CSCI-GA.2560-001, Artificial Intelligence March 21, 2022

Solutions to Problem 1 of Homework 5 (2 Points)

Name: Anav Prasad (ap7152) *Due: 5PM on Monday, March 21*

Collaborators:

Let D be a domain consisting of individual animals, species, and time periods. Let L be the first-order language over D with the following primitives:

- Animal(a) — Predicate. a is an animal.
- SpeciesOf(a,s) — Predicate. Animal a belongs to species s.
- Living(a,t) — Predicate. Animal a was alive at time t.
- Extinct(s,t) — Predicate. Species s was extinct at time t.
- Parent(p,c) — Predicate. Animal p is a parent of animal c.
- RinTinTin, Dog, Mammoth, 1918 — Constants with the obvious meanings.

Represent the following sentences in L:

1. If p is a parent of c and c belongs to species s then p also belongs to species s.
2. A species s is extinct at time t if and only if no animal belonging to s is alive at t.
3. No mammoths were alive in 1918.
4. Mammoths were extinct in 1918.

Solution:
So, the given sentences would be represented as follows:

1. $\forall p, c, s\ [Parent(p, c) \wedge SpeciesOf(c, s)] \implies SpeciesOf(p, s)$
2. $\forall s, t\ Extinct(s, t) \iff \neg[\exists a\ Animal(a) \wedge SpeciesOf(a, s) \wedge Living(a, t)]$
3. $\neg[\exists a\ Animal(a) \wedge SpeciesOf(a, Mammoth) \wedge Living(a, 1918)]$
4. $Extinct(Mammoth, 1918)$

$\square$

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Solutions to Problem 2 of Homework 5 (8 Points)

Name: Anav Prasad (ap7152) *Due: 5PM on Monday, March 21*

Collaborators:

Using resolution theorem proving, prove d. as a consequence of b., c.. You must show the intermediate stages of Skolemization. You must show all the clauses generated.

Solution:
The given sentences in L in the previous part were:

1. $\forall p, c, s\ [Parent(p, c) \wedge SpeciesOf(c, s)] \implies SpeciesOf(p, s)$
2. $\forall s, t\ Extinct(s, t) \iff \neg[\exists a\ Animal(a) \wedge SpeciesOf(a, s) \wedge Living(a, t)]$
3. $\neg[\exists a\ Animal(a) \wedge SpeciesOf(a, Mammoth) \wedge Living(a, 1918)]$
4. $Extinct(Mammoth, 1918)$

To Prove: $2. \wedge 3. \implies 4.$
Proof (by contradiction): Suppose the negation is true. That is to say, suppose the following holds. true:

$$\neg[2. \wedge 3. \implies 4.]$$

which is equivalent to

$$2. \wedge 3. \wedge \neg 4.$$

So, our new set of sentences are:

1. $\forall s, t\ Extinct(s, t) \iff \neg[\exists a\ Animal(a) \wedge SpeciesOf(a, s) \wedge Living(a, t)]$
2. $\neg[\exists a\ Animal(a) \wedge SpeciesOf(a, Mammoth) \wedge Living(a, 1918)]$
3. $\neg Extinct(Mammoth, 1918)$

Converting to CNF**...**

- **Before Step 1:** The given set of sentences:
    1. $\forall s, t\ Extinct(s, t) \iff \neg[\exists a\ Animal(a) \wedge SpeciesOf(a, s) \wedge Living(a, t)]$
    2. $\neg[\exists a\ Animal(a) \wedge SpeciesOf(a, Mammoth) \wedge Living(a, 1918)]$
    3. $\neg Extinct(Mammoth, 1918)$

- **Step 1:** $\iff$
    1. $\forall s, t\ (Extinct(s, t) \implies \neg[\exists a\ Animal(a) \wedge SpeciesOf(a, s) \wedge Living(a, t)]) \wedge$
       $(\neg[\exists a\ Animal(a) \wedge SpeciesOf(a, s) \wedge Living(a, t)] \implies Extinct(s, t))$
    2. $\neg[\exists a\ Animal(a) \wedge SpeciesOf(a, Mammoth) \wedge Living(a, 1918)]$
    3. $\neg Extinct(Mammoth, 1918)$

- **Step 2:** $\implies$

  1. $\forall s,t\ (\neg Extinct(s,t) \vee \neg[\exists a\ Animal(a) \wedge SpeciesOf(a,s) \wedge Living(a,t)]) \wedge$
     $(\neg\neg[\exists a\ Animal(a) \wedge SpeciesOf(a,s) \wedge Living(a,t)] \vee Extinct(s,t))$
  2. $\neg[\exists a\ Animal(a) \wedge SpeciesOf(a, Mammoth) \wedge Living(a, 1918)]$
  3. $\neg Extinct(Mammoth, 1918)$

- **Step 3:** $\neg$

  1. $\forall s,t\ (\neg Extinct(s,t) \vee [\forall a\ \neg Animal(a) \vee \neg SpeciesOf(a,s) \vee \neg Living(a,t)]) \wedge$
     $([\exists a\ Animal(a) \wedge SpeciesOf(a,s) \wedge Living(a,t)] \vee Extinct(s,t))$
  2. $\forall a\ \neg Animal(a) \vee \neg SpeciesOf(a, Mammoth) \vee \neg Living(a, 1918)$
  3. $\neg Extinct(Mammoth, 1918)$

- **Step 4:** $\exists$ (Skolemization)

