CSCI-GA.2560-001, Artificial Intelligence December 20, 2021

Solutions to Problem 1 of Final Examination

Name: Kumar Prasun (kp2692) *Due: December 20*

Collaborators:

Solution: a) With 2 bits we can represent at maximum 4 numbers. With 3 bits we can represent at maximum 8 numbers.
So we will use 3 bits fixed-width encoding. Since there 10,000 instances, total bits $3*10000 = 30,000$ bits.

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b) We want to minimise the number of bits used to encode the numbers. So based on frequencies we set the following prefix-free code:

Value	Encoding	Number of Bits
A	1110	4
B	10	2
C	0	1
D	110	3
E	1111	4

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c) The total number of bits used $= 2000 + 6000 + 4000 + 6000 + 2000 = 20000$ bits:

Value	Encoding	Number of Bits	Frequency	Total Bits
A	1110	4	0.05	4 * 0.05 * 10000 = 2000
B	10	2	0.3	2 * 0.3 * 10000 = 6000
C	0	1	0.4	1 * 0.4 * 10000 = 4000
D	110	3	0.2	3 * 0.2 * 10000 = 6000
E	1111	4	0.05	4* 0.05 * 10000 = 2000

For the decoding table, since we use a "." separator to separate 5 values, we'll need 4 "." separators. Assuming that for decoding 2 bits are required for each value, we have 4+2+1+3+4+4 = 14+4 = 18 bits with the form which would be as follows: 1110.10.0.110.1111.
So total decoding bits are $18 * 2 = 36$ bits.
Total bits $= 20000 + 36 = 20036$ bits.

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d) Code 0: The classifier gives the right answer.
Code 10: The classifier gives the wrong value. The true value is C.
Code 110: The classifier gives the wrong value. The true value is D.
Code 111: The classifier gives the wrong value. The true value is E.

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e) Because of no false negatives for A and B, the frequency of them appearing in classifier's wrong answer is 0.
Therefore,

- frequency of C $= 0.4/(0.4 + 0.2 + 0.05)$
- frequency of D $= 0.2/(0.4 + 0.2 + 0.05)$
- frequency of E $= 0.05/(0.4 + 0.2 + 0.05)$

Total bits are as follows:

- If classifier gives correct answers, it requires $0.8 * 10000 * 1 = 8000$ bits.
- If classifies gives wrong answer and true value is C, it requires $0.2 * 0.4/0.65 * 10000 * 2 = 1600 \approx 2461.53 \approx 2462$ bits.
- If classifies gives wrong answer and true value is D, it requires $0.2 * 0.2/0.65 * 10000 * 3 = 1200 \approx 1846.15 \approx 1846$ bits.
- If classifies gives wrong answer and true value is E, it requires $0.2 * 0.05/0.65 * 10000 * 3 = 300 \approx 461.53 \approx 462$ bits.
- We have k=1+2+3+3+5(for the separator)=14 bits which with the decoding table uses $2k = 28$ bits.
- The classifier requires 128 bits

The sequence : 0...10.110.111
Total bits $= 8000 + 2462 + 1846 + 462 + 128 + 28 = 12926$ bits. □

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Solutions to Problem 2 of Final Examination

Name: Kumar Prasun (kp2692) *Due: December 20*

Collaborators:

