# AM205 - HW2 - Exercise 4

October 12, 2021

[ ]:

```
# Start writing code here...
import numpy as np
import scipy.linalg as linalg
from numpy.linalg import norm as norm
import matplotlib.pyplot as plt
```

### 0.0.1 Exercise 4

Collaborated with Varshini

This function generates G

[ ]:

```
#Problem 4a:

n = 10

def generate_g(n):
    G = np.zeros ((n,n))
    np.fill_diagonal(G, 1)
    G[: , [-1]] = 1
    G
    for i in range(n):
        G[i, :i] = -1
    return G

generate_g(n)
```

[ ]:

```
array([[ 1.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  1.],
       [-1.,  1.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  1.],
       [-1., -1.,  1.,  0.,  0.,  0.,  0.,  0.,  0.,  1.],
       [-1., -1., -1.,  1.,  0.,  0.,  0.,  0.,  0.,  1.],
       [-1., -1., -1., -1.,  1.,  0.,  0.,  0.,  0.,  1.],
       [-1., -1., -1., -1., -1.,  1.,  0.,  0.,  0.,  1.],
       [-1., -1., -1., -1., -1., -1.,  1.,  0.,  0.,  1.],
       [-1., -1., -1., -1., -1., -1., -1.,  1.,  0.,  1.],
       [-1., -1., -1., -1., -1., -1., -1., -1.,  1.,  1.],
       [-1., -1., -1., -1., -1., -1., -1., -1., -1.,  1.]])
```

This finds the run time

[ ]:

```
#Problem 4b:
from time import process_time,time
import numpy as np
import scipy.linalg

def generate_g(n):
    G = np.zeros ((n,n))
    np.fill_diagonal(G, 1)
    G[: , [-1]] = 1
    for i in range(n):
        G[i, :i] = -1
    return G


n_list = list(range(10, 1510, 10))
#for n in n_list:
#    print(generate_g(n))

times_list = []


for n in n_list:
    start_time = process_time()
    generate_g(n)
    end_time = process_time()
    time = end_time-start_time
    times_list.append(time)


plt.plot(n_list, times_list)
```

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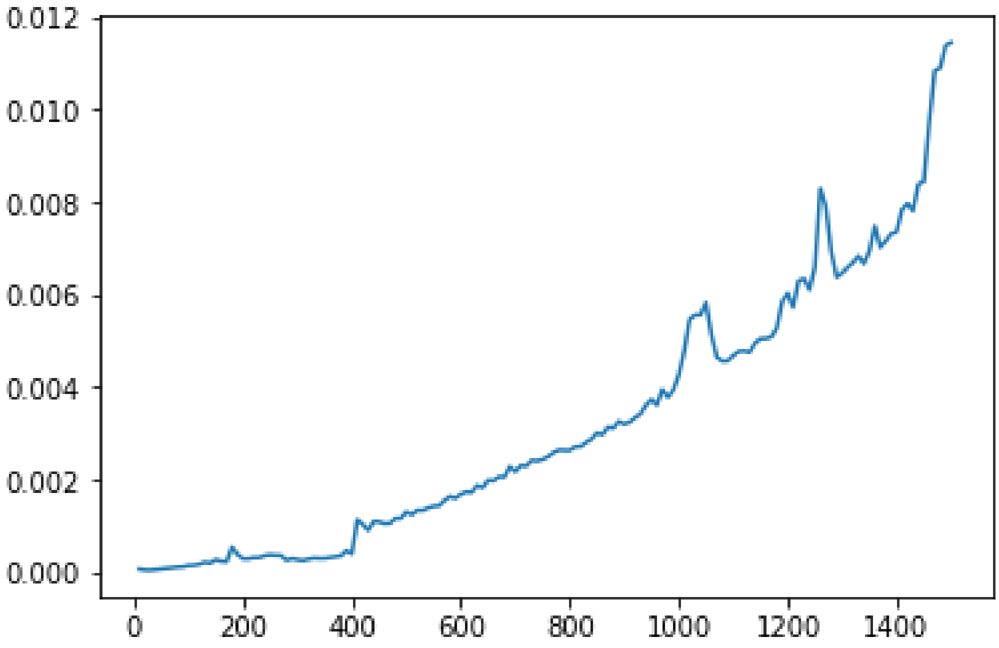
```
[<matplotlib.lines.Line2D at 0x7f0cffc834d0>]
```



[ ]:

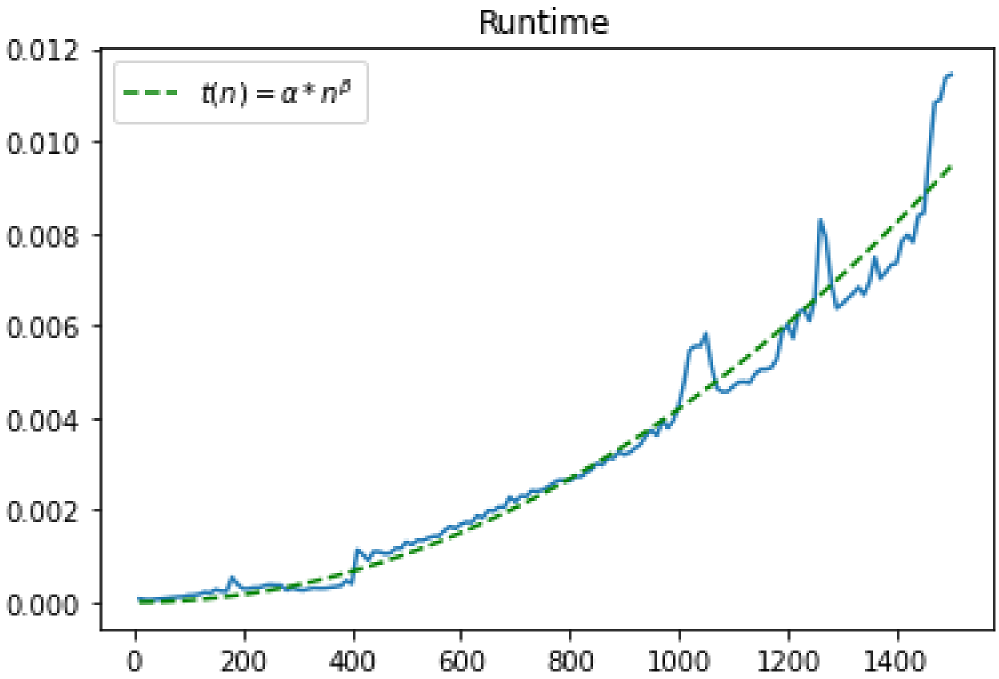
```
#4c

from scipy.optimize import curve_fit

def relationship(x, a, b):
  #return a*pow(x,b)
  return a*(x**b)

fitting_parameters, covariance = curve_fit(relationship, n_list, times_list)
a, b = fitting_parameters
plt.plot(n_list, times_list)
plt.plot(n_list, relationship(n_list, *fitting_parameters), 'g--', label='$t(n)␣
 ↪= \\alpha*n^{\\beta}$')
plt.legend()
plt.title("Runtime")
plt.draw()
plt.savefig("graph_4b.jpeg", dpi=300, bbox_inches='tight')
plt.show()


print(a)
print(b)
```



```
3.959739170518327e-09
2.0081546670375574
```

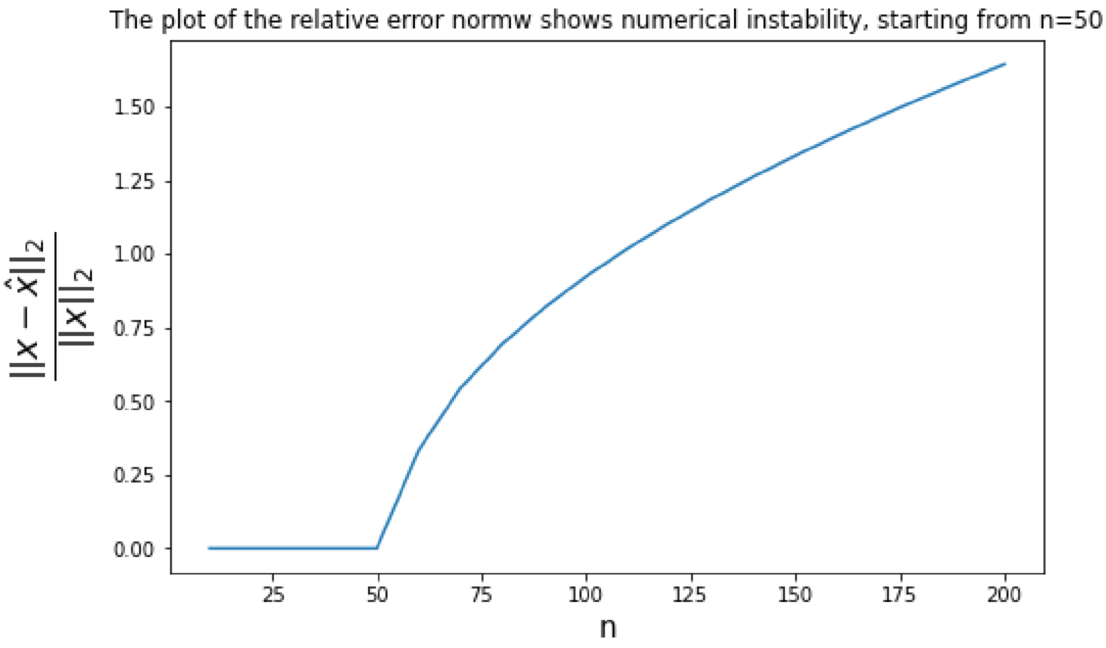
```
[ ]: n=4
     x = np.ones(n)
     print(x.shape)
     G_n = generate_g(n)
     #G_n.shape
     b = G_n@x
     b
```

```
(4,)
```

```
[ ]: array([ 2.,  1.,  0., -2.])
```

```
[ ]: ns = list(range(10, 210, 10))
     # Calculate the ||x-x_hat||_2
     relative_errors_2_norm = []
     for n in ns:
         x = np.ones(n)
         G_n = generate_g(n)
         b = G_n@x
         x_hat = np.linalg.solve(G_n, b)
         relative_error = (np.linalg.norm(x_hat-x) / np.linalg.norm(x_hat))
         relative_errors_2_norm.append(relative_error)
```

```
plt.figure(figsize=(8,5))
plt.plot(ns, relative_errors_2_norm)
plt.xlabel('n', fontsize=15)
plt.ylabel('$\\frac{||x-\hat{x}||_2}{||x||_2}$', fontsize=25)
plt.draw()
plt.title("The plot of the relative error normw shows numerical instability,␣
 ↪starting from n=50")
plt.savefig("graph_4c.jpeg", dpi=300, bbox_inches='tight')
plt.show()
```



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