CSCI-GA.2560-001, Artificial Intelligence February 28, 2022

Solutions to Problem 1 of Homework 4 (5 Points)

Name: Anav Prasad (ap7152) *Due: 5PM on Monday, February 28*

Collaborators:

Convert the above sentences into CNF using: $\sim$, $\vee$ as operators (you may also use ! and | if you want, just be consistent). Show each iteration of the algorithm, so step 1) eliminate if and only if, etc.

Solution:
Converting the given set of sentences to CNF using the resolution steps:

- Before Step 1: The given set of sentences:

$$\neg(P \vee \neg Q) \iff R$$
$$R \implies T \vee U$$
$$\neg U \vee Q \wedge R$$
$$(\neg V \vee R) \implies U$$
$$V \implies (\neg P \wedge \neg T)$$

- Step 1: $\iff$

$$(\neg(P \vee \neg Q) \implies R) \wedge (R \implies \neg(P \vee \neg Q))$$
$$R \implies T \vee U$$
$$\neg U \vee Q \wedge R$$
$$(\neg V \vee R) \implies U$$
$$V \implies (\neg P \wedge \neg T)$$

- Step 2: $\implies$

$$(\neg\neg(P \vee \neg Q) \vee R) \wedge (\neg R \vee \neg(P \vee \neg Q))$$
$$\neg R \vee (T \vee U)$$
$$\neg U \vee Q \wedge R$$
$$\neg(\neg V \vee R) \vee U$$
$$\neg V \vee (\neg P \wedge \neg T)$$

- Step 3: $\neg$

$$(P \vee \neg Q \vee R) \wedge (\neg R \vee (\neg P \wedge Q))$$
$$\neg R \vee T \vee U$$
$$\neg U \vee (Q \wedge R)$$
$$(V \wedge \neg R) \vee U$$
$$\neg V \vee (\neg P \wedge \neg T)$$

- Step 4: Distribution

$$(P \vee \neg Q \vee R) \wedge (\neg R \vee \neg P) \wedge (\neg R \vee Q)$$
$$\neg R \vee T \vee U$$
$$(\neg U \vee Q) \wedge (\neg U \vee R)$$
$$(V \vee U) \wedge (\neg R \vee U)$$
$$(\neg V \vee \neg P) \wedge (\neg V \vee \neg T)$$

- Step 5: Split

$$P \vee \neg Q \vee R$$
$$\neg R \vee \neg P$$
$$\neg R \vee Q$$
$$\neg R \vee T \vee U$$
$$\neg U \vee Q$$
$$\neg U \vee R$$
$$V \vee U$$
$$\neg R \vee U$$
$$\neg V \vee \neg P$$
$$\neg V \vee \neg T$$

The CNF obtained is as follows:

$$P \vee \neg Q \vee R$$
$$\neg R \vee \neg P$$
$$\neg R \vee Q$$
$$\neg R \vee T \vee U$$
$$\neg U \vee Q$$
$$\neg U \vee R$$
$$V \vee U$$
$$\neg R \vee U$$
$$\neg V \vee \neg P$$
$$\neg V \vee \neg T$$

□

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Solutions to Problem 2 of Homework 4 (5 Points)

Name: Anav Prasad (ap7152) *Due: 5PM on Monday, February 28*

Collaborators:

Perform a step-by-step execution of DPLL on the CNF output from the previous. As demonstrated in class, show each key progression, e.g. "Assign pure literal, propagate P=true, or guess W as true. When guessing in DPLL, choose the lowest unbound alphabetical and try True first.

Solution:
Solving the above set of CNF sentences obtained using DPLL

- Assign $= \{P : unbound,\ Q : unbound,\ R : unbound,\ T : unbound,\ U : unbound,\ V : unbound\}$

$$P \vee \neg Q \vee R$$
$$\neg R \vee \neg P$$
$$\neg R \vee Q$$
$$\neg R \vee T \vee U$$
$$\neg U \vee Q$$
$$\neg U \vee R$$
$$V \vee U$$
$$\neg R \vee U$$
$$\neg V \vee \neg P$$
$$\neg V \vee \neg T$$