Solution:
a) The CNF are as follows

$$\begin{aligned}
\boldsymbol{\neg(A \vee \neg B) \Leftrightarrow C} &= (\neg(A \vee \neg B) \Rightarrow C) \wedge \ (C \Rightarrow \neg(A \vee \neg B)) && (\alpha \Leftrightarrow \beta \equiv (\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) \\
&= (((\neg(A \vee \neg B)) \vee C) \wedge (\neg C \vee \neg(A \vee \neg B)) && (\alpha \Rightarrow \beta \equiv \neg\alpha \vee \beta) \\
&= ((A \vee \neg B) \vee C) \wedge (\neg C \vee \neg(A \vee \neg B)) && (\neg(\neg\alpha) \equiv \alpha) \\
&= (A \vee \neg B \vee C) \wedge (\neg C \vee (\neg A \wedge \neg(\neg B))) && \text{(De Morgan's)} \\
&= (A \vee \neg B \vee C) \wedge (\neg C \vee (\neg A \wedge B)) && (\neg(\neg\alpha) \equiv \alpha) \\
&= (A \vee \neg B \vee C) \wedge ((\neg C \vee \neg A) \wedge (\neg C \vee B)) && ((\alpha \wedge \beta) \vee \gamma \equiv (\alpha \vee \gamma) \wedge (\beta \vee \gamma)) \\
&= (A \vee \neg B \vee C) \wedge (\neg C \vee \neg A) \wedge (\neg C \vee B) && \text{(Simplified)} \\
\boldsymbol{C \Rightarrow (D \vee E)} &= ((\neg C) \vee (D \vee E)) && (\alpha \Rightarrow \beta \equiv \neg\alpha \vee \beta) \\
&= \neg C \vee D \vee E && \text{(Simplified)} \\
\boldsymbol{\neg E \vee (B \wedge C)} &= (\neg E \vee B) \wedge (\neg E \vee C) && ((\alpha \wedge \beta) \vee \gamma \equiv (\alpha \vee \gamma) \wedge (\beta \vee \gamma)) \\
\boldsymbol{(\neg F \vee C) \Rightarrow E)} &= \neg(\neg F \vee C) \vee E && (\alpha \Rightarrow \beta \equiv \neg\alpha \vee \beta) \\
&= (\neg(\neg F) \wedge (\neg C)) \vee E && \text{(De Morgan's)} \\
&= (F \wedge \neg C) \vee E && (\neg(\neg\alpha) \equiv \alpha) \\
&= (F \vee E) \wedge (\neg C \vee E) && ((\alpha \wedge \beta) \vee \gamma \equiv (\alpha \vee \gamma) \wedge (\beta \vee \gamma)) \\
\boldsymbol{F \Rightarrow (\neg A \wedge \neg D)} &= \neg F \vee (\neg A \wedge \neg D) && (\alpha \Rightarrow \beta \equiv \neg\alpha \vee \beta) \\
&= (\neg F \vee \neg A) \wedge (\neg F \vee \neg D) && ((\alpha \wedge \beta) \vee \gamma \equiv (\alpha \vee \gamma) \wedge (\beta \vee \gamma))
\end{aligned}$$

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b) The given CNF is: $(A \vee \neg B \vee C) \wedge (\neg C \vee \neg A) \wedge (\neg C \vee B) \wedge \ (\neg C \vee D \vee E) \wedge (\neg E \vee B) \wedge (\neg E \vee C) \wedge \ (F \vee E) \wedge (\neg C \vee E) \wedge (F \vee \neg A) \wedge (F \vee \neg D)$

To apply Davis Putnam(DPLL), we break this CNF into a tabular form of clauses:

1	$A \vee \neg B \vee C$
2	$\neg C \vee \neg A$
3	$\neg C \vee B$
4	$\neg C \vee D \vee E$
5	$\neg E \vee B$
6	$\neg E \vee C$
7	$F \vee E$
8	$\neg C \vee E$
9	$\neg F \vee \neg A$
10	$\neg F \vee \neg D$

Step 1: Hard guess $A = True$

2	$\neg C$
3	$\neg C \vee B$

4 $\neg C \lor D \lor E$
5 $\neg E \lor B$
6 $\neg E \lor C$
7 $F \lor E$
8 $\neg C \lor E$
9 $\neg F$
10 $\neg F \lor \neg D$

Step 2: We have easy cases B, C, F. We take $B = True$.

2 $\neg C$
4 $\neg C \lor D \lor E$
6 $\neg E \lor C$
7 $F \lor E$
8 $\neg C \lor E$
9 $\neg F$
10 $\neg F \lor \neg D$

Step 3: We have easy cases C, F. We take $C = False$.

6 $\neg E$
7 $F \lor E$
9 $\neg F$
10 $\neg F \lor \neg D$

Step 4: We have easy cases D, E, F. We take $D = False$.

6 $\neg E$
7 $F \lor E$
9 $\neg F$

Step 5: We have easy cases E, F. We take $E = False$.

7 F
9 $\neg F$

Step 6: We take easy case $F = True$. We get a contradiction. So we fail.

7 F
9 $\neg F$

Step 7: We now try hard guess $A = False$.

