CSCI-GA.2560-001, Artificial Intelligence March 7, 2022

Solutions to Problem 1 of Homework 5 (6 Points)

Name: Anav Prasad (ap7152) *Due: 5PM on Monday, March 7*

Collaborators:

Let D be the domain of people and animals. Let L be the first-order language over D with the following primitives:

- A(a) - a is an animal
- L(x, y) - person x loves y
- H(x, y) - person x hates y
- D(a) - a is a dog
- Fido: dog
- Jane, Jim: people

Represent the following sentences in L:

1. Everyone who loves all animals is loved by someone
2. Jane or Jim hates the dog, who is named Fido
3. Anyone who hates an animal is loved by no one.
4. A dog is an animal
5. Jane loves all animals
6. Jim hates Fido

Solution:
So, the given sentences would be represented as follows:

1. $\forall x\ [\forall y\ A(y) \implies L(x,y)] \implies [\exists z\ L(z,x)]$
2. $[H(Jane, Fido) \vee H(Jim, Fido)] \wedge D(Fido)$
3. $\forall x\ [\exists y\ A(y) \wedge H(x,y)] \implies [\forall z\ \neg L(z,x)]$
4. $\forall x\ D(x) \implies A(x)$
5. $\forall x\ A(x) \implies L(Jane, x)$
6. $H(Jim, Fido)$

$\square$

CSCI-GA.2560-001, Artificial Intelligence	March 7, 2022
Solutions to Problem 2 of Homework 5 (4 Points)	
Name: Anav Prasad (ap7152)	*Due: 5PM on Monday, March 7*
Collaborators:	

Convert the sentences from A into CNF. Show all steps including Skolemization.

Solution:
Converting the given set of sentences to CNF using the resolution steps:

- **Before Step 1:** The given set of sentences:

  1. $\forall x\ [\forall y\ A(y) \implies L(x,y)] \implies [\exists z\ L(z,x)]$
  2. $[H(Jane, Fido) \lor H(Jim, Fido)] \land D(Fido)$
  3. $\forall x\ [\exists y\ A(y) \land H(x,y)] \implies [\forall z\ \neg L(z,x)]$
  4. $\forall x\ D(x) \implies A(x)$
  5. $\forall x\ A(x) \implies L(Jane, x)$
  6. $H(Jim, Fido)$

- **Step 1:** $\iff$: No change here

  1. $\forall x\ [\forall y\ A(y) \implies L(x,y)] \implies [\exists z\ L(z,x)]$
  2. $[H(Jane, Fido) \lor H(Jim, Fido)] \land D(Fido)$
  3. $\forall x\ [\exists y\ A(y) \land H(x,y)] \implies [\forall z\ \neg L(z,x)]$
  4. $\forall x\ D(x) \implies A(x)$
  5. $\forall x\ A(x) \implies L(Jane, x)$
  6. $H(Jim, Fido)$

- **Step 2:** $\implies$

  1. $\forall x\ \neg[\forall y\ \neg A(y) \lor L(x,y)] \lor [\exists z\ L(z,x)]$
  2. $[H(Jane, Fido) \lor H(Jim, Fido)] \land D(Fido)$
  3. $\forall x\ \neg[\exists y\ A(y) \land H(x,y)] \lor [\forall z\ \neg L(z,x)]$
  4. $\forall x\ \neg D(x) \lor A(x)$
  5. $\forall x\ [\neg A(x) \lor L(Jane, x)]$
  6. $H(Jim, Fido)$

- **Step 3:** $\neg$

1. $\forall x\ [\exists y\ A(y) \wedge \neg L(x,y)] \vee [\exists z\ L(z,x)]$
2. $[H(Jane, Fido) \vee H(Jim, Fido)] \wedge D(Fido)$
3. $\forall x\ [\forall y\ \neg A(y) \vee \neg H(x,y)] \vee [\forall z\ \neg L(z,x)]$
4. $\forall x\ \neg D(x) \vee A(x)$
5. $\forall x\ \ [\neg A(x) \vee L(Jane, x)]$
6. $H(Jim, Fido)$

- **Step 4:** $\exists$ (Skolemization)

1. $\forall x\ [A(Sk0(x)) \wedge \neg L(x, Sk0(x))] \vee L(Sk1(x), x)$
2. $[H(Jane, Fido) \vee H(Jim, Fido)] \wedge D(Fido)$
3. $\forall x\ [\forall y\ \neg A(y) \vee \neg H(x,y)] \vee [\forall z\ \neg L(z,x)]$
4. $\forall x\ \neg D(x) \vee A(x)$
5. $\forall x\ \ [\neg A(x) \vee L(Jane, x)]$
6. $H(Jim, Fido)$

- **Step 5:** $\forall$

1. $[A(Sk0(x)) \wedge \neg L(x, Sk0(x))] \vee L(Sk1(x), x)$
2. $[H(Jane, Fido) \vee H(Jim, Fido)] \wedge D(Fido)$
3. $\neg A(b) \vee \neg H(a,b) \vee \neg L(c,a)$
4. $\neg D(d) \vee A(d)$
5. $\neg A(e) \vee L(Jane, e)$
6. $H(Jim, Fido)$

- **Step 6:** Distribution

1. $[A(Sk0(x)) \vee L(Sk1(x), x)] \wedge [\neg L(x, Sk0(x)) \vee L(Sk1(x), x)]$
2. $[H(Jane, Fido) \vee H(Jim, Fido)] \wedge D(Fido)$
3. $\neg A(b) \vee \neg H(a,b) \vee \neg L(c,a)$
4. $\neg D(d) \vee A(d)$
5. $\neg A(e) \vee L(Jane, e)$
6. $H(Jim, Fido)$

- **Step 7:** Split:

$$
\begin{aligned}
&A(Sk0(x)) \lor L(Sk1(x), x) \\
&\neg L(x, Sk0(x)) \lor L(Sk1(x), x) \\
&H(Jane, Fido) \lor H(Jim, Fido) \\
&D(Fido) \\
&\neg A(b) \lor \neg H(a, b) \lor \neg L(c, a) \\
&\neg D(d) \lor A(d) \\
&\neg A(e) \lor L(Jane, e) \\
&H(Jim, Fido)
\end{aligned}
$$

The CNF obtained is as follows:

$$
\begin{aligned}
&A(Sk0(x)) \lor L(Sk1(x), x) \\
&\neg L(x, Sk0(x)) \lor L(Sk1(x), x) \\
&H(Jane, Fido) \lor H(Jim, Fido) \\
&D(Fido) \\
&\neg A(b) \lor \neg H(a, b) \lor \neg L(c, a) \\
&\neg D(d) \lor A(d) \\
&\neg A(e) \lor L(Jane, e) \\
&H(Jim, Fido)
\end{aligned}
$$

$\square$