Omega(t)}{2}\hat{F}_x \\ &+ (\epsilon + 4E_L)(\hat{F}_z^2 - \mathbb{I}) + \Delta_0\hat{F}_z\end{aligned} \tag{8}$$

(check factors of 2!)

To get rid of the time dependence in the Hamiltonian and ultimately getting the 'tunable' spin-orbit coupling we can choose a transoformation of the Hamiltonian such that $\hat{U}^\dagger \frac{\partial \hat{U}}{\partial t} = -i\frac{\Omega(t)}{2}\hat{F}_x$. This will be satisfied for

$$\hat{U} = e^{-i\frac{\Omega}{2}\int_0^t \cos(\delta\omega t')dt'} = e^{-i\frac{\Omega}{2\delta\omega}\sin(\delta\omega t)}. \tag{9}$$

Under this time dependent transformation, the time evolution of the system will be given by the Hamiltonian

$$\begin{aligned}\hat{\tilde{H}} =& \hat{U}^\dagger \hat{H}(t)\hat{U} + i\hat{U}^\dagger \frac{\partial \hat{U}}{\partial t} \\ =& \frac{\hbar^2\hat{k}^2}{2m} + \alpha\hat{k}\hat{F}_z + 4E_L\mathbb{I} + \frac{\Omega_0}{2}\hat{F}_x \\ &+ \frac{\tilde{\Omega}}{2}\hat{F}_{xz} + (\tilde{\epsilon} + 4E_L)(\hat{F}_z^2 - \mathbb{I}) + \Delta\hat{F}_z\end{aligned} \tag{10}$$

which is exactly Eq. 2. To arrive to this final form we have transformed the operators that don't commute with $\hat{F}_x$ as

$$\begin{aligned}e^{i\theta\hat{F}_x}\hat{F}_z e^{-i\theta\hat{F}_x} &= \cos\theta\hat{F}_z + \sin\theta\hat{F}_y \\ e^{i\theta\hat{F}_x}\hat{F}_y e^{-i\theta\hat{F}_x} &= -\sin\theta\hat{F}_z + \cos\theta\hat{F}_y \\ e^{i\theta\hat{F}_x}\hat{F}_z^2 e^{-i\theta\hat{F}_x} &= \cos^2\theta\hat{F}_z^2 + \sin^2\theta\hat{F}_y^2 + \sin\theta\cos\theta(\hat{F}_z\hat{F}_y + \hat{F}_y\hat{F}_z).\end{aligned} \tag{11}$$

and neglected the terms oscillating at high frequency

$$\begin{aligned}\cos(\Omega/2\delta\omega\sin(\delta\omega t)) &= J_0(\Omega/2\delta\omega) + 2\sum_{n=1}^{\infty} J_{2n}(\Omega/2\delta\omega)\cos(2n(\delta\omega t)) \\ &\approx J_0(\Omega/2\delta\omega) \\ \sin(\Omega/2\delta\omega\sin(\delta\omega t)) &= 2\sum_{n=0}^{\infty} J_{2n+1}(\Omega/2\delta\omega)\sin((2n+1)(\delta\omega t)) \approx 0,\end{aligned} \tag{12}$$

(verify how large $\delta\omega$ needs to be for this approximation to be valid)

6.2. Effective two level system

When is it good to approximate the system as a two level system? Maybe I don't really care too much about it and can just mention the 2 limiting cases. Importance to effective mass.

6.3. Coupling strength and detuning calibrations

Both Raman lasers were generated by diode laser with frequency set at 790.024 nm, the magic wavelength of Rb^{87}. We offset their frequency using two acousto-optic modulators, one driven at a single frequency near 80 MHz, and the second one driven

with a superposition of up to 3 frequencies. We detune the Raman lasers by changing the driving frequency of the AOMs while keeping the bias magnetic field constant. We calibrated the Raman coupling strength Ω and the detuning from Raman resonance Δ by fitting the three-level Rabi oscillations between the $m_F = 0$ and $m_f = \pm 1$ states to the expected time dependent evolution from the Hamiltonian in Eq. 1.

6.4. Magnetic field stabilization

We stabilized the magnetic field and measured fluctuations from the set point by applying a pair of microwave pulses with frequencies close to resonance from the $5^2S_{1/2}$ $F = 2$ line, and imaging in-situ the population transfered by each pulse. For our protocol we first prepare atoms in $m_f = 0$ using an adiabatic rapid passage sequence and apply a 17.0556 G bias filed along the z axis. We then apply a pair of $250\mu s$ microwave pulses, each one blue (red) detuned from the $|F = 1, m_F = 0\rangle \leftrightarrow |F = 2, m_F = 1\rangle$ transition by $+(-)2$ kHz. We can non destructively image the atoms transfered into $F = 2$ and extract the fractional population imbalance, which gives us a 4 kHz wide error signal. We use the error signal both to feed forward our bias coils and actively stabilize the bias field, and also to post select that is 0.5 mG of the desired magnetic field set-point.