1 $\neg B \lor C$
3 $\neg C \lor B$
4 $\neg C \lor D \lor E$
5 $\neg E \lor B$
6 $\neg E \lor C$
7 $F \lor E$
8 $\neg C \lor E$
10 $\neg F \lor \neg D$

Step 7: We now try hard guess $B = True$.

1 C
4 $\neg C \lor D \lor E$
6 $\neg E \lor C$
7 $F \lor E$

8 $\neg C \vee E$
10 $\neg F \vee \neg D$

Step 8: We have easy cases C, E. We take $C = True$.

4 $D \vee E$
7 $F \vee E$
8 E
10 $\neg F \vee \neg D$

Step 9: We have easy cases E. We take $E = True$.

10 $\neg F \vee \neg D$

Step 10: We have easy cases D, F. We take $D = False$.
Step 10: F is unbounded, so we take it $F = False$. $\square$

Therefore, with the following as:

$$A = False, B = True, C = True, D = False, E = True, F = False$$

we can satisfy the CNF clauses.

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Solutions to Problem 3 of Final Examination

Name: Kumar Prasun (kp2692) *Due: December 20*

Collaborators:

Solution:
a) For this part we have
1. For initial centroids C1:(750,1500), C2:(2200,500), C3:(1250,1250):

We have the following clusters around the centroid:
$C1 = \{F, G\}$
$C2 = \{A, C, I\}$
$C3 = \{B, D, E, H, J, K, L\}$

We now calculate the new centroids for each cluster:
$C1 = (425.0, 912.5)$
$C2 = (2640.33, 1303.0)$
$C3 = (1564.42, 1335.42)$

In the next iteration, we have the following clusters around the centroid:
$C1 = \{B, F, G\}$
$C2 = \{A, C\}$
$C3 = \{D, E, H, I, J, K, L\}$

We now calculate the new centroids for each cluster:
$C1 = (584.33, 708.33)$
$C2 = (2947, 1397.5)$
$C3 = (1725, 1451.71)$

For the newly computed centroids C1:(584.33, 708.33), C2:(2947, 1397.5), C3:(1725, 1451.71), we get the same clusters for each centroid:
$C1 = \{B, F, G\}$
$C2 = \{A, C\}$
$C3 = \{D, E, H, I, J, K, L\}$
Therefore we stop the iteration here as no change in cluster representation was found. The final centroid values are -
$C1 = (584.33, 708.33), C2 = (2947, 1397.5), C3 = (1725, 1451.71)$

Calculating the distance of each cluster to every other cluster, we get the following table:

	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**
A	0	2490	1351	2420	1546	2806	3177	2364	1312	2283	1997	1740
B		0	3793	3228	1444	1184	997	1618	1938	2457	1529	1602
C			0	1069	2349	2837	4480	2175	1855	1336	2264	2191
D				0	1784	2272	3915	1610	1290	771	1699	1626
E					0	1260	2131	818	494	1013	451	194
F						0	1643	662	1494	1501	809	1066
G							0	2305	2625	3144	2216	2289
H								0	1052	839	367	624
I									0	971	685	428
J										0	928	855
K											0	257
L												0

Complete-Linkage:

1. The clusters E and L have the smallest maximum pairwise distance 194, so we merge them into $\{E, L\}$.
2. Now, the clusters H and K have the smallest maximum pairwise distance 367, so we merge them into $\{H, K\}$.
3. Now, the clusters I and $\{E, L\}$ have the smallest maximum pairwise distance 494, so we merge them into $\{E, I, L\}$.
4. Now, the clusters D and J have the smallest maximum pairwise distance 771, so we merge them into $\{D, J\}$.
5. Now, the clusters F and K have the smallest maximum pairwise distance 809, so we merge them into $\{F, K\}$.
6. Now, the clusters B and G have the smallest maximum pairwise distance 997, so we merge them into $\{B, G\}$.
7. Now, the clusters C and $\{D, J\}$ have the smallest maximum pairwise distance 1336, so we merge them into $\{C, D, J\}$.
8. Now, the clusters $\{E, I, L\}$ and $\{F, K\}$ have the smallest maximum pairwise distance 1494, so we merge them into $\{E, F, I, K, L\}$.
9. Now, the clusters A and $\{D, J\}$ have the smallest maximum pairwise distance 2283, so we merge them into $\{A, C, D, J\}$.