  1. $\forall s,t\ (\neg Extinct(s,t) \vee [\forall a\ \neg Animal(a) \vee \neg SpeciesOf(a,s) \vee \neg Living(a,t)]) \wedge$
     $([Animal(Sk0(s,t)) \wedge SpeciesOf(Sk0(s,t),s) \wedge Living(Sk0(s,t),t)] \vee Extinct(s,t))$
  2. $\forall a\ \neg Animal(a) \vee \neg SpeciesOf(a, Mammoth) \vee \neg Living(a, 1918)$
  3. $\neg Extinct(Mammoth, 1918)$

- **Step 5:** $\forall$

  1. $(\neg Extinct(s,t) \vee \neg Animal(a) \vee \neg SpeciesOf(a,s) \vee \neg Living(a,t)) \wedge$
     $([Animal(Sk0(s,t)) \wedge SpeciesOf(Sk0(s,t),s) \wedge Living(Sk0(s,t),t)] \vee Extinct(s,t))$
  2. $\neg Animal(a) \vee \neg SpeciesOf(a, Mammoth) \vee \neg Living(a, 1918)$
  3. $\neg Extinct(Mammoth, 1918)$

- **Step 6:** Distribution

  1. $[\neg Extinct(s,t) \vee \neg Animal(a) \vee \neg SpeciesOf(a,s) \vee \neg Living(a,t)] \wedge$
     $[Animal(Sk0(s,t)) \vee Extinct(s,t)] \wedge [SpeciesOf(Sk0(s,t),s) \vee Extinct(s,t)] \wedge$
     $[Living(Sk0(s,t),t) \vee Extinct(s,t)]$
  2. $\neg Animal(a) \vee \neg SpeciesOf(a, Mammoth) \vee \neg Living(a, 1918)$
  3. $\neg Extinct(Mammoth, 1918)$

- **Step 7:** Split:

  1. $\neg Extinct(s,t) \vee \neg Animal(a) \vee \neg SpeciesOf(a,s) \vee \neg Living(a,t)$
  2. $Animal(Sk0(s,t)) \vee Extinct(s,t)$
  3. $SpeciesOf(Sk0(s,t),s) \vee Extinct(s,t)$
  4. $Living(Sk0(s,t),t) \vee Extinct(s,t)$
  5. $\neg Animal(a) \vee \neg SpeciesOf(a, Mammoth) \vee \neg Living(a, 1918)$
  6. $\neg Extinct(Mammoth, 1918)$

The CNF obtained is as follows:

1. $\neg Extinct(s,t) \vee \neg Animal(a) \vee \neg SpeciesOf(a,s) \vee \neg Living(a,t)$
2. $Animal(Sk0(s,t)) \vee Extinct(s,t)$
3. $SpeciesOf(Sk0(s,t),s) \vee Extinct(s,t)$
4. $Living(Sk0(s,t),t) \vee Extinct(s,t)$
5. $\neg Animal(a) \vee \neg SpeciesOf(a, Mammoth) \vee \neg Living(a, 1918)$
6. $\neg Extinct(Mammoth, 1918)$

Now, following the resolution theorem:

- Substituting $a \rightarrow Sk0(s,t)$ in 1. and unifying 1. and 2. with $Animal(Sk0(s,t))$ as the unifier, we get:

$$\neg Extinct(s,t) \vee \neg SpeciesOf(Sk0(s,t),s) \vee \neg Living(Sk0(s,t),t) \vee Extinct(s,t)$$

  1. $SpeciesOf(Sk0(s,t),s) \vee Extinct(s,t)$
  2. $Living(Sk0(s,t),t) \vee Extinct(s,t)$
  3. $\neg Animal(a) \vee \neg SpeciesOf(a, Mammoth) \vee \neg Living(a, 1918)$
  4. $\neg Extinct(Mammoth, 1918)$
  5. $\neg Extinct(s,t) \vee \neg SpeciesOf(Sk0(s,t),s) \vee \neg Living(Sk0(s,t),t) \vee Extinct(s,t)$

- Refactor 5. and unify $\neg Extinct(s,t)$ and $Extinct(s,t)$:

  1. $SpeciesOf(Sk0(s,t),s) \vee Extinct(s,t)$
  2. $Living(Sk0(s,t),t) \vee Extinct(s,t)$
  3. $\neg Animal(a) \vee \neg SpeciesOf(a, Mammoth) \vee \neg Living(a, 1918)$
  4. $\neg Extinct(Mammoth, 1918)$
  5. $\neg SpeciesOf(Sk0(s,t),s) \vee \neg Living(Sk0(s,t),t)$

- Unify 1. and 5. with the unifier $SpeciesOf(Sk0(s,t),t)$:

  1. $Living(Sk0(s,t),t) \vee Extinct(s,t)$
  2. $\neg Animal(a) \vee \neg SpeciesOf(a, Mammoth) \vee \neg Living(a, 1918)$
  3. $\neg Extinct(Mammoth, 1918)$
  4. $Extinct(s,t) \vee \neg Living(Sk0(s,t),t)$

- Unify 1. and 4. with the unifier $Living(Sk0(s,t),t)$:

  1. $\neg Animal(a) \vee \neg SpeciesOf(a, Mammoth) \vee \neg Living(a, 1918)$
  2. $\neg Extinct(Mammoth, 1918)$
  3. $Extinct(s,t)$

- Substitute $s \to Mammoth,\ t \to 1918$ and unify 2. with 3. with $Extinct(Mammoth, 1918)$ as the unifier:

    1. $\neg Animal(a) \vee \neg SpeciesOf(a, Mammoth) \vee \neg Living(a, 1918)$
    2. $[null\ sentence]$

Thus, we have received a null sentence and proved our hypothesis by contradiction.
Hence Proved. $\square$