CSCI-GA.2560-001, Artificial Intelligence February 7, 2022

Solutions to Problem 1 of Homework 2 (4 Points)

Name: Anav Prasad (ap7152) *Due: 5PM on Monday, February 7*

Collaborators:

Trace the execution of hill climbing from a start state of A,D. Use basic hill climbing, so no sideways motion or restarts.

Solution:

```
Given T = 23, M = 13
Start: {A, D}, Total Value = 20, Total Weight = 15, Error = 5
Consider:
       SET          TOTAL VALUE     TOTAL WEIGHT    ERROR
    {A, B, D}           28              19            6
    {A, C, D}           31              22            9
    *{A, D, E}          25              17            4
    {A}                 13              11            10
    {D}                 7               4             16
    {B, D}              15              8             8
    {C, D}              18              11            5
    {D, E}              12              6             11
    {A, B}              21              15            4
    {A, C}              24              18            5
    {A, E}              18              13            5

Since there are two sets with lowest error (4), choosing the set
with higher total value: {A, D, E}
Consider:
       SET          TOTAL VALUE     TOTAL WEIGHT    ERROR
    {A, B, D, E}        33              22            8
    {A, C, D, E}        36              24            11
    {D, E}              12              6             11
    {A, E}              18              13            5
    {A, D}              20              15            5
    {B, D, E}           20              10            3
    *{C, D, E}          23              13            0
    {A, B, E}           26              17            4
    {A, C, E}           29              20            7
    {A, B, D}           28              19            6
    {A, C, D}           31              22            9

Choosing the set with the lowest error: {C, D, E}
Consider:
```

SET	TOTAL VALUE	TOTAL WEIGHT	ERROR
{A, C, D, E}	36	24	11
{B, C, D, E}	31	17	4
{D, E}	12	6	11
{C, E}	16	9	7
{C, D}	18	11	5
{A, D, E}	25	17	4
{B, D, E}	20	10	3
{A, C, E}	29	20	7
{B, C, E}	24	13	0
{A, C, D}	31	22	9
{B, C, D}	26	15	2

Return {C, D, E} since nothing better was found.

$\square$

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Solutions to Problem 2 of Homework 2 (3 Points)

Name: Anav Prasad (ap7152) *Due: 5PM on Monday, February 7*

Collaborators:

Also using basic hill climbing, trace the execution of hill climbing from a start state of A,B, and explain what was different from part 1.

Solution:

```
Given T = 23, M = 13
Start: {A, B}, Total Value = 21, Total Weight = 15, Error = 4
Consider:
       SET          TOTAL VALUE     TOTAL WEIGHT    ERROR
    {A, B, C}           32              22            9
    {A, B, D}           28              19            6
    *{A, B, E}          26              17            4
    {B}                 8               4             15
    {A}                 13              11            10
    {B, C}              19              11            4
    {B, D}              15              8             8
    {B, E}              13              6             10
    {A, C}              24              18            5
    {A, D}              20              15            5
    {A, E}              18              13            5

Since there are two sets with lowest error (4), choosing the set
with higher total value: {A, B, E}
Consider:
       SET          TOTAL VALUE     TOTAL WEIGHT    ERROR
    {A, B, C, E}        37              24            11
    {A, B, D, E}        33              22            8
    {B, E}              13              6             10
    {A, E}              18              13            5
    {A, B}              21              15            4
    *{B, C, E}          24              13            0
    {B, D, E}           20              10            3
    {A, C, E}           29              20            7
    {A, D, E}           25              17            4
    {A, B, C}           32              22            9
    {A, B, D}           28              19            6

Choosing the set with the lowest error: {B, C, E}
Consider:
```

SET	TOTAL VALUE	TOTAL WEIGHT	ERROR
{A, B, C, E}	37	24	11
{B, C, D, E}	31	17	4
{C, E}	16	9	7
{B, E}	13	6	10
{B, C}	19	11	4
{A, C, E}	29	20	7
{C, D, E}	23	13	0
{A, B, E}	26	17	4
{A, D, E}	25	17	4
{A, B, C}	32	22	9
{B, C, D}	26	15	2

`Return {B, D, E} since nothing better was found.`

So, to explain the difference between the results of Problem 1 and 2, first of all notice that there are multiple possible solutions as seen in both Problem 1 and 2. Specifically, there are at least 2 possible solutions: $\{C, D, E\}$, $\{B, D, E\}$.
Now, the difference between these two problems was that while both of the hill climbing solutions considered both of the sets of $\{C, D, E\}$ and $\{B, D, E\}$ as possible solutions, they encountered different possible solutions first because of different starting points. Problem 1's solution encountered $\{C, D, E\}$ first, whereas Problem 2's solution encountered $\{B, D, E\}$ first. Since we aren't considering sideways motion and both $\{C, D, E\}$ and $\{B, D, E\}$ have error 0, the two solutions returned different solutions. $\square$

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Solutions to Problem 3 of Homework 2 (3 Points)

Name: Anav Prasad (ap7152) *Due: 5PM on Monday, February 7*

Collaborators:

Consider now the general case where there are N objects under consideration (not the example above). What is the size of the state space? What is maximal number of neighbors of any state?

Solution:
The state space is the set of all possible subsets (not necessarily proper subsets) of the set of all `N` objects. As such, the size of the state space trivially becomes the count of all subsets of a set of size `N` which is 2^N.
Therefore:
Size of State space $= 2^N$

Now to find the maximal number of neighbors of any state, consider a state set of size x. As such, the number of it's neighbors would be the sum of the following:

- Count of neighbors with one object added $= N - x$. That is so because there are $N - x$ objects that are not in the start set and only one object is added to get a distinct neighbor.
- Count of neighbors with one object removed is, trivially, x.
- Count of neighbors with one object swapped with an object outside of the start set $= x \cdot (N - x)$. This is so because for every object in the start set (count $= x$) there are $N - x$ possible objects that are *outside* the start set that can be swapped in place of the object in the set to obtain a distinct neighbor.

Therefore, the number of neighbors of a set with x objects, say r, is:

$$r = N - x + x + (x \cdot (N - x))$$
$$\therefore r = N + N \cdot x - x^2$$
$$\therefore r = -x^2 + N \cdot x + N$$

Therefore, the maximal number of neighbors of any state is the maximized value of r over all possible values of x.

As such, to maximize r over x, differentiating r w.r.t. x and equating to 0 to obtain the point of maxima/minima:

$$\frac{dr}{dx} = 0$$
$$\therefore -2 \cdot x + N = 0$$
$$\implies x = \frac{N}{2}$$

Now, computing the double derivative at $x = \frac{N}{2}$ to confirm whether x is the point of minima or maxima

$$\begin{aligned}
\frac{d^2r}{dx^2}\bigg|_{x=N/2} &= (-2)\bigg|_{x=N/2} \\
&= -2
\end{aligned}$$

Therefore, we can conclude that $x = N/2$ is a point of maxima. Thus, the maximal value of r is:

$$\begin{aligned}
r\bigg|_{x=N/2} &= \left(-x^2 + N \cdot x + N\right)\bigg|_{x=N/2} \\
&= \frac{N^2}{2} - \frac{N^2}{4} + N \\
&= \frac{N^2}{4} + N \\
\implies r &= \frac{N}{4} \cdot (N + 4)
\end{aligned}$$

Therefore, the maximal number of neighbors is $\frac{N}{4} \cdot (N + 4)$. $\square$