Thus, the final 3 clusters are: $\{A, C, D, J\}$, $\{B, G\}$, and $\{E, F, H, I, K, L\}$.

In the extra credit, for the equation $3x + 4y = 10000$ will be a linear separator if:
For points that are label A: $3x + 4y > 10000$ and $3x + 4y <= 10000$ otherwise.
This is correct as all points that are labeled A are above the line, they are as follows: A, C, D, I, J.
So the line is a linear separator.
The line is not a SVM as the distance to the two closest points which are (H with distance of 57 units) and (I with distance 537 units) on each side is not maximised. $\square$

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Solutions to Problem 4 of Final Examination

Name: Kumar Prasun (kp2692) *Due: December 20*

Collaborators:

Solution:
We represent coin 1 as $C1$ and coin 2 as $C2$.

$\boldsymbol{P_{C1}(H)} = 0.7$ & $\boldsymbol{P_{C2}(H)} = 0.4$

a) Probability that it comes up head is:

$$\begin{aligned}\boldsymbol{P(H)} &= \frac{1}{2}(P_{C1}(H) + P_{C2}(H)) \\ &= \frac{1}{2}(0.7 + 0.4) = \frac{1}{2}(1.1) = 0.55\end{aligned}$$

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b) We have:

$\boldsymbol{P_{C1}(T)} = 0.3$ & $\boldsymbol{P_{C2}(T)} = 0.6$
$\boldsymbol{P(T)} = 1 - P(H) = 0.45$

Probability that the coin was $C1$ or $C2$ when it came up tails is:

$$Coin\ 1 : \boldsymbol{P(C1|T1)} = \frac{P(C1, T1)}{P(T1)} = \frac{P(C1) * P(T1|C1)}{P(T1)} = \frac{\frac{1}{2} * 0.3}{0.45} = \frac{0.15}{0.45} = \frac{1}{3} = 0.3333$$
$$Coin\ 2 : \boldsymbol{P(C2|T1)} = \frac{P(C2, T1)}{P(T1)} = \frac{P(C2) * P(T1|C2)}{P(T1)} = \frac{\frac{1}{2} * 0.6}{0.45} = \frac{0.3}{0.45} = \frac{2}{3} = 0.6666$$

Probability that the same coin will give heads is:

$$\begin{aligned}\boldsymbol{P(H2|T1)} &= P(H2, C1|T1) + P(H2, C2|T1) \\ &= P(H2|C1, T1) * P(C1|T1) + P(H2|C2, T1) * P(C2|T1) \\ &= P(H2|C1) * P(C1|T1) + P(H2|C2) * P(C2|T1) && \text{(Independent coin flips)} \\ &= (0.7) * (\frac{1}{3}) + (0.4) * (\frac{2}{3}) = 0.5\end{aligned}$$

c) We calculate the rest of the probabilities: Probability that the coin was $C1$ or $C2$ when it came up heads is:

$$Coin\ 1: \boldsymbol{P(C1|H1)} = \frac{P(C1,H1)}{P(H1)} = \frac{P(C1)*P(H1|C1)}{P(H1)} = \frac{\frac{1}{2}*0.7}{0.55} = \frac{0.35}{0.55} = \frac{7}{11} = 0.6363$$
$$Coin\ 2: \boldsymbol{P(C2|H1)} = \frac{P(C2,H1)}{P(H1)} = \frac{P(C2)*P(H1|C2)}{P(H1)} = \frac{\frac{1}{2}*0.4}{0.55} = \frac{0.2}{0.55} = \frac{4}{11} = 0.3636$$

Probability that the same coin will give heads is:

$$\begin{aligned}\boldsymbol{P(H2|H1)} &= P(H2,C1|H1) + P(H2,C2|H1)\\ &= P(H2|C1,H1)*P(C1|H1) + P(H2|C2,H1)*P(C2|H1)\\ &= P(H2|C1)*P(C1|H1) + P(H2|C2)*P(C2|H1) && \text{(Independent coin flips)}\\ &= (0.7)*(\frac{7}{11}) + (0.4)*(\frac{4}{11}) = 0.5909\end{aligned}$$

The other probabilities are:

$$\boldsymbol{P(T2|H1)} = 1 - P(H2|H1) = 0.4091$$
$$\boldsymbol{P(T2|T1)} = 1 - P(H2|T1) = 0.5$$