- $P : True$ (hard guess)
  Current assignment: $\{P : True,\ Q : unbound,\ R : unbound,\ T : unbound,\ U : unbound,\ V : unbound\}$

$$\neg R$$
$$\neg R \vee Q$$
$$\neg R \vee T \vee U$$
$$\neg U \vee Q$$
$$\neg U \vee R$$
$$V \vee U$$
$$\neg R \vee U$$
$$\neg V$$
$$\neg V \vee \neg T$$

- $Q : True$ (easy guess, pure literal)
  Current assignment: $\{P : True,\ Q : True,\ R : unbound,\ T : unbound,\ U : unbound,\ V : unbound\}$

$$\neg R$$
$$\neg R \vee T \vee U$$
$$\neg U \vee R$$
$$V \vee U$$
$$\neg R \vee U$$
$$\neg V$$
$$\neg V \vee \neg T$$

- $R : False$ (easy guess, unit-clause)
  Current assignment: $\{P : True,\ Q : True,\ R : False,\ T : unbound,\ U : unbound,\ V : unbound\}$

$$\neg U$$
$$V \vee U$$
$$\neg V$$
$$\neg V \vee \neg T$$

- $T : False$ (easy guess, pure literal)
  Current assignment: $\{P : True,\ Q : True,\ R : False,\ T : True,\ U : unbound,\ V : unbound\}$

$$\neg U$$
$$V \vee U$$
$$\neg V$$

- $U : False$ (easy guess: unit-clause) Current assignment: $\{P : True,\ Q : True,\ R : False,\ T : True,\ U : False,\ V : unbound\}$

$$V$$
$$\neg V$$

- $V : True$ (easy guess: unit-clause) Current assignment: $\{P : True,\ Q : True,\ R : False,\ T : True,\ U : False,\ V : True\}$

$$\{0\} \backslash\backslash \textit{ empty sentence}$$

- Thus, we have a failure. So, we go back to the first hard guess ($P : True$), and flip $P's$ assignment.
  State before $P's$ assignment: Current assignment: $\{P : unbound,\ Q : unbound,\ R : unbound,\ T : unbound,\ U : unbound,\ V : unbound\}$

$$P \vee \neg Q \vee R$$
$$\neg R \vee \neg P$$
$$\neg R \vee Q$$
$$\neg R \vee T \vee U$$
$$\neg U \vee Q$$
$$\neg U \vee R$$
$$V \vee U$$
$$\neg R \vee U$$
$$\neg V \vee \neg P$$
$$\neg V \vee \neg T$$

  $P : False$
  Current assignment: $\{P : False,\ Q : unbound,\ R : unbound,\ T : unbound,\ U : unbound,\ V : unbound\}$

$$\neg Q \vee R$$
$$\neg R \vee Q$$
$$\neg R \vee T \vee U$$
$$\neg U \vee Q$$
$$\neg U \vee R$$
$$V \vee U$$
$$\neg R \vee U$$
$$\neg V \vee \neg T$$

- $Q : True$ (hard guess)
  Current assignment: $\{P : False,\ Q : True,\ R : unbound,\ T : unbound,\ U : unbound,\ V : unbound\}$

$$R$$
$$\neg R \vee T \vee U$$
$$\neg U \vee R$$
$$V \vee U$$
$$\neg R \vee U$$
$$\neg V \vee \neg T$$

- $R : True$ (easy guess, unit-clause)
  Current assignment: $\{P : False,\ Q : True,\ R : True,\ T : unbound,\ U : unbound,\ V :$

$unbound\}$

$$T \vee U$$
$$V \vee U$$
$$U$$
$$\neg V \vee \neg T$$

- $U : True$ (easy guess, unit-clause)
  Current assignment: $\{P : False,\ Q : True,\ R : True,\ T : unbound,\ U : True,\ V : unbound\}$

$$\neg V \vee \neg T$$

- $T : False$ (easy guess, pure literal)
  Current assignment: $\{P : False,\ Q : True,\ R : True,\ T : False,\ U : True,\ V : unbound\}$
  No sentences left now
- Since the set of sentences is empty, assigning unbound assignments:
  $\therefore$ Final assignment: $\{P : False,\ Q : True,\ R : True,\ T : False,\ U : True,\ V : True\}$
  Return the final assignment

Thus, the successful assignment found by DPLL is as follows:
$\{P : False,\ Q : True,\ R : True,\ T : False,\ U : True,\ V : True\}$ □