

Prime numbers¹: a positive integer number P is prime, $P > 1$, if the only positive integers that evenly divide P are P and 1.e.g., 2, 3, 5, 7, 11, 13, ...

(The source of this discussion about prime numbers is the following: Kenneth Rosen, *Discrete Mathematics and Its Applications*, 7th Edition, McGraw-Hill, 2011)

Check if a given number P is prime:

- **Basic algorithm**

If $P = 2$, then P is prime.

Otherwise, if **none** of the integers in the interval $(1, P-1]$ divides P , then P is prime.

- **Better algorithm**

If $P = 2$, then P is prime.

Otherwise, if **none** of the integers in the interval $(1, \sqrt{P}]$ divide P , then P is prime.

Note: Why considering integers up to \sqrt{P} ?

if P is not prime, it can be factored into two factors a and b such that $P = a * b$.

If both a and $b > \sqrt{P}$, then $a*b > P$. So, either a or $b < \sqrt{P}$.

Therefore, we only need to check for values less than or equal to the square root.

- **Even Better algorithm**

If $P = 2$, then P is prime.

Otherwise, if 2 divides P , then P is **not** prime.

Otherwise, if **none** of the **odd** integers in the interval $(1, \sqrt{P}]$ divides P , then P is prime.

- **Even² Better algorithm**

If $P = 2$, then P is prime.

Otherwise, if **none** of the **prime** integers in the interval $(1, \sqrt{P}]$ divides P , then P is prime.

consider IsPrime2T.py and write the function is_prime to check whether an input, n , is prime.

input: positive integer

output: true or false whether the number is prime or not

process: check if the number is 2, if it is, return true

otherwise, check if the number is 1 or divisible by 2, then return false

otherwise, try to divide n by the numbers from 3 to $\text{sqrt}(n)+1$ with a step of 2 (to only consider odd numbers)

```
% python .\IsPrime2.py
% python
Python 3.6.7 .....
>>> import IsPrime2
>>> IsPrime2.is_prime(2)
True
>>>
```