For the chance nodes in the decision tree

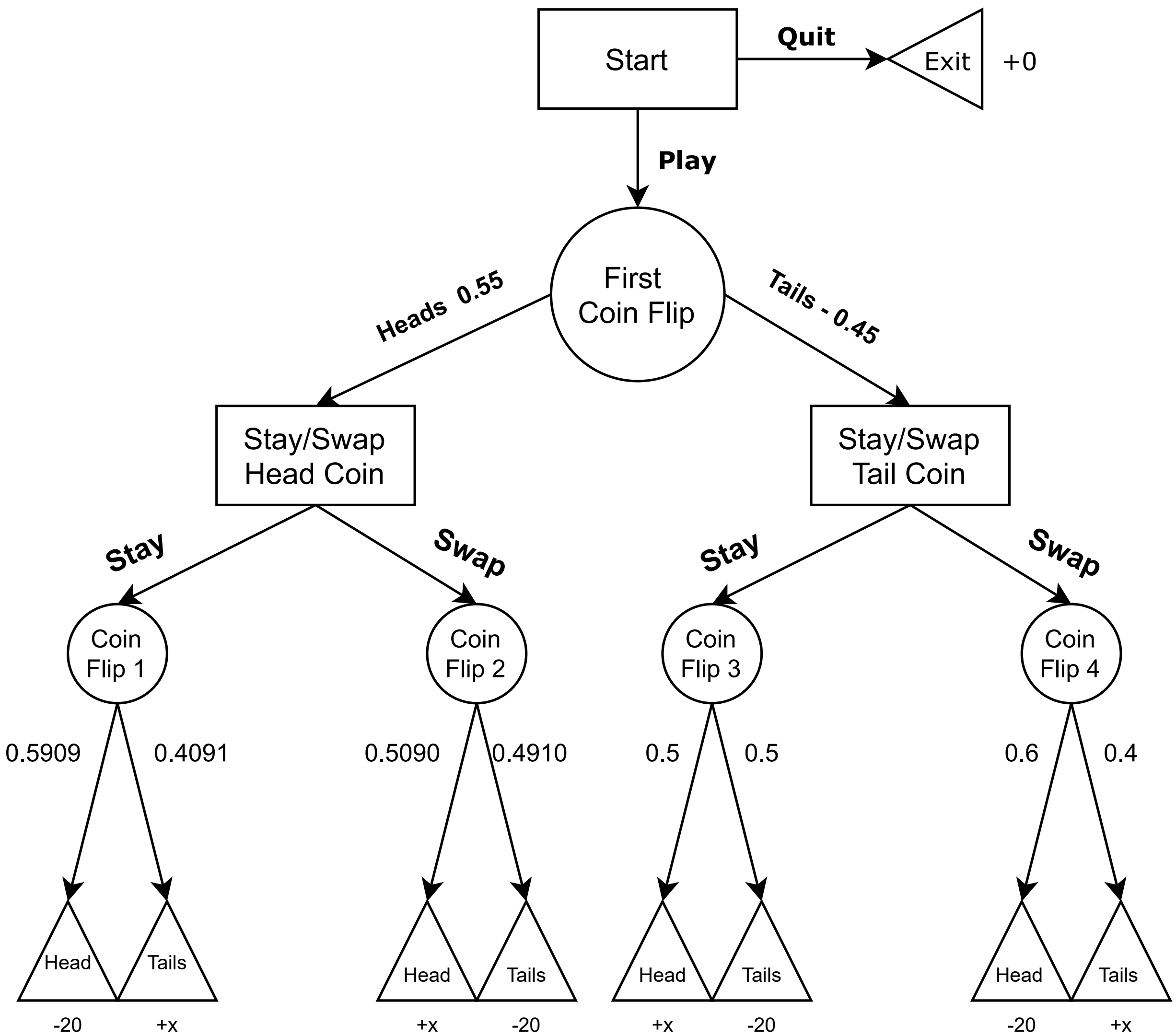
- For Chance(Coin Flip 1)
  Probability of getting heads again: $P(H2|H1) = 0.5909$
  Probability of getting tails after heads: $P(T2|H1) = 1 - P(H2|H1) = 0.4091$

- For Chance(Coin Flip 2)
  Probability that our coin is C1: $1 - P(C1|H1) = 1 - 0.6363 = 0.3636$
  Probability that our coin is C2: $1 - P(C2|H1) = 1 - 0.3636 = 0.6363$
  Probability of getting heads again: $P(H2|H1) = (0.7)*0.3636 + (0.4)*0.6363 = 0.5090$
  Probability of getting tails: $P(T2|H1) = 1 - P(H2|H1) = 0.4910$

- For Chance(Coin Flip3)
  Probability of getting heads after tails: $P(H2|T1) = 0.5$
  Probability of getting tails again: $P(T2|T1) = 1 - P(H2|T1) = 0.5$

- For Chance(Coin Flip4)

  Probability that our coin is C1: $1 - P(C1|T1) = 1 - 0.3333 = 0.6666$
  Probability that our coin is C2: $1 - P(C2|T1) = 1 - 0.6666 = 0.3333$
  Probability of getting heads: $P(H2|T1) = (0.7)*0.6666 + (0.4)*0.3333 = 0.6$
  Probability of getting tails again: $P(T2|T1) = 1 - P(H2|T1) = 0.4$


Start
Quit
Exit
+0
Play
First Coin Flip
Heads 0.55
Tails - 0.45
Stay/Swap Head Coin
Stay/Swap Tail Coin
Stay
Swap
Stay
Swap
Coin Flip 1
Coin Flip 2
Coin Flip 3
Coin Flip 4
0.5909
0.4091
0.5090
0.4910
0.5
0.5
0.6
0.4
Head
Tails
Head
Tails
Head
Tails
Head
Tails
-20
+x
+x
-20
+x
-20
-20
+x

This gives us the following Decision Tree:

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d) We now calculate the expected value of all the nodes.

- Coin Flip 1 $= 0.5909 * -20 + 0.4091 * x = 0.4091 * x - 11.818$
- Coin Flip 2 $= 0.5090 * x + 0.4910 * -20 = 0.5090 * x - 9.82$
- Coin Flip 3 $= 0.5 * x + 0.5 * -20 = 0.5 * x - 10$
- Coin Flip 4 $= 0.6 * -20 + 0.4 * x = 0.4 * x - 12$
- Decision Stay/Swap Head Coin $= max(CoinFlip1, CoinFlip2) = max(0.4091*x-11.818, 0.5090*x - 9.82)$
- Decision Stay/Swap Tail Coin $= max(CoinFlip3, CoinFlip4) = max(0.5*x-10, 0.4*x-12)$

For Decision Stay/Swap Head Coin, let us assume that we took Coin Flip 1.
Therefore, Coin Flip 1 $>$ Coin Flip2 which implies
$0.4091 * x - 11.818 > 0.5090 * x - 9.82$
This implies: $-1.998 > 0.0999 * x$ or $x < -20$
This is clearly not feasible, as we do not want a negative value for x. So we will always take Coin Flip 2.
Therefore, Decision Stay/Swap Head Coin is

$$max(CoinFlip1, CoinFlip2) = CoinFlip2 = 0.5090 * x - 8.02$$

Similarly if for Decision Stay/Swap Tail Coin, we took Coin Flip 4.
This implies $0.5 * x - 10 < 0.4 * x - 12$ or $0.1 * x < -2$ or $x < -20$.
As before, we do not want a negative value for x. So we will always take Coin Flip 3.
Therefore, Decision Stay/Swap Tail Coin is

$$max(CoinFlip3, CoinFlip4) = CoinFlip3 = 0.5 * x - 10$$

- First Coin Flip $= 0.55 * (0.5090 * x - 9.82) + 0.45 * (0.5 * x - 10) = 0.5049x - 9.901$
- Start Node $= max(0.5049x - 9.901, 0)$

For the start node, $0.5049x - 9.901 > 0$ gives us $x = 19.60$ which when rounded to the nearest integer gives us $x = 20$ for which the game is worth playing as we get an expected positive payout. $\square$

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Solutions to Problem 5 of Final Examination

Name: Kumar Prasun (kp2692) *Due: December 20*

Collaborators:

Solution:

A) The sentences are:

i) $\forall x[\forall y A(y) \Rightarrow L(x,y)] \Rightarrow [\exists z L(z,x)]$

ii) $[H(Jane, Fido) \vee H(Jim, Fido)] \wedge D(Fido)$

iii) $\forall x[\exists y A(y) \wedge H(x,y)] \Rightarrow [\forall z \neg L(z,x)]$

iv) $\forall x\ D(x) \Rightarrow A(x)$

v) $\forall a A(a) \Rightarrow\ L(Jane, a)$

vi) $H(Jim, Fido)$

B)

$\boldsymbol{\forall x[\forall y A(y) \Rightarrow L(x,y)] \Rightarrow [\exists z L(z,x)]}$

$= \forall x\ (\forall y\ \neg A(y) \vee L(x,y)) \Rightarrow [\exists z\ L(z,x)]$ $(Removing\ \ \Rightarrow)$

$= \forall x\ (\neg\forall y\ \neg A(y) \vee L(x,y)) \vee (\exists z\ L(z,x))$ $(Removing\ \ \Rightarrow)$

$= \forall x\ (\exists y\ \neg(\neg A(y) \vee L(x,y))) \vee (\exists z\ L(z,x))$ $(\sim\exists x\ \alpha = \forall x\ \sim\alpha)$

$= \forall x\ (\exists y\ A(y) \wedge \neg L(x,y)) \vee (\exists z\ L(z,x))$ $(DeMorgan's)$

$= \forall x\ (A(Sk0(x)) \wedge \neg L(x, Sk0(x))) \vee (L(Sk1(x), x))$ $(Skolemization)$

$= (A(Sk0(x)) \wedge \neg L(x, Sk0(x))) \vee (L(Sk1(x), x))$ $(Eliminate\ \forall)$

$= (A(Sk0(x)) \vee L(Sk1(x), x)) \wedge (\neg L(x, Sk0(x)) \vee L(Sk1(x), x))$ $(Distribute)$

$\boldsymbol{[H(Jane, Fido) \vee H(Jim, Fido)] \wedge D(Fido)}$

$= (H(Jane, Fido) \vee H(Jim, Fido)) \wedge (D(Fido))$ $(Simplify)$

$\boldsymbol{\forall x[\exists y A(y) \wedge H(x,y)] \Rightarrow [\forall z \neg L(z,x)]}$

$= \forall x(\neg\exists y A(y) \wedge H(x,y)) \vee (\forall z \neg L(z,x))$ $(Removing\ \ \Rightarrow)$

$= \forall x\ (\forall y\ \neg(A(y) \wedge H(x,y))) \vee (\forall z \neg L(z,x))$ $(\sim\exists x\ \alpha = \forall x\ \sim\alpha)$

$= \forall x\ (\forall y\ \neg A(y) \vee \neg H(x,y)) \vee (\forall z \neg L(z,x))$ $(DeMorgan's)$

$= (\neg A(y) \vee \neg H(x,y)) \vee (\neg L(z,x))$ $(Eliminate \forall)$

$= \neg A(y) \vee \neg H(x,y) \vee \neg L(z,x)$ $(Simplify)$

$\boldsymbol{\forall x\ D(x) \Rightarrow A(x)}$

$= \forall x\ (\neg D(x) \vee A(x))$ $(Removing\ \ \Rightarrow)$

$= \neg D(x) \vee A(x)$ $(Eliminate \forall)$

$\boldsymbol{\forall a A(a) \Rightarrow \ L(Jane, a)}$

$= \forall a \ (\neg A(a) \vee L(Jane, a)) \qquad (Removing \ \Rightarrow)$

$= \neg A(a) \vee L(Jane, a) \qquad (Eliminate \forall)$

$\boldsymbol{H(Jim, Fido)}$

$= H(Jim, Fido) \qquad (Simplify)$

The statement ϕ we wish to prove is:

$$\phi \equiv H(Jim, Fido)$$

Applying the resolution theorem proving procedure, we have

$$\Delta \equiv \sim H(Jim, Fido)$$

The CNF clauses from the previous question and Δ are:

1. $A(Sk0(x)) \vee L(Sk1(x), x)$
2. $\neg L(x, Sk0(x)) \vee L(Sk1(x), x)$
3. $H(Jane, Fido) \vee H(Jim, Fido)$
4. $D(Fido)$
5. $\neg A(y) \vee \neg H(x, y) \vee \neg L(z, x)$
6. $\neg D(x) \vee A(x)$
7. $\neg A(a) \vee L(Jane, a)$
8. $\neg H(Jim, Fido)$

Applying unification to 1 with 7 under the substitution $\sigma = \{a \rightarrow Sk0(x)\}$, picking $A(Sk0(x))$ as the unifier we get:

9. $L(Jane, Sk0(x)) \vee L(Sk1(x), x)$

Applying unification to 2 with 9 under the substitution $\sigma = \{x \rightarrow Jane\}$, picking $L(Jane, Sk0(Jane))$ as the unifier we get:

10. $L(Sk1(Jane), Jane) \vee L(Sk1(Jane), Jane)$

Applying factoring to 10 we get:

11. $L(Sk1(Jane), Jane)$

Applying unification to 5 with 11 under the substitution $\sigma = \{x \rightarrow Jane, z \rightarrow Sk1(Jane)\}$, picking $L(Sk1(Jane), Jane)$ as the unifier we get:

12. $\neg A(y) \vee \neg H(Jane, y)$

Applying unification to 4 with 6 under the substitution $\sigma = \{x \rightarrow Fido\}$, picking $D(Fido)$ as the unifier we get:

13. $A(Fido)$

Applying unification to 12 with 13 under the substitution $\sigma = \{y \rightarrow Fido\}$, picking $A(Fido)$ as the unifier we get:

14. $\neg H(Jane, Fido)$

Applying unification to 3 with 14 picking $H(Jane, Fido))$ as the unifier we get:

15. $H(Jim, Fido)$

Resolving 15 with 8, we get the desired null cause result. Hence, we get the statement $vi.$ as a consequence of $i. - v.$ from the question.

$\square$

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Solutions to Problem 6 of Final Examination

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Collaborators:

Solution: A) We have the following:

- State: A state is the sequence of tasks assigned to the different processors
- Start State: All tasks assigned to first processor
- Terminal State: All tasks processed
- Goal State: Processing time is less than D
- Operator: Move one task to another processor or swap tasks between two processors
- Branching Factor: $B = 4$

B)

- In the first state P1 = $T1, T2, T3, T4$, P2= []. Cost=45
- Then, P1 = $T2, T3, T4$, P2= $T1$, as lowest number movement. Cost=39
- Then, P1 = $T3, T4$, P2= $T1, T2$. Cost=10
- Then, P1 = $T4$, P2= $T1, T2, T3, T4$. Cost=1
- Then, P1 = [], P2= $T1, T2, T3, T4$. This branch is fully explored. Cost=19
- We continue this till we wil get P1 = $T1, T3$, P2 = $T2, T4$ and cost = -1.

C) Start state is P1 = $T1, T2, T3, T4$ with cost given as 45. P2 is empty. The next possible states are

P1	P2	Cost
T1,T2,T3	T4	max{(24+84+96)/4,108/6} = 51 - 33 = 18
T1,T2,T4	T3	max{(24+84+108)/4,96/4} = 54 - 33 = 21
T1,T3,T4	T2	max{(24+96+108)/4,84/6} = 57 - 33 = 24
T2,T3,T4	T1	max{(84+96+108)/4,24/6} = 72 - 33 = 39

We choose P1 = $T1, T2, T3$, P2 = $T4$ and cost = 18 as it is minimum cost.
We now repeat the process:

P1	P2	Cost	Action
T1,T2,T3	T4	max{(24+84+96)/4,108/6} = 51 - 33 = 18	Swap
T1,T2,T4	T3	max{(24+84+108)/4,96/4} = 54 - 33 = 21	Swap
T1,T3,T4	T2	max{(24+96+108)/4,84/6} = 57 - 33 = 24	Swap
T2,T3,T4	T1	max{(84+96+108)/4,24/6} = 72 - 33 = 39	Swap
T1,T2	T3,T4	max{(24+84)/4,(96+108)/6} = 34 - 33 = 1	Move
T1,T3	T2,T4	max{(24+96)/4,(84+108)/6} = 32 - 33 = -1	Move
T2,T3	T1,T4	max{(84+96)/4,(24+108)/6} = 45 - 33 = 12	Move
T1,T2,T3,T4	-	max{(24+84+96+108)/4,0} = 78 - 33 = 45	Move

We choose P1 = $T1, T3$, P2 = $T2, T4$ and cost = -1 as it is minimum cost.As it is less than 0 we stop here. □